Integrated Laycan and Berth Allocation Problem

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***Abstract*—Handling vessels within the agreed time limits at a port with an optimal exploitation of its quays plays an important role in the improvement of port effectiveness as it reduces the stay time of vessels and avoids the payment of contractual penalties to shipowners due to the overrun of laytimes. In this paper, we propose a mixed zero-one linear model for a new problem called the integrated Laycan and Berth Allocation Problem with dynamic vessel arrivals in a port with multiple continuous quays. The model aims, first, to achieve an optimal berth plan that reduces the late departures of chartered vessels by maximizing the difference between their despatch money and demurrage charges, considering water depth and maximum waiting time constraints and the productivity that depends on berth positions and, second, to propose laycans for new vessels to charter. Only one binary variable is used to determine the spatiotemporal allocations of vessels and the spatiotemporal constraints of the problem are covered by a disjunctive constraint. An illustrative example and several numerical tests are provided.**

Keywords—laycan allocation; berth allocation; mixed zero-one linear programming; spatiotemporal disjunctive constraints

I. INTRODUCTION

Ports play an important role in integrated supply chains requiring maritime transport. This is the case with OCP Group, world leader in the phosphate industry. Port management performance is related to both the respect of contractual clauses and the optimal use of port resources (quays, equipment, manpower, etc.). These two aspects are linked: the first comes from laycan negotiations between shipowners and charterers whose result becomes a constraint to the second.

Laycan is an abbreviation for the "Laydays and Canceling" clause in a charterparty (maritime contract between a shipowner and a charterer for the hire of a vessel). This clause establishes the earliest date, when the vessel is required by the charterer, and the latest date for the commencement of the charter when the charterers have the option of canceling the charter. Once the vessel arrives at the port of loading, the charterer should be ready to start loading its cargo in order not to exceed the laytime.

Laytime is the amount of time allowed by the shipowner to the charterer for loading and / or unloading the cargo. It equals the cargo volume divided by the contractual rate of loading or unloading. If the charterer exceeds the laytime, a predetermined penalty called "demurrage" is incurred. This penalty equals the time exceeded multiplied by the demurrage rate. Otherwise, if the whole period of laytime is not needed, a refund called "despatch" may be payable by the shipowner to the charterer. This refund equals the time advanced multiplied by the despatch rate. Despatch is normally paid at 50% of the demurrage rate "Despatch half Demurrage", but this depends on the terms of the charterparty. The vessel may thus be able to leave port early. These chartering terms are shown in Fig. 1.

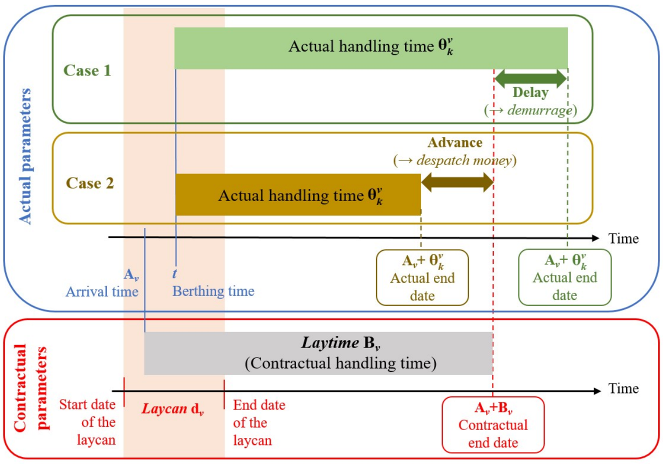

Fig. 1. Comparison between the contractual and the actual parameters

The Laycan Allocation Problem (LAP) refers to the problem of assigning berthing time windows to vessels within a medium-term planning (several weeks), by taking into consideration commercial, logistical and production constraints like sales forecasts, availability of cargo and production planning, due dates and quays availability, hence its interaction with the Berth Allocation Problem.

The Berth Allocation Problem (BAP) refers to the problem of assigning berthing positions and times to every vessel projected to be served within a short-term planning horizon (several days), such that a given objective function is optimized. The assignment must respect the constraints of the problem (vessels' drafts and lengths, expected arrival times and projected handling times, etc.).

