Dynamic blending as a source of flexibility and efficiency in managing a phosphate supply chain

Ahlam Azzamouri[1],Vincent Giard[2],

[1] PhD Student at EMINES, School of Industrial Management, University Mohammed VI – Polytechnique, BenGuerir, Morroco,
PhD Student at University Paris Nanterre, Nanterre, France
[2] OCP chair of Industrial Optimization of EMINES School of Industrial Management, University Mohammed VI - Polytechnique, BenGuerir, Morroco
Emeritus Professor at University Paris-Dauphine, PSL Research University, Paris, France

{Ahlam.azzamouri@emines.um6p.ma, azzamouri.ahlam@u-paris10.fr, vincent.giard@dauphine.fr, vincent.giard@emines.um6p.ma}

Abstract. This paper deals with the blending problem of extracted phosphate ores of different characteristics (phosphate, silica…), to produce required grades that respect range constraints for these characteristics. The data used in this paper is the actual data from an OCP site[1]; OCP is world leader of phosphate extraction and transformation (acids, fertilizers) industry. We have extended the traditional LP formulation of the blending problem in a dynamic version where the optimal blends of required grades vary in time and depend on initial stocks and possible feedings of extracted ores and on the set of grades to be produced. The performance of our dynamic blending model has led the OCP site concerned to develop and use an extended version of this model.

Keywords: Blending, Phosphate Supply Chain, Make-To-Order, sequential inputs supply.

1 Introduction

The OCP SA integrated Supply Chain (SC) links the different processes of ore extraction, ore blending, phosphoric acid and fertilizer production and export. It comprises three independent axes: the north, center and southern axes. This paper focuses on the Ben Guerir subset of the center axis, that includes a mine, from which 14 ores with different chemical characteristics, are extracted, and a blending plant which uses these inputs to produce 5 different blended ores (outputs called merchantable ores), whose chemical composition is constrained. The extraction process can be viewed as a *push system*. Merchantable ores are mainly produced to order so the blending process can be viewed as a *pull system*.

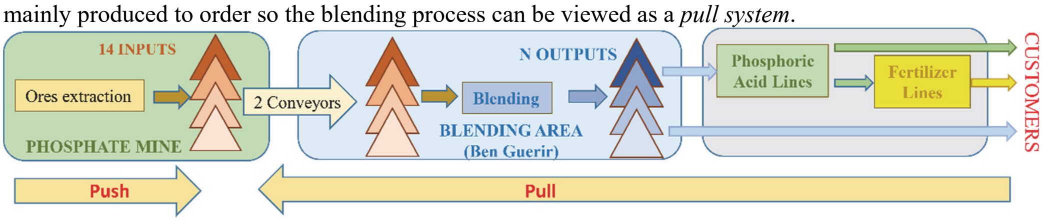

Figure 1 Ben Guerir Phosphate Supply Chain.

Blending cost is not very sensitive to grade requirements and many possible blends may be used to produce a particular merchantable ore defined by a specific quality chart (chemical characteristics). Prior to our research work, the blend for a required grade was selected from a set of a limited number of blends. More often than not, this solution generated additional costs due to emergency extraction and preserving stock (with surplus ores extracted to obtain the ores requiring to be stored temporarily). This paper shows how the blending unit can gain in efficiency and effectiveness by using an extended version of the blending model that defines jointly ores feedings from the mine and optimal blends of outstanding orders. It takes into account the following specificities that determine inputs availability:

- Blending inputs are not imported but extracted from layers of the ore deposits that belong to the company. To be able to extract ore from a layer at a particular place, the upper layers must previously be cleared; this implies that all 14 ores may not be available at a given time and that the availability of a particular extracted ore is quite irregular. Extracted ores may stay on the deposit site as long as the extraction program does not need to access the lower layer.

- Feeding input stocks to the blending area is performed by two conveyors. A particular extracted ore cannot be transported simultaneously by both conveyors. This sequential feeding limits the availability of required extracted ore.

- If an extracted ore can neither stay on the deposit, nor be stored in the blending area, it is temporarily stored in a remote dumping area, near the stone screening plant, before being retrieved when space has been freed up. These steps are costly and add no value.

- The chemical characteristics of extracted ores may vary slightly depending on layer location.

In section 2, we present our analysis of the literature. Our proposed blending models are described in Section 3. Section 4 illustrates the importance of blending flexibility and discusses some of the factors explaining why, for optimization reasons, blended ore composition cannot be set. We end with a short conclusion (Section 5).

2 Literature review

Defining the optimal composition of blended products designed to minimize product costs was the first problem to have been addressed by linear programming in the 1950's and this remains the single focus of industrial management and operation research textbooks.

