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Monitoring of the upstream part of a supply chain
dedicated to the customized mass production with a
revisited version of MRP

Abstract: *This article focuses on the monitoring of a supply chain dedicated to the mass production of strongly diversified products. Specifically, we are interested in the part of this chain that contributes to the production of a set of alternative modules assembled on a work station of one or several assembly lines, whose production levels are stable. The MRP approach is adopted for the monitoring of this chain. The distance between the production units leads to a mix between production to stock and production to order. In this article, we establish the relations that allow us to define, in a steady state, the quantities to produce that address the requirements of the Master Production Schedule and that are partially or completely random to limit the stockout risk to a very low predetermined level. We will distinguish two cases by accounting for, or not accounting for, problems that are related to quality.*

Keywords: *MRP, Make To Stock, Safety Stock, Supply Chain, Order Penetration Point.*

1 Introduction

In the real world, for Supply Chains (SCs) dedicated to the mass production of strongly diversified products that are characterized by a certain geographical dispersion of the production units of the SC, production is determined by several distant final assembly lines with a stable daily production (at least in the course of a few weeks). Diversity is provided mainly by the assembly of alternative modules (e.g., engines, gear boxes) on several workstations of the assembly line. Each station is dedicated to a different set of alternative modules, of which one must be mounted on the end product processed by this station. The optional modules (e.g., the sliding roof) can be treated as a specific case of the alternative modules. Several examples of this type of SC can be found in the automotive industry.

When the total daily production volume is certain, the daily requirement for all of the systematically assembled parts and the components that they use is also certain. The control of these flows is relatively easy in the absence of risk related to the quality of the components, the transportation time or the production time.

For the alternative modules and the components that they use, the flow control is more complex. We are interested here in piloting the production units of these modules and the components that they use under the assumption that customer orders to suppliers are addressed simultaneously with the same periodicity. The operating process used is Manufacturing Resource Planning (MRP), for which planned orders of the components are determined by a backward simulation that combines a mechanism of explosion of the Bill Of Material (BOM) with a mechanism of lead time offset. In MRP, the Master Production Schedule (MPS) plans the production of all of the supply chain units that manufacture the modules and components included in the end products.

After a brief review of the literature (§1), a revisited version of the MRP will be proposed (§2) and then numerically illustrated (§3). In conclusion (§4), we will describe the conditions that must be guaranteed for applying this approach efficiently.

2. Literature review

MRP was initially proposed to be implemented in a deterministic environment; no securing method was initially imagined. The confrontation of this idealized perception of planning with the real world highlighted the necessity of dealing with the uncertainty of certain information that is used within the framework of the MRP model. At the end of the 1970s, attempts to adapt the MRP method to an uncertain environment were suggested.

The first source of uncertainty that was studied was related to the demand. To address this concern, (Büchel, 1982) models uncertainty with random variables and calculates, on each level of the BOM, the variance of the demand for the components, leading to the possibility of calculating safety stocks. The problem of the components that are common to several end-products is managed by considering that their demands are random variables whose variances are the sums of the variances of the requests. The implicit assumption of demand independence does not hold if it concerns a component that is common to a subset of alternative modules belonging to a given set of alternative modules mounted at the same workstation of the final assembly line. Indeed, it is shown below (see §2) that the probability distribution of the demand for a component common to several components of a subset of alternative modules is a weighted sum of binomial distributions; this relationship invalidates the property of the variance additivity.

Based on a case study, (De Bodt & al., 1982) evaluate the effectiveness of the use of safety stocks in MRP with uncertain demands. Their level and localization result from a simple comparison between several scenarios: no determination method is presented in this article.

(Winjgaard & al., 1985) describe the principles and advantages of the use of the safety stock, of the safety lead time and of the increase in the quantities of the MPS. A dimensioning method for the safety stocks is provided for serial, convergent and divergent configurations of the supply chain. The authors only consider the case of components that are related to the end-product by a BOM coefficient equal to 1 without accounting for the commonality.

(Lagodimos & al., 1993) examine the problem of the localization of the safety stocks in the MRP of standardized products. Their objective is to maximize the service level by distributing among the various levels of the BOM a given value of safety stock by using an analytical approach. It is considered here that the service level is not a decision variable and that the holding costs are identical for each level of the BOM. This assumption is strong given that the value of a component usually decreases with its BOM level. The proposition goes in the direction of a placement of the safety stock on the level of the end-products for the serial structure of a supply chain. Nevertheless, the authors relativize their results by pointing out the limits imposed by the assumptions of their modeling.

Under the assumption that demands are usually distributed in the steady state, (Inderfurth & al., 1998) are interested in the double problem of the dimensioning and the localization of the safety stocks of components. The goal of this study is to reach a given service level (size and duration of stockout) at each level of the supply chain while minimizing the overall carrying costs. They propose a model of optimization that respects a service level that ensures the decoupling of the supply chain units. The requirements of a component come from the final MPS demand and could be related to one or more different end products. In this last case, correlations between demands are introduced to model the requirements of the studied component. This approach is not valid if that component is mounted in different alternative modules that belong to the same set of alternative modules.

On the basis of the holding cost and service rate, (Zhao & al., 2001) compare the performance of three dimensioning methods for calculating the safety stock. Accounting for the results realized by (Carlson & al., 1986), they decided to place the safety stock at the level of the end product. For each of the three methods, a different demand cover time is used for the calculation of the safety stock. No justification is given to defend the choices.

