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Rank changes in production/assembly lines: impact and analysis

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Abstract *: Modern production and assembly lines contain several inspection stations and the quality control tests made at such stations may lead to sending the tested unit to an off-line repair shop in order make the necessary repair operations before reinserting the unit back in the line. Consequently the order (sequence) at which processed units leaves the production/assembly line is different from its initial entering order. The consequences of these sequence or rank changes are quite important for customized mixed-model production lines such as motorcar assembly lines. In this paper we derive the probability density function of the rank change of a given unit, from the probability density function of repair time. Having the exact form of this function is a prerequisite for a better and complete analysis of this rank change phenomenon, of its economical impact as well as for the design of efficient solutions to avoid its disruptive effects.*

Keywords: *mixed-model asssembly line, motor car industry, probability analysis.*

Résumé *: Les exigences de qualité conduisent à effectuer des contrôles systématiques sur certains postes d'une ligne de fabrication. Le contrôle effectué sur un poste peut conduire à sortir de la ligne un article défectueux pour permettre sa mise en conformité, avant réinjection dans la ligne, à la sortie de ce point de contrôle. L'ordre des articles à la sortie de ce poste de contrôle diffère de celui à son entrée, en raison de ces perturbations. Les conséquences de ce changement sont importantes dans la production de masse de produits personnalisés par des options, comme c'est le cas dans l'industrie automobile. On a établi analytiquement la fonction de probabilité du décyclage (= avance ou retard d'un article quelconque sortant du poste de contrôle), à partir de celle du retard positif ou nul engendré au contrôle. La connaissance d'une telle fonction est un préalable aux analyses poussées du décyclage, des moyens à mettre en œuvre pour contrer la désorganisation qui s'ensuit et, ce faisant, pour fonder économiquement la fiabilisation des processus de production ou de remise en conformité.*

Mots clés: *lignes d'assemblage multi-modèles, industrie automobile, analyse de probabilité, décyclage.*

1 Introduction

Modern quality standards and quality requirements make it necessary to add a number of inspection or quality control stations to any production/assembly line. There are two possible outcomes of an inspection operation in such a context. Either the inspected unit passes the inspection test and goes to the next workstation, or it fails the test, which implies that this unit should be withdrawn and send to a repair shop or a repair line to be repaired. Afterwards, the unit returns to the main line (see Figure 1). The unit is brought back either to the inspection station that sent it for repairs or to the following workstation. This depends on whether an equivalent inspection is done in the repair shop or not. In many cases, it is required that the unit under

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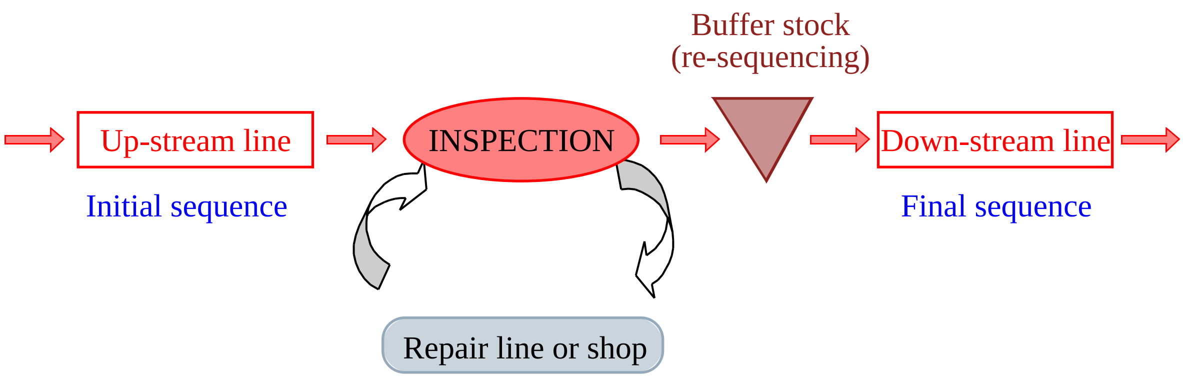
repair does not leave the repair shop until it is re-inspected and passes the inspection tests that caused its withdrawal from the main line.

In a mixed-model production line, different models are introduced to the line according to a predetermined sequence, which gives to each unit or model a rank in the sequence. The withdrawal and return of a unit implies a rank change for this unit as well as for some other units. Thus, in general, the final sequence is different from the initial sequence. If no other unit is withdrawn until the return of this unit, it will lose a number of positions in the sequence that depends on the repair time of the unit and the cycle time of the main production/assembly line. On the other hand, if some of the preceding and succeeding units are also withdrawn and require more repair time than our unit, the new position of the unit could be ahead of its old one. This rank change phenomenon is of interest for many industries that use mixed-model production/assembly lines particularly for the car manufacturing industry. That is why we will focus hereafter on the impact and the analysis of this problem in this industry.

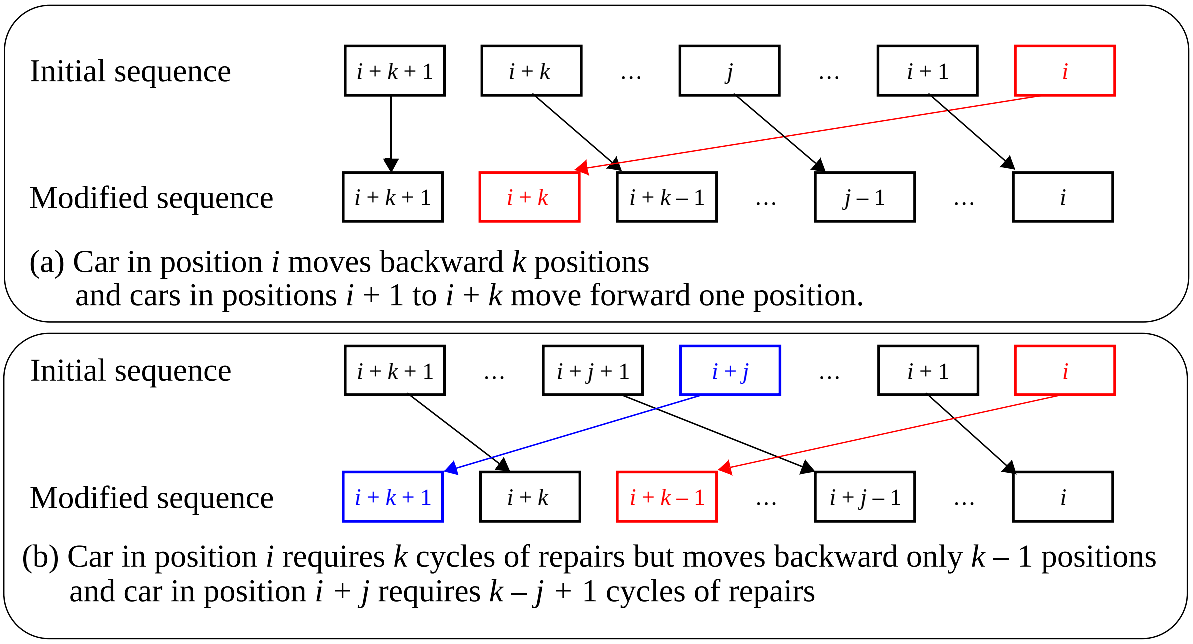
Figure 1: Mixed-model assembly lines with inspection stations