- The articles we analysed deal with blending issues in various sectors, namely mining [5] [9] [15], agrifoods [13] [14] [16], milling [3] [8], chemistry [1] [7] [6] [11], tanker [2] [4] [12] cementitious [10], metallurgy [17]. All the articles reviewed were evaluated mainly with reference to the following three criteria:
- **a) The type of input feeding:** there are two types of feeding: *i*) Blending units fed through single conveyors, which requires a ***sequential*** (successive) feeding of inputs. This is illustrated by the case of bulk grain blending [3], semi-continuous blending of materials [6] and blending of raw materials to produce raw cement meal [10]; *ii)* ***Parallel*** input feeding is used in fertilizer blending [1], crude oil [2] [12], sausages [13] coking coal [15] and ore blending [17].
- **b) Blending objective:** this defines whether blending is carried out under a: *i*) Pull flow which means that blending is performed in order to meet a specific need expressed by an internal or external client (*make-to-order*); this case is illustrated by articles [2] [3] [5] [8] [12] [13] [14] [15] [17]; or *ii)* Push flow which involves producing for stock or *make-to-stock*, which is the case of articles [1] [10].
- **c) The optimization criterion:** differs from article to article, depending on type of blending. Some articles use an optimization criterion for either production or running costs as illustrated by articles [1] [4] [9 [16] [17]. Others seek to minimize the cost of transport [3], or maximize profit as in articles [2] [12]. Other papers dealing with blending focus on quality [11] [7], while others combine cost and quality criteria [9] [8] [14] or the three criteria of cost, quality and profit [13]. Moreover, the minimization of penalties incurred due to any deviation from target values is also a resolution criterion addressed in certain articles [5] [6] [10].

Based on our analysis of the articles, we were able to confirm that our paper addresses a blending issue that combines several features which are not found in existing research: *i*) inputs extracted upstream in an integrated supply chain, with characteristics that differ over time due to location and depth changes of the blocks extracted, *ii*) varying availability of inputs due to accessibility of layers through time, *iii*) sequential feeding of input stocks with limited storage and transportation capacity. These constraints must be taken into account in order to satisfy demand requirements in terms of quality and quantity. The variation and uncertainty of the system features determine the specific nature of the problem. In addition to cost resolution criteria, our paper addresses the issue of scarcity, which roughly consists in the conservation of the Phosphate-rich layers.

3 Formulation of the Blending Problem

We first present a static version of the blending problem (section 3.1) before moving on to a dynamic version that includes inputs feedings (section 3.2).

3.1 The static blending model

The basic static model (section 3.1.1) comprises several variants used to understand blending flexibility and limits (section 3.1.2).

3.1.1 The basic model

The blending units can produce a set of J outputs $(j = 1..\mathrm{J})$. We address the problem of producing K orders k $(k = 1...\mathrm{K})$ during a single period, which involves the parallel use of K blending units. Order *k* relates to a single output $j = \lambda_k$ for quantity D_k. Production of order *k* is obtained by mixing several inputs from a set of I inputs $(i = 1...\mathrm{N})$, by using x_{ik} of input *i* in the blend of output λ_k required by order *k*. Relation (1) links inputs requirements to output demand types in the blending

$$\sum_i x_{ik} = \mathrm{D}_k, \forall k \tag{1}$$

The solution must take into account the availability S_i of input *i* (relation 2). One notes the absence of restriction of input availability if $\mathrm{S}_i = \sum_k \mathrm{D}_k$, but this limit may be lower.

$$\sum_k x_{ik} = \mathrm{S}_i, \forall i \tag{2}$$

Input and output are characterized by C components $(c = 1...\mathrm{C})$. The input structure α_{ci} of input *i* is known; it is defined as the percentage of the weight of component *c* in the total weight of input *i* (see example in table 1). It is important to note that the set of inputs changes over time, due to change in their components composition, as shown by additional core samples extracted from the mine locations that are scheduled to be extracted soon. The output structure β_{cj} of output *j* must comply with a quality chart ($\beta_{cj}^{\mathrm{Min}} < \beta_{cj} < \beta_{cj}^{\mathrm{Max}}$; see one such chart in Table 1).

Table 1. Composition α_{ci} of inputs and specifications β_{cj} of composition of some outputs

Share α_{ci} (%) of component *c* in the weight of input *i*

Component *c*	Input *i*													
	i=1 C3 sup	*i*=2 SA2	*i*=3 C3G	*i*=4 C1	*i*=5 C0	*i*=6 C4	*i*=7 C5	*i*=8 C2 sup	*i*=9 SB	*i*=10 SX	*i*=11 C3 inf	*i*=12 C1 Exp	*i*=13 C2 Exp	*i*=14 C6
c=1 BPL	50,0	54,9	56,0	59,5	59,5	61,0	59,0	60,0	61,5	63,0	64,0	65,5	65,5	65,7
c=2 CO_2	3,7	7,7	5,4	4,5	5,2	4,8	7,7	5,1	5,2	5,5	5,0	5,9	4,6	5,0
c=3 MGO	1,0	0,7	0,9	1,2	1,2	1,5	1,7	0,9	0,8	0,8	1,1	0,8	0,7	1,2
c=4 SIO_2	18,0	8,0	17,2	9,5	8,5	11,7	9,8	11,5	8,0	11,0	10,0	7,5	8,0	6,0
c=5 Cd/B (ppm)	24	16	8	11	8	10	14	12	10	10	5	12	13	9

Constraints on the share β_{cj} (%) of component *c* in the weight of output *j*

Component *c*	Output *j*				
	j=1 Tess	*j*=2 Stand	*j*=3 MT	*j*=4 BT	*j*=5 TBT
c=1 BPL	$57,9<\beta_{11}<61$	$57,9<\beta_{12}<61$	$\beta_{13}>64$	$57<\beta_{14}<60$	$54<\beta_{15}<56$
c=2 CO_2	$5,6<\beta_{21}<7$	$5,6<\beta_{22}<7$	$5<\beta_{23}<7$	$6<\beta_{24}<8$	$7<\beta_{25}<9$
c=3 MGO	$\beta_{31}<1,4$	$\beta_{32}<1,4$	$\beta_{33}<1$	$\beta_{34}<1,4$	$\beta_{35}<2$
c=4 SIO_2	$\beta_{41}<13,5$	$\beta_{42}<13,5$	$\beta_{43}<8$	$\beta_{44}<13,5$	$\beta_{45}<15$
c=5 Cd/B (ppm)	$\beta_{51}<10$	$\beta_{52}<12$	$12<\beta_{53}<18$	$\beta_{54}<20$	$\beta_{55}<26$