In a more recent article, (Persona & al., 2007) propose an analytical method for dimensioning safety stocks of components or modules in the case of a production or assembly to order. In their modeling, they consider a steady state in which the procurement lead times are certain and the demands for alternative components are correlated within the same family of alternative modules. The concept of correlation used in this article refers to the statistical link that exists between the demands relating to alternative modules of the same set \mathcal{E}_j . This relation is the result of a constraint based on the multinomial distribution and is not the result of a correlation between random variables. Correlations exist between alternative components that belong to different sets, but they are only important in the case of a component that is used to link two alternative modules that belong to two different sets of alternatives (junction components).

(Winjgaard & al., 1985), (Guerrero & al., 1986), (Buzacott & al., 1994), (Molinder, 1997) and (Chang, 1985) explored other strategies for protection against uncertainty in the MRP environment such as raising the requirements of the MPS or lengthening the lead times artificially. (Buzacott & al., 1994) are interested in the possibility of using safety lead times as a means of protection against uncertainty. They consider that a trade-off must be made between safety stocks and safety lead times according to the reliability of the forecasts, the production capacities, the holding costs and the stockout costs. They consider in their modeling only one MRP echelon operating in an uncertain environment. The conclusion from this study is that safety stocks reduce the total costs in the event of strong uncertainties on the demand. The safety lead time has the ability to anticipate a known demand to come and necessarily leads to an increase in a stock at a level that depends on the period length.

In a similar study, (Molinder, 1997) proposes rules for decision making to help choose between safety stocks and safety lead times. The parameters of the decision refer to the degrees of uncertainty of demand and lead times and the ratio between the stockout costs and holding costs. Its conclusions are somewhat different from those made by (Buzacott & al., 1994) in that he recommends the use of safety lead times in the event of strong uncertainties of both demand and lead times. One can note that the cost that is associated with a safety lead time is induced by an increase in the stocks and that consequently, the opposition between the safety stock and the safety lead time is not very clear.

Exploring other ways for dealing with uncertainty in MRP, (Zhao & al., 1993) propose to freeze the master production schedule in a high proportion according to parameters related to the planning horizon and the re-planning periodicity. This solution, conceivable in a make to stock context of standardized products, is unusable in a make to order production of diversified products.

Many literature review articles relating to MRP have been published in the 2000s. To clearly highlight fields not yet explored by the specialized literature and to clarify the positioning of our work compared to what has already been accomplished, it is interesting to check the analysis framework used by these literature reviews.

(Guide & al., 2000) are interested in the techniques used to counter uncertainty in MRP. The analysis framework comprises the following seven axes: the modeling process, the dimensioning method for the safety stocks, their localization, the definition of the safety lead time, the type of uncertainty accounted for in the model, the nature of the planning horizon (rolling or fixed), and finally, the key performance indicator that is retained. The examination of the literature in this article leads to the conclusion that several gaps still exist even if certain suggested solutions remain applicable, according to the

authors, under a set of assumptions that are sometimes debatable. These authors also regret the lack of methodology and the absence of precise rules that allow for the establishment of a compromise between the various suggested alternatives for protection.

Another categorization of the literature is proposed by (Koh & al., 2002), in which the analysis is performed on three axes. In the first axis, the authors distinguish between the inside risks that are related to the processes and the outside risks. The second axis is related to the point of occurrence of the risk, which can be upstream or downstream in relation to the productive process. Last, the third axis relates to the type of solution that is recommended to mitigate the risk through safety stocks or techniques, such as an increase in the quantities of the MPS. The interesting conclusion from this article is that very few studies address the interactions between several sources of risk.

More recently, (Dolgui & al., 2007) propose an analysis of articles that relate to the management of the risk in MRP. The articles selected are classified into three categories based on the type of the risk taken: the demand, the lead time and a combination of both. The authors briefly present some of the techniques that are commonly used to control the consequences of uncertainty, namely the frozen horizon, safety stock and safety lead times. Aside from the limitations highlighted by this study and those identified by (Guide & al., 2000) and (Koh & al., 2002), they emphasized that there are very few articles that address the MRP replanning in multi-echelon configurations.

The risk that is taken in the productive part of SC, treating the procurement of optional or alternative components (and the components that they use) for assembly lines dedicated to the mass production of customized products, presents several specific characteristics that have not yet been considered in the literature and that we will attempt to develop.

3. The MRP model revisited

3.1 Treatment uncertainty in the traditional approach of the MRP

The frozen horizon H_F is the initial part of the planning horizon H_P on which the demands of the first periods of the MPS (from t to $t + H_F - 1$) are intangible. This concept is related to that of the Order Penetration Point (OPP), which delimits what can be made to order from what is to make to stock (Vollmann & al., 1997). This separation, obvious in the SC convergent network, is less clear in a divergent network that characterizes an SC that has several distant assembly lines. In this case, a part mounted on those lines could be partly made-to-order (MTO) and partly made-to-stock (MTS), depending on the distance between the assembly lines and the plant that produces that component.

