Reverse Blending: an efficient answer to the challenge of obtaining required fertilizer variety

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Abstract. Blending problems aim to select a subset of inputs from a set of existing ones and to define the input quantities to be mixed in order to produce a particular output using a predefined set of outputs. The structure of these existing inputs, defined by the % of some components in their total weight, is known. The structure of an output varies according to the chosen mixture which is constrained by % ranges that must be respected for each component. Reverse blending is an extension of the blending problem in which the set of inputs does not preexist but must be defined to be able to produce any output. In this problem, never addressed to our knowledge, the number of inputs to be produced must be as small as possible and the composition of each input is to be defined. This problem arises in the context of a growing diversity of fertilizers to be managed both in production and in distribution logistics. A real-life application example of this new approach is provided.
Keywords: fertilizer, diversity, reverse blending, input creation.

1. Introduction

Mass customization, a major trend in modern economy, involves assembling components in discrete production, while, in continuous production, it consists in a sequence of batches of different products. This both reduces achievable diversity and multiplies transportation issues. This situation is addressed by the fertilizer industry which, to enable sustainable agriculture, must provide highly customized fertilizers. Sustainable agriculture aims to both increase agricultural yield and preserve soil fertility. To do so, the use of fertilizers is essential. In fact, of all the short-term factors that can rapidly increase agricultural production, chemical fertilizers are the most efficient in obtaining the highest yields and delivering the best return on investment (Pratt, 1965). However, these fertilizers must be used selectively using specifically adapted formulas whose nutrients and proportions differ according to the pedological characteristics and the crops involved. For a fertilizer manufacturer, this constraint implies developing complex fertilizer compounds, manufactured through raw material chemical reactions upstream of granulation. This involves managing a wide variety of distribution flows. An alternative is to produce fertilizers by blending simple or compound fertilizers that are already granulated or compacted. This alternative solution simplifies logistical problems somewhat but requires substantial investments, without actually meeting final demand. Our objective therefore, is to find a good compromise between producing a large variety of customized fertilizers and reducing logistics costs.

To this end, rather than blending ready-made fertilizers of known nutritional composition, this paper proposes a new approach, which involves delayed differentiation (performed near end-users, through small blending units). It is based on the chemical identification of a limited number of new inputs to be developed. Blends of these new products enable production of the widest variety of required custom-made fertilizers (outputs). This new approach, which we call **Reverse Blending** (RB), aims, through a parameterized quadratic program, to define the optimal specifications of a number N, which we seek to keep down to a minimum, of primary inputs whose blending enables production of the required diversity of outputs (fertilizer formulas). This problem, therefore aims to define input optimal composition, respect fertilizer components constraints and cater to any type of demand.

Thanks to this drastically new approach, we will be able to rely on small capacity blending units located close to actual end-use areas and fed by substantial flows. This approach is crucial to delayed differentiation since we aim to perform the blending as close as possible to local markets. This will lead to flow

consolidation to both streamline production and transportation management. Note that delayed differentiation does not take place at original production sites but, which is unusual, in the vicinity of customers. The management of diversity involving this type of remote differentiation is already performed in the automotive industry (e.g. Smart) or in the production of paints in department stores: these paints (over 2000 colors), listed in a color catalog, are produced by blending a very limited number of primary paints. This is exactly the approach that we seek to apply through remote blending, except that in the case of the paint industry, the ingredients enabling this diversity are already known, while our aim is to create them.
Section 2 is devoted to a brief literature review: we first discuss the blending problem to highlight the unique value of RB before explaining the need for fertilizer customization to achieve reasoned and sustainable agriculture. We then show that nutrient needs are highly diversified. In section 3, we describe our RB model and go on to illustrating it with a simplified real-life example in section 4.

2. Literature review

2.1 Blending Vs Reverse Blending

After years of using fertilizers made from substances extracted or isolated from natural sources, farmers had started using chemical fertilizers since 1849 when the first patent for the production of chemical fertilizers was issued (RC. Sheridan, 1979). Fertilizers are manufactured by chemical reactions from various raw materials that depend on the nutritive composition of the final product. Admittedly, chemical production enables obtaining the nutritional structures that a granule must present, but the variety generated by this mode of production encourages producers to opt for the alternative technique of 'fertilizer blending'.
Fertilizer blending problems have drawn a lot of attention, as demonstrated in the papers by Babcock, Rister *et al*. (1984), Minguez, Romero and Domingo (1988), Ashayeri and Eijs (1994), Traoré, Koulibaly and Dakuo (2007), Lima, Severino *et al*. (2011), Aldeseit (2014), Srichaipanya, Artrit, and Sangrung (2014), Cole and Bradshaw (2015), Loh, Cheong *et al*. (2015), and their respective references. These problems consist in determining the optimal quantities to be taken from a subset of inputs in order to produce one or more fertilizer formulas. The inputs used can be either nutritious raw materials (Barrie M. Cole & Steven Bradshawa, Artrit, and Sangrung, 2014) or fertilizers, obtained by chemical reaction or by blending, for which we know the percentage of each nutrient (Minguez, Romero and Domingo, 1988). In both cases, these inputs, either supplied or produced, are already available.
On the other hand, in the RB approach, consisting in determining the composition of a minimum of inputs enabling the production of a wide variety of outputs, inputs are not pre-existing but are to be created and their compositions have to be specified. After pursuing several research avenues, we wish to emphasize the originality of the RB approach because, as far as we know, all the inputs of blending problems addressed in the literature already exist. To reach this conclusion, we researched multiple scientific databases, included in the bib.cnrs.fr metabase, using several combinations of keywords (blending / inputs, blending / inputs properties, blending / inputs characteristics / modelling, blending / inputs specifications, blending / raw materials, blending / raw materials properties / modelling, mixture problems / identifying inputs, mixture problems / non-existing inputs ...). The papers resulting from this research all deal with blending that involves the mixing of inputs, the exact composition of which we admittedly may ignore, but which we know that they already exist. In this regard, it is important not to confuse the case of an unknown input with that of an existing one. The first case may correspond to inputs that exist, but whose properties change over time, are unmeasured, unknown or poorly known. The component percentages of this type of inputs are first retrieved or estimated before integrating them, as parameters, in the optimization model, which is not the case in RB where they become decision variables.