The transcription of these compositions by referring to orders *k* leads to output structure $\beta_{c\lambda_k}$ for order *k*, concerning output $j = \lambda_k$, obtained by the blending of x_{ik}, is $\beta_{c\lambda_k} = \sum_c \alpha_{ci} \cdot (x_{ik} / \mathrm{D}_k)$. Thus, the quality chart may be described by relation (3) in a problem formulation using an AML (Algebraic Modeling Language) which allows use of predicates in the generation of variables and constraints.

$$\sum_i \alpha_{ci} \cdot x_{ik} = \beta^{\mathrm{Eq}}_{c\lambda_k} \cdot \mathrm{D}_k, \forall k \mid \beta^{\mathrm{Eq}}_{c\lambda_k} \neq 0$$
$$\sum_i \alpha_{ci} \cdot x_{ik} > \beta^{\mathrm{Min}}_{c\lambda_k} \cdot \mathrm{D}_k, \forall k \mid \beta^{\mathrm{Min}}_{c\lambda_k} \neq 0 \quad (3)$$
$$\sum_i \alpha_{ci} \cdot x_{ik} < \beta^{\mathrm{Max}}_{c\lambda_k} \cdot \mathrm{D}_k, \forall k \mid \beta^{\mathrm{Max}}_{c\lambda_k} \neq 0$$

In this static modelling, input *i* has an acquisition cost γ_i and the problem is to find the blend that minimizes the cost of acquisition (relation 4). Since the problem variables are continuous, there is an infinite number of possible solutions or none, if the problem is unfeasible.

$$\mathrm{Min}(\sum_i \gamma_i \cdot \sum_k x_{ik}) \quad (4)$$

In the productive system under review, direct blending and extracting costs are not very sensitive to the solution. This leads us to look for the best solution while sparing future resources (criterion used in the field). In our numerical examples, we arbitrarily use the weighting system $\gamma_i = \alpha_{i1}$ (where index $c = 1$ corresponds to the BPL content) for the residual stock $S_i - \sum_k x_{ik}$ of inputs *i* after blending. Relation (5) is thus preferred to relation (4) in our extracted of blending problem.

$$\mathrm{Max}(\sum_i \alpha_{i1} \cdot (S_i - \sum_k x_{ik})) \quad (5)$$

3.1.2 Variants of the static model designed to explore the blending extent and limits of blending options

The literature on blending is focused on finding a solution rather than a range of possible solutions, unlike our purpose here. In order to assess the different blending options available, a number of variants of this general model are used. They include as many orders as there are available processors and they leave time considerations aside, which implies input feeding as stock S_i is deemed sufficient. *Variants A* to *D* concern the blending problem for a single order j', without any input availability issue; in that context J independent problems are successively studied. *Variant E* relates to multiple orders where there are availability constraints of inputs.

- *Variant A*: is a basic static model with a single order, using objective-function (5).
- *Variant B*: replaces successively objective-function (5) by the minimization $(Min(x_{i'j'} / \mathrm{D}_{j'}))$ or by the maximization $(Max(x_{i'j'} / \mathrm{D}_{j'}))$ of input i' $(i' = 1..\mathrm{N})$ percentage in the weight of j', which leads, in our example, to $2 \times 5 \times 14 = 140$ optimizations.
- *Variant C*: forcing the blending process to use a predetermined number H of inputs in the blending of output j'; in this context; the binary variable $z_{ij'} = 1$ if input *i* is used to produce output j'. This is obtained with relation (6) where M is a very large number and ε a very small one; the additional constraint (7) forces the number of inputs used for blending to be H.

$$\mathrm{M} \cdot z_{ij'} \geq x_{ij'}, \forall i$$
$$\varepsilon \cdot z_{ij'} \leq x_{ij'}, \forall i \quad (6)$$

$$\sum_i z_{ij'} = \mathrm{H} \quad (7)$$

- *Variant D*: sets to 0 the stock for the most requested input in the current solution, and identifies an alternative new blend; this process, which complies with the minima of *Variant B*, starts from the solution corresponding to the lowest value of H found in *Variant C* and goes on until it is no longer possible to find a solution.

- *Variant E*: also static, now covers several orders fulfilled simultaneously and assesses the impact of insufficient availability of inputs to achieve optimal solutions for *Variant A*.

3.2 Dynamic blending model of extracted ores

The particular context of the supply chain in which the blending problem occurs (sequential feeding of inputs, make-to-order) leads to a dynamic reformulation of this problem based on splitting time into periods $(t=1..T)$. The scheduling of a set of K production orders $(k=1..K)$ fulfilled in parallel blending processors is assumed to be already established and consistent with processor availability. This leads to ignoring the processors in our formulation. In the dynamic version studied here:

- The fulfilment of order *k* leads to the withdrawal of inputs during periods *t* such that $\delta_{kt}=1$ (otherwise, this Boolean equals 0); the withdrawal duration, used in relation (8), is noted ν_k $(\sum_t \delta_{kt}=\nu_k)$.

