An axiomatic approach to outranking relations using the concordance / non-discordance principle

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Introduction

Context
- preference modelling for MCDA

Two main traditions
- **Axiomatic**: conjoint measurement and additive value functions
  - firm theoretical background (Krantz et al., 1971)
  - implementation often delicate: requires a detailed analysis of preferences
- **Pragmatic**: dominance relation and refinements
  - outranking relations based on a concordance-discordance principle
  - intuitive... but often criticized for their lack of axiomatic foundations
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Outranking relations

Roy (1968), ELECTRE I

- alternative $x$ is “at least as good as” alternative $y$ if
  - **Concordance condition** the set of attributes for which $x$ is at least as good as $y$ is “sufficiently important”
  - **Non-discordance condition** there is no attribute on which $y$ is “far better” than $x$

This type of comparison:

- is, apparently, quite different from the one used in the additive value function model
- has a definite “ordinal” flavor
- may lead to intransitive/incomplete preference relations

$\Rightarrow$ usual conjoint measurement tools are not adequate
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\( \Rightarrow \) usual conjoint measurement tools are not adequate
Objectives

- Propose a general framework for conjoint measurement
  - simple and intuitive
  - nontrivial
  - having a numerical representation
  - tolerating incompleteness and intransitivity

- Put this framework to work
  - to characterize concordance relations (Tours talk)
  - to characterize outranking relations (Today’s talk)
Objectives

Propose a general framework for conjoint measurement

- simple and intuitive
- nontrivial
- having a numerical representation
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Put this framework to work

- to characterize concordance relations (Tours talk)
- to characterize outranking relations (Today’s talk)
Outline

1 Definitions and notation
   - Setting
   - Concordance relations
   - Outranking relations
   - Example

2 Conjoint measurement framework
   - Model
   - Axioms
   - Results

3 Results
   - Concordance relations
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4 Discussion
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Classical conjoint measurement setting

- $N = \{1, 2, \ldots, n\}$: set of attributes
- $X = \prod_{i=1}^{n} X_i$ with $n \geq 2$: set of alternatives
- notation: $(x_J, y_{-J})$ and $(x_i, y_{-i}) \in X$

$\succ$ asymmetric binary relation $X$ “strict preference”

Remark

- we only study today asymmetric relations $\succ$ à la TACTIC
- analysis can be extended to cover reflexive relations $\succeq$ à la ELECTRE I
  - the introduction of discordance raises tricky duality problems however
Framework

Classical conjoint measurement setting

- $N = \{1, 2, \ldots, n\}$: set of attributes
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Definition of strict concordance relations

**Strict concordance relations (SCR)**

\[ x \succ y \iff P(x, y) \triangleright P(y, x) \]

with \( P(x, y) = \{ i \in N : x_i P_i y_i \} \) and

- \( P_i \): asymmetric binary relation \( X_i \)
- \( \triangleright \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Definition of strict concordance relations

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\[
A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D
\]
Definition of strict outranking relations

\[ x \succ y \iff [P(x, y) \triangleright P(y, x) \text{ and } V(y, x) = \emptyset] \]

with \( P(x, y) = \{i \in N : x_i P_i y_i\} \) and \( V(y, x) = \{i \in N : y_i V_i x_i\} \)

- \( P_i \): asymmetric binary relation \( X_i \)
- \( V_i \): a binary relation on \( X_i \) such that \( V_i \subseteq P_i \)
- \( \triangleright \): binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Definition of strict outranking relations

Strict outranking relations (SOR)

\[ x \succ y \iff [P(x, y) \triangleright P(y, x) \text{ and } V(y, x) = \emptyset] \]

with \( P(x, y) = \{ i \in N : x_i P_i y_i \} \) and \( V(y, x) = \{ i \in N : y_i V_i x_i \} \)

- \( P_i \): asymmetric binary relation \( X_i \)
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\[ A \triangleright B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \triangleright D \]
Example

