A theoretical analysis of multi-agent patrolling strategies

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1. Introduction

A group of agents can be used to perform patrolling tasks in a variety of domains ranging from computer network administration to computer wargame simulations. The quality of the strategies used for patrolling may be evaluated using different measures. Informally, a good strategy is one that minimizes the time lag between two passages to the same place and for all places.

Recently, many different architectures of multi-agent systems have been proposed and evaluated on the patrolling problem [1]. In particular, different types of agents (reactive vs. cognitive), agent communication (allowed vs. forbidden), coordination scheme (central and explicit vs. emergent), agent perception (local vs. global), and decision-making mechanism (random selection vs. goal-oriented selection) were proposed.

This paper proposes a theoretical analysis of the patrolling problem addressing the following issues : Are there efficient algorithms generating near-optimal strategies ? Are patrolling strategies based on partitioning the territory good?

2. The patrolling task

As in [1], the territory on which agents patrol is represented by a graph $G(V, E)$, where $V = \{1 \dots n\}$ is the set of nodes and $E \subseteq V^2$ the set of edges of G. To each edge (i, j) will correspond a weight c_{ij} representing the distance between nodes. The time taken by an agent to move across an edge (i, j) will be exactly c_{ij}. When the patrolling task starts, agents will move simultaneously around the nodes and edges of the graph according to a predetermined strategy.

Definition. The *strategy* of an agent is a function $\pi : \mathbb{N} \rightarrow V$ such that $\pi(j)$ is the j^{th} node visited by the agent. A *multi-agent strategy* $\Pi = \{\pi_1 ... \pi_r\}$is simply defined as a set of r single-agent strategies.

Our main goal is to find good patrolling strategies. We thus need an evaluation criterion. We will use *idleness* criteria introduced in [1].

Definition. Let r agents patrol a graph G according to a multi-agent strategy Π. The *idleness* of a node i at time t is the amount of time elapsed since that node has received the visit of an agent. The idleness of all nodes at the beginning of the patrolling task is set to 0. The *worst idleness* is the biggest value of the idleness occurred during the entire patrolling process for all nodes. It is noted WI_{Π}.

3. The cyclic strategies

Consider a single agent patrolling over an area. The simplest strategy which comes to mind would be to find a cycle covering all the area, and then to make the agent travel around this cycle over and over. Single-agent strategies consisting in traveling indefinitely along a cycle which covers all nodes of G will be called *single-agent cyclic strategies*. With a single agent patrolling around a cycle s, the worst idleness will be equal to the length of s. This problem clearly relates to the well known *Traveling Salesman Problem (TSP)* [3], which consists in finding the shortest cycle covering all nodes of a graph. This problem is known to be NP-complete.

Let S_{TSP} denote the cycle being the optimal solution to the TSP, whereas S_{Chr} will denote the cycle obtained by the algorithm of Christofides[1] [3]. The following holds:

Theorem 1. For a single agent, the optimal strategy in terms of worst idleness is the cyclic-based strategy based on S_{TSP}.

Proofs for this theorem and all others in this paper can be found in [2]. One way to extend single-agent cyclic strategies to the multi-agent case is to arrange agents on the same cycle such that when they start moving through that path

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[1] an algorithm able to compute an approximation of the TSP such that $\mid S_{Chr} \mid \leq \frac{3}{2} \mid S_{TSP} \mid$

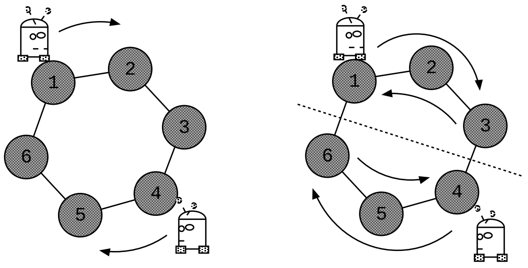

Figure 1. Left: cyclic strategy $\Pi_{cyc} = \{\pi_{cyc1} = 1, 2, 3, 4, 5, 6, 1, 2 \ldots, \pi_{cyc2} = 4, 5, 6, 1 \ldots\}$**. Right: partition based strategy** $\Pi_P = \{\pi_{P_1=1,2,3,2,1,2,\ldots}, \pi_{P_2} = 4, 5, 6, 5, 4, 5, \ldots\}$**. Here,** $WI_{\Pi_{cyc}} = 3$ **and** $WI_{\Pi_P} = 4$**.**

all in the same direction, they keep an approximately constant gap between them. More formally:

Definition. Let $S = s_0 ... s_m$ be a cycle visiting all nodes of a graph G. The strategy $\Pi = \{\pi_1 ... \pi_r\}$ is a *multi-agent cyclic strategy* based on S iff there exists $d_1 ... d_r \in \mathbb{N}$ such that $\pi_i(k) = s_{(k+d_i) \bmod m}$. The set of all multi-agent cyclic strategies will be referred to as $\mathbf{\Pi_{cyclic}}$.

The following theorem states that using a TSP approximation algorithm such as Christofides [3], we can obtain very good multi-agent strategies. From now on, *opt* will refer to the worst idleness of the optimal strategy.

Theorem 2. *Let G=(V,E) a connected metric graph and let r agents patrolling on it. Let Π_{Chr} be the multi-agent cyclic strategy based on S_{Chr}. We have* $WI_{\Pi_{Chr}} \leq 3 \times opt + 4 \times \max_{ij}\{c_{ij}\}$.

Note that if all edges of G have the same length, then $opt \geq \max_{ij}\{c_{ij}\}$, and therefor, $WI_{\Pi_{Chr}} \leq 7 \times opt$.

4. Partition-based strategies

Another very intuitive way to make r agents patrol over a territory would be make a partition of this territory into r disjoint regions, and to have each agent patrolling inside a single region.

From now on $P = \{P_1 ... P_r\}$ will denote a partition of V. Thus, $P_1 \cup \ldots \cup P_r = V$ and $P_i \cap P_j = \emptyset$.

Definition. A multi-agent strategy $\Pi = \{\pi_1 \ldots \pi_r\}$ is said to be based on a partition P iff each agent k following strategy π_k visits the nodes of a single region of P. The class of all strategies based on partition P is referred to as $\mathbf{\Pi_P}$.

Figure 1 illustrates how both strategies perform on a circular graph with two agents. On this example, the cyclic strategy wins. It seems that on graphs having “long corridors” connecting two sub-graphs, partition-based strategies could be better.

The following theorem compares the values of the worst idleness of the optimal cyclic strategy and the optimal partition-based strategy. Here $opt_{\Pi_{Cycle}}$ and opt_{Π_P} refer to the worst idleness of the optimal cyclic strategy and of the optimal strategy based on partition P.

Theorem 3. $opt_{\mathbf{\Pi_{cycle}}} \leq opt_{\mathbf{\Pi_P}} + 3 \times \max_{ij}\{c_{ij}\}$

Corrolary. Let $P = \{P_1, ..., P_r\}$ a partition of V. It is possible to compute in $O(n^3)$ a cyclic strategy Π_{Chr} such that $WI_{\Pi_{Chr}} \leq \frac{3}{2} opt_{\mathbf{\Pi_P}} + 4 \times \max_{ij}\{c_{ij}\}$.

5. Conclusion

We have presented various theoretical results for the patrolling problem. First, we showed that the single-agent patrolling problem could be solved with a TSP approach. Then, we defined the class of multi-agent cyclic strategies, based on an extension of this approach to more than one agent, for which we obtained an approximation result, stating that with the $O(n^3)$ Christofides algorithm, a close to optimal strategy could be obtained.

The strategies based on a partitioning of the graph were also studied. A surprising result was obtained : except when $\max_{ij}\{c_{ij}\}$ is big, the cyclic strategies based on TSP were shown to be better than any partition based strategy. However, when graphs have long “tunnels” separating regions, the cyclic strategies are not well suited.

Finally, some experiments were conducted to compare the state-of-the-art patrolling algorithms to the cyclic strategy. We used the freely available *Concorde* software package to compute more efficiently than with Christofides algorithm approximations of the TSP. The cyclic strategy was compared to all strategies described in [1] on six very different graphs. In each case, the cyclic strategy was at least as good as all the other strategies.

References

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