Conceptual Design and Implementation of the Fuzzy Semantic Model

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Outline

1. Introduction
2. Fuzzy Semantic Model
3. Schema Definition in FSM
4. Mapping FSM-Based Model
5. Extending Binary and Set Operators
6. Towards a Flexible Query Language
7. Conclusion and Future Works

Salem Chakhar and Abelkader Telmoudi
Several proposals to develop models that support **Vagueness**, **uncertainty** and **impreciseness** of real-world.

Vagueness, imprecision and uncertainty are introduced at different levels:

**Two levels within the relational database model**

- **Tuple level**: Tuples belong to their relations with given d.o.m.
- **Attribute level**: Attributes are authorized to take imprecise, uncertain and vague values.

**Three levels within object-oriented and semantic database models**

- **Class levels**: Classes, relationships and attributes domains have d.o.m in the model.
- **Entity level**: Entities/Objects and instances of relationships belong to their classes and relationships with a given d.o.m.
- **Attribute level**: Attributes are authorized to take imprecise, uncertain and vague values.
Introduction

Fuzziness within object-oriented/semantic database models

- There are several efforts to extend semantic and object-oriented database models to support fuzziness of real-world.
- Most of these extensions introduce fuzziness only at the attribute level and consider that entities are fully encapsulated into their classes, which means that they fully verify the properties of these classes.

However....

- In many applications it is difficult to assign an entity to a particular class, mainly when this entity verifies only partially the class properties.

Our objective

- Introduce FSM—Fuzzy Semantic Model:
  - FSM uses basic concepts of semantic modeling.
  - FSM supports fuzziness of real-world at all levels.
Basic idea

- **$E$:** is the space of entities, i.e. is the set of all entities of the interest domain.
- A **fuzzy entity** $e$ in $E$ is a natural or artificial entity that one or several of its properties are fuzzy.

**Definition**

A **fuzzy class** $K$ in $E$ is a collection of fuzzy entities:

$$K = \{(e, \mu_K(e)) : e \in E \land \mu_K(e) > 0\}$$

where:

- $\mu_K$ is a characteristic or **membership function**.
- $\mu_K(e)$ represents the **degree of membership** (d.o.m) of fuzzy entity $e$ in fuzzy class $K$. 
A fuzzy class is a collection of fuzzy entities having some **similar properties**.

We denote by $X_K = \{p_1, p_2, \ldots, p_n\}$ (with $n \geq 1$) the set these properties. $X_K$ is called **extent set**.

Extent properties may be derived from the **attributes** of the class or from **common semantics**.

We associate to each extent property $p_i$ a non-negative **weight** $w_i$ reflecting its importance in deciding whether or not an entity $e$ is a member of a given fuzzy class $K$.

We also impose that:

$$\sum_{i=1}^{n} w_i > 0.$$
Let $D^i$ be the basic domain of extent property $p_i$ values.
Let $P^i \subseteq D^i$ be the set of possible values for property $p_i$.

**Definition**

The partial membership function of an extent property $p_i$, $\rho_{P^i_K}$, is:

$$\rho_{P^i_K} : D^i \rightarrow [0, 1]$$

$$v_i \rightarrow \rho_{P^i_K}(v_i)$$

**Remark**

For extent properties based on common semantics, $v_i$ is a semantic phrase and the partial d.o.m $\rho_{P^i_K}(v_i)$ is supposed to be equal to 1 but the user may explicitly provide a value less than 1.
Global membership function

The **global membership function** of fuzzy class $K$ is:

$$
\mu_K: \quad E \rightarrow [0, 1] \\
e \rightarrow \mu_K(e)
$$

with:

$$
\mu_K(e) = \frac{\sum_{i=1}^{n} \rho_{P_i}^{K}(e \cdot v_i) \cdot w_i}{\sum_{i=1}^{n} w_i}
$$
Basic idea
Entity/Class membership function
Constructs of FSM