- The initial availability of input *i* at the beginning of period *t,* is increased by supply A_{it}, possibly null.

- The initial stock is now noted S_{i0}. Stock of input *i* at the end of period *t* is noted S_{it} $(S_{it}\geq 0)$ and depends on initial stock S_{i0}, increased by supplies $A_{it'}$ available at the beginning of period $t'\leq t$ and decreased by the withdrawals of this input during period $t'\leq t$ (which are x_{ik}/ν_k assuming $\delta_{kt'}=1$). It will be assumed here that withdrawal rate is constant over the ν_k withdrawal period (otherwise it would be necessary to introduce dated coefficients, without increasing the size of the problem). Relations (8) reflect the flow conservation constraint and (9) ensure that the stock is never depleted.

$$S_{i,t}=S_{i,t-1}+A_{it}-\sum_{k|\delta_{kt}=1} x_{ik}/\nu_k, \forall i, \forall t>1 \quad (8)$$

$$S_{it}\geq 0, \forall i, \forall t>1 \quad (9)$$

We define binary variable $\phi_{it}=1$ if input *i* withdrawal occurs during period *t*. As a given input *i* cannot be carried, during period *t* by more than one conveyor, those conveyors having the same withdrawal rate δ, the quantity input *i* to be conveyed to the blending zone is $A_{it}=\delta\cdot\phi_{it}$ and relation (8) can be replaced by relation (8')

$$S_{i,t}=S_{i,t-1}+\delta\cdot\phi_{it}-\sum_{k|\delta_{kt}=1} x_{ik}/\nu_k, \forall i, \forall t>1 \quad (8')$$

Decisions taken upstream of the extraction chain define possible input supplies and routings to the blending zone. As this information is available along with daily extraction outputs, we are able to define the cumulative availability B_{it} of input *i* at the end of period *t* upstream of the blending zone, regardless of withdrawals of input *i,* from initial stock B_{i0}. Existence of the order variable ϕ_{it} is subject to cumulative availability B_{it} $(B_{it}=0\rightarrow\phi_{it}=0)$. The inputs withdrawal process is constrained by the number R_t of conveyors available during period *t*, (relation 10) and by a sufficient available build up (relation 11).

$$\sum_i \phi_{it}\leq R_t, \forall t|B_{it}>0 \quad (10)$$

$$B_{it}\geq\delta\cdot\sum_{t'\leq t}\phi_{it}, \forall i,t \quad (11)$$

The remaining features of our model deal with stock considerations.

- The use of the model leads naturally to solutions with important input feedings, to improve the value of the objective-function. Thus one must introduce a constraint on the global storage, which cannot exceed σ, with relation (12). Storage capacity constraints at the blending zone also imply a maximum stock build up not to be exceeded S_i^{Max}, with relation (13).

$$\sum_i S_{it} < \sigma, \forall t \quad (12)$$

$$S_{it} < \mathrm{S}_i^{\mathrm{Max}}, \forall t, i \quad (13)$$

- We also have a security stock $\mathrm{S}_i^{\mathrm{Min}}$ to deal with unforeseen demand [19], which would be governed by $S_{it} - \mathrm{S}_i^{\mathrm{Min}} \geq 0, \forall i, t$ .This hard constraint, introduced for an hypothetical future problem, may prevent from finding a feasible solution of the current problem. Thus, it is preferable to use relation (14) where w_{it} is the lack of security stock and combine it with penalty θ_i introduced in objective-function (5), which leads to objective-function (16)

$$w_{it} \geq 0, \forall i, t$$
$$w_{it} \geq S_i^{\mathrm{Min}} - S_{it}, \forall i, t \quad (14)$$

- Dumping storage of input *i* takes place whenever stock *i* in the blending area exceeds threshold $\mathrm{S}_i^{\mathrm{Max}}$, involving a significant additional cost for the ore to be transferred later from the dumping storage to the stock to use in the blending area. Variable y_{it}, possibly null, determines this surplus by relations (15) and supports a cost η ; thus the dumping cost is proportional to both the duration of the dumping storage and to stored quantities. The system used to calculate the cost of dumping storage is irrelevant so long as dumping can be avoided (as the corresponding partial cost in the objective function is null), so we use a high fictitious cost η that serves to eliminate solutions that lead to dumping.

$$y_{it} > 0, \forall i, t$$
$$y_{it} > S_{it} - S_i^{\mathrm{Max}} \quad (15)$$

The objective function of that mixed linear problem is given by relation (16).

$$\mathrm{Min}(\sum_i \alpha_{i1} \cdot S_{i\mathrm{T}} + \sum_i \theta_i \cdot w_{it} + \eta \cdot \sum_t \sum_i y_{it}) \quad (16)$$

4 Illustration of the Blending Problem

We illustrate blending flexibility below, first in a static context (production during a single period), then in a dynamic context. Parametrized problems were defined by using the Algebraic Language Formulation (AGL) of Xpress-IVE [18] and solved with that software.

4.1 Blending flexibility in a static context

We first illustrate the importance of blending flexibility in the case of a single output (variants *A* to *D*) before moving on to multiple outputs (variant *E*). We use the data from Table 1 and for variants *A* to *C* we use $\mathrm{D}_j = 100, \forall j$ and $\mathrm{S}_{0i} = 100, \forall i$, to prevent the risk of stockout and enable expressing x_{ij} as a percentage.