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The integrated Laycan and Berth Allocation Problem (LBAP) considers the LAP and the BAP together. The combined problem aims to find an efficient schedule for berthing chartered vessels and new vessels to charter. It has to be noted that the freight transport is hardly predictable as many disturbances may occur (e.g., vessel delays, bad weather, etc.), thereby disabling the scheduled berth plan. Therefore, the LBAP should be solved on a rolling horizon. Indeed, each time a change occurs in the inputs of the problem (e.g., arrival and handling times of vessels, etc.), the model must be run again.

The paper is organized as follows. A literature review of the BAP and the LAP is presented in Section 2. The characteristics and the description of the mathematical model of the LBAP are presented in Section 3. An illustrative example and several numerical tests are provided in Section 4. Finally, in Section 5, we draw some conclusions and indicate future research.

II. Literature Review

The BAP has been widely studied by the scientific community. However, bulk terminals have received less attention than container terminals in the operational research literature. According to Bierwirth and Meisel [1], [2], the BAP models can be classified within four attributes: spatial, temporal, handling time and performance measure. We can add a fifth attribute that concerns the modeling of the BAP like the type of spatial and temporal constraints and the number and the type of variables used in the model (discrete, continuous, binary or a mix of these variables).

Most of authors solve the BAP using exact methods ranging from MILP formulations combined with standard solvers to highly sophisticated branching-based algorithms, heuristics like Genetic and Evolutionary Algorithms [3], and metaheuristics like Tabu Search [4] and Simulated Annealing [5].

Regarding the LAP, Lorenzoni et al. [6] proposed a tool based on a mathematical model of the LAP as a multi-mode resource-constrained scheduling problem. The tool determines laycans to vessels under the condition that once the vessels have arrived at the port, they have to be attended in first come first served order. They solved the problem using a heuristic procedure based on the Differential Evolution Algorithm. However, they only proposed a temporal allocation of port resources in general.

To the best of our knowledge, in the previous studies, the LAP and the BAP are solved separately. In this paper, we propose a mixed zero-one linear model for solving a new problem that combines the LAP and the BAP: the LBAP.

III. Mathematical Model for the Integrated Laycan and Berth Allocation Problem

A. *Characteristics of the Model*

1) *Spatial attribute*

We consider a continuous berth layout (partitioned into a set of short length sections) with water depth restrictions (i.e. all the sections of a quay can have the same water depth or the water depth increases seaward). We also take into consideration the technical constraints of vessels that prohibit their berthing at some quays or oblige them to berth at a specific quay.

2) *Temporal attribute*

We assume dynamic vessel arrivals with a maximum waiting time in harbor for each vessel (i.e. maximum berthing date of the vessel after its arrival to the port).

3) *Handling time attribute*

Handling times of vessels depend on their berthing positions. This variation can be due to the characteristics of the available equipment at the occupied sections (quay cranes, conveyors for bulk, internal transfer vehicles for containers, etc.). If a quay is divided into zones that have equipment with different productivities, we will consider that a vessel must berth at sections with equal productivities. In this case, the berth layout becomes almost hybrid in the sense that more than one vessel can berth at the same zone of the quay (without overlapping) but a vessel cannot berth at two different zones at the same time. This simplifying assumption can be subsequently withdrawn.

The laytime of a vessel is considered as its longest handling time in the port. The vessels which are already berthed in the port have fixed berthing times and positions and residual handling times.

4) *Performance measure attribute*

The objective is to achieve an optimal berth plan that reduces the late departures of chartered vessels by maximizing the difference between their despatch money and demurrage charges, while favoring their berthing as close as possible to the port yard. Furthermore, laycans are proposed for the new vessels to charter by making them leave the port as early as possible and berth as close as possible to the port yard without impacting the economic results of the chartered vessels.

B. *Description of the Model*

1) *Input data*

We consider a planning horizon divided into T periods $(t = 1,...,\mathrm{T})$ (e.g., days) and a port with Q quays $(q = 1,...,\mathrm{Q})$ partitioned into a set of short length sections (e.g., 10 m) in order to be in the continuous berth layout case. Each quay has a length of S_q sections $(s_q = 1,...,\mathrm{S}_q)$; by convention, the first section is the section that is closest to the port yard.

A vessel can berth at a section if the water depth is greater than its draft. We hence define H water depth and draft classes $(h = 1,..., \mathrm{H})$; by convention, the water depth of sections increases as h increases. The water depth of the section s_q is $\mathrm{J}_q^{s_q}$.

