Synchronization and decoupling of plants piloting in a supply chain dedicated to customized mass production

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SUMMARY: The synchronization of the production of a manufacturing supplier, who makes alternate components assembled on his industrial customer's work station with this client's production specialized in mass production of highly diversified products, must take into account the improvement of their knowledge of the final demand (displacement of the Order Penetration Point) and the distance of some of the suppliers. The customer periodically forwards firm orders to his supplier calculated so as to preclude any production line stoppage. It is necessary that the supplier honor them to ensure the decoupling of the control of these two entities in the supply chain and define the efficiency of synchronization. In the considered context, the supplier also receives all available projected information from the industrial customer (final orders, firm on the short term, and structural characteristics of the final demand beyond). The efficiency of the supplier depends on the proper use of all information, notably when the production cycle of alternate components is longer than the demand cycle. In the study of the customer's requirements, it is necessary to take into account the batch constraints linked to transportation, which compels the customer to hold safety stocks even though the set up organization guarantees that orders will be duly honored. The determinants of these stocks will be put in evidence. Similarly at the supplier, safety stocks will be necessary if the production process involves grouping in batches.

KEY-WORDS: mass production of customized products, optimal use of customer's information, safety stocks.

For Kouvelis *et al.* (2006) "actions or approaches which lead supply chain partners to act in ways that are best for the chain as a whole are known as supply chain coordination."

The two main modes of supply chain coordination are contracts and information sharing.

- Contracts determine the exchange conditions: quantities, prices, dates, return conditions (which depend on the speed of obsolescence). Our study pertains essentially to contracts concerning the downstream supply chain (between retailers and the producer). In a Vendor-Managed-Inventory contract (Disney & Towill 2003), the supplier chooses how many units to deliver to his customer, but he is paid only for the sold quantities. In profit sharing contracts (Cachon & Lariviere 2005), a share of the retailer's earnings returns to the supplier. VMI and profit sharing contracts can be coupled. The wholesale-price-based contracts take place at two times: first the supplier chooses the price and then the retailer chooses the quantity (Gerchak & Wang 2004). To coordinate a supply chain, it is necessary to associate buybacks, which are unsold quantities returned to the supplier who buys them back at a predetermined lower price (Emmons & Gilbert 1998). Supply chain coordination by contract supposes that the supplier has the capacity to produce the quantities required by his customer. No problems will occur if the supplier works systematically in sub-capacity, but they will if the supplier's production capacity is barely sufficient to cover the customer's demand and if this demand concerns different products.
  We will retain as a principle of supply chain coordination the use and transmission of information to the upstream part of a supply chain (i.e. starting from the last production process).
- In information sharing models, various degrees of sharing exist. The spectrum goes from the successive supply chain customers' order history (Axsäter 1993) to the real-time transmission of inventories positions and final demands (Cachon & Fisher 1997), passing by the final sales

forecasts (Forslund & Jonnson 2007) or the customer's order policy parameters and the final demand distribution (Gavirneni *et al*. 1999). The purpose of information sharing is to reduce the bullwhip effect (Viswanathan *et al.* 2007) and some authors consider that the supply chain approach was initially imagined to protect oneself from this effect (Medan & Gratacap 2008). The base stock strategies implemented within this framework are thus based upon two main ideas: the repercussion along the supply chain of all information available and the determination of the orders based on inventory position instead of actual inventory level in every network node. The shared information allows for a decrease in carrying costs and stock out costs in the supply chain (Hariharan & Zipkin 1995), because it aims at improving the supplier's decisions on order quantities (Lee & Tang 2000) and the product allowance between retailers (Chen & Samroengraja 2000). The problem of earnings sharing has been widely developed in literature, for example by Lee & Tang (2000), but is ignored here.

Customer orders are firm information for the supplier; on the one hand they can't be based on a known final demand because of insufficient anticipation if the supplier is distant, on the other hand, the supplier can improve his efficiency by exploiting all his customer's known information, beyond those of the firm orders (demand pattern); the proper use of this information will be approached here. To our knowledge, only Bourland *et al*. (1996) consider the case where customers communicate their orders to their supplier every week for the following two or three weeks, as well as their forecasts for the five or six weeks beyond. They explain why the plants need stocks to counter the effects of uncertain orders and deliveries, but they only consider the case of a single good. The transmission of firm information is only approached under the Production To Order and the OPP (Order Penetration Point) analysis (Giard & Mendy 2008). We will show that the upstream flow of information along the supply chain entails the creation of a plurality of OPPs that are locally defined.

The impact of batch constraints on the supply chain control doesn't seem to be approached in literature. The lot-sizing problem is taken into account through scheduling as well as its impact upon production capacity due to set-up times (White & Wilson 1977). Optimal batch sizes are the result of a minimized cost function which integrates both carrying and set-up costs. The ex-ante determination of the batch size can be related to packaging constraints or to storage constraints near a work station, preventing from having the variety required by the consumed components. Packaging is also approached in transportation problems through its influence upon transportation capacities, but it is not taken into account in the determination of orders to be sent to the supplier. Here we will examine the impact of batch constraints on supply chain flow controls, respective to both the transformation of the orders to be made and the necessity of holding a safety stock.

It is necessary to accurately describe the chosen context. We are interested in an elementary supply chain consisting in a production unit (indicated in this article as the customer) configured as an assembly line allowing mass production of diversified products (an automotive production line, for example) and another unit (indicated as being the supplier) producing alternate components (e.g. car engines) or optional components (e.g. sunroof) assembled on a work station of this line and contributing to the required diversity (Anderson & Pine 1997). From the point of view of the analyzed problem, the case of the optional components is a particular case of the alternate components. Figure 1 describes the problem parameters.

After a few weeks, the daily production of the customer (= n) is predetermined by the opening duration of the line (stable on this horizon) and the line's cycle time. The total daily demand of the alternate components to be ordered from the supplier is thus known. From the final demand (vehicle orders), the industrial customer determines his production schedule on a horizon of K days, which in turn determines, on this horizon, the ordered list of the alternate components to assemble each day on the assembly line work station where they are mounted. Beyond this horizon K, the customer only has information about the average structure of the demand. At the beginning of every day t, the schedule of days t to $t+K-1$ is kept and the new orders of day $t+K$ are sequenced according to the logic of revolving planning. The schedule's update is immediately transmitted to the supplier.