Global membership function

Example

Class: YoungPers
- \( X_{\text{YoungPers}} = \{ \text{age, height} \} \)
- \( w = \{ w_{\text{age}}, w_{\text{height}} \} = \{0.8, 0.3\} \)

Representation of fuzzy properties of “being young” and “having average height”

We have:
- \( \rho_{p_{\text{age}}}^{\text{YoungPers}}(e.\text{age}) = 0.53 \)
- \( \rho_{p_{\text{height}}}^{\text{YoungPers}}(e.\text{height}) = 0.9 \)

\[ \Rightarrow \quad \mu_{\text{Young}}(e) = \frac{0.53 \cdot 0.8 + 0.9 \cdot 0.3}{0.8 + 0.3} = 0.630. \]
Two categories of classes

- A class $K$ is an **exact class** iff $\mu_K(e) = 1 \forall e \in K$.
- A class $K$ is a **fuzzy class** iff i.e., $\exists e \in K$ such that $\mu_K(e) < 1$.

$\alpha$-MEMBERS

- $\alpha$-MEMBERS denotes the set $\{e : e \in K \land \mu_K(e) \geq \alpha\}$; where $\alpha \in [0, 1]$.
- $\alpha$-MEMBERS $\subseteq \beta$-MEMBERS for all $\alpha$ and $\beta$ in $[0, 1]$ and verifying $\alpha \geq \beta$.
- 1-MEMBERS may also be refereed to true or exact members

Remark

The concept of $\alpha$-MEMBERS may be mapped to the concept of $\alpha$-cut associated with fuzzy sets and which is defined for a fuzzy subset $F$ as the set $F_\alpha = \{x : \mu_F(x) \geq \alpha\}$ with $0 \leq \alpha \leq 1$. 

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Constructs of FSM

Relationships

FSM supports four different relationships:

- **Property**: relates fuzzy classes to domain classes. Each property relationship creates an attribute and each attribute has a unique datatype and may be exact or fuzzy. Also, attributes may single-valued, unknown, undefined, null or multi-valued.

- **Decision-rule**: implements the extents of fuzzy classes, i.e., the set of properties-based rules used to assign fuzzy entities to fuzzy classes.

- **Membering**: relates fuzzy entities to fuzzy classes through the definition of d.o.m.

- **Interaction**: relates members of one fuzzy class to other members of one or several fuzzy classes.
Constructs of FSM

Complex fuzzy classes

FSM contains several complex classes:

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction fuzzy class</td>
<td>A fuzzy class that describes the interaction of two or more fuzzy classes</td>
</tr>
<tr>
<td>Fuzzy superclass</td>
<td>A generalization of one or many, simple or complex, fuzzy classes</td>
</tr>
<tr>
<td>Fuzzy subclass</td>
<td>A specialization of one or many, simple or complex, fuzzy classes</td>
</tr>
<tr>
<td>Grouping fuzzy class</td>
<td>A fuzzy class that its members are homogenous collection of members from the same fuzzy class</td>
</tr>
<tr>
<td>Aggregate fuzzy class</td>
<td>A fuzzy class that its members are heterogeneous collection from several fuzzy classes</td>
</tr>
<tr>
<td>Composite fuzzy class</td>
<td>A fuzzy class that has other fuzzy classes as members</td>
</tr>
</tbody>
</table>
### Definition of d.o.m