Because the vessel handling times depend on the vessel berthing positions, we also define K productivity classes to sections $(k = 1,..., \mathrm{K})$; by convention, the productivity of sections increases as k increases. The productivity of the section s_q is $\mathrm{L}_q^{s_q}$.

We consider V_1 berthed vessels $(v = 1,...,\mathrm{V}_1)$, V_2 chartered vessels to berth $(v = \mathrm{V}_1 + 1,...,\mathrm{V}_1 + \mathrm{V}_2)$ and V_3 new vessels to charter (vessels with unfixed laycans) $(v = \mathrm{V}_1 + \mathrm{V}_2 + 1,...,\mathrm{V})$, where $\mathrm{V}=\mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3$. The berthing of a vessel v at a quay q is subject to its technical constraints defined by the Boolean parameter F_q^v (1 if vessel v can berth at quay q, 0 otherwise). Each vessel is characterized by an estimated time of arrival A_v (expressed as a number of periods), a length λ_v (expressed as a number of sections) and a draft I_v.

The handling time θ_k^v of a vessel v depends on the productivity class $k = \mathrm{L}_q^{s_q}$ of the berthing position of its bow at the section s_q ($\theta_{\mathrm{L}_q^{s_q}}^v$). The V_1 berthed vessels have residual handling times, and fixed berthing times and positions. Each of the V_2 chartered vessels has a laytime $\mathrm{B}_v = \max_k(\theta_k^v)$, a daily despatch rate α_{1v}, a daily demurrage rate α_{2v}, and a maximum waiting time in harbor a_v provided that: $\mathrm{A}_v + \mathrm{a}_v + \mathrm{B}_v - 1 \le \mathrm{T}$. The role of this latter parameter is to reduce the solution space of berthing times in the planning horizon T. Therefore, the length of T does not influence the computation time of the model and its results.

Finally, the V_3 vessels with unfixed laycans are handled as follows in the model: we assume that they have estimated times of arrival equal to availability dates of cargo to be exported. The number of days d_v of the laycan of vessel v is included in its handling time. They also have fictitious daily despatch and demurrage rates equal to one and high maximum waiting times in harbor, so as not to affect the economic results of the V_2 chartered vessels (despatch and demurrage).

2) *Decision variables*

Each vessel v arriving in the port can wait in the harbor before berthing at a time and a position. So we define the binary decision variable $x_{vtq}^{s_q}$ that equals one if the vessel v berths at the beginning of the period t and occupies the sections of the quay q from s_q to $s_q + \lambda_v - 1$, where s_q is the section occupied by its bow. The existence of the decision variable $x_{vtq}^{s_q}$ is subject to five conditions:

- The vessel v should berth after its estimated time of arrival A_v within the maximum waiting time in harbor, denoted a_v: $\mathrm{A}_v \le t \le \mathrm{A}_v + \mathrm{a}_v$.
- The vessel v should be able to berth at the quay q: $\mathrm{F}_q^v = 1$.
- The length of the vessel v, denoted λ_v, should not exceed the limits of the quay q: $s_q \le \mathrm{S}_q - \lambda_v + 1$.
- The draft of the vessel v, denoted I_v, should not exceed the water depth of the section of the berthing position of its bow s_q: $\mathrm{I}_v \le \mathrm{J}_q^{s_q}$. If this condition is verified for the first section s_q, it will be implicitly verified for the other sections occupied by the vessel because the water depth of sections increases seaward.
- All sections occupied by the vessel v should have the same productivity class. Hence, the productivity classes of the two sections that have the berthing positions of the vessel's bow and stern should be equal: $\mathrm{L}_q^{s_q} = \mathrm{L}_q^{s_q + \lambda_v - 1}$.

The logical condition of the existence of the decision variable $x_{vtq}^{s_q}$ is the following one:

$$\mathrm{A}_v \le t \le \mathrm{A}_v + \mathrm{a}_v \wedge \mathrm{F}_q^v = 1 \wedge s_q \le \mathrm{S}_q - \lambda_v + 1 \wedge \mathrm{I}_v \le \mathrm{J}_q^{s_q} \wedge \mathrm{L}_q^{s_q} = \mathrm{L}_q^{s_q+\lambda_v-1}, \forall v \in \mathscr{V}$$

Conditioning the existence of the decision variable $x_{vtq}^{s_q}$ to the respect of the five conditions described above improves significantly the computational performance of the model since it is no longer necessary to introduce them as constraints in the model.

