ECONOMICAL ANALYSIS OF PRODUCT STANDARDIZATION

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Abstract: The standardization of partially substitutable components destined to meet a set of needs is studied. The monocriterion approach to standardization proposed in the last century is highlighted by taking into account several criteria and an economic valorization through modeling by linear programming. The notion of diversity is analyzed in this perspective and some cost problems of this method are discussed. *Copyright © 2000 IFAC.*

Keywords: standardization, product design, mass production, linear programming.

Calibration was the main standardization effort in the era of craftsmanship. It dealt with the establishment of measures (monetary, weight, length, etc.) in order to determine the exchange of goods and services on comparable bases and to progressively build the related sciences. One has to wait for the industrial revolution, which came about through the evolution of knowledge, before the use of standardized processes allowed production of substitute products, enabling the production of ever more complex finished products manufactured from identical components made on machines, which are also becoming increasingly sophisticated. This product standardization is compatible with a systematic production of "custom" component most often used in a single product. The *design rationalization of a range of homogeneous and partially interchangeable products, designed to meet various needs*, presented here as the modern definition of product standardization, is rapidly felt. We begin with the relationships between diversity and standardization before proposing a lead to optimize this standardization.

1. DIVERSITY AND STANDARDIZATION

After an overview of standardization fundamentals, the diversity concept will be examined in detail

1.1. Standardization fundamentals

Standardization efforts are undertaken in all industrialized countries in the first half of the nineteenth century. In France, it seems that modeling of this rationalization dates back to the mid 1800s with the work of Renard on French navy ropes: increasing ship sophistication, with custom-designed rigging, and colonial expansion led to a rapidly unmanageable proliferation of rope stocks. The idea is quite simple. In this problem, the main characteristic of a rope is the maximum tensile strength Y that a rope of diameter X can withstand before rupturing. A test on ropes of various diameters helped establish Figure 1. Then, subdivide the axis of ordinates into a definite number of disjoint intervals and associate, to any y_k tension request, the diameter associated with the upper limit of the interval that contains y_k (i.e., in the example on Figure 1, a rope of diameter x_2, for a limit traction before rupture between y_1 and y_2).

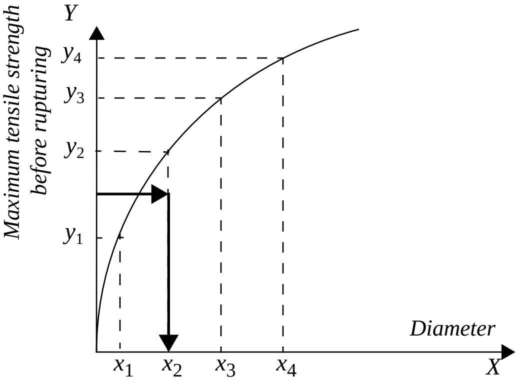

Fig. 1. Analysis of traction limit before rope rupture, by Renard

This approach to the problem can limit the number of item-types to produce and manage, but the number of intervals and their definition remain to be solved. To do so, empirical work proposed solutions (and gave birth namely to the "Renard series" still used in the industry). Before proposing an economic modeling of this problem under a multicriteria generalization approach, it could be useful to put the problem in a larger perspective of diversity and its cost, in an economic context that further aims at interesting the client through custom production of goods and services.

1.2. Diversity Analysis

The global perception of finished products by the consumer or the potential client is not discussed here. That perception is conditioned by numerous factors; several depend on the mix-media selected in marketing and are of no concern to us. We only consider the visible production diversity on components of a finished product satisfying needs of an identical nature, the presence of one of these components being mandatory. Doing so, the supplementary diversity obtained by adding optional components is not analyzed[1].

Three types of custom production must be differentiated (See Anderson and Pine, 1997). Some product characteristics can be customized by the client for a perfect suitability with his tastes or needs (*customizable products*). For example, the owner of a stereo sound system can set up certain characteristics of his tuner so the sound of his stereo system matches his preferences, the driver of certain makes of automobiles can adjust the height and inclination of the steering wheel, or the PC user can modify the parameters of his work environment. Other products (*customizing products*) are self-adapting, meaning they automatically adjust to the context of their utilization, which is namely to ensure the function for which they are designed. This product category can be illustrated by the electric shaver with floating heads or by washing machines using fuzzy logic to determine the best wash cycle (this is progress compared to washing machines customized with a few dozen washing programs, between which the user often has trouble deciding). Finally, some products are customized by the manufacturer (and, in some cases, by the salesperson). Several non-exclusive solutions can describe this *customization*.