The delivery request, transmitted by the customer for the beginning of day t, is determined by the assembly plan of the alternate components. If the delivery time λ does not exceed the horizon K, then the demand is transmitted no later than the beginning of day t-λ. Otherwise, the delivery request is a periodic replenishment policy which will be presented in this article. Whatever the determination made by the customer (firm demand and/or structure of forecast demand), the requisitions that are transmitted are firm demands for the supplier. *If the organization relating to the supplier's production and transportation guarantees that deliveries are in conformity with requisitions, there is decoupling between the customer and the supplier in the supply chain*

Customer orders, which are firm information for the supplier, determine an OPP in his process. This supplier in turn sends requisitions to his own supplier that are firm orders to the supplier's supplier, creating a new OPP. All along the supply chain, upstream information transmission creates local OPPs disconnected from final demand.

In the case where the articles are bulky, trucking capacity can lead to split the delivery into several successive shipments throughout the day. Conversely, this delivery can cover the needs of day t and θ-1 following days, if the interval between two deliveries is of θ days; in that case, the demand concerns a production schedule transmitted no later than at the beginning of day t-λ+1-θ.

The alternate components are most often manufactured with common production means which can require specific equipment. For efficiency reasons (amount of the set-up costs), the supplier may have an interest in successively producing the alternate references on a cycle of H days (H can be equal to 1). The periodic production of the references must take into account the fact that daily deliveries made by the supplier integrate all references. The delay separating the date t' of the beginning of the production of reference i from its delivery is noted L_i.

This article deals at first with the optimal use of available information by the customer and the supplier to guarantee the supply of the customers' assembly work station on the manufacturing line. As the daily delivery is defined to avoid stockout, respect of these requests by the supplier is a constraint for his organization (efficiency criterion). The intelligent use of all the information provided allows him to produce in an efficient and effective way.

In a second section, we shall see how the batch constraint for the alternate components to be delivered (impossibility of mixing different alternate components within the same container) compels the customer to send requests to the supplier who takes these parameters into account. It follows that the customer must arrange for safety stocks of these alternate components in spite of the fact that the requests sent must be considered as firm and that no risk relates to the deliveries.

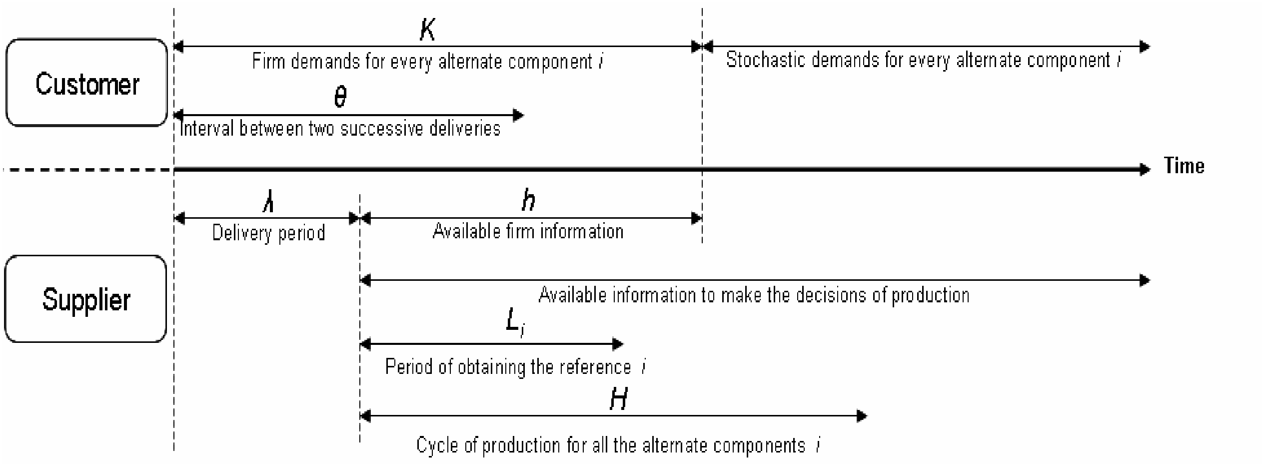


Figure 1. Problem parameters.

1. Determination of the periodic replenishment policies of customer demand and supplier production

We rely on the example of motor assembly line in which a work station takes up the engine desired by the final customer; this one has the choice between six engines (alternate components). The daily production of this assembly line is 962 vehicles.

Table 1. Distribution of engines' demand.

Engine i	1	2	3	4	5	6
p_i	54.46 %	13.29 %	3.58 %	21.51 %	5.13 %	2.03 %

The observed structure on any day differs necessarily from this average structure, because we are in presence of a realization of the Multinomial distribution with parameters $\{n = 962; p_i\}$ and $\sum_{i=1}^{6} X_i = 962$, where X_i represents the daily demand of the requested engine i. For the determination of a confidence interval of X_i, we use binomial law $\mathscr{B}(n, p_i)$ because the analysis focuses on this reference against the set of all other references.

Determination of the order-up-to level of a periodic replenishment policy

The demand X_{iD} of reference i on D consecutive days follows the law $\mathscr{B}(nD, p_i)$ that we can approximate by the Normal distribution $\mathscr{N}(nDp_i, \sqrt{nDp_i(1-p_i)})$, due to the high value of nD.