**TACTIC (Vansnick, 1986)**

\[ x \succ y \iff \left\{ \sum_{i \in P(x,y)} w_i > \rho \sum_{j \in P(y,x)} w_j + \varepsilon \right. \]

and

\[ V(y, x) = \emptyset \]

with:

- \( \rho \geq 1 \) and \( \varepsilon \geq 0 \)
- \( P_i \): semiorder
- \( V_i \subseteq P_i \): semiorder
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Conjoint measurement framework

**Model (M)**

\[ x \succ y \iff F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \quad (M) \]

with

- \( p_i \) skew symmetric \( (p_i(x_i, y_i) = -p_i(y_i, x_i)) \)
- \( F \) is odd \( (F(x) = -F(-x)) \)
- \( F \) is nondecreasing in all its arguments

**Interpretation**

- \( p_i \) measures preference differences between levels on attribute \( i \in N \)
- \( F \) synthesizes these preference differences

Many variants of model (M) not studied here
Conjoint measurement framework

Model (M)

\[ x \succ y \iff F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \quad (M) \]

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- \( p_i \) skew symmetric \( (p_i(x_i, y_i) = -p_i(y_i, x_i)) \)
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Interpretation
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Axioms

\[(x_i, a_i) \succ (y_i, b_i) \quad \text{and} \quad (z_i, c_i) \succ (w_i, d_i) \quad \Rightarrow \quad \begin{cases} (x_i, c_i) \succ (y_i, d_i) \\ (z_i, a_i) \succ (w_i, b_i) \end{cases} \]

ARC1\textsubscript{i}

\[(x_i, a_i) \succ (y_i, b_i) \quad \text{and} \quad (y_i, c_i) \succ (x_i, d_i) \quad \Rightarrow \quad \begin{cases} (z_i, a_i) \succ (w_i, b_i) \\ (w_i, c_i) \succ (z_i, d_i) \end{cases} \]

ARC2\textsubscript{i}

ARC1 iff ARC1\textsubscript{i}, \forall i \in N
ARC2 iff ARC2\textsubscript{i}, \forall i \in N
Axioms

\[
(x_i, a_{-i}) \succ (y_i, b_{-i}) \\
and \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\]
\[\implies\]
\[
(x_i, c_{-i}) \succ (y_i, d_{-i}) \\
\quad \text{or} \quad \\
(z_i, a_{-i}) \succ (w_i, b_{-i})
\]

\[\text{ARC1}_i\]

\[
(x_i, a_{-i}) \succ (y_i, b_{-i}) \\
and \\
(y_i, c_{-i}) \succ (x_i, d_{-i})
\]
\[\implies\]
\[
(z_i, a_{-i}) \succ (w_i, b_{-i}) \\
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\]

\[\text{ARC2}_i\]

\[ARC1 \text{ iff } ARC1_i, \ \forall i \in N\]
\[ARC2 \text{ iff } ARC2_i, \ \forall i \in N\]
Axioms

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (z_i, c_{-i}) \succ (w_i, d_{-i}) \implies \begin{cases} (x_i, c_{-i}) \succ (y_i, d_{-i}) \\ (z_i, a_{-i}) \succ (w_i, b_{-i}) \end{cases} \quad ARC1_i\]

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (y_i, c_{-i}) \succ (x_i, d_{-i}) \implies \begin{cases} (z_i, a_{-i}) \succ (w_i, b_{-i}) \\ (w_i, c_{-i}) \succ (z_i, d_{-i}) \end{cases} \quad ARC2_i\]

\[ARC1 \iff ARC1_i, \forall i \in N\]
\[ARC2 \iff ARC2_i, \forall i \in N\]
Theorem (B&P, 2002, JMP)

[When each $X_i$ is at most countably infinite]
A binary relation $\succ$ on $X$ has a representation in model (M) iff

