Markowitz’s MVO model

The Markowitz’s MVO model is a convex quadratic programming formulation aiming at finding a minimum variance portfolio of \( n \) securities that guarantees a minimum desired target return \( R \):

\[
\begin{align*}
\min & \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij} x_i x_j \\
\text{s.t.} & \quad \sum_{i=1}^{n} \mu_i x_i \geq R \\
& \quad \sum_{i=1}^{n} x_i = 1 \\
& \quad x_i \geq 0, \quad i = 1, \ldots, n
\end{align*}
\]

where \( Q \) is a positive semidefinite matrix, representing the covariance of the security returns.

Case Study – Investigate the performance of the model

- Consider the securities and the historical data on the web page \texttt{http://www.lamsade.dauphine.fr/~furini/}. Set up and solve the Markowitz’s MVO model for different levels \( R \) of expected target return. Compute the efficient frontier and the composition of the portfolios in terms of different securities, solving this problem for values of \( R \) ranging between \( R_{\text{min}} \) and \( R_{\text{max}} \).

- Discuss the advantages and the disadvantages of the portfolios constructed in this manner and propose possible solutions for the principal problems. Investigate how sensitive the optimal portfolios are to small changes in the input data.

- Determine an optimal portfolio maximizing the Sharpe Ratio using different possible real-world risk-less assets. Compare the performances of the computed portfolio against the average performances of the market.

- Two of the main disadvantages of the model is the possibility of creating not well-diversified portfolios and/or portfolios which contains a very high number of securities.
In order to limit these problem set up and solve the Markowitz’s MVO model with Minimum (and/or Maximum) Transaction Levels:

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij} x_i x_j \quad (5)
\]

\[
\sum_{i=1}^{n} \mu_i x_i \geq R \quad (6)
\]

\[
\sum_{i=1}^{n} x_i = 1 \quad (7)
\]

\[
x_i \geq l_{i\text{min}} u_i \quad i = 1, \ldots, n \quad (8)
\]

\[
x_i \leq l_{i\text{max}} u_i \quad i = 1, \ldots, n \quad (9)
\]

\[
x_i \geq 0 \quad i = 1, \ldots, n \quad (10)
\]

\[
u_i \in \{0, 1\} \quad i = 1, \ldots, n \quad (11)
\]

Compute the new efficient frontier and the composition of the portfolios in terms of different securities, solving this problem for values of \( R \) ranging between \( R_{\text{min}} \) and \( R_{\text{max}} \). Compare these portfolios with the previous ones in terms of composition and portfolio variance.

**Business game:**

- Construct a portfolio consisting of one of the national markets (S&P 500 index, NASDAQ index) using the Markowitz’s MVO model. Determine the maximum number of securities and time steps that can be treated by the Markowitz’s MVO model. What is the impact on the CPU time of increasing the number of securities and what is the impact of increasing the number of time steps?

- Determine for this new real-world dataset on optimal portfolio and present its features. Try to convince the management that your portfolio is (or maybe is not) a good strategy of investment. Discuss the results as you were in a financial consulting lab trying to convince the management to adopt or not the proposed portfolios.

**Things to keep in mind:**

- Write a clear and self-contained document of maximum 30 pages, where you motivate your choices and ideas.

- Do not include the source code in your document but only the results and the discussion of the results. Describe only the setting of the experiments and the optimization software used.