- *Variant A***.** the optimal blending solutions were obtained successively and independently, using objective-function (5) for each one of the products. The optimal independent solutions are given in Table 2. The use of that objective-function leads to prefer inputs whose BPL content is in the medium range when the BPL range in the output composition is not high. Thus, the choice of the objective-function has an important impact on blend definition.

Table 2: Optimal composition of each output *j*

Output *j*	1	2	3	4	5
Input *i* : %	**1**:5.37%; **3**:43.02%; **5**:12.05%; **6**:12%; **7**:18.88%; **11**:7.77%	**1**:16.05%; **3**:35.25%; **7**:25.49%; **11**:8.24%; **14**:14.87	**1**:10.23%; **12**:51.52%; **14**:38.26%	**1**:4.31%; **2**:74.12%; **14**:21.56%	**1**:18.25%; **2**:81.60%; **12**:0,16%

- *Variant B*. The minimal and maximal weight shares of the different inputs used in manufacturing each output, where all inputs are deemed available and using objective function (5) are given in Table 3. One observes that: output 1cannot be produced without inputs 3 and 7; output 2 requires input 7; output 3 cannot be produced without input 12; output 4 requires input 2 while output 5 has no essential input. Thus stocks of inputs 2, 3, 7 and 12 are required if one wants to be able to produce outputs 1, 2, 3 and 4. The lack of other inputs does not prevent the production of these outputs. The problem of the safety stocks of inputs used in blending is different from that normally encountered in the assembly of discrete products: it is studied in another paper submitted to ILS 2018 [19].

Table 3: Minimal and Maximal % of inputs *i* in each output *j*

Output *j*	Input *i*: $i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	$i=12$	$i=13$	$i=14$
$j=1$	0-11	0-15	10-57	0-32	0-43	0-48	10-46	0-27	0-20	0-28	0-49	0-19	0-15	0-32
$j=2$	0-20	0-22	0-57	0-43	0-43	0-60	10-62	0-44	0-26	0-35	0-49	0-24	0-22	0-32
$j=3$	0-10	0-15	0-12	0-26	0-26	0-18	0-22	0-27	0-40	0-34	0-33	12-90	0-88	0-60
$j=4$	0-0	22-80	0-10	0-33	0-50	0-18	0-22	0-21	0-70	0-21	0-26	0-48	0-48	0-47
$j=5$	0-18	0-90	0-31	0-23	0-24	0-18	0-49	0-22	0-17	0-15	0-12	0-16	0-11	0-12

- *Variant C*. Table 3 describes successively for each output the sets of inputs that are adequate to produce a given output (sets ranked by their increasing cardinalities), using objective function (5). It leads to several conclusions. The minimum number of inputs required to produce an output may be more or less than that found in the optimal solution for *variant A*, at the cost of undermining performance; it can be equal but the list of chosen inputs may differ (e.g. output 1) for each output. The maximum number of inputs indicated in this table cannot be exceeded without violating the composition specifications for that output. We must stress that the objective-function chosen may change the composition of the sets but not their cardinalities, assuming inputs are available.

Table 4: Possible blends of some outputs, depending on a compulsory number H of inputs to be used

j = 1

Compulsory number of inputs to be used \ *i*	3	4	5	6	7	8	9	10	11	14
2	Impossible									
3	50	-	-	-	39	-	-	-	11	-
4	50	-	5	-	37	-	-	-	8	-
5	50	-	5	5	35	-	-	-	6	-
6	49	-	6	5	31	5	-	-	5	-
7	48	5	5	5	27	5	-	-	5	-
8	42	5	5	7	26	5	-	5	5	-
9	39	5	5	5	26	5	-	5	5	5
10	32	5	5	11	22	5	5	5	5	5
11	Impossible									
12	Impossible									

j = 2

Compulsory number of inputs to be used \ *i*	1	2	3	4	5	6	7	8	9	10	11	12	14
2	Impossible												
3	-	-	38	-	-	-	57	5	-	-	-	-	-
4	5	-	35	-	-	-	50	-	-	10	-	-	-
5	5	-	37	-	-	-	47	-	-	5	-	5	-
6	7	-	36	-	-	5	42	-	-	5	-	5	-
7	8	-	36	-	-	5	37	5	-	5	-	5	-
8	11	-	34	-	-	5	31	5	-	5	5	5	-
9	12	-	31	5	-	5	26	5	-	5	5	-	5
10	8	5	26	5	5	5	31	5	-	5	5	-	-
11	8	5	22	5	5	5	30	5	5	5	5	-	-
12	Impossible												

j = 3

Compulsory number of inputs to be used \ *i*	2	7	8	9	10	11	12	13	14
2	-	22		-	-	-	78	-	-
3	5	15		-	-	-	80	-	-
4	5	15	-	-	-	-	53	27	-
5	6	12	-	-	5	-	72	5	-
6	5	13	-	-	5	-	57	15	5
7	7	6	5	-	5	-	67	5	5
8	5	5	5	5	5	-	64	5	5
9	5	5	5	5	5	5	46	10	14
10	Impossible								
11	Impossible								
12	Impossible								

j = 4

Compulsory number of inputs to be used \ *i*	2	4	5	7	9	11	12	13	14
2	80	-	-	-	-	-	-	20	-
3	77	-	-	5	-	-	18	-	-
4	75	-	-	9	-	-	12	-	5
5	75	-	-	8	-	-	7	5	5
6	75	-	-	5	5	-	5	5	5
7	70	-	5	5	5	-	5	5	5
8	62	5	5	5	5	-	5	5	8
9	52	5	5	5	5	5	5	5	13
10	Impossible								
11	Impossible								
12	Impossible								

j = 5

Compulsory number of inputs to be used \ *i*	1	2	3	6	7	8	10
2	18	82	-	-	-	-	-
3	18	77	-	-	5	-	-
4	15	75	5	-	5	-	-
5	12	73	5	-	5	5	-
6	9	71	5	-	5	5	5
7	6	69	5	5	5	5	5
8	Impossible						
9	Impossible						
10	Impossible						
11	Impossible						
12	Impossible						

