An axiomatic approach to outranking relations using the concordance / non-discordance principle

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Lille — ROADEF'2006 — 2006

Introduction

Context

- preference modelling for MCDA

Two main traditions

- **Axiomatic**: conjoint measurement and additive value functions
  - firm theoretical background (Krantz et al., 1971)
  - implementation often delicate: requires a detailed analysis of preferences
- **Pragmatic**: dominance relation and refinements
  - outranking relations based on a concordance-discordance principle
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Outranking relations

Roy (1968), ELECTRE I

- alternative x is "at least as good as" alternative y if

  Concordance condition: the set of attributes for which x is at least as good as y is "sufficiently important"

  Non-discordance condition: there is no attribute on which y is "far better" than x

This type of comparison:

- is, apparently, quite different from the one used in the additive value function model
- has a definite "ordinal" flavor
- may lead to intransitive/incomplete preference relations

$\Rightarrow$ usual conjoint measurement tools are not adequate

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Objectives

Propose a general framework for conjoint measurement

- simple and intuitive
- nontrivial
- having a numerical representation
- tolerating incompleteness and intransitivity

Put this framework to work

- to characterize concordance relations (Tours talk)
- to characterize outranking relations (Today's talk)

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   - Setting
   - Concordance relations
   - Outranking relations
   - Example
2. Conjoint measurement framework
   - Model
   - Axioms
   - Results
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Framework

Classical conjoint measurement setting

- $N = \{1, 2, \ldots, n\}$: set of attributes
- $X = \prod_{i=1}^{n} X_i$ with $n \geq 2$: set of alternatives
- notation: (x_J, y_{-J}) and $(x_i, y_{-i}) \in X$
- $\succ$ asymmetric binary relation X "strict preference"

Remark

- we only study today asymmetric relations $\succ$ à la TACTIC
- analysis can be extended to cover reflexive relations $\succsim$ à la ELECTRE I
  - the introduction of discordance raises tricky duality problems however

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Definition of strict concordance relations

Strict concordance relations (SCR)

$$x \succ y \Leftrightarrow P(x, y) \rhd P(y, x)$$

with $P(x, y) = \{i \in N : x_i \; P_i \; y_i\}$ and

- P_i: asymmetric binary relation X_i
- $\rhd$: binary relation between disjoint subsets of attributes that is increasing w.r.t. inclusion

$$A \rhd B, C \supseteq A \text{ and } B \supseteq D \Rightarrow C \rhd D$$

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Example

TACTIC (Vansnick, 1986)

$$x \succ y \Leftrightarrow \begin{cases} \displaystyle\sum_{i \in P(x,y)} w_i > \rho \sum_{j \in P(y,x)} w_j + \varepsilon \\ \text{and} \\ V(y,x) = \varnothing \end{cases}$$

with:

- $\rho \geq 1$ and $\varepsilon \geq 0$
- P_i: semiorder
- $V_i \subseteq P_i$: semiorder

Outline

Conjoint measurement framework

Model (M)

$$x \succ y \Leftrightarrow F(p_1(x_1, y_1), p_2(x_2, y_2), \ldots, p_n(x_n, y_n)) > 0 \qquad \text{(M)}$$

with

- p_i *skew symmetric* ($p_i(x_i, y_i) = -p_i(y_i, x_i)$)
- F is *odd* ($F(\mathbf{x}) = -F(-\mathbf{x})$)
- F is *nondecreasing* in all its arguments

Interpretation

- p_i measures *preference differences* between levels on attribute $i \in N$
- F synthesizes these preference differences

Many variants of model (M) not studied here

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[When each X_i is at most countably infinite]
A binary relation $\succ$ on X has a representation in model (M) *iff*

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Remark

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Model (M) contains as particular cases

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$$x \succ y \Leftrightarrow \sum_{i=1}^{n} u_i(x_i) > \sum_{i=1}^{n} u_i(y_i)$$

- the additive difference model:

$$x \succ y \Leftrightarrow \sum_{i=1}^{n} \Phi_i(u_i(x_i) - u_i(y_i)) > 0$$

Coming next...

- model (M) also contains concordance relations
- model (M) also contains outranking relations

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Concordance relations

Observations

- if $\succ$ is a SCR, it satisfies $ARC1$ and $ARC2$
- if $\succ$ has a representation in model (M) in which each p_i takes at most three distinct values $(-k_i, 0, +k_i)$, it is a SCR

Consequences

- model (M) offers an adequate framework for characterizing concordance relations
- the distinctive feature of concordance relation is that they induce comparisons of preference differences that are coarse (at most three classes of preference differences)

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Outranking relations

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- if $\succ$ is an outranking relation, it satisfies $ARC1$ and $ARC2$
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- if the preference difference (x_i, y_i) is larger than (y_i, x_i), it may happen that (y_i, x_i) is not the smallest possible preference difference

Consequences

- keep $ARC1$ and $ARC2$
- keep $Maj1$
- relax $Maj2$ in order to allow for veto effects
  - at most five classes of preference differences, the last one playing a very special rôle

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Summary

Model (M)

- is quite flexible but nontrivial
- has a simple and intuitive interpretation using preference differences
- has a simple axiomatic characterization
- allows to understand the main distinctive characteristics of concordance and outranking relations
    - in Tours we showed that the use of Fishburn's "noncompensation" condition was not adequate to characterize concordance relations
    - the extension to outranking relations would have been impossible using the "noncompensation track"

Discussion

What about SOR in which S_i and V_i have nice transitivity properties?

- add additional axioms
- these additional axioms are independent from the previous ones
- underlying model

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with $\varphi_i(\nearrow, \searrow)$

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Extensions and future research

Reflexive outranking relations à la ELECTRE I

- no major problem: Bouyssou & Pirlot (2005)
  - duality: “veto” and “bonus” effects

ELECTRE TRI

- extension to sorting models: Bouyssou & Marchant (2005)

New models?

- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
- examples: models with “sophisticated discordance”

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- no major problem: Bouyssou & Pirlot (2005)
  - duality: "veto" and "bonus" effects

ELECTRE TRI

- extension to sorting models: Bouyssou & Marchant (2005)

New models?

- models using preference differences:
  - not as rich as in the additive value functions model
  - not as coarse as in outranking relations
- examples: models with "sophisticated discordance"

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