The application in the cascade of the BOM explosion results in finding a_{ik} units of reference i , of the BOM level n , included in the reference k pertaining to the subset \mathcal{E}_{ik} of the MPS (level 1 of the BOM). In addition, the application in the cascade of the mechanism of the lead times offsets leads to a lag λ_{ik} between the period t of launching into production, the reference i and the period $t + \lambda_{ik}$ for the use of the reference k in the MPS. This scenario makes it possible to bind the gross requirement GR_{it} to a reference i of level n of the nomenclature at period t , with the demands $MPS_{kt'}$ of reference k (level 1 of the nomenclature) at periods $t' > t$ and not with the planned orders of the references of level $n' < n$ of the nomenclature, which use that reference i directly (which is typical in an MRP).

To establish this relation, it is advisable to notice that, in a deterministic universe (non-revisable MPS, absence of quality problems, deterministic lead times), the on-hand balance OHB_{it} at the end of the period t for any reference i must be null because, in this deterministic context, a stock generates a cost without any useful counterpart. Consequently, the planned order PO_{it} for the reference i at the beginning of the period t is necessarily equal to the quantity $q_{i,t+L_i}$, which is delivered at the beginning of period $t + L_i$, taking into account the lead time L_i of this reference i . In its turn, $q_{i,t+L_i}$ is necessarily equal to the net requirement $NR_{i,t+L_i}$ for this period, which is equal to the gross requirement $GR_{i,t+L_i}$ for this same period. These various values are linked to the requirements of the MPS by the relation (1).

$$\begin{aligned} PO_{it} &= q_{i,t+L_i} = NR_{i,t+L_i} = GR_{i,t+L_i} \\ &= \sum_{\mathcal{E}_{ik}} a_{ik} \cdot MPS_{k,t+\lambda_{ik}} \end{aligned} \quad (1)$$

The reference i is entirely made-to-order if $\text{Max}_k(\lambda_{ik}) \leq H_F$, which corresponds to the “regular” case of MRP use. The reference i is entirely made-to-stock if $\text{Min}_k(\lambda_{ik}) > H_F$ because the MPS requirements used in this relation are unknown. Last, a mix of MTO and MTS occurs when the reference i is linked with at least a reference k such as $\lambda_{ik} < H_F$ and with at least a reference k' such as $\lambda_{ik'} \geq H_F$. The latter case, which is not treated, to our knowledge, in the literature on the OPP or on MRP, is important to consider because it is quite common in the modern world SC, such as in the automotive industry.

A company can use an MTO policy to control the production of a set of references that are manufactured on various sites belonging to its SC only if it has an OPP that is defined on a frozen horizon, such as $H_F \geq \text{Max}_{i,k}(\lambda_{ik})$. By increasing the lags λ_{ik} , the replacement of nearby suppliers by remote suppliers can lead insidiously to the non-observance of that condition

and, thus, can trigger a loss of flow control in the SC, unless this reference is partially (or completely) made-to-stock with appropriate rules to avoid stockouts.

In this type of SC, the diversity of the end-products is too important to allow the MPS to be defined at the level of the end-product. The MPS must be defined at the level of the alternative modules. The planning BOM \bar{c} of a set of K alternate modules describes the average structure for the use of these alternative modules by a set of end-products that are assembled on the same line. It is necessary to define as many planning BOMs as there are assembly lines using these alternative components because of the difference in the structure of the demand that must be met. The elements of this vector \bar{c} correspond to the average proportions c_k of the consumption of the modules K ; obviously, $\sum_{k=1}^K c_k = 1$.

This vector \bar{c} can be regarded as the vector of probabilities for the use of the alternative modules in the steady state to characterize the demands \bar{X} of the alternative modules beyond the frozen horizon. The random demand of the alternative modules \bar{X} follows the Multinomial distribution $\mathcal{M}(N, \bar{c})$, in which N is the daily production of the line. The daily demand X_k of the module k follows the Binomial distribution $\mathcal{B}(N, c_k)$, in which the complementary event to “use module k ” is to “use another alternative module”.

3.2 MRP in an MTO/MTS environment without a quality problem

The flow synchronization in the type of SC studied here can depend on the MPR if its rules can be adapted to calculate the planned orders of a component i , which depends on MPS requirements going beyond the frozen horizon ($\exists \lambda_{ik} \geq H_F, k \in \mathcal{E}_{ik}$). When there are lags of these alternative modules k such as $\lambda_{ik} \geq H_F$, the part of the production of component i that is induced by these requisitions must be made-to-stock. It is advisable, then, to replace the notation $MPS_{k,t+\lambda_{ik}}$, which is suitable for a known demand of the alternative module k for the period $t + \lambda_{ik}$, by the random variable $X_{k,t+\lambda_{ik}}$, which is replaced by X_k if the characteristics of the steady state remain stable over the planning horizon. In that case, the gross requirement $GR_{i,t+L_i}$ is defined by relation (2), as follows:

$$GR_{i,t+L_i} = \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F} a_{ik} \cdot MPS_{k,t+\lambda_{ik}} + \sum_{\mathcal{E}_{ik} | \lambda_{ik} \geq H_F} a_{ik} \cdot X_{k,t+\lambda_{ik}} \quad (2)$$