In the general case of a random variable X according to a Normal distribution of parameters $\bar{x}$ and σ, the value R of X such as P $(X > R) = \alpha$ is $R = \bar{x} + t_\alpha \sigma$, where α is the accepted risk and t_α, the value of the variable T according to the law $\mathscr{N}(0, 1)$ such as $P(T > t_\alpha) = \alpha$. It follows that the value R_{iD} such as $P(X_{iD} > R_{iD}) = \alpha$ is given by the relation 1.

$$R_{iD} = nDp_i + t_\alpha \sqrt{nDp_i(1-p_i)} = nDp_i(1 + t_\alpha \sqrt{\frac{1-p_i}{nDp_i}}) \qquad \textbf{Relation 1}$$

The periodic replenishment policy of the supply of a product i is characterized by making an order q_{it} for the component i at the beginning of the period t, calculated as the difference between its order-up-to level R_i and its inventory position P_{it} at the beginning of the period t; the interval between two successive decisions being θ. The determination of the optimal value of R_i is economically based on a trade-off between a carrying cost and a stockout cost, leading to the determination of an optimal value of risk α. As we look at the supply of alternate components to be taken up on an assembly line, the cost of a component shortage triggering a line stoppage is much greater than the carrying cost. It is then acceptable to use a very low risk α, for instance 0.01%. In the relation 1, R_{iD} is analyzed as an order-up-to level and $t_\alpha \sqrt{nDp_i(1-p_i)}$, as a safety stock. Contrary to what some practitioners recommend, this safety stock cannot be defined as a constant percentage of the average demand, it depends clearly on 4 parameters: α, n, p and D.

If the delivery delay λ is not null (the assumption made here), we have to consider the random demand expressed between t and $t+\theta+\lambda$, in order to determine the order-up-to level (noted $R_{i,\theta+\lambda}$). If the unsatisfied demands are delayed and/or if the probability of stock shortage is negligible (assumptions made here), then demands over two different periods are independent. The demand over the period $\theta+\lambda$ then follows the law $\mathscr{N}(n(\theta+\lambda)p_i, \sqrt{n(\theta+\lambda)p_i(1-p_i)})$, the order-up-to $R_{i,\theta+\lambda}$ is defined for the risk α as follow:

$$R_{i,\theta+\lambda} = n(\theta+\lambda)p_i + t_\alpha \sqrt{n(\theta+\lambda)p_i(1-p_i)} \qquad \textbf{Relation 2}$$

Under the conditions selected, when an order is placed, the stock position P_{it} is the sum of the observed stock, during this time, and of the expected deliveries ($k = \lfloor \lambda / \theta \rfloor$, where $\lfloor A \rfloor$ represents the lower roundness of A). Then, we obtain the following relation:

$$P_{it} = S_{it} + \sum_{j=1}^{k} q_{t-j\theta} \qquad \textbf{Relation 3}$$

Periodic demands of the customer to his supplier

Periodically (interval θ), the customer transmits to his supplier the specifications of the next delivery. We suppose that this requisition is daily ($\theta = 1$), without loss of generality (θ becoming the unit of time). This customer orders to his supplier, at the beginning of the day t, q_{it} units of the component i to be delivered at the beginning of the day $t+\lambda$.

If $\lambda \le K$, we obtain relation 4; the supplier can mobilize the techniques of the synchronous production (Giard & Mendy 2008) under certain conditions ($H \leq 2h$):

$$q_{it} = x_{i,t+\lambda} \;\; \left(\sum_i q_{it} = \sum_i x_{i,t+\lambda} = n\right) \qquad \textbf{Relation 4}$$

Otherwise ($\lambda > K$; $h=0$), this demand q_{it} is determined by relation 5 which is an adaptation of (the) relations 2 and 3.

$$q_{it} = [n(1+\lambda)p_i + t_\alpha \sqrt{n(1+\lambda)p_i(1-p_i)}] - [S_{it} + \sum_{j=1}^{\lambda} q_{i,t-j}] \qquad \textbf{Relation 5}$$

The stockout probability being negligible, the average residual stock before delivering (SM_{it}) is equal to the safety stock. Then, we obtain relation 6:

$$SM_{it} = t_\alpha \sqrt{n(1+\lambda)p_i(1-p_i)} \qquad \textbf{Relation 6}$$

With $R_{i,1+\lambda} = q_{it} + [S_{it} + \sum_{j=1}^{\lambda} q_{i,t-j}] = q_{i,t-1} + [S_{i,t-1} + \sum_{j=1}^{\lambda} q_{i,t-1-j}]$ and $S_{it} = S_{i,t-1} - x_{i,t-1} + q_{i,t-1-\lambda}$, we obtain the valid relation 7 in steady state under the stated conditions:

$$q_{it} = x_{i,t-1} \;\; \left(\sum_i q_{it} = \sum_i x_{i,t-1} = n\right) \qquad \textbf{Relation 7}$$

However, at the initialization and every time we take into account a change of characteristics of the steady state, relation 5 must be used.

In summary, if the delivery period is lower than the customer's programming horizon, the demand pertains to the firm final demand ahead (relation 4); otherwise, it corresponds to the previous period's consumption in order to reduce the stock position to the order-up-to level (relation 7). As it involves alternate components taken up on the same work station of the assembly line and all provided within the same delivery period at the same supplier, the level of the total daily order of these alternate components is constant (= n) because we are in the presence of a Multinomial distribution.

Table 2 illustrates the order of engine 2 on the assumption of a distant supply ($\lambda > K$) from day 100. The demands were generated randomly and the starting inventory at the beginning of day 100 was arbitrarily fixed (using relation 5 for day 100). For the following days, the use of relations 5 and 7 leads to the same results.

Table 2. Example of periodic replenishment policy.

Day	96	97	98	99	**100**	101	102	103	104	105	106	107	108	Daily demand:
Initial stock					125	101	95	90	74	77	80	100	115	$\mathcal{M}$ (962; 13,29%)
Order	111	126	132	118	115	135	132	137	134	112	132	112	122	$\lambda = 4$
Delivery					111	126	132	118	115	135	132	137	134	Demand on 5 days:
Demand					135	132	137	134	112	132	112	122	146	$\mathcal{N}$ (639.25, 23.54)
Final stock					101	95	90	74	77	80	100	115	103	α = 0,01% → R = 727

Periodic programming of the supplier

The supplier must make sure that the daily deliveries required by his customer are honored in a nearly certain way. He organizes the production of alternate components on a cycle of H days during which each component is successively produced. If H is lower or equal to $2h$ (see Figure 1), then the supplier can produce during a cycle the exact quantities which will be sent during the following cycle and can work in synchronous production. Otherwise, which is our interest here, he must make the best use of the firm information that he received ($x_{i,t+\lambda+1}$ to $x_{i,t+\lambda+h}$) and of the structural information (probability p_i and level n of the customer's daily production).