<table>
<thead>
<tr>
<th>Construct</th>
<th>Extent set and Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Attribute</strong></td>
<td>• $\rho_{p_i}^{K}(v_i)$</td>
<td>The partial d.o.m of a fuzzy entity $e$ for extent property $p_i$ and fuzzy class $K$.</td>
</tr>
<tr>
<td><strong>Entity/Class relationship</strong></td>
<td>• $X_K { p_1, p_2, \ldots, p_n }$ • $\mu_K(e) = \frac{\sum_{i=1}^{n} \rho_{p_i}^{K}(v_i) \cdot w_i}{\sum_{i=1}^{n} w_i}$</td>
<td>The global d.o.m of fuzzy entity $e$ in fuzzy class $K$.</td>
</tr>
<tr>
<td><strong>Interaction relationship</strong></td>
<td>• $\mu_I(e) = \prod_{i=1}^{m} \mu_{K_i}(e_i)$</td>
<td>The d.o.m of a member $e$ of a fuzzy interaction class $I$ relating $m$ members $e_1, e_2, \ldots, e_m$ from $m$ fuzzy classes $K_1, K_2, \ldots, K_m$.</td>
</tr>
<tr>
<td><strong>Entity from a subclass in its superclass</strong></td>
<td>• $X_{S_i} = { p_1, \ldots, p_k, p_{k+1}, \ldots, p_m }$ • $X_{S_j} = { p_1, \ldots, p_k, p_{k+1}', \ldots, p'<em>m, p</em>{m+1}, \ldots, p_n }$ • $\mu_{S_i}(e/S_j) = \frac{\sum_{i=1}^{k} \rho_{p_i}(v_i) \cdot w_i + \sum_{j=k+1}^{m} \rho_{p_j}(v'<em>j) \cdot w'<em>j}{\sum</em>{i=1}^{k} w_i + \sum</em>{j=k+1}^{m} w'_j}$</td>
<td>The d.o.m of a fuzzy entity $e$ from fuzzy subclass $S_j$ in its fuzzy superclass $S_i$; where $p'<em>{k+1}, \ldots, p'<em>m$ are overridden from $p</em>{k+1}, \ldots, p_m$ and $p</em>{m+1}, \ldots, p_n$ are specific for $S_j$.</td>
</tr>
<tr>
<td><strong>Subclass/Superclass relationship</strong></td>
<td>• $\mu(S_i, S_j) = \frac{\sum_{e_\alpha \in S_j} \mu_{S_i}(e_\alpha/S_j) \cdot \mu_{S_j}(e_\alpha)}{\sum_{e_\alpha \in S_j} \mu_{S_j}(e_\alpha)}$</td>
<td>The d.o.m of a fuzzy subclass $S_j$ in its fuzzy superclass $S_i$.</td>
</tr>
</tbody>
</table>
Definition of fuzzy classes

CLASS <class-name> WITH DOM OF <dom> {
    1 SUPERCLASS:
        OF <class-name> WITH DOM OF <dom>
        . . .
    2 INTERACTION CLASS OF <class-list>
    3 EXTENT:
        <ext-pr> WITH WEIGHT OF <w_i> DECISION RULE IS ((<attr-name><op><value>))| is-a <s-phrase>)
        . . .
    4 ATTRIBUTES:
        <attr-name>: [FUZZY] DOMAIN <domaine>: TYPE OF <type> WITH DOM OF <dom>:
        [REQUIRED][UNIQUE][MULTI-VALUED]
        . . .
    5 CONTENTS:
        [ENUMERATED COMPOSITION FROM (<class-name:members-list>)]
        [SELECTED COMPOSITION ON ATTRIBUTES <attr-list> FROM <class-list>]
        [AGGREGATION OF (<class-name:members-list>)]
        [GROUPING FROM <class-name:members-list>]
    6 INTERACTION:
        <inter-name> WITH (<class-name> INVERSE IS <inter-name> | <class-list>) [CLASS IS <class-name>]
        . . .
}
Definition of fuzzy classes

Example

CLASS star WITH DOM OF dom
{
  SUPERCLASS:
  OF supernova WITH DOM OF dom
  OF nova WITH DOM OF dom

  EXTENT:
  \( p_1 \) WITH WEIGHT OF 0.8 DECISION RULE IS \( \text{luminosity} \geq 0.5L_s \)
  \( p_2 \) WITH WEIGHT OF 0.3 DECISION RULE IS \( \text{weight} \geq 0.05W_s \)