It is the usual practice to have a buffer stock after each inspection station. This is needed to prevent line stoppage when a car is withdrawn for repairs and also allows re-sequencing the flow in order to overcome some of the negative effects of sequence or rank changes. These effects will be discussed later but let us first give the definition of some terms that will be used in this paper. We define the *rank* of a car under assembly at the entrance of a given workstation as its order number in the production sequence at this entrance point. Thus the *initial rank* of a car is its order number or position in the sequence at the entrance of the first workstation of the assembly line. Similarly, the *final rank* of a car is its rank at the entrance of the final stocking area at the end of the line. Assuming that our assembly line has a constant cycle time and that the line has never been stopped for any reason, the rank of a car among those to be introduced during a given time period (e.g. a day) at the entrance of a given workstation equals the number (starting the count at the beginning of the time period) of the production cycle where this workstation processes the considered car. The difference between the rank of a given car at a workstation B and its rank at a preceding workstation A, will be referred to as the *rank change* of this car between A and B. The *overall rank change* is the difference between the final rank and the initial rank. A rank change could be positive, negative or nil. A positive rank change (called *lag*) means that the car will come out of the line later than initially planned and a negative rank change (called *advance*) means that it will come out earlier. In this paper the rank change between A and B will also be called the «*decycling*» between A and B.

Obviously, there are no rank changes between a workstation A and a following workstation B if there is no inspection station in between and no withdrawal is allowed over this line segment. The rank change of a given car at a given inspection station, which is measured by the difference between its rank at the entrance of the following workstation and its rank at the entrance of the given inspection station, depends not only on what happens to this car but also on what happens to some of the succeeding and preceding cars. Figure 2-a shows that if: the car in the ith position fails the test losing k positions (due to its repair time), and all the other cars pass the test, then each of the cars in positions $i+1, \ldots, j, \ldots, i+k$ will get a negative rank change (advance) of 1 position (i.e., moves forward one position). Figure 2-b presents a situation where the car in position i needs repairs that take k cycles but loses only $k-1$ positions (positive rank change of $k-1$) as the car in position $i+j$ (with $j<k$) requires repairs for more than $k-j$ cycles (in the figure it require $k-j+1$ cycles).

Figure 2: Rank change depends on repair time of the car and repair time of its neighbours



The consequences of rank changes are discussed in detail in Danjou, Giard and Le Roy (2000,[4]). Hereafter we give a short discussion of these consequences. But to better understand rank changes effects, let us recall that modern assembly lines generally adopt a just-in-time policy for the delivery of components to the line. Also notice that the initial production sequence is mainly made in a way that uniformly disperses the cars of the same model or having a component requiring large installation time. This is usually called the model or option spacing constraints. Uniformly spacing cars that require higher operations time at a given workstation helps the workers at this station to follow the line pace. Actually, if a model requiring more time than the line cycle time, is followed by a sufficient number of models that require less time than the cycle time, then the line balance will be maintained. Furthermore, one of the objectives we pursue in determining the initial production sequence is to reduce or to minimize the number of tools and machines set-ups.

Rank changes have three main effects. First, the just-in-time, components-supply plan, which is established in terms of the initial production sequence, can no longer guarantee the continuous functioning of the line. Some components will stay longer before they are used and some others will not be available when needed. Consequently, rank changes force the holding of a larger safety stock of components. Second, rank changes make it harder for some workstations to keep up the pace. Consequently, additional workers should be sent to such workstations from time to time in order to help rebalance the line. Finally, rank changes usually lead to an increase in the number of tool and machine set-ups.

The contribution of this research is twofold. First we study the following important and fundamental problem. Given the probability density function of R, the number of cycles a car needs for repairs (R depends directly on repair times and the line cycle-time), determine the probability density function of Δ, the rank change between the entrance to an inspection station and the entrance to the following station (recall that usually we have a buffer stock in between). Thus we seek to determine $P(\Delta = \delta)$ as a function of $P(R = r)$. Second, we provide a preliminary analysis of some related problems such as the determination of the size of the buffer stock that follows an inspection station, how to determine the probability distribution of rank changes for lines with multiple inspection stations, and how to organize the repair line or the repair shop.

Obviously, determining the form and parameters of the probability density function of Δ is a prerequisite for a precise analysis of the economic impact of decycling or rank changes. In other words, knowing the form and parameters of the probability function of rank changes, $P(\Delta = \delta)$, is needed to calculate the resulting increase in components safety stock, the resulting increase in work force cost and the additional tool and machine set-ups. Knowing the form and the parameters of this function is also necessary if we want to evaluate the impact of a specific action to improve the production process. This is because production process improvements should lead to a modification of the probability density function $P(R = r)$, and consequently, will modify the function $P(\Delta = \delta)$. Hopefully, this will lower the cost of the resulting components safety-stock, of the resulting additional work force and of the additional set-ups induced by rank changes.

The form and parameters of the function $P(\Delta = \delta)$ can be found empirically by simulation. This is the only available method if the buffer stock, between the inspection station and the following workstation, is not a «first in first out (FIFO)» stock (see Danjou, Giard and Le Roy, 2000). Hereafter, we assume that the flow through this buffer stock is FIFO and we shall derive the exact form of $P(\Delta = \delta)$. Knowing the exact analytical form of this function is much more helpful than just having an approximate empirical form if we need to study the response of the manufacturing system to any improvement action.

In addition to the above-mentioned assumption, we also assume that:

- The repaired cars do not come back to the main line until all the defects that caused their withdrawal are completely corrected. When they return, they are directly inserted into the buffer stock that follows the inspection station.
- The number of cars in this buffer stock is sufficiently large to prevent any line stoppage due to withdrawal of cars. Actually, to prevent such line stoppage, the number of cars to place in the buffer stock should equal r, the maximum number of cycles that a car may need for repairs. However, we may accept placing fewer cars in the buffer if we can accept a limited probability of line stoppage. This point will be studied in more detail in § 3, page 8.
- There is complete independence between the event «a car needs repair» and the event «the following car needs repair».
- The repair line or shop has sufficient capacity. Consequently, there is no waiting time in this shop or line. The repair time depends only on the needed repairs and does not depend on the number of cars under repair.
- The production/assembly line is in a steady state. We are not studying the transition state of the line

Although several contributions to mixed-model production/assembly line problems have been proposed in the literature over the last three decades, to the best of our knowledge, the rank change problem and its economic impact have never been the subject of any publication. In the available literature, for example, Thomopoulus (1970, [14]), Chakravarty and Shtub (1985, [3]), Miltenburg and Sinnamon (1989, [13]), Kim and Kwak (1993, [9]), Askin and Zhou (1997, [1]), Gokeen and Erel (1997, [7]) and McMullen and Frazier (1997, [11]), studied the problem of mixed-model line balancing. Miltenburg (1989, [12]), Miltenburg and Sinnamon (1989, [13]), Inman and Buffin (1991, [8]), Kubiak and Sethi (1991, [10]) and Danjou *et al* (2000, [4]) studied the problem of production sequencing. Bolat *et al* (1994, [2]) and Giard (1997, [5]) studied the problem of real-time line rebalancing where such rebalancing is needed. A review of some of this research work can be found in Yano and Bolat (1989, [16]).