- *Variant D*. This represents successively the impact of zero stock for some inputs for each output (the positive minima of variant *C* being retained). This places us before a highly combinatorial problem (C_{14}^{k} problems). Therefore, we have opted for a scenario where *i*) we start from the optimal solution found with stocks set to minimum in Table 3, the others being at 100; *ii*) we keep the minima and set to zero initial stock for the most requested input in the previous optimal solution; *iii)* we look for the new optimal solution, and if one is found, we return to *ii*), if not we stop. Analysis of results shows that for example:

- *output 1*: putting inputs 3 and 7 together at the minima (10,10) and the rest of the inputs at 100, does not yield any solution; putting first only input 3 at the minima (10) it also precludes yielding a solution. However, by setting second input 7 at the minima (10); output 1 can be produced with: 8.47% of input 2; 41.61% of input 3; 3.79% of input 5; 36.13% of input 6; 10% of input 7. Thus, we conclude that the new minima for input 3 to produce output 1 is 42 which yields a feasible solution.
- *output 2:*can be produced with the minimum of input 7 (10) (+ inputs 1, 2, 3, 6 and 14) but cannot be produced if, in case input 6 is missing;
- *output 3:*can be produced with the minimum of input 12 (12) (+ inputs 1, 2, 13 and 14); it has substitutable inputs but cannot be produced where for example all of inputs 2, 13 and 14 are missing.

This analysis shows that when mandatory inputs are present *a minima*, another input must be available to be able to obtain an output matching the blending constraints. To conclude this static - mono-product analysis, *output production is still possible where some inputs* are not *available.*

- *Variant E*. Scenario 1 (see Table 5) deals with three orders fulfilled simultaneously, for equal quantities of 100, with each input availability remaining set at 100. The aggregation of input consumptions for the solutions found in variant *A* leads to stock-out of input 2. The introduction of constraint (15) leads to new blends for which the selected optimization criteria involve input 2 shortage for output 5 and 4. In scenario 2, input allocations are not identical; we obtain blends that are very different from those of scenario 1. To conclude, *the blends of outputs produced simultaneously depend on input availability*.

Table 5: Factors impacting blends (D_k and S_{i0}) in the "Mono-period / Multi-product" problem

Scenario 1 — $D_j = 100, \forall j$

Input i	Free optima of *Variant A* – Output j: 3	4	5	Sum	Optima of *Variant E* $S_{i0} = 100, \forall i$ – Output j: 3	4	5	Sum
1	10	4	18	33	10	0	18	28
2	0	74	82	**156**	-	64	36	100
3	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-
5	-	-	-	-	-	29	-	29
6	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	46	46
8	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-
12	52	-	-	52	52	-	-	52
13	-	-	-	-	-	-	-	-
14	38	22	-	60	38	7	-	46
Sum	100	100	100	300	100	100	100	300

Scenario 2 - S_{i0} and D_j not identical

Input i	S_{i0}	Free optima of *Variant A* – Output j: 3	4	5	Sum	Optima x_{ij} of *Variant E* – Output j: 3	4	5	Sum	Optimal blend % of variante E – Output j: 3	4	5
1	50	12	3	9	24	1	-	2	3	1	-	4
2	70	5	70	41	**116**	-	32	38	70	-	32	76
3	0	-	-	-	-	-	-	-	-	-	-	-
4	30	-	-	-	-	28	-	-	28	19	0	-
5	50	-	12	-	12	-	50	-	50	-	50	-
6	30	-	-	-	-	13	-	10	23	8	-	20
7	0	-	-	-	-	-	-	-	-	-	-	-
8	0	-	-	-	-	-	-	-	-	-	-	-
9	0	-	-	-	-	-	-	-	-	-	-	-
10	0	-	-	-	-	-	-	-	-	-	-	-
11	0	-	-	-	-	-	-	-	-	-	-	-
12	40	40	-	-	40	36	4	-	40	24	4	-
13	50	43	-	-	43	35	2	-	36	23	2	-
14	50	50	16	-	**66**	38	12	-	50	25	12	-
Sum		150	100	50	301	150	100	50	300	100	100	100

4.2 Blending flexibility in a dynamic context

Let us arbitrarily consider input blending for 6 orders fulfilled by 2 blending units running simultaneously. The time split selected is 16 half-days. The processor speed is 10 units / half day. We consider that during the last scheduled period of an order there is no input supply as this final period is dedicated to the completion of the final mixture. Table 6 presents the data for the production sequence problem: initial stocks of inputs, orders to be fulfilled and the cumulative availability B_{it} of input *i* at the end of period *t* upstream of the blending zone.