In the logic of the MRP, the planned delivery $q_{i,t+L_i}$ (equal to the planned order LP_{it}) corresponds to the net requirement $NR_{i,t+L_i}$ of this reference i for this period $t + L_i$, which is equal to the gross requirement $GR_{i,t+L_i}$

for this period, minus the projected available inventory $AI_{i,t+L_i-1}$ at the end of the previous period $t + L_i - 1$. This projected inventory $AI_{i,t+L_i-1}$ depends on the on-hand balance $OHB_{i,t-1}$ at the end of the period $t - 1$ and on the gross requirements and planned deliveries of the periods t to $t + L_i - 1$:

$$AI_{i,t+L_i-1} = OHB_{i,t-1} + \sum_{t'=t}^{t+L_i-1} [q_{i,t'} - GR_{i,t'}] \quad (3)$$

Then, in the determination of the planned delivery $q_{i,t+L_i}$, it is necessary to integrate uncertainty relating to the gross requirement $GR_{i,t+L_i}$ and to the projected inventory $AI_{i,t+L_i-1}$. The quantity to be delivered at the beginning of the period is then the sum of a known quantity $\sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F} a_{ik} \cdot MPS_{k,t+\lambda_{ik}}$ and a quantity defined as the difference between an order-up-to level $R_{i,t+L_i}$ and a deterministic projected available inventory $AI'_{i,t+L_i-1}$, which is derived from relation (3) in which only the deterministic part of $GR_{i,t'}$ is used. Then, $R_{i,t+L_i}$ corresponds to the fractile, which is associated to the stockout risk α of a probability distribution of a demand $Y_{i,t+L_i}$ ($P(Y_{i,t+L_i} > R_{i,t+L_i}) = \alpha$). The demand $Y_{i,t+L_i}$ corresponds to the sum of the random requirements of the periods $t+1$ to $t+L_i$, with the gross requirement GR_{it} of the first period being necessarily known. Given that the MPS requirements of the module k for the period $t + \lambda_{ik} - h$ are unknown, if $\lambda_{ik} - h \geq H_F$, then $Y_{i,t+L_i}$ is defined by relation (4), as follows:

$$Y_{i,t+L_i} = \sum_{h=0}^{h=L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} \geq H_F+h} a_{ik} \cdot X_{k,t+\lambda_{ik}-h} \quad (4)$$

Therefore, the planned order PO_{it} of reference i , which is equal to the planned delivery $q_{i,t+L_i}$ at period $t + L_i$, is the sum of the following:

- The deterministic requirement for period $t + L_i$, i.e., $\sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F} a_{ik} \cdot MPS_{k,t+\lambda_{ik}}$,
- The difference between $R_{i,t+L_i}$ and $AI'_{i,t+L_i-1}$.

Then, the planned order PO_{it} is defined by relation (5), as follows:

$$PO_{it} = q_{i,t+L_i} = \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F} a_{ik} \cdot MPS_{k,t+\lambda_{ik}} + R_{i,t+L_i} - [OHB_{it} + \sum_{h=0}^{h=L_i-1} q_{i,t+h} - GR_{it} - \sum_{h=1}^{h=L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F+h} a_{ik} \cdot MPS_{k,t+\lambda_{ik}-h}] \quad (5)$$

The production of the entire made-to-stock is a special case of relation (5), in which $\forall k \in \mathcal{E}_{ik}, \lambda_{ik} \geq H_F + L_i - 1$, to make random all of the gross requirements for the period $t+1$ to $t+L_i$. In that case, no deterministic requirement of MPS stands, and the relation (6) is to be used.

$$PO_{it} = q_{i,t+L_i} = GR_{it} + \left[R_{i,t+L_i} - OHB_{it} - \sum_{h=0}^{L_i-1} q_{i,t+h} \right]$$

with $\lambda_{ik} \geq H_F + L_i - 1, \forall k \in \mathcal{E}_{ik}$ (6)

In the steady state, $R_{it'} = R_i, \forall t'$. R_i can be drawn from relation (5), for period t , with $q_{it'} = LP_{i,t'-L_i}, \forall t'$ and can be replaced by that expression in that relation for period $t+1$. After simplification, one obtains relations (7) and (8), which are more convenient to use in the steady state than the relations (5) and (6). Because the deterministic part of the MPS requirements vary with the used horizon, a superscript is necessary to notate precisely at what initial period the analysis of the future requirements is performed.

$$PO_{it} = q_{i,t+L_i} = GR_{it} + \sum_{h=0}^{L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F + h} a_{ik} \cdot MPS_{k,t-1+\lambda_{ik}-h}^t - \sum_{h=0}^{L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F + h} a_{ik} \cdot MPS_{k,t-1+\lambda_{ik}-h}^{t-1}$$
 (7)

$$PO_{it} = q_{i,t+L_i} = GR_{it}$$

(Steady state and $\forall k \in \mathcal{E}_{ik}, \lambda_{ik} \geq H_F + L_i - 1$) (8)

A change of the steady state at period $t + \text{Max}_k(\lambda_{ik})$ modifies the order-up-to level. This change is progressive and lasts for $\text{Max}_k(\lambda_{ik}) - H_F$ periods. During that transitory state, the difference between the new value for the order-up-to level and the previous value is to add to the relations (7) and (8) for calculating the planned order.

3.3 MRP in an MTO/MTS environment with a quality problem

Let us assume that no quality problem occurs for all of the components that include component i of level n , until the modules k of level 0, and that the quality is checked before the departure of the deliveries.