The organization of this paper is as follows. In § 2, page 5, we analyse the rank change problem under the special assumption of identical repair time for all cars. In § 3, page 8, we discuss some related problems such as how to determine the size of the buffer stock that follows an inspection station, how to determine the probability distribution of rank changes for lines with multiple inspection stations, and how to organize the repair line or the repair shop. In § 4, page 9, we generalize the model to consider the case where cars may have different repair times.

§ 5, page 13 deals with the question of how to model the case where cars may require more than one visit to the repair shop. Finally, the conclusions of this research are drawn in § 6, page 13.

2 A simplified rank-change probability function

Let us consider the simplified situation where R, the number of cycles required for repairs, takes either the value 0 (the car passes the inspection test) or a fixed value ρ. This means that the repair time is the same for all cars that fail the inspection test. This situation occurs for example if the repairs are done on a paced secondary or parallel line with a fixed number of workstations

Let us also assume that the repair shop will repair all defects and consequently a repaired car will always be able to pass the inspection test with success. Thus a repaired car will be sent to the buffer stock following the inspection station and will not be sent back to the inspection station. In the next section we will show how to deal with the case where we cannot make this assumption, i.e., how to deal with the case where a car requires several visits to the repair shop or the repair line before it becomes acceptable.

Under the above assumptions, the car in position j either passes the inspection and immediately enters the buffer stock or fails the test and is sent to the repair line to come back and enters the stock ρ cycles later. When it comes back, its position in the sequence depends on x, the number of cars, among those originally in positions $j + 1$ through $j + \rho$, that failed the inspection test. Precisely its new position will be $j + \rho - x$.

Rank change d is negative (the car moves to a forward position), nil or positive (the car moves to a backward position). Rank change is negative ($\delta < 0$; also called position advance) if the car passes the inspection and one or more of the preceding r cars fail ($R = 0$ and $x > 0$). Rank change is nil ($\delta = 0$) if $R = 0$ (the car passes) and $x = 0$ (no one of the preceding ρ cars fail) or if $R = \rho$ (the car fails) and $x = \rho$ (all the succeeding ρ cars fail). Finally, rank change is positive ($\delta > 0$; also called position lag) if $R = \rho$ and some but not all the succeeding cars fail the inspection ($x < \rho$).

Let us call po the probability that a given car passes the inspection and $P(X = x)$ the probability that x among ρ successive cars fail the inspection. Then we have $P(R = 0) = p_0$, $P(R = \rho) = 1 - p_0$ and $P(X=x) = C_\rho^x (1-p_0)^x p_0^{\rho - x}$; $0 \le x \le \rho$ (Recall that we assumed that the events «a car fails the test» and «the succeeding car fails the test» are independent; thus $P(X = x)$ is a binomial distribution).

It is easy to see that if $R = 0$, which has a probability $P(R = 0) = p_0$, and if $x > 0$, which has a probability $C_\rho^x (1-p_0)^x p_0^{\rho - x}$, we obtain a negative rank change ($\delta < 0$) with a probability $P(\Delta = \delta) = p_0 C_\rho^x (1-p_0)^x p_0^{\rho - x}$. Also, $R = \rho$, which has a probability $P(R = \rho) = 1 - p_0$, and $\rho > x \ge 0$, which has a probability $C_\rho^x (1-p_0)^x p_0^{\rho - x}$, leads to a positive rank change ($\delta > 0$) with a probability $P(\Delta = \delta) = (1-p_0) C_\rho^x (1-p_0)^x p_0^{\rho - x}$. Finally, the probability $P(\Delta = 0)$ equals the sum of $P(R = 0).P(X = 0)$ and $P(R = \rho).P(X = \rho)$. Thus $P(\Delta = 0) = p_0^{\rho+1} + (1-p_0)^{\rho+1}$. In summary we have:

$$P(\Delta = \delta) = C_\rho^x (1-p_0)^x p_0^{\rho - x + 1} \; ; \; \delta < 0$$

$$P(\Delta = \delta) = p_0^{\rho+1} + (1-p_0)^{\rho+1} \; ; \; \delta = 0$$

$$P(\Delta = \delta) = C_\rho^x (1-p_0)^{x+1} p_0^{\rho - x} \; ; \; \delta > 0$$

As an illustration, Table1, page 7, gives the probability P(Δ = δ) for ρ = 10 and some values of po ranging from 0.80 to 0.98. Figure 3, page 6, gives the general form of this probability density function. This figure shows that P(Δ = δ) is, in general, not symmetrical. This can be explained by examining the probability function. It implies that P(Δ = δ; δ < 0) = $C_\rho^x(1-p_0)^x p_0^{\rho-x+1}$ while P(Δ = δ; δ > 0) = $C_\rho^x(1-p_0)^{x+1}p_0^{\rho-x}$ and if we agree that p_0 have in general a value larger than 0.50, we can see that P(δ < 0) is generally higher than P(δ > 0). However, this probability density function becomes symmetrical in the case where $p_0 = 0.50$.

Table2, page 7, and Figure 4 show the effect of r on the probability P(Δ = δ) for a given value of p_0 (p_0 = 0.98). Obviously, the standard deviation of rank changes decreases as ρ decreases. This may significantly reduce the economic impact of decycling.