Taking into account cumulative availability B_{it} (Table 6) of each input *i,* using the objective-function (16) and the following costs: η=200000 , $\theta_i = 200000, \forall i$, the model allows us to define the feeds that satisfy demand and comply with all the predefined constraints (Table 7). The resulting blend compositions are shown in Table 7 which highlights the differentiation of blends implemented from one order to another in terms of inputs consumed and their structure (%). For example: for the three orders $k = 1,3,4$ using the same output $\lambda_k = 5$, we observe that: $k = 1$ uses mainly 57% of input 1, $k = 3$ uses mainly only 34% of input 3 but $k = 4$ is uses mainly 33% of input 4.

Table 6: Problem data: initial Inventories S_{i0} , order specifications and cumulative availability B_{it}

Input *i*	*i*=1	*i*=2	*i*=3	*i*=4	*i*=5	*i*=6	*i*=7	*i*=8	*i*=9	*i*=10	*i*=11	*i*=12	*i*=13	*i*=14
S_{i0}	20	30	30	30	30	30	10	30	30	30	10	10	10	10

Orders specifications

Orders *k*	Processor	Output λ_k	Order Quantity	Production periods	
				Start	End
1	1	5	40	1	5
2	1	3	30	6	9
3	1	5	50	10	15
4	2	5	60	1	7
5	2	4	40	8	12
6	2	3	30	13	16

B_{it}	Input *i* / *t*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	i=1	-	-	-	-	-	-	-	-	10	20	20	20	20	20	20	20
	i=2	-	-	-	10	20	20	20	20	20	20	20	20	20	20	20	20
	i=3	-	10	20	20	20	20	20	20	20	20	20	20	20	20	20	20
	i=4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=8	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
	i=9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	i=12	-	-	-	-	-	10	20	30	30	30	30	30	30	30	30	30
	i=13	-	-	-	-	-	-	-	-	-	-	-	-	10	20	20	20
	i=14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10	20

Table 7: Results: feeding result $A_{it} = \delta \cdot \phi_{it}$ of input stocks and blend compositions

Inputs consumptions

Input i	Orders (Outputs) $k=1$ ($\lambda_k=5$)	$k=2$ ($\lambda_k=3$)	$k=3$ ($\lambda_k=5$)	$k=4$ ($\lambda_k=5$)	$k=5$ ($\lambda_k=4$)	$k=6$ ($\lambda_k=3$)	Sum
1	6	-	16	14	-	4	40
2	-	11	-	19	-	10	40
3	23	-	17	-	-	-	40
4	-	-	-	20	-	-	20
5	11	-	9	-	-	-	20
6	-	-	-	6	9	5	20
7	-	-	-	1	9	-	10
8	-	-	8	-	22	-	30
9	-	-	-	-	-	10	10
10	-	19	-	-	-	1	20
11	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-
Sum	40	30	50	60	40	30	250

Blend structure (%) of inputs

Input i	Orders (Outputs) $k=1$ ($\lambda_k=5$)	$k=2$ ($\lambda_k=3$)	$k=3$ ($\lambda_k=5$)	$k=4$ ($\lambda_k=5$)	$k=5$ ($\lambda_k=4$)	$k=6$ ($\lambda_k=3$)
1	16%	-	33%	23%	-	12%
2	-	37%	-	32%	-	33%
3	57%	-	34%	-	-	-
4	-	-	-	33%	-	-
5	27%	-	18%	-	-	-
6	-	-	-	10%	22%	17%
7	-	-	-	2%	22%	-
8	-	-	15%	-	56%	-
9	-	-	-	-	-	33%
10	-	63%	-	-	-	4%
11	-	-	-	-	-	-
12	-	-	-	-	-	-
13	-	-	-	-	-	-
14	-	-	-	-	-	-
Sum	100%	100%	100%	100%	100%	100%

A_{it} / Input i	t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	$i=1$	-	-	-	-	-	-	-	-	-	10	-	10	-	-	-	-
	$i=2$	-	-	-	-	-	-	10	-	-	-	-	-	10	-	-	-
	$i=3$	-	-	-	-	10	-	-	-	-	-	-	10	-	-	-	-
	$i=8$	-	-	-	-	-	-	-	-	-	10	-	-	-	-	-	-

5 Conclusion

An extended version of the "multi-product/multi-period model" (detailed in §3.3) was benchmarked during summer 2017 at the Ben Guerir site. These extensions involve integrating the choice of inputs to be transferred to the blending area and use of target values to determine output components. As this model clearly outperforms current practices, it has been adopted at the site and it is being considered for use at other OCP sites. The dynamic blending method will be coupled with an upstream extractive process scheduling model, which together will deliver an intelligent match between supply from the layers (pushed flows) and production of the blends (pull flows), to help address the relative unpredictability of the extractive process to the fullest possible extent.