To determine if a demurrage is incurred or a despatch money is to be collected, we need to know if a vessel is late or in advance, regarding its contractual departure time. Therefore, we introduce two variables: u_v for the delay of a vessel v (i.e., end of handling time > end of laytime) and w_v for its advance (i.e., end of handling time < end of laytime).

3) *Constraints*

If a vessel v berths at the quay q, it can only have one berthing time t and one berthing position of its bow s_q. Constraint (1) makes it possible that the problem might not necessarily lead to a solution where all vessels berth, as a strict equality enforces the berthing of all vessels.

$$\sum_{t|A_v \le t \le A_v + \mathrm{a}_v} \sum_{q|\mathrm{F}_q^v = 1} \sum_{s_q|s_q \le \mathrm{S}_q - \lambda_v + 1 \wedge \mathrm{I}_v \le \mathrm{J}_q^{s_q} \wedge \mathrm{L}_q^{s_q} = \mathrm{L}_q^{s_q+\lambda_v-1}} x_{vtq}^{s_q} \le 1, \forall v \in \mathscr{V} \quad (1)$$

As mentioned before, the V_1 berthed vessels will have residual handling times and fixed berthing times $(t = 1)$ and positions with $x_{vtq}^{s_q} = 1, \forall v \in \mathscr{V}_1$.

If a vessel v berths at the beginning of the period t, at the quay q and its bow occupies the section s_q that has a productivity class $k = \mathrm{L}_q^{s_q}$ ($x_{vtq}^{s_q} = 1$), this vessel will occupy the sections from $s'_q = s_q$ to $s'_q = s_q + \lambda_v - 1$ during the periods from $t' = t$ to $t' = t + \theta_{\mathrm{L}_q^{s_q}}^v - 1$.

The constraint expressed in (2) and illustrated in Fig. 2, on the next page, is a spatiotemporal disjunctive constraint. It guarantees that a section cannot be occupied by more than one vessel at the same time by preventing overlap among the spatiotemporal rectangles representing vessels, which are located between $s'_q = s_q - \lambda_v + 1$ and $s'_q = s_q$ on the spatial dimension, and between $t' = t - \theta_{\mathrm{L}_q^{s_q}}^v + 1$ and $t' = t$ on the temporal dimension.

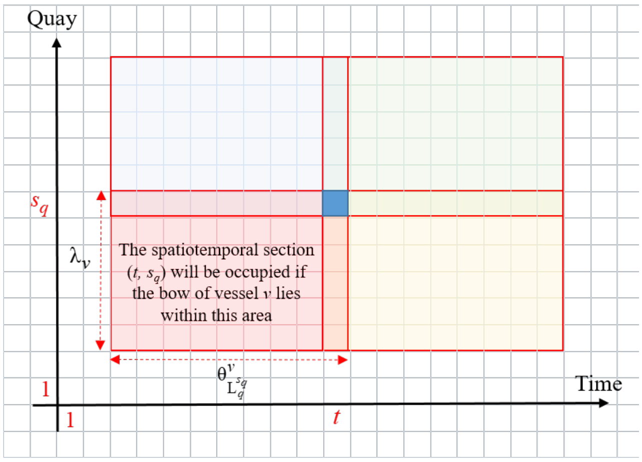

Fig. 2. Illustration of constraint (2).

To define the constraints of the variables u_v and w_v, we use an intermediate variable, τ_v, which gives the expected end of handling time for a vessel v proposed by the model:

$$\tau_v = \sum_{t|A_v \le t \le A_v + a_v} \sum_{q|F_q^v=1} \sum_{s_q|s_q \le S_q - \lambda_v + 1 \wedge I_v \le J_q^{s_q} \wedge L_q^{s_q} = L_q^{s_q+\lambda_v-1}} x_{vtq}^{s_q} \cdot (t + \theta^v_{L_q^{s_q}} - 1)$$

The difference between the end of handling proposed by the model and the contracted handling deadline for vessel v can be written as $\tau_v - (A_v + B_v - 1)$. The variables u_v and w_v should verify the constraints (3), (4) and (5) in order to determine the delay or the advance of each vessel.