Figure 4 : The simplified probability distribution P(Δ=δ) as a function of ρ (p_o = 0.98)

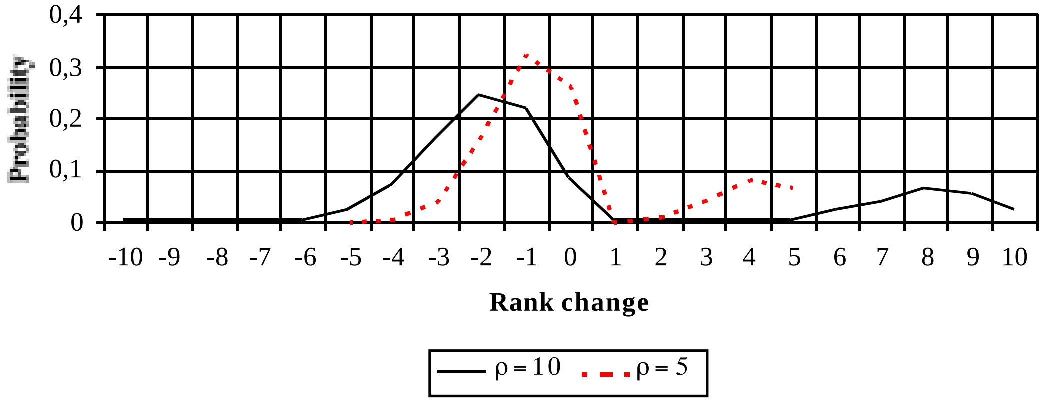


Figure 3 : Probability density function P(Δ = δ), in case of identical repair times (ρ = 10)

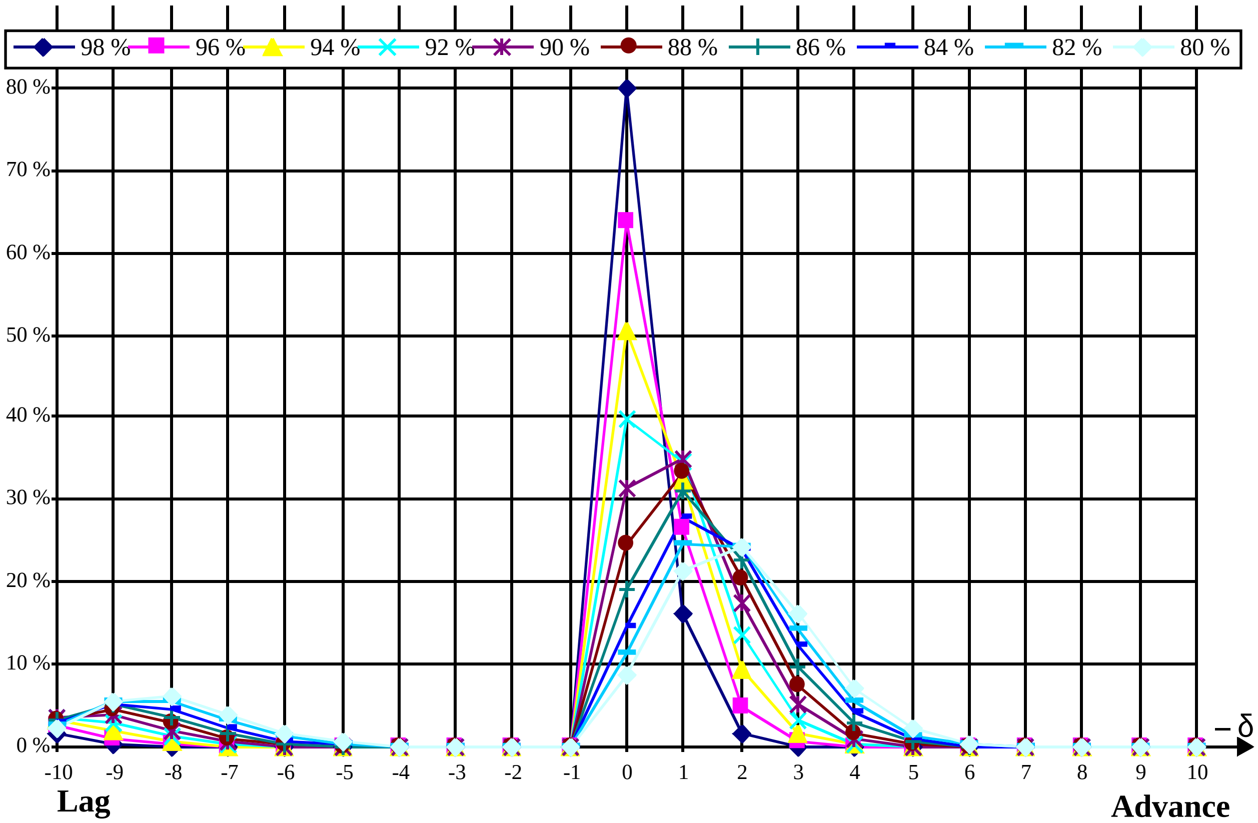


Table 1: The probability $P(\Delta=\delta)$ for $\rho = 10$

δ		p_0 (= probability of not passing the inspection test with success)									
		98%	**96%**	**94%**	**92%**	**90%**	**88%**	**86%**	**84%**	**82%**	**80%**
Advance	– 10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	– 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	– 8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
	– 7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.06%
	– 6	0.00%	0.00%	0.00%	0.00%	0.01%	0.03%	0.07%	0.15%	0.26%	0.44%
	– 5	0.00%	0.00%	0.01%	0.05%	0.13%	0.29%	0.55%	0.93%	1.45%	2.11%
	– 4	0.00%	0.04%	0.18%	0.48%	1.00%	1.78%	2.81%	4.06%	5.50%	7.05%
	– 3	0.08%	0.55%	1.58%	3.15%	5.17%	7.46%	9.85%	12.18%	14.31%	16.11%
	– 2	1.50%	4.99%	9.28%	13.60%	17.43%	20.51%	22.70%	23.99%	24.44%	24.16%
	– 1	16.34%	26.59%	32.32%	34.75%	34.87%	33.42%	30.98%	27.98%	24.74%	21.47%
0		80.07%	63.82%	50.63%	39.96%	31.38%	24.51%	19.03%	14.69%	11.27%	8.59%
Lag	1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.03%	0.06%	0.11%
	5	0.00%	0.00%	0.00%	0.00%	0.01%	0.04%	0.09%	0.18%	0.32%	0.53%
	6	0.00%	0.00%	0.01%	0.04%	0.11%	0.24%	0.46%	0.77%	1.21%	1.76%
	7	0.00%	0.02%	0.10%	0.27%	0.57%	1.02%	1.60%	2.32%	3.14%	4.03%
	8	0.03%	0.21%	0.59%	1.18%	1.94%	2.80%	3.69%	4.57%	5.36%	6.04%
	9	0.33%	1.11%	2.06%	3.02%	3.87%	4.56%	5.04%	5.33%	5.43%	5.37%
	10	1.63%	2.66%	3.23%	3.48%	3.49%	3.34%	3.10%	2.80%	2.47%	2.15%

Table 2: The simplified probability distribution $P(\Delta=\delta)$ as a function of ρ