- $\succ$ is asymmetric
- $\succ$ satisfies ARC1 and ARC2

Remark
- can be generalized to sets of arbitrary cardinality
Theorem (B&P, 2002, JMP)

[When each $X_i$ is at most countably infinite]
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- can be generalized to sets of arbitrary cardinality
Remark

Model (M) contains as particular cases

- the additive value function model:

\[ x \succ y \Leftrightarrow \sum_{i=1}^{n} u_i(x_i) > \sum_{i=1}^{n} u_i(y_i) \]

- the additive difference model:

\[ x \succ y \Leftrightarrow \sum_{i=1}^{n} \Phi_i(u_i(x_i) - u_i(y_i)) > 0 \]

Coming next...

- model (M) also contains concordance relations
- model (M) also contains outranking relations
Remark

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### Observations

- If $\succ$ is a SCR, it satisfies $ARC1$ and $ARC2$.
- If $\succ$ has a representation in model (M) in which each $p_i$ takes at most three distinct values ($-k_i$, $0$, $+k_i$), it is a SCR.

### Consequences

- Model (M) offers an adequate framework for characterizing concordance relations.
- The distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences).
Concordance relations

Observations
- If $\succ$ is a SCR, it satisfies $ARC_1$ and $ARC_2$.
- If $\succ$ has a representation in model (M) in which each $p_i$ takes at most three distinct values ($-k_i$, 0, $+k_i$), it is a SCR.

Consequences
- Model (M) offers an adequate framework for characterizing concordance relations.
- The distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences).
Axioms

\[
\begin{align*}
(x_i, a_{-i}) &\succ (y_i, b_{-i}) \\
\text{and} \\
(z_i, a_{-i}) &\succ (w_i, b_{-i}) \\
\text{and} \\
(z_i, c_{-i}) &\succ (w_i, d_{-i})
\end{align*}
\]

\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) &\succ (x_i, b_{-i}) \\
\text{or} \\
(x_i, c_{-i}) &\succ (y_i, d_{-i})
\end{cases}
\]

\[
Maj1_i
\]

\[
\begin{align*}
(x_i, a_{-i}) &\succ (y_i, b_{-i}) \\
\text{and} \\
(w_i, a_{-i}) &\succ (z_i, b_{-i}) \\
\text{and} \\
(y_i, c_{-i}) &\succ (x_i, d_{-i})
\end{align*}
\]

\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) &\succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) &\succ (w_i, d_{-i})
\end{cases}
\]

\[
Maj2_i
\]

- \(Maj1\) if \(Maj1_i, \forall i \in N\)
- \(Maj2\) if \(Maj2_i, \forall i \in N\)
- \(RC1, RC2, Maj1\) and \(Maj2\) are independent conditions
Axioms

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (z_i, a_{-i}) \succ (w_i, b_{-i}) \quad \text{and} \quad (z_i, c_{-i}) \succ (w_i, d_{-i})\]

\[\Rightarrow \begin{cases} (y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(x_i, c_{-i}) \succ (y_i, d_{-i}) \end{cases} \implies \text{Maj}_1^{i} \]

\[(x_i, a_{-i}) \succ (y_i, b_{-i}) \quad \text{and} \quad (w_i, a_{-i}) \succ (z_i, b_{-i}) \quad \text{and} \quad (y_i, c_{-i}) \succ (x_i, d_{-i})\]

\[\Rightarrow \begin{cases} (y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i}) \end{cases} \implies \text{Maj}_2^{i} \]

- \text{Maj}_1 \text{ if } \text{Maj}_1^{i}, \forall i \in N
- \text{Maj}_2 \text{ if } \text{Maj}_2^{i}, \forall i \in N
- \text{RC}_1, \text{RC}_2, \text{Maj}_1 \text{ and } \text{Maj}_2 \text{ are independent conditions}
Axioms