If the gross requirement $GR_{i,t+L_i}$ to be met by the planned order PO_{it} is known ($\lambda_{ik} < H_F, \forall k$), then it is possible to protect against a stock shortage by the possibility that each produced part has the probability π_i of nonconformity, through a safety stock – or Target Stock – TS_i , defined as the fractile $U_{i,t+L_i}^{\alpha'}$ specified for the stockout risk α' of the random variable $Z_{i,t+L_i}$, which follows the Negative Binomial distribution $\mathcal{NB}(GR_{i,t+L_i}, \pi_i)$.

$$Z_{i,t+L_i} \sim \mathcal{NB}(\sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F} a_{ik} \cdot MPS_{k,t+\lambda_{ik}}, \pi_i)$$

$$\rightarrow TS_i = U_{i,t+L_i} \text{ with } P(Z_{i,t+L_i} > U_{i,t+L_i}) = \alpha' \quad (9)$$

Obviously, it is the variation of this target-stock $U_{i,t+L_i} - U_{i,t+L_i-1}$ that is added to the gross requirement $GR_{i,t+L_i}$ to calculate the planned delivery $q_{i,t+L_i}$, if $L_i > 1$; otherwise, $AOH_{i,t-1}$ replaces $U_{i,t+L_i-1}$. Then, relation (10) is to be used, as follows:

$$PO_{it} = q_{i,t+L_i} = \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F} a_{ik} \cdot MPS_{k,t+\lambda_{ik}} + U_{i,t+L_i} - U_{i,t+L_i-1} \quad (10)$$

The name “target-stock” is used here in preference to that of safety stock, which is defined classically as the difference between an order-up-to level and an average demand (which is not the case here). In the absence of defective pieces in the production that are launched in the period t , the projected inventory $AI_{i,t+L_i}$ at the end of period $t+L_i$ (after control and before expedition) is equal to this target-stock $U_{i,t+L_i}$.

The use of these relations implies that a calculation of the target-stock is made at each decision-making point. If the probability of nonconformity is weak, then a value for the stock-target will be valid for a relatively broad range of values of Gross Requirements, which allows an easy construction of a decision table that avoids repeating the calculation each time.

If the gross requirement $GR_{i,t+L_i}$ is completely or partially random, then the first argument of the Negative binomial distribution becomes random. The risk of nonconformity must be taken into account in the definition of the order-up-to level $R_{i,t+L_i}$ because only one safety stock can be used. This risk sharing, by the simultaneous consideration of uncertainties on the demand and quality, leads to don't separate the origin of the stockout. Then, the order-up-to level $R_{i,t+L_i}$ is defined by relation (11) and is calculated by using the Monte Carlo method, as follows:

$$P(W_{i,t+L_i} > R_{i,t+L_i}) = \alpha \text{ where } W_{i,t+L_i} = Y_{i,t+L_i} + Z_{i,t+L_i},$$

$$Y_{i,t+L_i} = \sum_{h=0}^{L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} \geq H_F + h} a_{ik} \cdot X_{k,t+\lambda_{ik}-h} \text{ and}$$

$$Z_{i,t+L_i} \sim \mathcal{NB}(Y_{i,t+L_i} + \sum_{h=0}^{L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F + h} a_{ik} \cdot MPS_{k,t+\lambda_{ik}-h}, \pi_i) \quad (11)$$

The exact solutions of relations (10) and (11) oblige us to compute the probability distributions $Z_{i,t+L_i}$ or $W_{i,t+L_i}$ at every decision-making point. In the steady state, if π_i is weak, then it is simpler to regard Z_i or W_i as random variables in which all of the requisitions of the MPS intervening in the calculation of the gross

requirement are random; then, relation (12) can be used, as follows:

$$\begin{aligned}
 P(W_i > R_i) &= \alpha \text{ with } W_i = Y_i + Z_i \\
 Z_i &\sim \mathcal{AB}(Y_i + Y'_i, \pi_i) \\
 Y_i &= \sum_{h=0}^{h=L_i-1} \sum_{\mathcal{E}_{ik} | \lambda_{ik} \geq H_F + h} a_{ik} \cdot X_k \\
 Y'_i &= \sum_{h=0}^{h=L_i} \sum_{\mathcal{E}_{ik} | \lambda_{ik} < H_F + h} a_{ik} \cdot X_k \quad (12)
 \end{aligned}$$

The generalization of the approach that accounts for the problems of quality in the production of the components j of level $n+1$ used by component i of level n is rather simple at the level of the principles. This approach is, however, complicated to implement in practice because it does not lead to analytical relations as simple as those found above. It is necessary to continue the calculation of planned orders $PO_{it'}$ for the periods $t' > t$ using the same approach as for the calculation of PO_{it} by distinguishing the deterministic part from the random part. After that, it is necessary to calculate planned orders of the components j by adapting the approach that led to the relation (11).

4 Numerical example

Let us illustrate the use of these analytical relations through a fictitious example. We consider the productive part of an automotive supply chain that leads to two cars' assembly factories (A and B) belonging to the same company. These two factories have a common engine supplier, which does not have other customers. The engine plant is supplied by a unique supplier of pistons and is its unique customer. The piston plant buys the piston crowns from a unique supplier, and it is also the supplier's unique customer. It is assumed that these different plants use an MRP-I approach that shares a common periodicity (a time bucket of two days) for launching production and receiving deliveries.