$$u_v \ge \tau_v - (A_v + B_v - 1) \quad (3)$$
$$u_v \ge 0$$
$$w_v \ge -(\tau_v - (A_v + B_v - 1)) \quad (4)$$
$$w_v \ge 0$$
$$u_v - w_v = \tau_v - (A_v + B_v - 1) \quad (5)$$

4) Objective function

If $\tau_v - (A_v + B_v - 1) > 0$, the vessel is overdue, the charterer will have to pay demurrage to the shipowner that is equal to $\alpha_{2v} \cdot u_v$; otherwise, the vessel is in advance, the shipowner will have to pay despatch money to the charterer that is equal to $\alpha_{1v} \cdot w_v$. Indeed, the charterer can benefit from a despatch money if the vessel berths as early as possible at sections with high productivity. In this case, the vessel's handling time will be lower than its laytime.

The objective function expressed in (6) at the bottom of this page aims to maximize the difference between the despatch money and the demurrage of each chartered vessel, favors their berthing as close as possible to the port yard and proposes laycans for new vessels to charter by making them leave the port as early as possible and berth as close as possible to the port yard without impacting the economic results of the chartered vessels.

The role of $Max\left\{\sum_{v \in \mathscr{V}_2 \cup \mathscr{V}_3} (\alpha_{1v} \cdot w_v - \alpha_{2v} \cdot u_v)\right\}$ is to favor despatch money over demurrage charges for the V_2 chartered vessels and to make the V_3 new vessels to charter leave the port as early as possible. The role of Z in $Max\left\{\sum_{v \in \mathscr{V}_2 \cup \mathscr{V}_3} \sum_{t|A_v \le t \le A_v + a_v} \sum_{q|F_q^v=1} \sum_{s_q|s_q \le S_q - \lambda_v + 1 \wedge I_v \le J_q^{s_q} \wedge L_q^{s_q} = L_q^{s_q+\lambda_v-1}} x_{vtq}^{s_q} \cdot (Z + 1/s_q)\right\}$ is to force berthing of all vessels (if possible), and the role of $1/s_q$ is to make vessels berth as close as possible to the port yard in order to select one of the optimal economical solutions.

The laycan of the vessel v has a start date equal to its berthing time t proposed by the model and an end date equal to $(t + d_v - 1)$: $laycan = [t, t + d_v - 1]$. The maximum waiting time in harbor a_v to negotiate with the shipowner should be greater than or equal to the waiting time in harbor proposed by the model: $a_v \ge t - A_v$. Thereafter, we can assign daily demurrage and despatch rates not equal to one to each new vessel to charter in order to see their impact on the economic criteria of the objective function. Therefore, it's the decision-maker who will see how much he can accept the deterioration of the economic results.

IV. Illustrative Example and Numerical Tests

A. Illustrative Example

We consider a planning horizon divided into $T = 50$ days and a port of $Q = 3$ quays. Each quay has a maximum length ($S_1 = 40$, $S_2 = 50$ and $S_3 = 60$ sections of 10 meters each). We define $K = 3$ productivity classes to sections and $H = 3$ draft and water depth classes to vessels and sections.

We consider $V_1 = 2$ berthed vessels ($x_{v=1,t=1,q=1}^{s_q=1} = 1$ and $x_{v=1,t=1,q=3}^{s_q=21} = 1$), $V_2 = 16$ vessels to berth and $V_3 = 2$ vessels with unfixed laycans. The daily despatch rates are half the daily demurrage rates ($\alpha_{1v} = \alpha_{2v}/2$) and the constant $Z =$ 10 000. The detailed characteristics of sections and vessels are shown in Table I and Table II on the next page.