δ		p_0 = 98%		p_0 = 96%		p_0 = 94%		p_0 = 92%		p_0 = 90%		p_0 = 88%		p_0 = 86%		p_0 = 84%		p_0 = 82%		p_0 = 80%	
		ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5	ρ=10	ρ=5
Avance	– 10	0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00	
	– 9	0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00	
	– 8	0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.00		0.01	
	– 7	0.00		0.00		0.00		0.00		0.00		0.00		0.01		0.02		0.03		0.06	
	– 6	0.00		0.00		0.00		0.00		0.01		0.03		0.07		0.15		0.26		0.44	
	– 5	0.00	0.00	0.00	0.00	0.01	0.00	0.05	0.00	0.13	0.00	0.29	0.00	0.55	0.00	0.93	0.01	1.45	0.02	2.11	0.03
	– 4	0.00	0.00	0.04	0.00	0.18	0.01	0.48	0.02	1.00	0.04	1.78	0.08	2.81	0.14	4.06	0.23	5.50	0.35	7.05	0.51
	– 3	0.08	0.01	0.55	0.06	1.58	0.18	3.15	0.40	5.17	0.73	7.46	1.18	9.85	1.75	12.18	2.43	14.31	3.22	16.11	4.10
	– 2	1.50	0.37	4.99	1.36	9.28	2.81	13.60	4.58	17.43	6.56	20.51	8.64	22.70	10.72	23.99	12.75	24.44	14.65	24.16	16.38
	– 1	16.34	9.04	26.59	16.31	32.32	22.02	34.75	26.36	34.87	29.52	33.42	31.66	30.98	32.93	27.98	33.46	24.74	33.37	21.47	32.77
0		80.07	88.58	63.82	78.28	50.63	68.99	39.96	60.64	31.38	53.14	24.51	46.44	19.03	40.46	14.69	35.13	11.27	30.40	8.59	26.22
Retard	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02	0.00	0.04	0.00	0.08	0.00	0.13
	2	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.03	0.00	0.08	0.00	0.16	0.00	0.28	0.00	0.46	0.00	0.71	0.00	1.02
	3	0.00	0.01	0.00	0.06	0.00	0.18	0.00	0.40	0.00	0.73	0.00	1.18	0.00	1.75	0.00	2.43	0.01	3.22	0.02	4.10
	4	0.00	0.18	0.00	0.68	0.00	1.41	0.00	2.29	0.00	3.28	0.00	4.32	0.01	5.36	0.03	6.37	0.06	7.32	0.11	8.19
	5	0.00	1.81	0.00	3.26	0.00	4.40	0.00	5.27	0.01	5.90	0.04	6.33	0.09	6.59	0.18	6.69	0.32	6.67	0.53	6.55
	6	0.00		0.00		0.01		0.04		0.11		0.24		0.46		0.77		1.21		1.76	
	7	0.00		0.02		0.10		0.27		0.57		1.02		1.60		2.32		3.14		4.03	
	8	0.03		0.21		0.59		1.18		1.94		2.80		3.69		4.57		5.36		6.04	
	9	0.33		1.11		2.06		3.02		3.87		4.56		5.04		5.33		5.43		5.37	
	10	1.63		2.66		3.23		3.48		3.49		3.34		3.10		2.80		2.47		2.15	
Standard deviation		1.47	0.77	2.06	1.07	2.49	1.30	2.85	1.49	3.15	1.64	3.41	1.78	3.64	1.90	3.84	2.01	4.03	2.10	4.20	2.19

3 Preliminary analysis of some related problems

In this section we study three problems related to the repair and rank change problems: (§ 3-1, page 8) the determination of the size of the buffer stock that follows an inspection station, (§ 3-2, page 8) how to determine the probability distribution of rank changes for lines with multiple inspection stations, and (§ 3-3, page 9) how to organize the repair line or the repair shop.

3-1 Buffer size

As mentioned an inspection station is usually followed by a buffer stock. This is essential to avoid line stoppage if a car is withdrawn for repairs. Instead of sending the cars that pass the inspection directly to the following station, we send them to a buffer stock that is used to feed the following station. Assuming that the assembly line is in a steady state, then the number of cars in the buffer at any moment equals its size (i.e. the maximal number of cars that it can hold) minus the number of cars in the repair line or shop at that moment. So, to avoid line stoppage, the buffer size should be larger than or equal to the maximum number of cars that could be found in the repair line or shop at any moment. Under our assumptions, it is easy to see that this number equals the maximum number of cycles a car may need for repairs. Thus, to never empty the buffer, which leads to line stoppage, the size of the buffer should equal ρ, the maximum number of cycles a car may need for repairs.

We have to mention that the buffer stock is also needed to avoid stoppage of the down stream part of the line (stations following the buffer stock) if the up-stream part (the part preceding the buffer) stops as a result of any technical problem or any material supply problem. Hereafter, we deal only with line stoppage because of cars withdrawn for repairs.

Holding a large number of cars in the buffer leads to an increased inventory holding cost. But as the probability that r cars be simultaneously in repair is quite small, we may decide to hold a smaller number of cars, say *S*, in the buffer stock. This decision should be made taking into consideration the inventory holding cost and the cost of stopping the line. This later cost mainly depends on the probability of having more than *S* cars withdrawn for repairs. But as shown in the previous section, *P(X=x)*, the probability that *x* among ρ cars fail the inspection test, is given by a binomial probability function. Consequently, The probability of having more than *S* cars withdrawn at a given moment, noted *P(X>S)* can be obtained by:

$$P(X = S) = \sum_{x = S+1}^{\rho} C_{\rho}^{x}(1-p_0)^x p_0^{\rho - x + 1}$$

Table 3, gives some values of P($X > S$) for ρ =10 and po ranging from 0.80 to 0.98. This table shows that for ρ =10 and p_0 =0.96 the probability of having more than 2 cars in repair is only 0.62 %. For the same value of ρ but with p_0 = 0.80, the probability of having more than 5 cars in repair is almost at the same level (0.64 %)

Table 3: The probability of having more than S cars in repair.

S	p_0, the probability that a car passes the inspection test									
	98%	96%	94%	92%	90%	88%	86%	84%	82%	80%
2	0.09%	0.62%	1.88%	4.01%	7.02%	10.87%	15.45%	20.64%	26.28%	32.22%
4	0.00%	0.00%	0.02%	0.06%	0.16%	0.37%	0.73%	1.30%	2.13%	3.28%
5	0.00%	0.00%	0.00%	0.00%	0.01%	0.04%	0.10%	0.20%	0.37%	0.64%

3-2 Lines with multiple inspection stations

Let us consider a production/assembly line with m inspection stations and assume that each of these stations is followed by a buffer stock of sufficient capacity to prevent any line stoppage.

Also we assume that the flow through these buffers is FIFO (no re-sequencing is done) and complete independence between the events (pass or fail) at different inspection points. Then the probability density function of the overall rank change between the initial and final sequence can be determined easily as the overall rank change δ will be the sum of successive rank changes $\delta_1, \delta_2, \ldots, \delta_m$. The probability density function is the sum of m binomial functions, which, based on the central limit theorem and assuming that m is sufficiently large, can be approximated by a normal probability density function.

3-3 Repair activities organization

We have mentioned that repairs are usually done either in a paced secondary line with a number of workstations or in a repair shop with a sufficient number of parallel and identical repair stations. In this second case each car is assigned to one and only one repair station, which will perform all the necessary repairs. The number of repair stations should be sufficiently large in order to avoid waiting times. This implies that, in order to be able to treat cars as soon as they arrive in the repair shop, the number of parallel stations should be greater than or equal to ρ, the maximum number of cars that could be found in the repair shop simultaneously. However, we may decide to use a fewer number of repair stations, say S stations. Then the probability that a car will wait for repairs equals the probability $P(X > S)$. This probability can be determined as shown in the previous section.