This periodic replenishment policy takes into account an average achievement delay L_i for component i, with production being considered available only at the end of this period. Without loss of generality, we will assume that L_i is an integer number of days and that during one day several references are successively produced, a common lead time being then shared by those references. In a cycle beginning at t, and if the other components were produced earlier during this cycle, the earliest date of production of component i is $t' > t$ (the previous launch in production of this component had begun at $t'-H$).

The quantity launched in production has to guarantee that between two deliveries there is almost no chance of being out of stock. The order-up-to level would cover the unknown demand.

Two rules to determine production $q_{it'}$ are possible, the first one uses all the available information at t'_i (relations 8)[1]:

Rule 1 **Relations 8**

$$R_{i,H-h+L_i} = n(H-h+L_i)p_i + t_\alpha\sqrt{n(H-h+L_i)p_i(1-p_i)}$$
$$q_{it'_i} = \sum_{j=1}^{h} x_{i,t'_i-1+j} + R_{i,H-h+L_i} - S_{it'_i} = \sum_{j=-(H-h)}^{h-1} x_{i,t'_i+j}$$

This relation establishes that during the steady state, the quantity to be launched is the sum of h next deliveries and H-h last ones. These have unknown values during the last launch decision taken at the beginning of day $t'_i - H$. The safety stock that the supplier establishes for the same risk α for each of the references is then:

$$SS_{i,H-h+Li} = t_\alpha\sqrt{n}\sum_i\sqrt{(H-h+L_i)p_i(1-p_i)}$$ **Relation 9**

This rule has the effect of leading to a variable total quantity from one cycle to another, which complicates the organization of production, while the total quantities shipped each day are constant. The quantities produced correspond to the sum of quantities shipped on H consecutive days. However, this set of consecutive days is not the same for all the references. In order to stabilize production, this set of consecutive days should be the same for all the references, that which is obtained with rule 2 which determines production at the beginning of period t regardless of the information available between t and t'_i. Considering J_i to be the period for obtaining the produced quantity of component i from t, we obtain relation 10 by modifying both relations 8 and 9. The counterpart of having constant production is an increase of the safety stock.

Rule 2 **Relation(s) 10**

$$R_{i,H-h+J_i} = n(H-h+J_i)p_i + t_\alpha\sqrt{n(H-h+J_i)p_i(1-p_i)}$$
$$q_{it'_i} = \sum_{j=1}^{h} x_{i,t-1+j} + R_{i,H-h+J_i} - S_{it} = \sum_{j=-(H-h)}^{h-1} x_{i,t+j}$$
$$SS_{i,H-h+Ji} = t_\alpha\sqrt{n}\sum_i\sqrt{(H-h+J_i)p_i(1-p_i)}$$

With rule 2, the sum of stocks at t is constant because on the one hand the daily total demand is constant (n), on the other hand the previous relation allows one to write:

$$\sum S_{it} = \sum\sum_{j=1}^{h} x_{i,t-1+j} + \sum R_{i,H-h+J_i} - \sum\sum_{j=-(H-h)}^{h-1} x_{i,t+j} = hn + \sum R_{i,H-h+J_i} - Hn = \sum R_{i,H-h+J_i} - (H-h)n$$

As long as no delivery is made, the sum of all stocks remains constant and continues at the beginning of the subsequent periods.

Table 3 illustrates the application of rules 1 and 2, assuming that λ=2, K=5 or 6 (h=3 or 4), n=962, H=5. In rule 1, safety stocks vary in the opposite direction of h. In the example, the fact that we increase the firm request visibility for one day allows the supplier to win 16 % on the level of safety stocks. This provides elements for the evaluation of the value of the information transmitted to the supplier. The change from rule 1 to rule 2 leads to an increase of 26 % for the safety stock: the

[1] The second formulation of $q_{it'_i}$ is obtained by taking into account the fact that:

$R_{i,H-h+L_i} = q_{it'_i} + [S_{it'_i} - \sum_{j=1}^{h} x_{i,t'_i-1+j}] = q_{i,t'_i-H} + [S_{i,t'_i-H} - \sum_{j=1}^{h} x_{i,t'_i-1-H+j}]$ and $S_{it'_i} = S_{i,t'_i-H} - \sum_{j=1}^{H} x_{i,t'_i-1-H+j} + q_{i,t'_i-H}$

increase of the carrying cost can be compensated with the savings brought by the passage to constant daily production.

A simulation of rules 1 and 2 is proposed in tables 4 and 5, for h = 3.

Table 3. Parameters of the tested periodic replenishment policies, order-up-to levels, and safety stocks.

Engine i	p_i	L_i	Delivery day J_i	Rule 1 • h=3			Rule 1 • h=4			Rule 2 • h=3		
				$H+L_i-h$	$R_{i,H-h+L_i}$	$SS_{i,H-h+L_i}$	$H+L_i-h$	$R_{i,H-h+L_i}$	$SS_{i,H-h+L_i}$	$H+J_i-h$	$R_{i,H-h+J_i}$	$SS_{i,H-h+J_i}$
1	54,46%	2	2	4	2211	115,38	3	1672	100,28	4	2211	115,38
2	13,29%	1	4	3	452	68,45	2	312	56,30	6	864	96,90
3	3,58%	1	4	3	141	37,68	2	100	31,12	6	260	53,36
4	21,51%	1	3	3	703	82,22	2	481	67,15	5	1141	106,37
5	5,13%	1	4	3	193	44,95	2	135	36,30	6	359	62,90
6	2,03%	1	4	3	87	28,41	2	63	23,94	6	158	40,83
					Σ	377,09		Σ	315,09		Σ	475,74

Table 4. Simulation of the use of rule 1, with h = 3.