- The best known approach is the use of an *appropriate combination of modules*, each module being selected from a limited number of interchangeable modules. This solution, largely used in the automotive industry, not only implies a standardized production process (to minimize settings and tool changes) and a total substitutability of modules from a same family during assembly, but also the design of standardized assembly supports called product platform (Meyer and Lehner, 1998) to limit the “system risks” of malfunction, assured of sufficient lastingness and robust, i.e. accepting a large spectrum of constraints (electrical, mechanical, etc.).
- The use of *adjustable components with reversible settings* leads to immediate and cost-efficient customization of a component by different means (switch, software…) to activate a set of functions selected from a predetermined list; this decision could be reversed. This technique of often selected in the design of electronic or electric components.
- The use of *adjustable components with irreversible adjustments* corresponds to an immediate and cost-efficient adaptation of a component to the needs either by additional machining operations, or by a chemical treatment, both irreversible. Such products are found in the cutting trade, eyewear, custom bicycle manufacturing, or replacement doors and windows. Deferred differentiation often arises from this logic.

There are thus several ways to reach diversity of a finished product that avoid referring to systematic and costly customization. Two standpoints determine their *economical analysis*.

- A component is not a priori designed to be solely used in one finished product due to design, manufacturing and distribution synergies. The question of product design rationalization of a company must therefore be treated from a standpoint of technologically similar product families rather than from each individual product, taking into account that the industrial classification analysis of complex products with a same basic component is normally found in several different aggregate products (commonality). The question of product design rationalization deals with meeting a set of basic functional needs through a range of physically interchangeable components, each component with a limited spectrum for each of the functional needs selected. This approach to standardization makes it one component of company flexibility, complementing resources (equipment, personnel, procedures).
- This component variety must be detectable by the client and he must consider it an added value. There are numerous examples of diversity detectable by the client but of no added value for him. For example, Nissan decided to rationalize its supplies in 1993; it had more than 300 ashtrays for its vehicles (stated in Anderson & Pine, p. 95). Product internal diversity not detectable by the client is even worse (screw-products are a classical example of this type of diversity). Not only is it of no added value for the client, but it often generates additional costs (due to additional logistics problems, increased tooling diversity, etc.).

More often than not, a technical analysis of the classification of production and supply item-types will show a significant portion of components with identical or very similar characteristics. This mushrooming effect has several and well known explanations: misunderstanding of the impact of mushrooming item-types on production and product support service, the NIH syndrome (Not Invented Here), arbitrary design decisions, misleading argument about the “proper weight, etc.”, failure of the information system – all leading to the faster creation of a new item-type rather than looking up an existing one. It is clear that the economic approach to standardization that will be developed hereunder is of interest only if causes for mushrooming are not detected and progressively eliminated.

Searching to optimize standardization supposes on one hand that *functional needs* are adequately assessed and *translated into technical specifications*, which is facilitated by the QFD approach (Quality Function Deployment[2]), and on the other hand that a coherent and relevant reflection is carried out on all solutions considered adequate. It is obvious that the optimum solution of a poorly stated problem is not of a great interest (it even could reflect discredit on the method and not on its misuse). Determination of the solution portfolio to examine is based on a study eliminating a priori none of the custom modes described above. What already exists (internal manufacturing or supply) must of course be part of the alternatives studied and the new products planned must take into account the constraints of the existing production system or the one undergoing transformation[3]. Optimizing this standardization not only normally results in a decrease of production costs, but also in an

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[1] This type of diversity generates additional costs when the product is assembled on an assembly line due to the work variability caused by the assembly of optional components on some workstations, creating specific line balancing and scheduling problems (See Danjou, Giard and Le Roy, 2000). It may be economically interesting to systematically offer a popular option and if demand focuses on a few alternative options, it is then possible to relate to the method described here. Inversely, one can question the economic interest of least popular options.

[2] The Quality Function Deployment is a value chain optimization technique, from expression of needs to product support service. It originated in Japan and has been used for over 20 years by major companies, especially with the first step of the approach to switch from functional needs to technical specifications. An overview of these techniques is found in Revelle, Moran and Cox, 1997.

increase flexibility, the company could then react more rapidly and easily to the conjunctural and structural transformations of demand[4].

2. TOWARD OPTIMIZING STANDARDIZATION

Once the general modeling of the problem stated by linear programming, we will examine a few methodological problems stated by this economic rationalization, whether we call on the proposed optimization approach or not.