In this example, the LBAP model uses 4 826 variables and 5 750 constraints. It's computation time with Xpress in a PC of these characteristics (Intel® Xeon® CPU E3-1 240 v5 @ 3.50 GHz - 64 Go RAM) is 1 s.

$$\sum_{v \in \mathscr{V}|F_q^v=1} \sum_{s'_q = s_q - \lambda_v + 1 | s'_q \ge 1 \wedge I_v \le J_q^{s'_q} \wedge L_q^{s'_q} = L_q^{s'_q+\lambda_v-1}}^{s'_q = s_q | s'_q \le S_q - \lambda_v + 1 \wedge I_v \le J_q^{s'_q} \wedge L_q^{s'_q} = L_q^{s'_q+\lambda_v-1}} \sum_{t' = t - \theta^v_{L_q^{s_q}} + 1 | t' \ge A_v}^{t'=t | t' \le A_v + a_v} x_{vt'q}^{s'_q} \le 1, \forall t, \forall q, \forall s_q \quad (2)$$

$$Max\left\{\sum_{v \in \mathscr{V}_2 \cup \mathscr{V}_3} \left[\alpha_{1v} \cdot w_v - \alpha_{2v} \cdot u_v + \sum_{t|A_v \le t \le A_v + a_v} \sum_{q|F_q^v=1} \sum_{s_q|s_q \le S_q - \lambda_v + 1 \wedge I_v \le J_q^{s_q} \wedge L_q^{s_q} = L_q^{s_q+\lambda_v-1}} x_{vtq}^{s_q} \cdot (Z + 1/s_q)\right]\right\} \quad (6)$$

TABLE I. CHARACTERISTICS OF SECTIONS

Class	Water depth $J_q^{s_q}$ 1	2	3	Productivity $L_q^{s_q}$ 1	2	3
Range of sections of quay q=1	1 - 10	11 - 25	26 - 40	1 - 13	14 - 27	28 - 40
Range of sections of quay q=2	1 - 15	16 - 30	31 - 50	16 - 35	36 - 50	1 - 15
Range of sections of quay q=3	1 - 20	21 - 40	41 - 60	41 - 60	1 - 20	21 - 40

TABLE II. CHARACTERISTICS OF VESSELS

v	A_v	a_v	α_{2v}	λ_v	l_v	B_v	d_v	θ_k^v k=1	k=2	k=3	F_q^v q=1	q=2	q=3
01	1	0	x	8	1	x	x	10	8	7	1	0	0
02	1	0	x	10	1	x	x	7	6	5	0	0	1
1	1	4	121	17	1	11	x	11	9	8	1	1	1
2	1	4	35	7	1	10	x	10	8	7	1	1	1
3	2	3	26	14	2	7	x	7	6	5	1	0	1
4	2	4	26	16	3	9	x	9	8	7	1	1	1
5	3	4	131	18	1	9	x	9	8	7	1	1	1
6	3	4	79	15	1	9	x	9	8	7	1	1	1
7	3	4	63	11	1	10	x	10	8	7	1	1	1
8	4	5	84	9	2	13	x	13	11	9	1	1	1
9	6	4	53	13	1	10	x	10	8	7	1	1	1
10	6	4	81	14	1	10	x	10	8	7	1	1	1
11	7	4	55	9	3	9	x	9	8	7	1	1	1
12	8	3	102	12	2	7	x	7	6	5	1	1	1
13	9	4	104	10	3	9	x	9	8	7	0	1	1
14	9	4	134	11	1	9	x	9	8	7	1	1	1
15	10	4	35	10	1	10	x	10	8	7	1	1	1
16	11	4	96	13	3	9	x	9	8	7	1	1	0
001	8	20	1	9	1	11	2	11	9	8	1	1	1
002	12	20	1	18	1	10	4	10	8	7	1	1	1

For the V_2 chartered vessels, the sum of demurrage equals 442 against a sum of despatch equal to 843.5 and the sum of $1/s_q$ is 5.4599. For the V_3 vessels with unfixed laycans, the sum of $\alpha_{1v} \cdot w_v - \alpha_{2v} \cdot u_v$ is -2 and the sum of $1/s_q$ is 0.09375.

Fig. 3 shows the disposition of the V_1 berthed vessels, the V_2 chartered vessels, and the V_3 vessels with unfixed laycans, and the detailed results are shown in Table III.