Period	Engine 1			Engine 2			Engine 3			Engine 4			Engine 5			Engine 6			$\sum_i x_{it}$	$\sum_i S_{it}$	$\sum_i q_{it}$
t	S_{1t}	q_{1t}	x_{1t}	S_{2t}	q_{2t}	x_{2t}	S_{3t}	q_{3t}	x_{3t}	S_{4t}	q_{4t}	x_{4t}	S_{5t}	q_{5t}	x_{5t}	S_{6t}	q_{6t}	x_{6t}			
1	1200	2555	**520**	600	0	137	200	0	32	700	0	204	350	0	46	90	0	23	962	3140	
2	680	0	**508**	463	0	121	168	0	35	496	0	233	304	0	49	67	0	16	962	2178	
3	2727	0	**516**	342	0	146	133	0	38	263	1062	**192**	255	0	51	51	0	19	962	3771	4649
4	2211	0	531	196	628	**137**	95	150	**30**	1133	0	**205**	204	146	**46**	32	108	**13**	962	3871	
5	1680	0	511	687	0	**109**	215	0	**36**	928	0	**225**	304	0	**64**	127	0	**17**	962	3941	
6	1169	2594	**518**	578	0	**126**	179	0	**38**	703	0	210	240	0	**47**	110	0	**23**	962	2979	
7	651	0	**534**	452	0	133	141	0	36	493	0	203	193	0	44	87	0	12	962	2017	
8	2711	0	**500**	319	0	137	105	0	35	290	1058	**224**	149	0	52	75	0	14	962	3649	4779
9	2211	0	514	182	628	**107**	70	174	**38**	1124	0	**228**	97	239	**50**	61	86	**25**	962	3745	
10	1697	0	550	703	0	**119**	206	0	**36**	896	0	**193**	286	0	**50**	122	0	**14**	962	3910	
11	1147	2653	**524**	584	0	**132**	170	0	**29**	703	0	213	236	0	**43**	108	0	**21**	962	2948	
12	623	0	**555**	452	0	120	141	0	19	490	0	203	193	0	49	87	0	16	962	1986	
13	2721	0	**510**	332	0	138	122	0	33	287	1021	**205**	144	0	56	71	0	20	962	3677	4819
14	2211	0	526	194	646	**132**	89	164	**31**	1103	0	**211**	88	246	**43**	51	89	**19**	962	3736	
15	1685	0	544	708	0	**117**	222	0	**41**	892	0	**189**	291	0	**52**	121	0	**19**	962	3919	
16	1141	2625	**509**	591	0	**139**	181	0	**40**	703	0	213	239	0	**46**	102	0	**15**	962	2957	
17	632	0	**526**	452	0	134	141	0	33	490	0	206	193	0	42	87	0	21	962	1995	
18	2731	0	**520**	318	0	126	108	0	35	284	1037	**212**	151	0	48	66	0	21	962	3658	4796
19	2211	0	549	192	641	**134**	73	178	**23**	1109	0	**201**	103	213	**39**	45	102	**16**	962	3733	
20	1662	0	527	699	0	**129**	228	0	**47**	908	0	**205**	277	0	**37**	131	0	**17**	962	3905	
21	1135	2656	**510**	570	0	**118**	181	0	**40**	703	0	220	240	0	**47**	114	0	**27**	962	2943	
22	625	0	**521**	452	0	119	141	0	28	483	0	222	193	0	53	87	0	19	962	1981	

Table 5. Simulation of the use of rule 2, with h = 3.

Period	Engine 1			Engine 2			Engine 3			Engine 4			Engine 5			Engine 6			$\sum_i x_{it}$	$\sum_i S_{it}$	$\sum_i q_{it}$
t	S_{1t}	q_{1t}	x_{1t}	S_{2t}	q_{2t}	x_{2t}	S_{3t}	q_{3t}	x_{3t}	S_{4t}	q_{4t}	x_{4t}	S_{5t}	q_{5t}	x_{5t}	S_{6t}	q_{6t}	x_{6t}			
1	1200	2555	**520**	600	668	**137**	200	165	**32**	700	1070	**204**	350	155	**46**	90	126	**23**	962	3140	
2	680	0	**508**	463	0	**121**	168	0	**35**	496	0	**233**	304	0	**49**	67	0	**16**	962	2178	
3	2727	0	**516**	342	0	**146**	133	0	**38**	263	0	**192**	255	0	**51**	51	0	**19**	962	3771	4739
4	2211	0	531	196	0	137	95	0	30	1141	0	205	204	0	46	32	0	13	962	3879	
5	1680	0	511	727	0	109	230	0	36	936	0	225	313	0	64	145	0	17	962	4031	
6	1169	2594	**518**	618	642	**126**	194	175	**38**	711	1067	**210**	249	253	**47**	128	79	**23**	962	3069	
7	651	0	**534**	492	0	**133**	156	0	**36**	501	0	**203**	202	0	**44**	105	0	**12**	962	2107	
8	2711	0	**500**	359	0	**137**	120	0	**35**	298	0	**224**	158	0	**52**	93	0	**14**	962	3739	4810
9	2211	0	514	222	0	107	85	0	38	1141	0	228	106	0	50	79	0	25	962	3844	
10	1697	0	550	757	0	119	222	0	36	913	0	193	309	0	50	133	0	14	962	4031	
11	1147	2653	**524**	638	616	**132**	186	155	**29**	720	1042	**213**	259	248	**43**	119	96	**21**	962	3069	
12	623	0	**555**	506	0	**120**	157	0	**19**	507	0	**203**	216	0	**49**	98	0	**16**	962	2107	
13	2721	0	**510**	386	0	**138**	138	0	**33**	304	0	**205**	167	0	**56**	82	0	**20**	962	3798	4810
14	2211	0	526	248	0	132	105	0	31	1141	0	211	111	0	43	62	0	19	962	3878	
15	1685	0	544	732	0	117	229	0	41	930	0	189	316	0	52	139	0	19	962	4031	
16	1141	2625	**509**	615	648	**139**	188	180	**40**	741	1031	**213**	264	231	**46**	120	95	**15**	962	3069	
17	632	0	**526**	476	0	**134**	148	0	**33**	528	0	**206**	218	0	**42**	105	0	**21**	962	2107	
18	2731	0	**520**	342	0	**126**	115	0	**35**	322	0	**212**	176	0	**48**	84	0	**21**	962	3770	4810
19	2211	0	549	216	0	134	80	0	23	1141	0	201	128	0	39	63	0	16	962	3839	
20	1662	0	527	730	0	129	237	0	47	940	0	205	320	0	37	142	0	17	962	4031	
21	1135	2656	**510**	601	604	**118**	190	178	**40**	735	1049	**220**	283	219	**47**	125	104	**27**	962	3069	
22	625	0	**521**	483	0	**119**	150	0	**28**	515	0	**222**	236	0	**53**	98	0	**19**	962	2107	