2.1. Baseline model

For simple and non-expensive products (for example screw-products or the above-mentioned Nissan ashtrays), a sophisticated analysis is generally not mandatory as long as the reduction in the number of components for a function becomes self-evident from a classification simplification standpoint as well as for production and supply processes.

In other cases, the Renard approach is used to rationalize the range of relatively simple products, generally characterized by a unique quantitative criterion. Pareto analyses on need distribution based on this quantitative criterion (See, for example Anderson & Pine, chap. V, 1997) establish empirical approaches to determine a range of products, but the action is based on intuition and barely takes into account the economic standpoint. Product complexity also ensures that the technical analysis cannot have only one criterion. Formulation of this problem by linear programming using integers brings relevant response elements to these concerns[5]; the matrix generators approach tested can easily implement them[6].

The model will be presented with an example on engines. From a technical standpoint, we have n possible variants of a product[7], whether these variants are presently produced or only under study. These different variants will be identified with the subscript *j*. Moreover, we presume the demand analysis identified m segments, identified with the subscript *i* and characterized by a demand d_i. From an existing situation, we must normally have $n \leq m$, i.e. the variety of the commercial offer is lower than the variety of the demand.

The problem at hand is first related to an eventual decrease in the number of variants (with no economic vision of the problem). A first technical analysis must be performed to characterize the various engines. Selected criteria relate to technical characteristics that are more or less well perceived by demand (power, pollution, consumption…) and to characteristics of no interest for the client but essential in managing interfaces (weight, overall dimensions, engine mounting method on the chassis…). This technical analysis is performed on a table similar to Table 1, with a literal in each table cell to indicate the precise positioning of each engine.

Table 1: Technical characteristics of engines under study

Engines under study	Characteristics			
	1	2	…	p
1				
2				
j				
n				

A second analysis is to be carried out on technical characteristics of requested engines (Table 2). For each previously determined criterion, an interval of admissible values (minimum power, for example) is then defined based on the type of characteristic or a list of acceptable occurrences is determined if the characteristic is qualitative (such as mounting mode, for example). A difficulty encountered is determining the "appropriate need" aimed at pleasing, as mentioned above.

Table 2: Technical characteristics of requested engines

Requested engines	Characteristics				Demand
	1	2	…	p	
1					d_1
2					d_2
i					d_i
m					d_m

Comparing Tables 1 and 2 establishes Table 3, in which coefficients a_{ij} take the value 1 if demand in segment *i* can be met by engine *j*, or 0 if it cannot be met (the numerical illustration being fully arbitrary).

Table 3: Possibilities of satisfying the demand with the offer

Market segment	Variants of engines studied						
	1	2	3	…	*j*	…	n
1	1	0	0	…	0	…	0
2	1	1	0	…	0	…	0
3	1	0	1	…	0	…	0
…	…	…	…	…	…	…	…
i	0	1	1	…	1	…	0
…	…	…	…	…	…	…	…
m	0	0	1	…	1	…	1

It seems realistic to impose that the entire demand segment be satisfied by a same variant. Consequently, we define the binary variable x_{ij}, which will have the value 1 if demand of segment *i* is met totally by an engine *j*, and 0 if it is not. It is of course useless to create a variable x_{ij}

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[3] This category of concern is taken into account in DFA (Design For Assembly) literature; refer to Nof, Wilhelm and Warnecke, 1997, chap. III, and to Redford and Chal, 1994.

[4] Company flexibility (See Tarondeau, 1993 & 1997, Cohendet and Llerenna, 1989, and Reix, 1997) is classically seen through mobilized resources (equipment, tooling, personnel and procedures); product-related flexibility is generally highlighted but, however, economic instruments for its improvement have not been studied much to our knowledge, and not in the perspective treated here.

[5] In an previous research document (Giard, 1990), we presented briefly a similar model in a more restrictive perspective. In a recent publication (Giard, 1998), we proposed a system approach to production process modeling through mathematical programming indicating the mutual connections between various models found in the literature on operational research and explaining the complex model creation mechanisms using a set of base components. Formally, the problem here combines a transposition of a client assignment model to production centers (Giard, 1998, p. 46) and the recognition of non-linear costs with fixed costs that vary by brackets (Giard, 1998, p. 97), except that the total production of an item-type here is the sum of productions requested by various segments.