TABLE III. RESULTS OF THE LBAP

v	A_v	t	B_v	θ_k^v	A_v+B_v -t-θ_k^v	q	s_q	λ_v	Demurrage vs Despatch	Laycan
01	1	1	x	10	x	1	1	8	x	x
3	2	2	7	6	1		14	14	13	x
7	3	3	10	7	3		28	11	94.5	x
12	8	8	7	6	1		14	12	51	x
15	10	11	10	10	-1		1	10	-35	x
16	11	11	9	7	2		28	13	96	x
2	1	1	10	8	2	2	36	7	35	x
5	3	3	9	9	0		16	18	0	x
6	3	3	9	7	2		1	15	79	x
10	6	10	10	7	-1		1	14	-81	x
13	9	9	9	8	1		36	10	52	x
002	12	12	10	10	0		16	18	x	[12,15]
02	1	1	x	5	x	3	21	10	x	x
1	1	1	11	9	2		1	17	121	x
4	2	2	9	9	0		41	16	0	x
8	4	4	13	9	4		32	9	168	x
9	6	10	10	8	-2		1	13	-106	x
11	7	11	9	9	-4		41	9	-220	x
14	9	9	9	7	2		21	11	134	x
001	8	13	11	8	-2		32	9	x	[13,14]

B. *Numerical Tests*

Most instances for the BAP published in the literature consider just one quay and do not have the same problem characteristics mentioned here. Frojan *et al.* [7], for instance, solved the BAP with multiple quays, but they considered fixed handling times for vessels and they did not consider water depth restrictions. Due to such differences, the comparison with the existing literature is difficult and can only be done for the simplified settings, which would lack relevance regarding this study. In order to evaluate the quality of the LBAP model, we have generated a set of instances with different sizes: $V_2 = \{20,30,40,50\} | V_1 = 2 \wedge V_3 = \{0,2\}$ $(V_3 = 0 \rightarrow \text{BAP})$ and $Q = \{1,3,5\}$. For all the instances, we consider a planning horizon $T = 60$ days and quays with different lengths discretized in units of 10 meters.

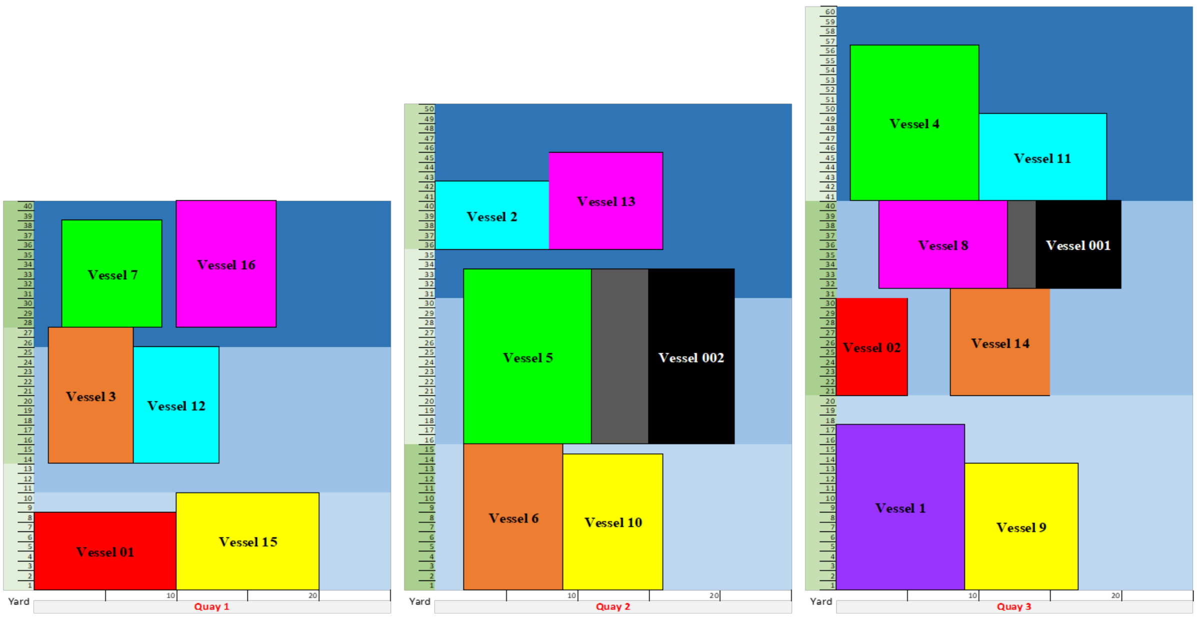

Fig. 3. Disposition of vessels at quays 1 to 3.