2. Determination of safety stocks allowing to counter rank-change induced disturbances

Impact of lot-sizing on demand and respect for demand

The customer's demand for alternate components is expressed in the form of a continuously updated ordered list of components. The order of M components placed earlier by the customer (gap θ) is delivered periodically (periodicity λ). Packaging constraints can lead to mandatory grouping of the references delivered by batches of m identical components, which will lead to a delivery of $y= M/m$ batches.
Lot-sizing has two consequences. At the arrival of each delivery one must perform a reconstruction of the sequence of M alternate components respecting the order of assembly. The batch of M delivered components has practically no chance to coincide with the batch which will be assembled; the missing number of components corresponds to the number of surplus components. As a result, it will be necessary to have a safety stock available for each component. To fully understand the results, it is useful to clarify the rank-change probability distribution of a component, as the studied variable corresponds to the number of ranks gained or lost at delivery for any alternate component, with regard to the demand of assembly (a similar phenomenon has been studied by Giard *et al* (2001a, 2001b) in the off-line treatment of quality problems in an automotive industry production line).

Before starting that study, it should be noted that an identical mechanism can be noticed at the supplier's. His demand to be fulfilled is γ batches. Some technical constraints can lead to a programming of the supplier's production in a succession of sequences of v batches, not necessarily based on the same alternate component but all sharing an identical characteristic such as including an identical elementary component (for example an identical crankcase used by several engines). This form of lot-sizing is a little more complex, since it does not imply homogeneity but, as previously, it involves rank-changes and thus the necessity for the supplier to this time get a safety stock allowing him to meet the demand.

In both cases, batch rules are taken into account when a deterministic demand is to satisfy leads to rank-changes (Figure 2) and it is necessary to set up safety stocks to avoid stockout.

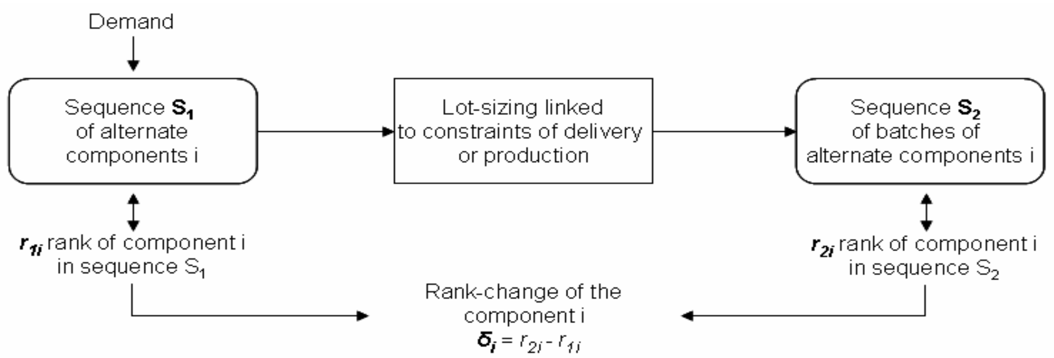


Figure 2. Origin of operations of rank-change

It is important to state that a rank-change does not systematically entail a stockout of stock and then the constitution of a safety stock: the supplier's deliveries to the customer are performed in rounds (deliveries of γ batches at interval θ), which implies that being out of stock is not determined by sequence S_2 but by the constitution of the group formed by the γ batches produced and in stock, and permutations within this are possible without any consequences.

For any component i, when using the known demand in set S_1, the last batch of m components may be incomplete (K_i elements < m). One can decide to only use the firm demand and wait for the demand of following days (ω components) to get the missing item m-K. The probability of filling up the batch is given by the Negative Binomial distribution $\mathcal{NB}(m- K_i, \omega, p_i)$. The risk of being unable to complete a batch may be significant, for instance, in the case of engine 4 for which p_4=0,9%, with

ω = 1,000 and K_i = 1, the risk increases by 11,3 %). The other solution consists in deciding to launch a full batch as soon as a component is required in S_1. This second solution, adopted henceforth, is equivalent to considering a set S_1 of infinite size.

Analysis of rank-change operations inferred by lot-sizing

The simulations of this second industrial example are performed with Simul8 software and concern six million engines distributed according to table 1. The 12,000 first components are excluded from the simulation since they represent the transient state before the steady state that interests us. The rule used to create S_2 is the following one, and deals with the treatment of vehicle y in sequence S_1.

- If the required engine for vehicle y is in stock, that engine is taken away from the stock, whose level falls by 1.
- Otherwise, a batch of six engines for that required engine is launched; that engine is taken from the stock, whose level drops to 5.
- Let's move on to vehicle y+1 in S_1.

Table 6 summarizes the simulation results for each engine E_i having the probability of use p_i, their average $\overline{\delta_i}$ of rank-changes, the standard deviations $\sigma_{\delta i}$ of the rank-changes and the safety stocks SS_i, required to avoid stockout:

Table 6. Rank change(s) expectations – standard deviations of rank-changes– and safety stocks per engine.