[6] This MGs approach is described in Murphy, Stohr and Asthana, 1992, Rosenthal, 1996, Jacquet-Lagrèze, 1997 and Giard, 1998. The technical and economic viability of this method is proven in numerous achievements (for example, two important industrial applications of these methods were carried out at the IAE in Paris within the scope of research contracts; they are described in Giard, Triomphe and André, 1997, and in Giard and Triomphe, 1996).

[7] Renault's Cléon plant, for example, manufactures more than 400 possible engine variants.

if the corresponding parameter a_{ij} is null[8]. Moreover, if we decide that this demand can be met by several variants, then variable x_{ij} can have any value between 0 and 1. To force the demand of segment *i* to be met, we must establish the constraint of Relation 1 (which leads, for binary variables, to a non null value).

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ , for } i = 1, \ldots, m \text{ (meeting the demand)} \qquad \textit{Relation 1}$$

Under these conditions, production y_j of item-type *j* is then the sum of productions carried out for each segment (demand d_i), this production could be null. This constraint is defined by Relation 2:

$$y_j = \sum_{i=1}^{m} d_i \cdot x_{ij} \text{ , for } j = 1 \ldots n \text{ (production of item-type } j\text{)} \qquad \textit{Relation 2}$$

To establish the annual production cost, we must introduce explicit hypotheses on the item-type costs function form. First, we will assume the cost functions are independent. This restrictive hypothesis will be removed later on. We will also assume variation in annual fixed costs per bracket, and that for each bracket the variable unit cost may vary but remains constant over disjoint value intervals; these very general hypotheses lead to a cost function of the type described in Figure 2.

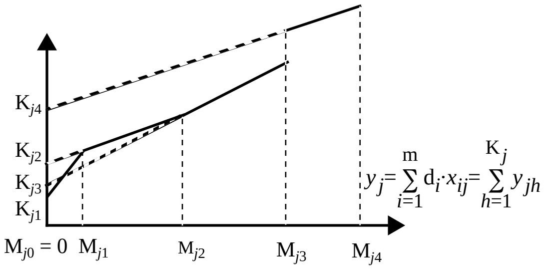

Fig. 2. Production cost function

This cost function is neither concave (which would imply that the mean production cost never increases when y_j increases) nor convex (which would imply that the mean production cost never decreases when y_j increases) and that in the selected example it takes into account the phenomena of scale diseconomies occurring at the saturation limit. This non-linear cost function can easily be taken into account in the objective function of a linear program by introducing as many dummy y_{jk} productions as there are K_j value intervals (comprised between $M_{j,k-1}$ and $M_{j,k}$, the upper threshold only is included in the interval, with $M_{j0} = 0$) for which the variable cost c_{jk} is constant, and are all null except for the one encompassing production y_j in its value interval and that is, of course, equal to this production ($y_j = y_{jh}$). From the example in Figure 2, the objective function becomes, for the section related to production cost y_j:

Min *z*, with $z = (c_{j1}y_{j1} + K_{j1}z_{j1}) + (c_{j2}y_{j2} + K_{j2}z_{j2}) + (c_{j3}y_{j3} + K_{j3}z_{j3}) + (c_{j4}y_{j4} + K_{j4}z_{j4}) + \ldots$

with the following list of constraints[9], added to those of Relations 1 and 2:

$$z_{j1} + z_{j2} + z_{j3} + z_{j4} \le 1$$
$$0 \le y_{j1} < M_{j1} z_{j1}$$
$$M_{j1} z_{j2} \le y_{j2} < M_{j2} z_{j2}$$
$$M_{j2} z_{j3} \le y_{j3} < M_{j3} z_{j3}$$
$$M_{j3} z_{j4} \le y_{j4} < M_{j4} z_{j4}$$

The first constraint ($z_{j1} + z_{j2} + z_{j3} + z_{j4} \le 1$) activates at most one of the fixed costs ($K_{jk} z_{jk}$), the one for which zk = 1, providing that if this item-type is not produced, all z_{jk} are null. The following constraints force a 0 value on y_{jk} productions of intervals that are not selected for the two limits are nul when $z_{jk} = 0$, forcing the corresponding production x_k to be null, and vice versa[10].

Generalizing the reasoning to all variants leads to the Relation 3 objective function and to the constraints described by Relations 4 and 5:

$$\text{Min } z \text{, with } z = \sum_{j=1}^{n} \left(\sum_{k=1}^{K_j} (c_k y_{jk} + K_{jk} z_{jk}) \right) \qquad \textit{Relation 3}$$

$$\sum_{k=1}^{K_j} z_{jk} \le 1 \text{ , for } j = 1 \ldots n \qquad \textit{Relation 4}$$

$$M_{jk-1} z_{jk} \le y_{jk} < M_{jk} z_{jk} \text{ , for } k = 1 \ldots K_j \text{ and } j = 1 \ldots n, \text{ with } M_{j0} = 0 \qquad \textit{Relation 5}$$

This formulation must be adapted to take into account positive or negative synergy effects related to simultaneous production of two or more engines on a production site. To do so, relevant and easy modeling tools have to be implemented[11]. Let us examine a few examples.