  ATTRIBUTES:
  star-name: TYPE OF string WITH DOM OF 1.0: REQUIRED
  \( \text{type-of-star} \): TYPE OF symbolic(nova, supernova) WITH DOM OF 1.0: REQUIRED
  age: FUZZY DOMAIN \{very young, young, old, very old\}: TYPE OF integer WITH DOM OF 1.0: REQUIRED
  weight: FUZZY DOMAIN \([0.01W_s \rightarrow 100W_s]\): TYPE OF real WITH DOM OF 1.0: REQUIRED
}
Definition of fuzzy subclasses

**SUBCLASS** `<class-name>` WITH DOM OF `<dom>`

{  
  SPECIALIZATION:  
  OF `<class-name>` WITH DOM OF `<dom>`:
  [BY ENUMERATION `<members-list>`]
  [ON ATTRIBUTES `<attr-list>`]
  [BY INTERSECTION WITH `<class-list>`]
  [BY DIFFERENCE WITH `<class-name>`]
  . . .  
  SUPERCLASS:  
  . . .  
  INTERACTION CLASS OF  
  . . .  
  EXTENT:  
  . . .  
  ATTRIBUTES:  
  . . .  
  CONTENTS:  
  . . .  
  INTERACTION:  
  . . .  
}

**Example**

**SUBCLASS** `supernova` WITH DOM OF `dom`

{  
  SPECIALIZATION:  
  OF `star` WITH DOM OF `dom`:
  ON ATTRIBUTES `type-of-star`
  . . .  
  EXTENT:  
  `p_6` WITH WEIGHT OF 0.6 DECISION RULE IS `luminosity` ≥ `high`
  `p_7` WITH WEIGHT OF 0.5 DECISION RULE IS `weight` ≥ `1W_s`
  ATTRIBUTES:  
  `snova-name`: TYPE OF `string` WITH DOM OF 1.0: REQUIRED UNIQUE  
  `type-of-snova`: TYPE OF `symbolic(Ia, Ib, Ic, Ib/c, Ic/b, II-P, II-L)` WITH DOM OF 1.0: REQUIRED  
  `luminosity`: FUZZY DOMAIN `{high, very high}`: TYPE OF `real` WITH DOM OF 1.0: REQUIRED  
  `weight`: FUZZY DOMAIN `[1W_s - 100W_s]`: TYPE OF `real` WITH DOM OF 1.0: REQUIRED
}

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FRO supports several types of imperfect information.

In fuzzy database literature:

- Uncertainty and imprecision are often represented through fuzzy sets and possibility distribution.
- Vagueness is often represented through fuzzy set theory, similarity, proximity and/or resemblance relations.
- The combination of several approaches is also frequent.

Adopted representation

To facilitate data manipulation and for computing efficiency, the different types of attributes values (crisp, imprecise, uncertain, fuzzy, unknown, undefined or null) are uniformly represented through possibility distribution.
Representation of imperfect information

Fuzzy range

\[ \mu_A(z) = \begin{cases} 
1, & \text{if } \beta \leq z \leq \gamma; \\
\frac{\lambda - z}{\lambda - \gamma}, & \text{if } \gamma < z < \lambda; \\
\frac{z - \alpha}{\beta - \alpha}, & \text{if } \alpha < z < \beta; \\
0, & \text{otherwise.}
\end{cases} \]

Approximate value

\[ \mu_A(z) = \begin{cases} 
1, & \text{if } z = c; \\
\frac{c^+ - z}{c^+ - c}, & c < z < c^+; \\
\frac{z - c^-}{c^- - c}, & c^- < z < c; \\
0, & \text{Otherwise.}
\end{cases} \]

Linguistic label

\[ \mu_A(z) = \begin{cases} 
\frac{1}{1 + \frac{z - a_1 - b_1}{b_1}}, & \text{if } z < a_1 + b_1; \\
\frac{1}{1 + \frac{z - a_2 + b_2}{b_2}}, & \text{if } z > a_2 - b_2; \\
1, & \text{if } a_1 + b_1 \leq z \leq a_2 - b_2;
\end{cases} \]
Meta-relation **ATTRIBUTES**

Stores the specificity of all the attributes:

- **attribute-id**: it uniquely identifies each attribute.
- **attribute-name**: it stores the name of an attribute.
- **class-name**: denotes the fuzzy class to which the attribute belongs.
- **data-type**: which is a multi-valued attribute that stores the attribute type.