Engine E_i	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%	0.4%	0.9%	3.0%	6.5%	6.1	3.5	6.1%	11.3%	2.6%
$\overline{\delta_i}$	-98,73	6,99	-65,20	-230,57	-230,57	39,40	29,44	-145,72	-230,45	-576,22	-586,58	-229,12	-36,57	8,95	7,52	-24,64	6,40	25,20	-48,29
σ_{δ_i}	134.62	36.88	103.10	257.68	258.46	9.13	16.75	178.10	259.09	585.75	588.51	256.36	76.30	34.62	36.12	65.44	36.80	20.18	87.62
SS_i	5	10	6	4	4	33	17	5	4	3	3	5	8	11	11	8	10	16	8

With the notations of Figure 2, a negative value of δ_i means that the engine which has just arrived is early with regard to the vehicle on which it will be assembled; a positive value means that the engine came late with regard to the vehicle on which it has to be assembled, which implies a withdrawal from the safety stock. In a deterministic universe with deliveries in accordance with requirements and lot-sizing, it is thus necessary to anticipate rank-changes with safety stocks. A safety stock is made up beforehand for each engine to avoid any line stoppage; this stock is fed by engines coming too early (which reduces the required safety stock) and by engines coming too late (which restores the safety stock). The weighted sum of rank-change $\overline{\delta_i}$ by probability p_i is null[2]. The engine rank-change curves are not identical: the mathematical expectation of the rank-change of engines varies in the same way as their use probabilities, whereas the standard deviations of engine rank-change vary in inverse order of their use probability (Figure 3). Therefore, those engines in low demand are delivered somewhat ahead of schedule, whereas the delivery of engines in high demand is somewhat delayed. It ensues that engines without great demand are delivered somewhat ahead of schedule, but the extent of their dispersion leads to the creation of safety stocks; engines in high demand are delivered somewhat late, but their small dispersion limits the need for safety stocks.

The rank-change function of an engine i depends both on the number I of engines ($i=1,..,I$) and on the distribution of the demand p_i. Figure 4 illustrates the case of an engine in low demand and the most demanded engine; the scales used not being identical.

[2] In steady state, there is inevitably compensation between the won ranks and the lost ranks. On a sample, the average observed (here $\overline{\delta_i}$) coincides only exceptionally with the mathematical hope (0).

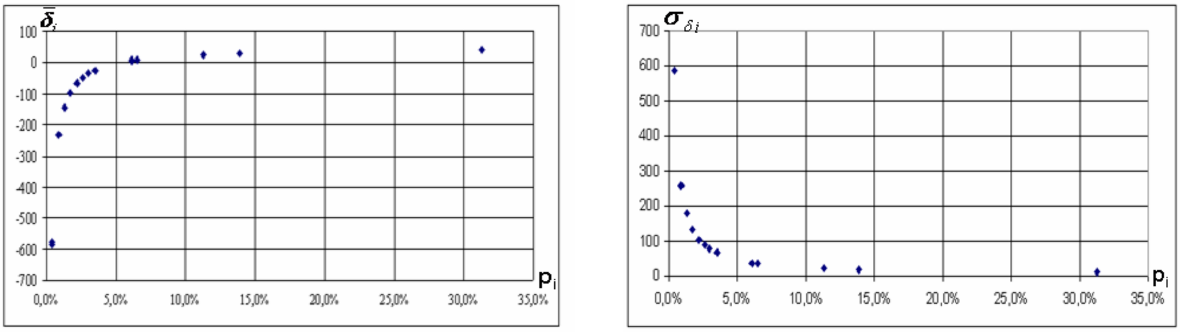


Figure 3. Evolution of the mathematical expectation and the standard variation of rank-change of engines according to their probabilities p_i of assembly

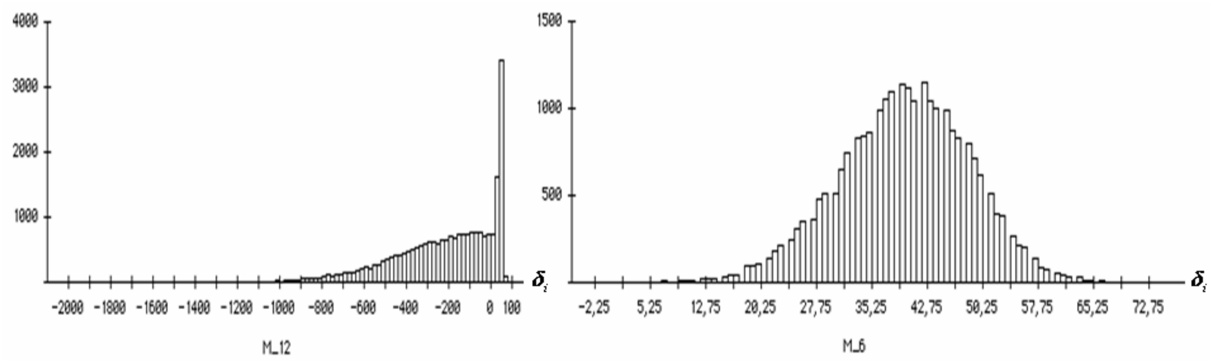
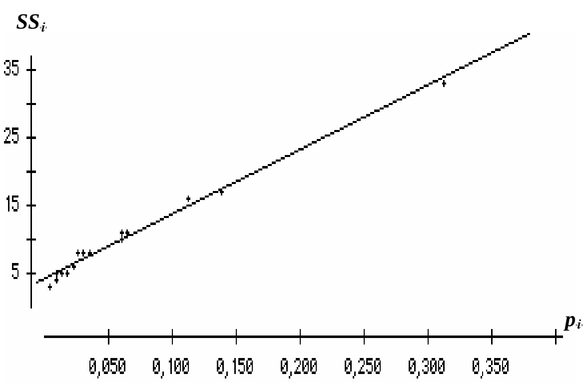


Figure 4. (Curves of r)Rank(s)-change(s) curves of the engines (p_{12} = 0,9% ; $\overline{\delta_{12}} = -229$) and E_6 (p_6 = 31,3% ; $\overline{\delta_6} = 39{,}4$)

Determination of safety stocks intended to counter the effects of lot-sizing

In deterministic universe, safety stocks depend on the rank-changes and on the lead time θ. The rank-changes themselves depend on the range *I* of engines and on their assembly probabilities p_i. An evaluation of their appropriate level can be performed only through an approach using simulation of the steady state. At the beginning of the simulation, the available quantities of alternate components at the customer's are fixed to a value *W*. The first delivery is made immediately before the first component of sequence S_1 is taken. This first delivery corresponds to the θ first components of sequence S_2 and the first withdrawn component corresponds to the first component of sequence S_1. Each simulation concerned the assembly of 6 million components so as to empirically obtain safety stocks SS_i having an insignificant probability to lead to a stockout. The safety stock level is calculated as the difference between *W* and the lowest inventory level during the simulation, since *W* must be big enough to avoid an empty stock. The safety stock of components varies in the same way as their use probabilities (Figure 5), which was not obvious a priori. In that industrial example, an approximately linear relation can be noticed ($\hat{\rho}^2$ = 0.983; SS_i=0.95889 p_i+3.008).