Furthermore, the repair shop can be considered as a queuing system with S parallel and identical servers. This allows us to determine the average waiting time and the average sojourn time.

4 A general rank-change probability function

In § 2, page 5, we derived the probability density function of rank changes in the simple case where R, the number of cycles required to perform the necessary repairs, takes either the value 0 or ρ. In this section we consider the case where R can take any integer value between 0 and ρ. In such a case, the rank change δ, for the car in position j and requiring ρ cycles for its repairs, will be $r - x$ where x is the number of cars with an initial rank between $j + r - 1$ and $j + r - \rho$ that are withdrawn for repairs and reintroduced in the main line after the considered car.

As $\delta = r - x$, $0 \le r \le \rho$ and $0 \le x \le \rho$, then $-\rho \le \delta \le \rho$. Also it is obvious that the same rank change δ can be the result of a number of combinations of r and x. For example a positive rank change value δ can be obtained by any value of r in the range $\delta \le r \le \rho$ and $x = r - \delta$. On the other hand a negative rank change δ can be obtained by any value of r in the range $0 \le r \le \rho - \delta$ and $x = r + \delta$. Thus, to generalize, a rank change δ (positive or negative) can be obtained by any value of r in the range $Max(0, \delta) \le r \le Min(\rho, \rho + \delta)$ and $x = r - \delta$.

Consequently the probability $P(\Delta = \delta)$ can be calculated by:

$$P(\Delta = \delta) = \sum_{Max(0,\delta)}^{Min(\rho,\rho+\delta)} P(R = r) \quad P(X = r - \delta)$$

As $P(R = r)$ is known, to calculate the probability $P(\Delta = \delta)$ we need to calculate the probability $P(X = x)$. This is more complicated. In the following we start by showing how to calculate the probabilities $P(X = 0)$, $P(X = 1)$, $P(X = 2)$ and $P(X = 3)$. Then, by induction, we derive the general formula for $P(X = x)$.

Recall that we are considering the car in position j and our objective is to calculate the probability that x among the cars in positions between $j + r - 1$ and $j + r - \rho$ come back to the line behind the considered car. Let the probability that the car in position $j + r - i$ require less than i cycles for its repairs be noted $P(R_{j+r-i} < i)$.

4-1 The probability P(*X*=0)

In this case none of the cars in positions between $j + r - 1$ and $j + r - \rho$ come back after the car having the initial position j. The probability of this combined event is:

$P(X = 0) = P(R_{j-1}<1).\ P(R_{j-2}<2)....\ .\ P(R_{j-\rho} <\rho)$

But assuming that the probability functions of R for all cars are identical and independent, we can write:

$$P(X = 0) = \prod_{i=1}^{\rho} P(R < i)$$

Now putting $\Pi = \prod_{i=1}^{\rho} P(R < i)$ we can write:

$$P(X = 0) = \Pi$$

4-2 The probability P(*X*=1)

In this case only one car among those initially in positions $j + r - 1$ and $j + r - \rho$ come back after the car having the initial position j. All other cars that are withdrawn for repairs come back to the main line in position preceding this car. The probability of this event is:

$$\begin{aligned} P(X = 1) = {} & P(R_{j-1}\geq 1).\ P(R_{j-2}< 2)....\ .\ P(R_{j-\rho} < \rho) \\ & + P(R_{j-1}< 1).\ P(R_{j-2}\geq 2)....\ .\ P(R_{j-\rho} < \rho) \\ & + \ldots \\ & + P(R_{j-1}< 1).\ P(R_{j-2}< 2)....\ .\ P(R_{j-\rho} \geq \rho) \end{aligned}$$

Again, assuming that the probability functions of *R* for all cars are identical and independent; then multiplying and dividing each term by P($R < i$), we obtain:

$$P(X = 1) = \left\{ \prod_{k=1}^{\rho} P(R < k) \right\} \sum_{i=1}^{\rho} \frac{P(R \geq i)}{P(R < i)}$$

Putting $g_i = \frac{P(R \geq i)}{P(R < i)}$ and $\sum_{i=1}^{\rho} g_i$ we can write:

$$P(X = 1) = \Pi \quad s_1$$

4-3 The probability P(*X*=2)

In this case exactly two of the cars in positions $j + r - 1$ and $j + r - \rho$ come back after the car having the initial position j and all other cars that are withdrawn for repairs come back to the main line in a preceding position. Let these two cars be those in position $j + r - i$ and $j + r - k$ with $i < k \leq \rho$. The probability of this situation is:

$$P(X = 2) = \sum_{i=1}^{\rho-1} \sum_{k=i+1}^{\rho} P(R_{j+r-1} < 1)\ldots P(R_{j+r-i} \geq i)\ldots P(R_{j+r-k} \geq k)\ldots P(R_{j+r-\rho} < \rho)$$

As before, we assume that the probability functions of *R* for all cars are identical and independent. Multiplying and dividing each term by P($R < i$), we obtain:

$$P(X = 2) = \prod_{k=1}^{\rho} P(R < k) \quad \sum_{i=1}^{\rho-1} \frac{P(R \geq i)}{P(R < i)} \quad \sum_{j=i+1}^{\rho} \frac{P(R \geq j)}{P(R < j)}$$

Modifying the limits of the first and second summations, we can write:

$$P(X=2) = \frac{1}{2} \prod_{k=1}^{\rho} P(R<k) \sum_{i=1}^{\rho} \frac{P(R\ge i)}{P(R<i)} \sum_{j\ne i} \frac{P(R\ge j)}{P(R<j)}$$

Which can be written:

$$P(X=2) = \frac{1}{2!}\ \Pi \sum_{i=1}^{\rho} (g_i \sum_{j\ne i} g_j) = \frac{1}{2!}\ \Pi \sum_{i=1}^{\rho} \{g_i\ (s_1 - g_i)\}$$

Putting $s_2 = \sum_{i=1}^{\rho}(g_i)^2$ and $N_2 = \sum_{i=1}^{\rho} (g_i \sum_{j\ne i} g_j) = (s_1)^2 - s_2$, the formula above becomes:

$$P(X=2) = \frac{N_2}{2!}\ \Pi$$

Similarly, we can rewrite P(X = 1) and P(X = 0). Let $N_1 = \sum_{i=1}^{\rho} (g_i) = s_1$ and $N_0 = 1$ then:

$$P(X=1)=\frac{N_1}{1!}\ \Pi \quad \text{and} \quad P(X=0)=\frac{N_0}{0!}\ \Pi$$

Notice that we can write $N_1 = s_1\ N_0$ and $N_2 = s_1N_1 - s_2N_0 = s_1N_1 - 1!s_2N_0$.