**Remark**

For crisp attributes, attribute "**data-type**" works as in conventional databases. For fuzzy attributes, the "**data-type**" attribute stores the fuzzy data type itself and the basic crisp data type on which the fuzzy data type is based.

**Example**

<table>
<thead>
<tr>
<th>attribute-id</th>
<th>attribute-name</th>
<th>class-name</th>
<th>data-type</th>
</tr>
</thead>
<tbody>
<tr>
<td>attr-15</td>
<td>star-name</td>
<td>STAR</td>
<td>{string}</td>
</tr>
<tr>
<td>attr-16</td>
<td>type-of-star</td>
<td>STAR</td>
<td>{symbolic}</td>
</tr>
<tr>
<td>attr-17</td>
<td>age</td>
<td>STAR</td>
<td>{linguistic label, real}</td>
</tr>
<tr>
<td>attr-18</td>
<td>luminosity</td>
<td>STAR</td>
<td>{linguistic label, real}</td>
</tr>
<tr>
<td>attr-20</td>
<td>weight</td>
<td>STAR</td>
<td>{interval, real}</td>
</tr>
</tbody>
</table>
Meta-relation PARAMETERS

Stores the parameters associated with different fuzzy datatypes:

- **attribute-id**: references one attribute that appears in ATTRIBUTES.
- **label**: stores a linguistic term belonging to the attribute domain.
- **parameters**: is a multi-valued attribute used to store the parameters required for generating the possibility distribution of the linguistic term.

**Remark**

Attributes with no parameters, will not be included in PARAMETERS meta-relation.

**Example**

<table>
<thead>
<tr>
<th>attribute-id</th>
<th>label</th>
<th>parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>attr-17</td>
<td>very young</td>
<td>{0.0, 0.0, 0.5, 1}</td>
</tr>
<tr>
<td>attr-17</td>
<td>young</td>
<td>{0.8, 1.7, 2, 2.5}</td>
</tr>
<tr>
<td>attr-17</td>
<td>old</td>
<td>{2.3, 5, 10, 15}</td>
</tr>
<tr>
<td>attr-17</td>
<td>very old</td>
<td>{12, 17, 50, 60}</td>
</tr>
</tbody>
</table>
Meta-relation A-DECISION-RULES

Stores attribute-based extent properties:

- **extent-property**: stores the name of the extent property.
- **class-name**: stores the name of the class for which the extent property is defined.
- **based-on**: references the "attribute-id" on which the extent property is based.
- **decision-rule**: is a composite attribute defined as follows:
  - **operator**: contains a binary or a set operator.
  - **right-hand-operand**: is a crisp or fuzzy value from the attribute domain.
- **weight**: stores the weight of the extent property.

### Example

<table>
<thead>
<tr>
<th>extent-property</th>
<th>class-name</th>
<th>based-on</th>
<th>decision-rule operator</th>
<th>right-hand-operand</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>ext-star-1</td>
<td>STAR</td>
<td>attr-17</td>
<td>≥, 0.005L_s</td>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td>ext-star-2</td>
<td>STAR</td>
<td>attr-18</td>
<td>≥, 0.5W_s</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>ext-snova-1</td>
<td>SUPERNOVA</td>
<td>attr-50</td>
<td>=, high</td>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td>ext-snova-2</td>
<td>SUPERNOVA</td>
<td>attr-51</td>
<td>≥, 1W_s</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>
### Meta-relation S-DECISION-RULES

Stores extent properties based on common semantics:

- **extent-property**: stores the name of the extent property.
- **class-name**: denotes the name of the fuzzy class for which the extent property is defined.
- **decision-rule**: is a composite attribute defined as follows:
  - **operator**: is an "is-a" operator.
  - **right-hand-operand**: is a semantic phrase.
- **weight**: stores the weight of the extent property.