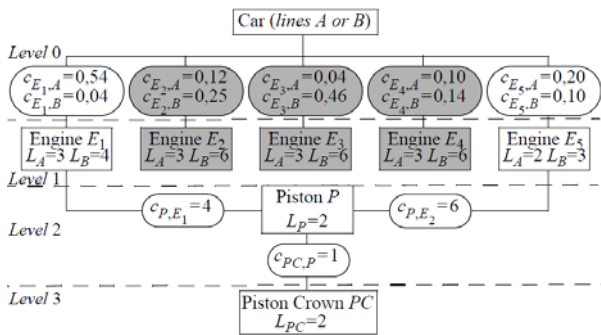


Figure 1: BOM and lead time definition

Figure 1 describes the relevant extract of BOM for the studied subset and shows the lead times. At level 0, one finds that the vehicles manufactured by the two distant assembly plants are dedicated to different markets that do not have the same structure of demand. At level 1, one finds the planning BOM that is associated with each one of these lines; among the 5 alternative modules of the set \mathcal{E} , only the engines E_1 and E_5 use the pistons that are of interest here. The piston crowns are specific to

each type of piston. Lead times integrate times of production and transportation to the customer. The distances between the assembly factories and the engines plant are such that it is necessary to isolate a transportation lead time in the mechanism of MRP.

The MPS of factories A and B are defined at the level of the alternative modules (engines), in other words, at level 1 of the BOM. These data, as well as the initial on-hand balance and planned deliveries, are provided in table 1 (to see at the end of this article). In this example, it is easy to calculate the λ_{ik} values for the pistons, for example, for the piston crowns (see table 2). All of the calculations in the table below were performed by considering that the frozen horizon length is 9 periods, with the result that one is under the normal conditions of use for MRP.

Alternative module $k \in \mathcal{E}_{ik}$	M_{1A}	M_{5A}	M_{1B}	M_{5B}
λ_{ik} , for $i = \text{piston crown}$	7	6	8	7
λ_{ik} , for $i = \text{piston}$	5	4	6	5

Table 2: λ_{ik} values for the pistons and piston crowns

The analytical solutions suggested for determining the order-up-to level or the target-stock will be illustrated with a very low level of risk. This choice is justified by the seriousness of the economic consequences of stock-outs, which encourages taking expensive measures for meeting the demands to avoid the propagation of disturbances along the supply chain.

4.1 Production partly to stock in the absence of a quality problem

This scenario occurs when reference i is linked with at least a module k such as $\lambda_{ik} < H_F$ and with at least a reference k' such as $\lambda_{ik'} \geq H_F$. With a frozen horizon of 7 periods, pistons are made-to-order, and piston crowns are made partly to order and partly to stock (see table 3). In table 1, information on the MPS corresponds to the expected values of the engine demands, beyond this 7th period. Table 3 links the gross requirements of the piston crowns to the engine demands.

Alternative components $k \in \mathcal{E}_{ik}$	λ_{ik}	Origins of requirements		
		deterministic	stochastic	
Gross Requirements at $t=2$	$E_{1,A}$	7	$MPS_{E_{1,A},7} = 1036 \times 4 = 4144$	-
	$E_{5,A}$	6	$MPS_{E_{5,A},6} = 90 \times 6 = 540$	-
	$E_{1,B}$	8	-	$X_{E_{1,B},8} \rightarrow 4 \times \mathcal{D}(960; 0, 20)$
Gross Requirements at $t=3$	$E_{3,B}$	7	$MPS_{E_{3,B},7} = 100 \times 6 = 600$	-
	$E_{1,A}$	7	-	$X_{E_{1,A},8} \rightarrow 4 \times \mathcal{D}(1840; 0, 54)$
	$E_{5,A}$	6	$MPS_{E_{5,A},7} = 86 \times 6 = 516$	-
	$E_{1,B}$	8	-	$X_{E_{1,B},9} \rightarrow 4 \times \mathcal{D}(960; 0, 20)$
$E_{3,B}$	7	-	$X_{E_{3,B},8} \rightarrow 6 \times \mathcal{D}(960; 0, 10)$	

Table 3: Decomposition of the requirements of the piston crown by engines MPS

The deterministic parts of the gross requirements of piston crowns (reference i) for periods 2 and 3 are 5284 and 516, respectively; $GR_{i1}(=5812)$ is completely deterministic (a firm order sent by the pistons factory). According to the relation (4), with $L_i = 2$, the demand Y_{i3} corresponds to the sum of the random requirements of the periods 2 to 3: $Y_{i3} = 4 \times \mathcal{B}(960;0,2) + 4 \times \mathcal{B}(1840;0,54) + 4 \times \mathcal{B}(960;0,2) + 6 \times \mathcal{B}(960;0,1)$. This distribution is a weighted sum of Binomial distributions and does not have an analytical solution. It is determined numerically without difficulty using the Monte Carlo simulation method. With a risk of 0.01%, $P(Y_3 > 6548) = 0.0001$ and the order-up-to level R_{i3} is 6548.