- Let us assume that manufacturing more than κ variants translates into an increase Γ^+ of annual fixed costs; we then have to create the γ^+ binary variable, add to the objective function the term $\gamma^+\Gamma^+$, and add Relation 6 constraints to force γ^+ to take the value 1 if at least κ variants are produced (the objective function is designed to bring γ^+ to equal 0).

  $$\sum_{j=1}^{n} \left(\sum_{k=1}^{K_j} z_{jk} \right) < \kappa + n\gamma^+ \qquad \textit{Relation 6}$$

  Of course, this relation can easily be adapted to a subset of engines or to several engine subassemblies. In the latter, the subsets can be disjoint, which will occur when technically specialized tool sets are used in the production of different engine sets. The same engine set can also be found in several constraints of this type. This allows defining the fixed costs as a step function of the number of item-types produced and not as a function of the total quantity produced.
- Let us assume, on the contrary, that manufacturing more than κ variants means a decrease Γ^- of annual fixes costs; we then have to create the γ^- binary variable, subtract the term $\gamma^-\Gamma^-$ from the objective function and add Relation 7 to constraints to force γ^- to only take the value 1 if at least κ variants are produced (the objective function is designed to bring γ^- to equal 1).

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[8] This special feature, easy to take into account in the problem description by a GM (see, for example, Giard, 1998, and Jaquet-Lagreze, 1997) helps in significantly limiting the size of the problem. This convention makes Relation 1 useless in the form $\sum_{j=1}^{n} a_{ij} x_{ij} = 1$

[9] As variables are not negative, the first double constraint is in reality reduced to $x_1 \le M_1 y_1$; the selected formulation has the sole advantage of allowing rationale generalization.

[10] If the cost function is concave (non-decreasing total cost function), then the “straight section” of double unequations is useless.

[11] See Giard, 1998, chapter III.

$$\sum_{j=1}^{n}\left(\sum_{k=1}^{K_j} z_{jk}\right) > \kappa\gamma^{-} \qquad \textit{Relation 7}$$

Here again, this approach can easily be adapted to a subset of variants or to several variant subsets. Among others, nothing precludes positive or negative synergy effects to occur simultaneously for different variant subsets or not, as a generalization of the previous comments.

- We can assume finally that certain fixed costs vary per bracket with relation to produced quantities over a set of item-types, notwithstanding the possibility offered for each item-type to include in its cost function its own variation of fixed costs per bracket. In this case, the Ω engine subset being affected by variations of fixed costs, we simply have to adapt the formulation as follows:
  - create variable ω corresponding to the total production of the Ω engine subset, which leads to Relation 8:

$$\omega = \sum_{j \subseteq \Omega} y_j = \sum_{j \subseteq \Omega} \sum_{i=1}^{m} d_i \cdot x_{ij} \qquad \textit{Relation 8}$$

  - add to the objective function the impact of variations in fixed costs $K_{\omega k}$, following the same method used for item-type $\sum_{k=1}^{K_\omega} K_{\omega k} z_{\omega k}$;
  - then adapt Relations 2, 4 and 5, which leads to Relations 9 to 11

$$\omega \le \sum_{k=1}^{K_\omega} \omega_k \qquad \textit{Relation 9}$$

$$\sum_{k=1}^{K_\omega} z_{\omega k} \le 1 \qquad \textit{Relation 10}$$

$$M_{\omega k-1} z_{\omega k} \le \omega_k < M_{\omega k} z_{\omega k}\text{, for } k = 1 \ldots K_\omega\text{, with } M_{\omega k} = 0 \qquad \textit{Relation 11}$$

2.2. Method problems created by using this optimization approach

The relevance of this modeling depends both on its the use to describe alternate scenarios and on the costs used in the economic function.

First, we are working at a detail level too coarse to pretend it accurately represents the processes used, the effective demand on the production system, with its seasonal and random fluctuations, or the robustness of the production system when faced with problems. This comment can be made for most help tools on strategic decisions: the important aspect is the relevance of the order of magnitude of collected data in volume or in value. This brings us back to valorization problems.