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
\text{and} \\
(z_i, a_{-i}) & \succ (w_i, b_{-i}) \\
\text{and} \\
(z_i, c_{-i}) & \succ (w_i, d_{-i})
\end{align*}
\]

\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(x_i, c_{-i}) \succ (y_i, d_{-i})
\end{cases}
\]

\[
Maj 1_i
\]

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
\text{and} \\
(w_i, a_{-i}) & \succ (z_i, b_{-i}) \\
\text{and} \\
(y_i, c_{-i}) & \succ (x_i, d_{-i})
\end{align*}
\]

\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{cases}
\]

\[
Maj 2_i
\]

- **Maj 1** if Maj 1\(_i\), \(\forall i \in N\)
- **Maj 2** if Maj 2\(_i\), \(\forall i \in N\)
- **RC1, RC2, Maj 1** and Maj 2 are independent conditions
Result model (M)

Theorem (B&P, 2002, JMP)

A binary relation $\succ$ on $X$ has a representation in model (M) iff

- $\succ$ is asymmetric
- $\succ$ satisfies ARC$_1$ and ARC$_2$

Remark

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
**Theorem (B&P, 2005, EJOR)**

A binary relation $\succ$ on $X$ is a **SCR** iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_2$

**Remark**

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Result \text{SCR}

\textbf{Theorem (B&P, 2005, EJOR)}

A binary relation $\succ$ on $X$ is a SCR iff
\begin{itemize}
  \item $\succ$ is asymmetric
  \item $\succ$ satisfies $ARC_1$ and $ARC_2$
  \item $\succ$ satisfies $Maj_1$ and $Maj_2$
\end{itemize}

\textbf{Remark}

- model (M) can be used to analyze other types of models (e.g., additive value functions or additive differences)
Outranking relations

Observations

- if $\succ$ is an outranking relation, it satisfies $ARC_1$ and $ARC_2$
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it is the largest possible preference difference, so that $Maj_1$ holds
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it may happen that $(y_i, x_i)$ is not the smallest possible preference difference

Consequences

- keep $ARC_1$ and $ARC_2$
- keep $Maj_1$
- relax $Maj_2$ in order to allow for veto effects
  - at most five classes of preference differences, the last one playing a very special rôle
Outranking relations

Observations

- if $\succ$ is an outranking relation, it satisfies $ARC1$ and $ARC2$
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it is the largest possible preference difference, so that $Maj1$ holds
- if the preference difference $(x_i, y_i)$ is larger than $(y_i, x_i)$, it may happen that $(y_i, x_i)$ is not the smallest possible preference difference

Consequences

- keep $ARC1$ and $ARC2$
- keep $Maj1$
- relax $Maj2$ in order to allow for veto effects
  - at most five classes of preference differences, the last one playing a very special rôle
Axiom $Maj2$

\[
\begin{align*}
(x_i, a_{-i}) &\succ (y_i, b_{-i}) \\
\text{and} \\
(w_i, a_{-i}) &\succ (z_i, b_{-i}) \\
\text{and} \\
(y_i, c_{-i}) &\succ (x_i, d_{-i})
\end{align*}
\]

\[
\Rightarrow \begin{cases} 
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i}),
\end{cases}
\]

- $Maj3$ if $Maj3_i$, $\forall i \in N$
- $Maj2_i$ implies $Maj3_i$
- an outranking relation satisfies $Maj3$
- $RC1$, $RC2$, $Maj1$ and $Maj3$ are independent conditions
- condition $Maj3$ is inspired by GMS (2001)
Axiom \( \text{Maj}3 \)

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
& \text{and} \\
(w_i, a_{-i}) & \succ (z_i, b_{-i}) \\
& \text{and} \\
(y_i, c_{-i}) & \succ (x_i, d_{-i}) \\
& \text{and} \\
(z_i, e_{-i}) & \succ (w_i, f_{-i})
\end{align*}
\]

\[
\Rightarrow \left\{ \begin{array}{l}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{array} \right\}
\]