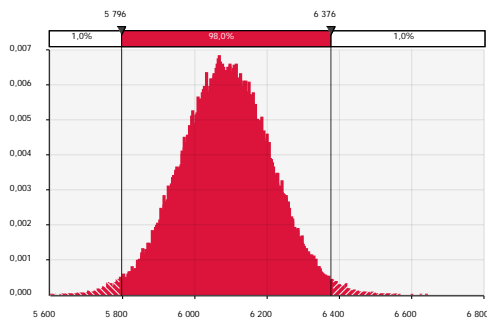


Figure 2: Distribution of Y_{i3}

The use of R_{i3} is illustrated in table 4 (see the end of this article), which shows, for a horizon passing from 9 to 7 periods, the calculation of the planned order of the piston crowns, the planned orders of the pistons and the engines that remain unchanged. The available inventory $AI_{i2} = 1014$ at the end of period 2 (and thus at the beginning of period 3) is the difference between the sum of the initial on-hand inventory (450) and the expected deliveries (5870+5790) and the deterministic part of the gross requirements of periods 1 to 2 (5812+5284). The planned order $PO_{i1} = q_{i3} = 6050$ is the sum of the deterministic part of the gross requirement of period 3 (516) and of $R_{i3} - AI_{i2} = 6548 - 1014 = 5534$.

After having illustrated the use of relation (5), let us next illustrate relation (7), which is valid in the steady state. By performing the calculations for MRP one period later (see table 5), the decision to be made at the beginning of period 2 depends now on the engine demands known for the periods 2 to 8. Let us suppose that, for this last period, the demands for the engines are $MPS_{1A,8} = 991$, $MPS_{5A,8} = 109$, $MPS_{1B,8} = 214$ and $MPS_{5B,8} = 84$. In period 1, the deterministic part of the gross requirements of periods 2 and 3 was $5284 + 516 = 5800$; in period 2, the deterministic part of the gross requirements of periods 3 and 4 is $4984 + 612 = 5596$. In application of relation (7), the planned order PO_{i2} is equal to

$6140 + (5596 - 5800) = 5936$ (the application of the relation (5) yields the same result $= 612 + 6548 - 1224$).

4.2 Production in the presence of a quality problem

To illustrate relation (9), it will be assumed that problems of quality are encountered in the manufacturing of the pistons, with a risk of $\pi = 0.1\%$ that a piston is regarded as defective after quality control. With a gross requirement of 6050 units for period 3 (see table 1), the target-stock is 17, as defined by the fractile of $Z_{pistons,3} \sim \mathcal{NB}(6050;0,1\%)$ for the risk $\alpha = 0.01\%$. If the quality control, performed before sending the delivery of 5780 pistons at the beginning of period 1, rejects 7 pistons, then the available inventory at the end of period 2 is decreased by 7, and the initial planned order (5812) is increased by $17 + 7 = 24$ to reach 5836 units. It is interesting to note that the target-stock of 17 units is associated with a range of gross requirements from 5507 to 6269, which suggests the possibility of avoiding calculations by using decision tables that are easy to establish.

To illustrate relation (11), we suppose now that the production of engines and pistons are without defects and that the defect risk for a piston crown is still $\pi = 0.1\%$. The deterministic part of the gross requirement remains $5812 + 5284 + 516 = 11612$. The definition of Y_{i3} corresponds to the same sum of the random requirements of periods 2 to 3. Then, the distribution of Z_{i3} is $\sim \mathcal{NB}(11612 + Y_i; 0,1\%)$, and the distribution of $W_{i3} = Y_{i3} + Z_{i3}$ is obtained using a Monte Carlo simulation, from which one obtains $P(W_{i3} > 6556) = 0.01\%$. This scenario yields the new order-up to level that protects from the two sources of randomness.

5 Conclusions

Thus, it is possible to guarantee a good performance in piloting a supply chain dedicated to mass-customized production using a revisited approach of MRP that combines MTO and MTS, accounting for the risks of the demand fluctuation and the quality and allowing the use of different frozen horizons depending on the knowledge of the customers' demands. Two observations must be made with regard to the underlying assumptions that place conditions on the effectiveness of this approach.

The use of the planning BOM \vec{c} of a set of K alternative modules assumes its stability. In the opposite case, it is necessary to anticipate the dated transformations of this structure that are induced by changes in the customers' expectations or by marketing actions (limited series, promotions) of the company or its competitors. A regular and rigorous analysis of these upcoming changes is essential; otherwise, the piloting of the supply chain will not remain under control. The costs of this informational monitoring are low, taking into consideration those induced by a pilot and not ensuring decisional coher-

ence in the supply chain. The organization and the follow-up of this vigilance place a condition on the performance, even perennity, of the supply chain.

On the principles level, the mechanisms proposed are relatively easy to implement because it is a question of transmitting order-ups to levels and target-stocks. The implementation is more difficult when the concerned plants do not belong to the same company. A contractual agreement must then be proposed, accepted by all and implemented. An additional difficulty is related to the fact that a supplier of the supply chain can belong to several supply chains (Renault and PSA supply chains share many suppliers). Technically, the generalization of the suggested approach could be immediate if the same principles of piloting are retained by all, which makes it possible to pool the risk; however, this strategy then poses some problems in the contractual formulation because the safety stocks are calibrated for aggregated demands. A less efficient solution consists of regarding the customers of the same supplier as independent. In addition, it is advisable to not underestimate the problems of confidentiality that are involved by the sharing of information that is coming from several supply chains.