#### Example

<table>
<thead>
<tr>
<th>extent-property</th>
<th>class-name</th>
<th>decision-rule</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>ext-person</td>
<td>PERSON</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ext-galaxy</td>
<td>GALAXY</td>
<td>{is-a, person}</td>
<td>1.0</td>
</tr>
<tr>
<td>ext-scientist</td>
<td>SCIENTIST</td>
<td>{is-a, galaxy}</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>{is-a, scientist}</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Other Meta-relations

There are several other meta-relations:

**PROXIMITY**: Stores the proximity relations associated with some datatypes (e.g. linguistic label).

**SUB-SUPER-COMP**: Stores information concerning fuzzy subclass/superclass and composition relationships.

**GROUPING**: Stores information concerning fuzzy grouping and aggregation relationships.

**INTERACTION**: Stores information concerning fuzzy interaction relationships.
Extent definition of fuzzy classes

At the extent definition of the fuzzy class, each fuzzy attribute is mapped into a new composite one composed of:

- **attr-value**: stores the value of the attribute as provided by the user.
- **data-type**: stores the data type of the value being inserted.
- **parameters**: is a multi-valued attribute used to store parameters associated with the attribute value that are used to generate its possibility distribution.

### Example

<table>
<thead>
<tr>
<th>Luminosity</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>attr-value</td>
<td>data-type</td>
</tr>
<tr>
<td>{high, less than linguistic label, {25,28}}</td>
<td>0.1\text{L}_S, \text{real}, {\text{nil}}</td>
</tr>
<tr>
<td>{0.1\text{L}_S, \text{real}, {\text{nil}}}</td>
<td>{12\text{W}_S-15\text{W}_S}, \text{interval}, {12,15}}</td>
</tr>
<tr>
<td>{more than 10\text{L}_S, more than linguistic label, {7.5\text{L}_S,10}}</td>
<td>{about 17\text{W}_S, \text{approximate value}, {15,17,18}}</td>
</tr>
</tbody>
</table>
Simple classes transformation

- Each fuzzy class in the FSM model is mapped into a relation in the database level.
- The fuzzy attributes are mapped into composite ones. The crisp attributes are treated as in conventional databases.
- An additional non printable attribute, `dom`, used to store the global d.o.m is systematically added into the new relation.
- The information relative to the extent properties of the fuzzy class are automatically introduced in the A-DECISION-RULES and/or S-DECISION-RULES meta-relations.

<table>
<thead>
<tr>
<th>star-name</th>
<th>type-of-star</th>
<th>luminosity attr-value</th>
<th>data-type</th>
<th>parameter</th>
<th>dom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vega</td>
<td>NOVA</td>
<td>{low, linguistic label, {2.5, 2}}</td>
<td></td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>3C 58</td>
<td>SUPERNOVA</td>
<td>{0.1Ls, real, {nil}}</td>
<td></td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Proxima Centauri</td>
<td>SUPERNOVA</td>
<td>{more than 10Ls, more than linguistic label, {7.5Ls, 10}}</td>
<td></td>
<td></td>
<td>0.75</td>
</tr>
</tbody>
</table>
Subclass/Superclass relationships transformation

A fuzzy subclass $B$ of a fuzzy superclass $A$ is mapped into a relation which inherits all the attributes of the relation transformed from $A$ (the relational object database model allows inheritance).

In addition to the attribute "dom", the relation $B$ contains a new attribute, denoted by $\text{dom-A}$, which is used to store the d.o.m of one entity from fuzzy subclass $B$ in its fuzzy superclass $A$.

The same reasoning is used for fuzzy subclasses with more than one fuzzy superclass. Note particularly that the relation mapped from fuzzy class $B$ will contain several "dom-A", one for each fuzzy superclass.