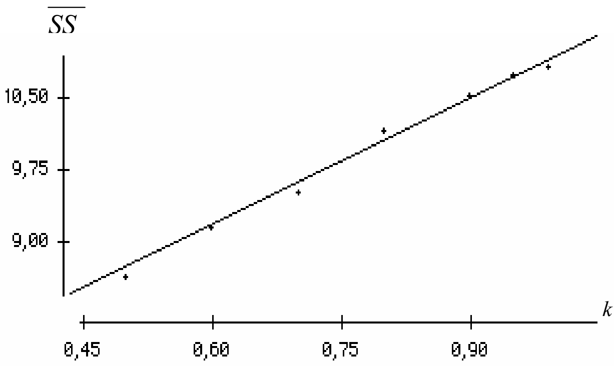
Dependent variable is: SSi
No Selector
R squared = 98,4% R squared (adjusted) = 98,3%
s = 0,9364 with 19 - 2 = 17 degrees of freedom

Source	Sum of Squares	df	Mean Square	F-ratio
Regression	891.094	1	891.094	1.02e3
Residual	14.9061	17	0.87683	

Variable	Coefficient	s.e. of Coeff	t-ratio	prob
Constant	3.95323	0.2669	14.8	≤ 0,0001
pi	95.8887	3.008	31.9	≤ 0,0001

Figure 5. Evolution of the safety stock SS_i according to the probabilities p_i of the assembly of engines

We would think that the dispersion of probabilities influences the level of safety stocks. Thus, new simulations have been performed by replacing the probabilities of Table 6 by distributions of type $p_i = p_1 \times k^{i-1} + b$ with b = 0.005 (asymptote), k being the coefficient of decreasing under control (the closer it is to 1 k, the more equal probabilities of component demands there is). An approximately linear relation can be noticed ($\hat{\rho}^2$=0.991; $\overline{SS}$ =0.0438 k+0.1579).



Dependent variable is: SS bar
No Selector
R squared = 99,2% R squared (adjusted) = 99,1%
s = 0,09063 with 8 - 2 = 6 degrees of freedom

Source	Sum of Squares	df	Mean Square	F-ratio
Regression	6.31521	1	6.31521	769
Residual	0.0492802	6	0.00821336	

Variable	Coefficient	s.e. of Coeff	t-ratio	prob
Constant	6.5549	0.1196	54.8	≤ 0,0001
k	4.37857	0.1579	27.7	≤ 0,0001

Figure 6. Evolution of the average safety stock $\overline{SS}$, according to k

Then, the impact of the periodicity of supplying was studied by performing successively the same simulation with various values of θ. This analysis was lead through three structures of demand with 5, 10 and 19 alternate components and equal probabilities of component demands. We finally made a more detailed analysis of the impact of the range I on the average safety stock, for equal probabilities of component distributions.

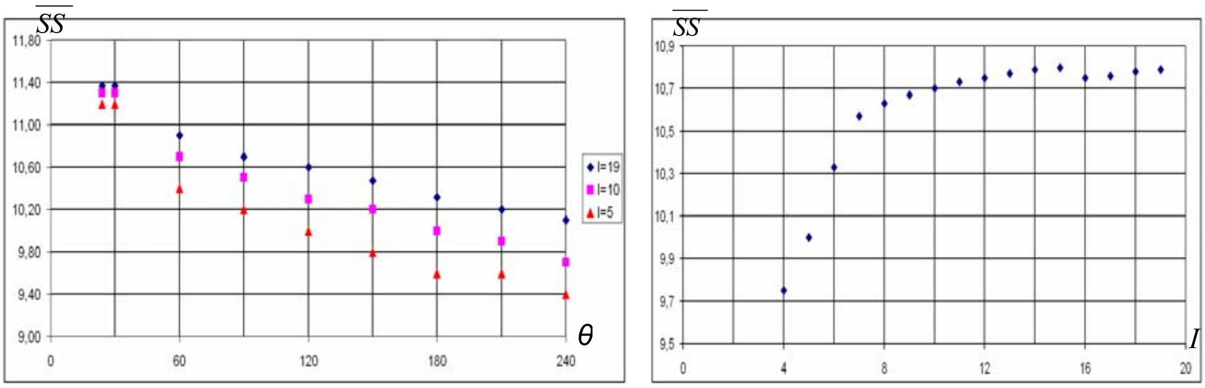


Figure 7. Evolution of the average safety stock $\overline{SS}$ according to θ for various values of I

Safety stocks also depend on other risks such as quality problems in a production context, variation of transportation time or modification of the definite sequence of final demand. A combination of disruptions leads to risk pooling; thus the safety stock needed to face several disruptions is lower than the sum of necessary safety stocks if one disruption is considered regardless of the other.

3. Conclusion

We have studied synchronization and decoupling of the control of the last two links of a supply chain dedicated to customized mass production. Knowledge of the production sequence set in response to a known final demand as well as of the demand structure modifies the traditional policies of piloting the flow, making improvements in effectiveness and efficiency possible. Taking into account lot-sizing entails the creation of safety stocks; the explanatory factors of their importance were analyzed. All of these elements take on significance in the context of the increasing geographical dispersion of the links of large worldwide logistic chains.

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