- \( \text{Maj}3 \) if \( \text{Maj}3_i, \ \forall i \in N \)
- \( \text{Maj}2_i \) implies \( \text{Maj}3_i \)
- an outranking relation satisfies \( \text{Maj}3 \)
- \( RC1, RC2, \text{Maj}1 \) and \( \text{Maj}3 \) are independent conditions
- condition \( \text{Maj}3 \) is inspired by GMS (2001)
Axiom \textit{Maj3}

\[
\begin{align*}
(x_i, a_{-i}) & \succ (y_i, b_{-i}) \\
& \text{and} \\
(w_i, a_{-i}) & \succ (z_i, b_{-i}) \\
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(y_i, c_{-i}) & \succ (x_i, d_{-i}) \\
& \text{and} \\
(z_i, e_{-i}) & \succ (w_i, f_{-i})
\end{align*}
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\[ \Rightarrow \left\{ \begin{array}{l}
(y_i, a_{-i}) \succ (x_i, b_{-i}) \\
\text{or} \\
(z_i, c_{-i}) \succ (w_i, d_{-i})
\end{array} \right. \]

\textit{Maj3} if \textit{Maj3} \_{i}, \forall i \in N

\textit{Maj2} \_{i} \text{ implies } \textit{Maj3} \_{i}

an outranking relation satisfies \textit{Maj3}

\textit{RC1}, \textit{RC2}, \textit{Maj1} and \textit{Maj3} are independent conditions

condition \textit{Maj3} is inspired by GMS (2001)
Theorem (B&P, 2002, JMP)

A binary relation $\succ$ on $X$ has a representation in model (M) iff

- $\succ$ is asymmetric
- $\succ$ satisfies ARC1 and ARC2
Result **SCR**

**Theorem (B&P, 2005, EJOR)**

A binary relation $\succ$ on $X$ is a **SCR** iff

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- $\succ$ satisfies $ARC1$ and $ARC2$
- $\succ$ satisfies $Maj1$ and $Maj2$
Theorem (B&P, 2005, *EJOR WP*)

A binary relation $\succ$ on $X$ is a **SCR SOR** iff

- $\succ$ is asymmetric
- $\succ$ satisfies $ARC_1$ and $ARC_2$
- $\succ$ satisfies $Maj_1$ and $Maj_2$ $Maj_3$
Theorem (B&P, 2005, WP)

A binary relation $\succ$ on $X$ is a SOR iff

- $\succ$ is asymmetric
- $\succ$ satisfies ARC1 and ARC2
- $\succ$ satisfies Maj1 and Maj3
Definitions and notation

Setting
Concordance relations
Outranking relations
Example

Conjoint measurement framework

Model
Axioms
Results

Results

Concordance relations
Outranking relations

Discussion
Summary

Model (M)

- is quite flexible but nontrivial
- has a simple and intuitive interpretation using preference differences
- has a simple axiomatic characterization
- allows to understand the main distinctive characteristics of concordance and outranking relations
  - in Tours we showed that the use of Fishburn’s “noncompensation” condition was not adequate to characterize concordance relations
  - the extension to outranking relations would have been impossible using the “noncompensation track”
What about SOR in which $S_i$ and $V_i$ have nice transitivity properties?

- add additional axioms
- these additional axioms are independent from the previous ones
- underlying model

$$x \succ y \iff F(\varphi_1(u_1(x_1), u_1(y_1)), \ldots, \varphi_n(u_n(x_n), u_n(y_n))) > 0$$

with $\varphi_i(\nearrow, \searrow)$

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### Extensions and future research

**Reflexive outranking relations à la ELECTRE I**
- no major problem: Bouyssou & Pirlot (2005)
  - duality: “veto” and “bonus” effects

**ELECTRE TRI**
- extension to sorting models: Bouyssou & Marchant (2005)

**New models?**
- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
- examples: models with “sophisticated discordance”
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