Example

<table>
<thead>
<tr>
<th>snova-name</th>
<th>type-of-snova</th>
<th>...</th>
<th>dom</th>
<th>dom-star</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN1987a</td>
<td>IIb</td>
<td>...</td>
<td>0.95</td>
<td>1.0</td>
</tr>
<tr>
<td>SN1604</td>
<td>Ic</td>
<td>...</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>SN1006</td>
<td>Unknown</td>
<td>...</td>
<td>0.7</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Extending binary and set operators

We need to extend the binary and the set operators to apply to imperfect information.

**Fuzzy "=" operator**

Models the **equality** concept for **precise** as well as **imprecise** data values:

<table>
<thead>
<tr>
<th>$\mu(\bar{x}, \bar{y})$</th>
<th>$\mu(\bar{x}, \bar{y})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{cases} 1, &amp; x = y; \ 0, &amp; \text{otherwise.} \end{cases}$</td>
<td>$\sup_{(x,y) \in X \times Y} \min[p(x,y), \pi_{\tilde{x}}(x), \pi_{\tilde{y}}(y)]$</td>
</tr>
</tbody>
</table>

where: $p(x,y)$: proximity relation; $\pi_{\tilde{x}}, \pi_{\tilde{y}}$: possibility distributions.

**Fuzzy "≃" operator**

Gives the **degree** in which two **exact numbers** are **approximately** equal:

$$\mu(\bar{x}, \bar{y}) = \begin{cases} 0, & |x - y| > c; \\ 1 - \frac{|x - y|}{c}, & |x - y| \leq c. \end{cases}$$

We suppose that the parameters $c^+$ and $c^-$ of an approximate value are the same and equal to $c$. 
Syntax of retrieve queries

\[
\text{[FROM } \{ (\text{<perspective-class-name>} \quad [\text{WITH DOM} \quad <\text{op}_1> <\text{class-level}>] \mid \<\alpha-MEMBERS\; \text{OF} \; \text{perspective-class-name}> ) \} \}\text{]}
\]
\text{RETRIEVE} \; <\text{target-list}> \\
\text{[ORDER BY} \; <\text{order-list}>\] \\
\text{[WHERE} \; <\text{selection-expression}> \quad [\text{WITH DOM} \; <\text{op}_2> <\text{attr-level}>] \}\]

Example

Retrieve the name and type of supernova that have global d.o.m equal to or greater than 0.7 and have luminosity greater than 15L_{\odot} with partial d.o.m equal to or greater than 0.9.

\text{FROM} \; \text{supernova} \; \text{WITH DOM} \; \geq \; 0.7 \\
\text{RETRIEVE} \; \text{snova-name, type-of-snova} \\
\text{WHERE} \; \text{luminosity} \; > \; 15L_{\odot} \; \text{WITH DOM} \; \geq \; 0.9
There are three phases

- **Phase 1**: Syntactic analysis.
- **Phase 2**: Verifying the conditions specified in the **FROM** statement. It returns, for each tuple, a global satisfaction degree $d_g \in [0, 1]$ measuring the level to which the tuple satisfies the **class-level conditions**. The tuples for which $d_p > 0$ represent the input for the next phase.
- **Phase 3**: Associate to each tuple a partial satisfaction degree $d_p \in [0, 1]$ measuring the level to which tuples satisfy the **entity-level conditions**.

The overall satisfaction $d_o \in [0, 1]$ is computed as follows:

$$d_o = d_g \times d_p$$
Conclusion and Future Works

Compared to several other proposals, FSM
- Semantically richer.
- Supports fuzziness within all of its constructs and within all levels.

The mapping approach
- supports a rich set of fuzzy, imprecise and uncertain data types ensuring its high flexibility.
- guarantees a high level of data/application independency thanks to its relational part.
- supports all the constructs of FSM.

Future works
- Full implementation of FRO.
- Implement the query language.