MPS of engines E1 et E5 defined at the beginning of period 1											
Period		1	2	3	4	5	6	7	8	9	
A	MPS(engine 1)	993	984	978	1001	979	976	1036	994	994	
	MPS(engine 5)	97	93	97	112	107	90	86	92	92	
B	MPS(engine 1)	171	198	183	184	193	188	205	192	192	
	MPS(engine 5)	117	94	82	105	113	114	100	96	96	
TRANSPORT: 2 days from the engine assembly plant to the vehicle assembly plant A 4 days from the engine assembly plant to the vehicle assembly plant B											
Period		1	2	3	4	5	6	7	8	9	
Delivery at plant A	Delivery (engine 1)	993	984	978	1001	979	976	1036	994	994	
	Delivery (engine 5)	97	93	97	112	107	90	86	92	92	
Delivery at plant B	Delivery (engine 1)	171	198	183	184	193	188	205	192	192	
	Delivery (engine 5)	117	94	82	105	113	114	100	96	96	
Departures to plant A	Expeditions (engine 1)	984	978	1001	979	976	1036	994	994	994	
	Expeditions (engine 5)	93	97	112	107	90	86	92	92	92	
Departures to plant B	Expeditions (engine 1)	183	184	193	188	205	192	192	192	192	
	Expeditions (engine 5)	82	105	113	114	100	96	96	96	96	
Engine 1 L=2 OHB(0)=30 Delivery (1)=1190 Delivery (2)=1200											
Period		0	1	2	3	4	5	6	7	8	9
Gross requirement (E1 to be sent to A)			984	978	1001	979	976	1036	994	994	994
Gross requirement (E1 to be sent to B)			183	184	193	188	205	192	192	192	192
Total gross requirement			1167	1162	1194	1167	1181	1228	1186	1186	1186
Planned delivery (beginning of period)			1190	1200							
Available inventory (end of period)	30		53	91	-1103						
Net requirement					1103	1167	1181	1228	1186	1186	1186
Planned order			1103	1167	1181	1228	1186	1186	1186	1186	1186
Engine 5 L=1 OHB(0)=15 Delivery (1)=190											
Period		0	1	2	3	4	5	6	7	8	9
Gross requirement (E5 to be sent to A)			93	97	112	107	90	86	92	92	92
Gross requirement (E5 to be sent to B)			82	105	113	114	100	96	96	96	96
Total gross requirement			175	202	225	221	190	182	188	188	188
Planned delivery (beginning of period)			190								
Available inventory (end of period)	15		30	-172							
Net requirement					172	225	221	190	182	188	188
Planned order			172	225	221	190	182	188	188	188	188
Pistons L=2 OHB(0)=20 Delivery (1)=5780 Delivery (2)=5900											
BOM coefficient 4 pistons for engine 1 6 pistons for engine 5											
Period		0	1	2	3	4	5	6	7	8	9
Gross requirement from E1			4412	4668	4724	4912	4744	4744	4744	4744	4744
Gross requirement from E5			1032	1350	1326	1140	1092	1128	1128	1128	1128
Total gross requirement			5444	6018	6050	6052	5836	5872	5872	5872	5872
Planned delivery (beginning of period)			5780	5900							
Available inventory (end of period)	20		356	238	-5812						
Net requirement					5812	6052	5836	5872	5872	5872	5872
Planned order			5812	6052	5836	5872	5872	5872	5872	5872	5872
Piston crown L=2 OHB(0)=480 Delivery (1)= 5870 Delivery (2)= 5790											
BOM coefficient 1 Piston crown per piston											
Period		0	1	2	3	4	5	6	7	8	9
Gross requirement			5812	6052	5836	5872	5872	5872	5872	5872	5872
Planned delivery (beginning of period)			5870	5790							
Available inventory (end of period)	450		508	246	-5590						
Net requirement					5590	5872	5872	5872	5872	5872	5872
Planned order			5590	5872	5872	5872	5872	5872	5872	5872	5872

XXX Pegging of pistons

XXX Pegging of piston crown

Table 1: Illustration of the MRP calculus in a deterministic universe ($H_F = 9$)

Piston crown L=2 OHB(0)=450 Delivery (1)= 5870 Delivery (2)= 5790											
BOM coefficient 1 Piston crown per piston											
Period		0	1	2	3	4	5	6	7	8	9
Gross requirement			5812	6052	5836	5872	5872	5872	5872	5872	5872
Deterministic gross requirement			5812	5284	516						
Order-up-to level R					6548						
Planned delivery (beginning of period)			5870	5790							
Available inventory (end of period)	450		508	1014	-4822						
Net requirement					6050	5872	5872	5872	5872	5872	5872
Planned order			6050								

Table 4: Calculus of the planned order of piston crowns of period 1 ($H_F = 7$ - no quality problem)

Piston crown	L =2		OHb(1)=508		Delivery (1) = 5790		Delivery (2) = 6050			
BOM coefficient	1 Piston crown per piston									
Period	0	1	2	3	4	5	6	7	8	9
Gross requirement			6140	5752	5932	5872	5872	5872	5872	5872
Deterministic gross requirement			6140	4984	612					
Order-up-to level R					6548					
Planned delivery (beginning of period)			5790	6050						
Available inventory (end of period)		508	158	1224	-4708					
Net requirement					5936	5872	5872	5872	5872	5872
Planned order			5936	5872	5872	5872	5872	5872	5872	

Table 5: Calculus of the planned order of piston crowns of period 2 ($H_F = 7$ - no quality problem)

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