4-4 The probability P(X=3)

In this case exactly three of the cars in positions $j + r - 1$ and $j + r - \rho$ come back after the car having the initial position j and all other cars that are withdrawn for repairs come back to the main line in a preceding position. Let these three cars be those in position $j + r - i$, $j + r - k$ and $j + r - l$ with $i < k < l \le \rho$. The probability of this event is:

$$P(X=3) = \sum_{i=1}^{\rho-2}\sum_{k=i+1}^{\rho-1}\sum_{l=k+1}^{\rho} P(R_{j+r-1}<1)\ldots P(R_{j+r-i}\ge i)\ldots P(R_{j+r-k}\ge k)\ldots P(R_{j+r-l}\ge l)\ldots P(R_{j+r-\rho}<\rho)$$

Here also we assume that the probability functions of R for all cars are identical and independent. Multiplying and dividing each term by P(R < i), we obtain:

$$P(X=3) = \prod_{k=1}^{\rho} P(R<k) \sum_{k=1}^{\rho-2}\frac{P(R\ge k)}{P(R<k)} \sum_{i=k+1}^{\rho-1}\frac{P(R\ge i)}{P(R<i)} \sum_{j=i+1}^{\rho}\frac{P(R\ge j)}{P(R<j)}$$

Modifying the limits of the three summations, we can write:

$$P(X=3) = \frac{1}{6} \prod_{k=1}^{\rho} P(R<k) \sum_{k=1}^{\rho}\frac{P(R\ge k)}{P(R<k)} \sum_{i\ne k}\frac{P(R\ge i)}{P(R<i)} \sum_{j\ne i\ne k}\frac{P(R\ge j)}{P(R<j)}$$

Which can be written:

$$P(X=3) = \frac{1}{3!}\ \Pi \sum_{k=1}^{\rho} (g_k \sum_{i\ne k} g_i(\sum_{j\ne i\ne k} g_j)) = \frac{1}{3!}\ \Pi \sum_{k=1}^{\rho} (g_k \sum_{i\ne k} g_i(s_1 - g_k - g_i))$$

And leads to:

$$P(X=3)=\frac{1}{3!}\ \Pi \left[s_1\ ((s_1)^2 - s_2) - 2\sum_{k=1}^{\rho}\sum_{i\ne k}(g_k)^2\ g_i\right]$$

Putting $s_3 = \sum_{i=1}^{\rho}(g_i)^3$ and $N_3 = \sum_{k=1}^{\rho} g_k(\sum_{i\ne k} g_i \sum_{j\ne i\ne k} g_j) = (s_1)^3 - 3s_1s_2 + 2s_3$, the formula above becomes:

$$P(X = 3) = \frac{1}{3!} \sqcap \; [(s_1)^3 - 3s_2 s_2 + 2s_3] \; = \frac{N_3}{3!} \sqcap$$

Notice that we can write $N_3 \; = \; s_1 N_2 - 2s_2 N_1 + 2s_3 N_0 \; = \; s_1 N_2 - 2! s_2 N_1 + 2! s_3 N_0$

4-5 The probability P(X=x)

In this case exactly *x* cars will come back after the car originally in position *j*. By mathematical induction we can write:

$$P(X = x) = \frac{N_x}{x!} \sqcap$$

Where: $N_x \; = \sum_{k=1}^{\rho} g_k \left(\sum_{i \neq k} g_i \sum_{j \neq i \neq k} g_j (\sum \;) \right) = \sum_{t=1}^{x} (-1)^{t-1} \frac{(x-1)!}{(x-t)!} s_t N_{x-t}$

As an illustration, Table5, page 13, and Figure 5 give the probability P(Δ=δ) for four different probability distribution functions P(*R*=*r*). For these distributions ρ =10, p_o = P(*R* = 0) is either 0.80 or 0.90 and the conditional probability P(*R* = *r* /*R* > 0) is either skewed to the left or to the right as shown in Table 4.

Figure 5 : The general probability distribution P(Δ=δ).

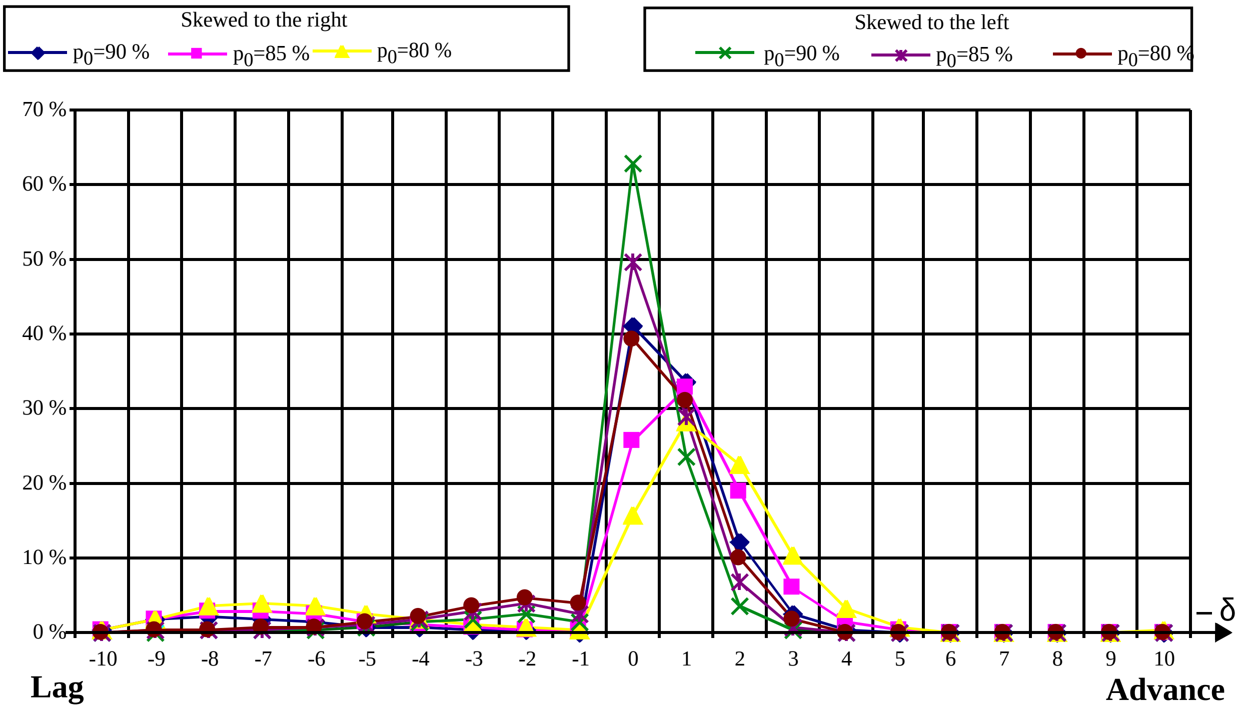


Table 4: Conditional probability P(R = r/R > 0)