The most important problems we may meet here relate to time or, more specifically, correctly defining the interdependence through time of decisions made within the cost accounting system.

In the rare case where the studies alternatives relate to new production from new equipment, it is easy to see the use of a dynamic version of the model proposed, as the decision to meet a demand segment by a given engine is taken for all periods[12]; other hypotheses can be used, but seem more difficult to justify. The separation between fixed costs and variable costs helps isolate start-up investments (and thus avoid the problem of determining the depreciation) from direct fixed costs (especially personnel) that could vary per bracket with relation to quantities produced and variable direct costs (materials, etc.). As this information used in the economic function is dated[13], we would have to call upon actualization to correctly weight the monetary flows generated at different periods. Classic problems are encountered when this approach is applied to compare investment alternatives and, more specifically, to determine the determination of the interest rate used in VAN calculus and the problem of comparing solutions having different useful lives.

When the problem relates to a set of item-types that at least partially exists, when it is the object of external supply or an internal production in a production system which could eventually be only marginally modified, several methodology problems arise.

- Certain key components of a certain complexity are designed to be used in several finished products, some of which do not yet exist; the term “main sub-assembly” projects is used in the automotive industry. The transfer price of such components creates challenging methodological projects[14]. A decision on standardization that questions the economic hypotheses used to launch such components must rely on a cost function that guarantees decision coherence in time and between strategic and tactical decisions (See Giard, 1988 and 1992).
- Within this framework and more generally, the decision to stop production of a item-type (or of a group of item-types) can lead to support an “item withdrawal cost”. This impact can easily be taken into account by the objective function[15].
- Existing standard costs are only relevant on a certain range of quantities produced or supplied; before applying the method, thorough investigations are needed to reconstitute the cost function and, for supply, carry out a prior consultation with suppliers on the basis of volume scenarios that could substantially deviate from the current solution.
- The standardization problem can occur within a multilevel bill of materials. Decisions taken at a detailed level then are based on hypotheses of direct demand and demand originating from other item-types (using MRP type mechanisms) that are themselves the object

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[12] Which can be translated by maintaining Relation 1 and the following adaptation of Relation 2 which becomes: $y_{jt} = \sum_{i=1}^{m} d_{it} \cdot x_{ij}$, Relations 3 and 5 being modified by the inclusion of the period index, Relation 3 integrating actualization coefficients.

[13] With an appropriate time sub-division, this helps take in to account possible learning effects on recurring costs, with values that can be considered reasonably stable over each time interval. .

[14] For a more complete presentation of problems related to economic management of products over their life cycle, refer to Gautier and Giard, 2000.

[15] Using notations of Relation 4, for a cost of Φ, simply add the term $\left\{1 - \sum_{k=1}^{K_j} z_{jk}\right\}\Phi$ in the objective function. Knowing that $\sum_{k=1}^{K_j} z_{jk} = 1$ if item-type *j* is produced, the cost Φ will not be supported in this case alone. Generalization of a set Ψ of ψ item-types is immediate $\{1 - t\}\Phi$, with $\sum_{j \in \Psi} \sum_{k=1}^{K_j} z_{jk} - \psi < t$.

of a standardization optimization. An independent analysis of these different problems cause the problem of elementary component standardization to depend on hypothetical aggregate component demands and cause the problem of aggregate component standardization to depend on hypothetical costs of elementary components. Convergence toward a set of coherent solutions can empirically be assured through a certain number of iterations, but this model could also be adapted to take into account the Bill Of Materials[16], which could lead to a model of undue size.

- The creation of new components induces management costs related to increased diversity that are difficult to evaluate. In economic calculations made at the design stage, certain companies such as Intel or Renault apply different cost rates to new components and to reused existing components. This incentive to reduce diversity is astute but must be used with great caution within a complete reconsideration required when applying the approach suggested in this paper.

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[16] Simply replace Relation 2 ($y_j = \sum_{i=1}^{m} \mathrm{d}_i \cdot x_{ij}$), which defines production y_j, as being equal to the sum of demands d_i related to it by binary variables x_{ij}, with the following Relation: $y_j = \sum_{i=1}^{m} \mathrm{d}_i \cdot x_{ij} + \sum_{h=1}^{m} \mathrm{a}_{hj} \cdot y_h \cdot x_{hj}$, where item-type h is related to item-type j due to the production of 1 item-type h unit requires a_{hj} unit of item-type j. (This item-type h is related to final demands through Relation 2).