The data relating to each vessel are drawn randomly from uniform distributions as follows: $\mathscr{U}[1,30]$ for arrival times, $\mathscr{U}[7,20]$ for lengths, $\mathscr{U}[7,13]$ for laytimes, $\mathscr{U}[2,4]$ for laycan periods, $\mathscr{U}[1,3]$ for drafts and $\mathscr{U}[20,150]$ for daily demurrage rates. The Boolean parameter $F_q^v = 0$ for some vessels. The maximum waiting time in harbor is determined by applying this criterion: $a_v = 0.5 \cdot \min_k \theta_k^v$, rounded up to the next integer (inspired by the criterion of the desired departure time sets by Bierwirth and Meisel [8]). The V_3 vessels with unfixed laycans have fictitious despatch and demurrage rates equal to one and high maximum waiting times in harbor ($a_v = 20$). The handling time of vessels decreases by 20% from the productivity class k to $k+1$, rounded up to the next integer.

TABLE IV. RESULTS OF THE NUMERICAL TESTS

		V_1=2, V_3=0 **(BAP)**			V_1=2, V_3=2 **(LBAP)**		
Q	V_2	CPU[a] (s)	Gap[b] (%)	Traffic density[c]	Time (s)	Gap (%)	Traffic density
1	20	1.1	0	0.54	2.5	0	0.57
	30	9.5	0	0.91	14.3	0	0.94
	40	300	2.02	1.15	300	1.72	1.19
	50	300	3.18	1.43	300	2.67	1.46
3	20	0.6	0	0.29	1.2	0	0.30
	30	1.7	0	0.49	2.6	0	0.50
	40	11	0	0.61	13.1	0	0.63
	50	19.9	0	0.76	26.4	0	0.78
5	20	1.1	0	0.17	2.2	0	0.18
	30	2.8	0	0.28	4.5	0	0.29
	40	8	0	0.35	8.8	0	0.36
	50	26.9	0	0.44	22	0	0.45

a. Computation time is limited to 300 s.

b. $\text{Gap} = (ub - lb) \cdot 100 / ub$, where ub is the value of the best upper bound obtained by considering all the decision variables as continuous, and lb is the value of the objective function corresponding to the best integer solution achieved within the time limit.

c. Traffic density = $(\sum_v \lambda_v \cdot B_k) / (T \cdot \sum_q S_q)$. This indicator measures the maximum spatiotemporal occupations of vessels within the planning horizon and quay spaces (but does not include the arrival times of vessels which have a significant impact on the computation time). A traffic density higher than one means that the port cannot handle all vessels during the predefined planning horizon.

V. CONCLUSION

In this paper, we propose a mixed zero-one linear model to solve a new problem called the integrated Laycan and Berth Allocation Problem. So, we combined the Laycan Allocation Problem and the Berth Allocation Problem which is one of the most important problems confronted at the quayside of ports. We apply the model in a port with multiple quays. Each quay has a continuous berth layout. We take into consideration quays' water depths and vessels' drafts and their technical constraints that prohibit the berthing of vessels in some quays or oblige them to berth at a specific quay. We also consider dynamic arrival times of vessels with a maximum waiting time in harbor for each vessel. Handling times of vessels depend on their berthing positions and the objective function aims to achieve an optimal berth plan that reduces the late departures of chartered vessels by maximizing the difference between their despatch money and demurrage charges, while favoring their berthing as close as possible to the port yard, and to propose laycans for the new vessels to charter by making them leave the port as early as possible and berth as close as possible to the port yard without impacting the economic results of the chartered vessels.

The LBAP model uses one binary variable to determine the spatiotemporal allocations of vessels and two continuous variables that have integer values to determine if a demurrage is incurred or a despatch money is to be collected, and the spatiotemporal constraints are covered by a disjunctive constraint. The model is applied on a dataset where all the constraints are verified and its solving with Xpress is fast. This preliminary test enables us to validate the model. Other numerical experiences on instances with different sizes were done in order to evaluate the performance of the model and identify its limits.

It has to be noted that the optimal solutions proposed by the model may be unfeasible because of the unavailability of cargo to be exported in vessels (phosphate and its derivatives in the case of OCP Group): in practice, there is a strong interaction between vessels' loading and production, unless an important decoupling of these problems is done by high stock levels, which is an expensive solution. So as a perspective, we will develop a decision support system (DSS) to integrate the different port problems of allocation and scheduling, all in taking into account the constraints of the upstream supply chain. This DSS would follow an approach that combines optimization and simulation.

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