Probabilité conditionnelle P(*R* = *r* / *R* > 0)	***r* = 1**	***r* = 2**	***r* = 3**	***r* = 4**	***r* = 5**	***r* = 6**	***r* = 7**	***r* = 8**	***r* = 9**	***r* = 10**
Skewed to the left	10%	29%	22%	14%	10%	5%	4%	3%	2%	1%
Skewed to the right	1%	2%	3%	4%	5%	10%	14%	22%	29%	10%

Tableau 5 : The probability density function P(Δ=δ) in the general case

		Skewed to the left			Skewed to the right		
p_0		90%	85%	80%	90%	85%	80%
Advance	– 10	0.46%	0.45%	0.39%	0.07%	0.09%	0.09%
	– 9	1.70%	1.89%	1.82%	0.17%	0.22%	0.26%
	– 8	2.23%	3.01%	3.44%	0.27%	0.37%	0.46%
	– 7	1.89%	2.98%	3.96%	0.37%	0.52%	0.66%
	– 6	1.37%	2.34%	3.43%	0.47%	0.67%	0.86%
	– 5	0.87%	1.58%	2.49%	0.84%	1.17%	1.44%
	– 4	0.56%	1.02%	1.65%	1.26%	1.78%	2.23%
	– 3	0.39%	0.68%	1.08%	1.94%	2.73%	3.41%
	– 2	0.28%	0.48%	0.74%	2.65%	3.79%	4.81%
	– 1	0.18%	0.32%	0.51%	1.55%	2.61%	3.77%
0		41.16%	25.65%	15.73%	62.99%	49.68%	39.18%
Lag	1	33.74%	33.00%	28.13%	23.63%	28.84%	31.10%
	2	12.24%	18.83%	22.48%	3.53%	6.69%	9.95%
	3	2.58%	6.24%	10.44%	0.27%	0.79%	1.62%
	4	0.35%	1.33%	3.11%	0.01%	0.05%	0.15%
	5	0.03%	0.19%	0.62%	0.00%	0.00%	0.01%
	6	0.00%	0.02%	0.08%	0.00%	0.00%	0.00%
	7	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%
	8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	10	0.00%	0.02%	0.37%	0.00%	0.00%	0.00%

5 Repeated failure to pass inspection tests

To simplify our model we assumed that each repaired car does not leave the repair shop or line until all the defects that caused its withdrawal from the main line are correctly repaired. In the opposite case where a car fails the inspection several times our results remain valid but we have to use a modified probability function of *R*..

Consider a car that requires r_1 cycles for its first visit to the repair line or shop and requires r_2 cycles for its second visit. Assuming that the number of repair cycles of the two visits have the same probability function (i.e., $P(R_1 = r) = P(R_2 = r),\ \ r$), then the probability of requiring a total of *r* cycles for the two visits (with $2\rho \geq r \geq r_1$) can be obtained by:

$$P(R = r) = \sum_{r_1 = 0}^{Min(r,\rho)} P(R_1 = r_1)P(R_2 = r - r_1)\ ;\ 0 \leq r \leq 2\rho$$

In the case where the car requires three visits the probability is given by:

$$P(R = r) = \sum_{r_1 = 0}^{Min(r,\rho)} P(R_1 = r_1) \sum_{r_2 = 0}^{Min(r - r_1,\rho)} P(R_2 = r_2)P(R_3 = r - r_1 - r_2)\ ;\ 0 \leq r \leq 3\rho$$

And we can generalize this function in a very classical way to the case of a number of visits ν. Then we can derive the probability density function of rank changes (i.e., P(Δ=δ)) using the generalized probability function of *R*.

6 Conclusions

In this paper we showed that it is possible to derive and we derived some probability density functions of rank changes in mixed-model production/assembly lines. The results obtained open the door for a more rigorous analysis of the rank changes phenomenon and its economic impact. A preliminary analysis of some of its consequences is provided here but it is obvious that a deeper investigation is needed. This will be the subject of some forthcoming reports.

7 References

[1] R. G. **Askin** & M. **Zhou**, 1997, «A parallel station heuristic for the mixed-model production line balancing problem», *International Journal of Production Research*, vol. 35, n° 11, p. 3095-3105.

[2] A. **Bolat**, M. **Savsar**, M. **Al-Fawzan**, 1994, «Algorithms for real-time scheduling of jobs on mixed model assembly lines», *Computer Operations Research*, vol. 21, n° 5, p. 487-498

[3] A. K. **Chakravarty** & A. **Shtub**, 1985, «Balancing mixed model lines with in process inventory», *Management Science*, vol. 31, n° 9, p. 1161-1174.

[4] F. **Danjou**, V. **Giard** & E. **Le Roy**, 2000, «Analyse de la robustesse des ordonnancements/réordonnancements sur ligne de production et d'assemblage dans l'industrie automobile», *Revue Française de Gestion Industrielle*, 19, 1, 17-36.

[5] V. **Giard**, 1997, *Processus productifs et programmation linéaire*, Economica.

[6] V. **Giard**, 2000, *Gestion de production*, Economica, 3° édition.

[7] H. **Gokcen** & E. **Erel**, 1997, «A goal programming approach to mixed model assembly line balancing problem», *International Journal of Production Economics*, vol. 48, n° 2, p. 177-185.

[8] R. R. **Inman** & R. L. **Bulfin**, 1991, «Sequencing JIT mixed-model assembly lines», *Management Science*, Vol. 37, n° 7, p. 901-904.

[9] Y. G. **Kim** & J.-S. **Kwak**, 1993, «Mixed model assembly line balancing with the related task consideration», *Journal of the Korean OR-MS Society*, vol. 18, n° 2, p. 1-22.

[10] W. **Kubiak** & S. **Sethi**, 1991, «A note on «Level Schedules for mixed-models assembly lines in just-in-time production system», *Management Science*, vol. 37, p. 121-122.

[11] P. R. **McMullen** & G. V. **Frazier**, 1997, «A heuristic for solving mixed model line balancing with stochastic durations», *International Journal of Production Economics*, vol. 51, n° 3, p. 177.190.

[12] J. **Miltenburg**, 1989, «Level Schedules for mixed-models assembly lines in just-in-time production system», *Management Science*, vol. 35, p. 192-207.

[13] J. **Miltenburg** & G **Sinnamon**, 1989, «Scheduling mixed-model multi-level just-in-time production system», *International Journal of Production Research*, vol. 27, n° 9, p. 1487-1509.

[14] N.T. **Thomopoulos**, 1967, «Line balancing - sequencing for mixed model assembly», *Management Science*, vol. 14, n° B, p. 59-75.

[15] N.T. **Thomopoulos**, 1970, «Mixed-model balancing with smoothed station assignments», *Management Science*, vol. 19, n° 9, p. 593-603.

[16] C. A. **Yano** & A. **Bolat**, 1989, «Survey, development and application of algorithms for sequencing paced assembly lines», *Journal of Manufacturing and Operations Management*, vol 3, p. 172-198.

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Rank changes in production/assembly lines: impact and analysis

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