Acknowledgements*: Section II and IV were written in collaboration with Meryem Bamoumen, Mouna Bamoumen, Najat Bara, Latifa Benhamou, Hajar Hilali and Asma Rakiz, all students of the EMINES PhD program*

6 References

1. Ashayeri, J., van Eijs, A. G. M., Nederstigt, P.: Blending modelling in a process manufacturing: A case study. European Journal of Operational Research 72, 460–468 (1994)
2. Bengtsson, J., Bredström, D., Flisberg, P., Rönnqvist, M.: Robust planning of blending activities at refineries. The Journal of the Operational Research Society 64, 848–863 (2013)
3. Bilgen, B., Ozkarahan, I.: A mixed-integer linear programming model for bulk grain blending and shipping. International Journal of Production Economics 107, 555–571 (2007)
4. Cain, M., Price, M. L. R.: Optimal Mixture Choice. Journal of the Royal Statistical Society. Series C (Applied Statistics) 35, 1–7 (1986)
5. Chanda, E. K. C., Dagdelen, K.: Optimal blending of mine production using goal programming and interactive graphics systems. International Journal of Surface Mining, Reclamation and Environment 9, 203–208 (1995)

6. Goldsmith, P. L.: The Theoretical Performance of a Semi-Continuous Blending System. Journal of the Royal Statistical Society. Series D (The Statistician) 16, 227–251 (1966)
7. Jonuzaj, S., Adjiman, C. S.: Designing optimal mixtures using generalized disjunctive programming: Hull relaxations. Chemical Engineering Science 159, 106–130 (2017)
8. Karmarkar, U. S., Rajaram, K.: Grade Selection and Blending to Optimize Cost and Quality. Operations Research 49, 271–280 (2001)
9. Kumral, M.: Application of chance-constrained programming based on multi-objective simulated annealing to solve a mineral blending problem. Engineering Optimization 35, 661–673 (2003)
10. Kural, A., Özsoy, C.: Identification and control of the raw material blending process in cement industry. Int. J. Adapt. Control Signal Process. 18, 427–442 (2004)
11. Montante, G., Coroneo, M., Paglianti, A.: Blending of miscible liquids with different densities and viscosities in static mixers. Chemical Engineering Science 141, 250–260 (2016)
12. Oddsdottir, T. A., Grunow, M., Akkerman, R.: Procurement planning in oil refining industries considering blending operations. Computers, Chemical Engineering 58, 1–13 (2013)
13. Steuer, R. E.: Sausage Blending Using Multiple Objective Linear Programming. Management Science 30, 1376–1384 (1984)
14. Stokes, J. R., Tozer, P. R.: Optimal Feed Mill Blending. Review of Agricultural Economics 28, 543–552 (2006)
15. Williams, K. B., Haley, K. B.: A practical application of linear programming in the mining industry. Journal of the Operational Research Society 10, 131–137 (1959)
16. Yoon, W. B., Park, J. W., Kim, B. Y.: Linear Programming in Blending Various Components of Surimi Seafood. Journal of Food Science 62, 561–564 (1997)
17. Zhang, R., Lu, J., Zhang, G.: A knowledge-based multi-role decision support system for ore blending cost optimization of blast furnaces. European Journal of Operational Research 215, 194–203 (2011)
18. FICO Xpress Optimization Suite, http://www.fico.com/en/products/fico-xpress-optimization-suite
19. Bamoumen, M., Hilali, H., Hovelaque, V., Giard, V.: Flexi-security stocks, a new approach for semi-raw materials used in blending. Paper submitted to the 7th International Conference on Information Systems, Logistics and Supply Chain, ILS Conference 2018, July 8-11, Lyon, France (2018)

7 Table of notations

i	Index of ore $(i=1..\mathrm{N})$
j	Index of a blended outputs $(j=1..\mathrm{J})$
c	Index of chemical component $(c=1..C)$
t	Index of a time period $(t=1..\mathrm{T})$
α_{ci}	Share (%) of component c in the weight of input i
k	Index of order to satisfy $(k=1..\mathrm{K})$
H	Number of inputs used in the blending
γ_i	An acquisition cost of input i
η	Excess storage cost
θ_i	Fictitious penalty induced by lack of security stock
δ	Flow rate of the conveyors
β_{cj}	Output structure of output j
$\beta_{cj}^{\mathrm{Min}}$	Lower bound of each grade's component in the output j
$\beta_{cj}^{\mathrm{Max}}$	Upper bound of each grade's component in the output j
β_{cj}^{Eq}	Component c constrains the composition of output j in the case of equality
ν_k	Withdrawal duration
R_t	Number of conveyors available during period t
B_{it}	Cumulative availability of input i at the end of period t upstream of the blending zone
S_{i0}	The initial stock of input i

S_i^{Max}	Storage capacity of input *i*
S_i^{Min}	Security stock
S_{it}	Stock of input *i* at the end of period *t*
σ	Capacity on the global storage
$A_{it'}$	Supplies available at the beginning of the period $t' \leq t$
D	Demand of unique output
D_k	Demand of order k
λ_k	The output of order k
x_i	Order variable corresponds to the quantity of input *i* used to obtain a unique output
x_{ik}	Order variable corresponding to the weight of input *i* included in the weight D_k of order *k* dealing with the output λ_k
$z_{ij'}$	Binary variable, $z_{ij'} = 1$ if input i is used to produce output j', otherwise zero
δ_{kt}	Binary variable = 1 if The fulfilment of order *k* leads to the withdrawal of inputs during periods *t* such as $\delta_{kt} = 1$ (otherwise, this Boolean equals 0)
ϕ_{it}	Binary variable equals to 1 if input *i* withdrawal occurs during period *t*
B_{it}	Cumulative availability of input *i* at the end of period *t* upstream of the blending zone
y_{it}	Dumping storage of input *i* takes place as soon as the stock exceeds threshold S_i^{Max}. The variable y_{it}, possibly null, determines this surplus
w_{it}	Variable defined the lack of security stock