2.2 Customized fertilizers: a prerequisite of reasoned agriculture

In order to feed a global population of 9.1 billion people by 2050, food production will have to increase by about 70% by 2050 from its 2005 level (FAO, 2009). Rising to this challenge demands rational fertilization to provide plants with needed nutrients in the most appropriate way. However, as a result of low fertilizer utilization and high nutrient extraction rates, soils are often unable to provide these nutrients without recourse to supplementary ingredients (Fixen *et al*., 2015). These ingredients must contain the appropriate

proportions of a number of nutrients, the most important of which are Nitrogen (N), Phosphorus (P) and Potassium (K).
- Nitrogen plays a key role in plant growth and crop yield (Hirel *et al*., 2007; Krapp *et al*., 2014; Ruffel *et al*., 2014; Vidal *et al*., 2014; Wang *et al*., 2012). Its availability and internal concentration affect the distribution of biomass between roots and shoots (Bown *et al*., 2010) as well as metabolism, physiology and plant development (O'Brien *et al*., 2016).
- Phosphorus is an essential nutrient for root development and nutrient availability (Jin *et al*., 2005). It is essential for cell division, reproduction and metabolism of plants and allows to store energy and regulate its use (Epstein and Bloom, 2004).
- Potassium plays a major role in regulating the opening and closing of stomata which is necessary for photosynthesis, the transport of water and nutrients and the cooling of plants (Kalavati and Modi, 2012).

Recommending fertilizer blends of these three elements requires a good knowledge of the different aspects of fertilization including objective yield, crop nutrient need and nutrient supply by the soil (Cottenie, 1978). To quantify this supply, farmers must perform soil tests to manage nutrients and avoid long-term nutritional and health problems (Watson *et al*., 2007). These tests are required at least every three years (Warncke *et al*., 2000) as soil properties vary over time in response to changes in land management practices and inherent soil characteristics (Jenny, 1941). As a matter of fact, soils undergo multiple processes: biological, physical, chemical and human. Thus, not a single hectare of cultivated soil is completely homogenous from a pedological standpoint. This results in a very large variety of fertilizer needs, hence the need to produce customized fertilizer formulas.
Such customization can encourage farmer loyalty and therefore increase market share, provided it is affordable, which is practically impossible where multiple fertilizers are to be produced and transported. The RB approach addresses this problem of effective and efficient delayed differentiation in a radically different way. It aims to produce a very large number of fertilizers (outputs) by combining a very limited number of inputs that are not ready-made ones but inputs the optimal composition of which we are attempting to define.

3. Modelling

3.1 Classical blending

In the traditional formulation of a blending problem, a set of N possible inputs ($i=1..\mathrm{N}$) is available. Any input i is characterized by a set of C components ($c=1..\mathrm{C}$) (N, P, K…). The relative weight of component c in input i is α_{ci} and their values are known. They satisfy relation (1) that parameters must comply with.

$$\sum_c \alpha_{ci} = 1, \forall i \quad (1)$$

Blending aims at defining the optimal mixture of selected inputs taken in quantities x_{ij} (**order variables**) to obtain quantity D_j, requested quantity of output j (j=1..J), which complies with constraint (2).

$$\mathrm{D}_j = \sum_i x_{ij}, \forall j \quad (2)$$

The relative weight of component c in output j obtained by blending $\beta_{cj} = \sum_i \alpha_{ci} \cdot x_{ij} / \mathrm{D}_j$ may have to belong to a range of values flowing from constraints (3).

$$\beta_{cj}^{\mathrm{Min}} \le \sum_i \alpha_{ci} \cdot x_{ij} / \mathrm{D}_j \le \beta_{cj}^{\mathrm{Max}}, \forall i, j \quad (3)$$

The inputs requirements x_{ij} must respect availability A_i of each input (if $\mathrm{A}_i \ge \sum_j \mathrm{D}_j$ implies that input i can match any output request).

$$\sum_j x_{ij} \le \mathrm{A}_i, \forall i \quad (4)$$

Input i has an acquisition cost γ_i and the problem of traditional blending is generally to identify the blends that minimize acquisition cost (4).

$$Max_{i,j}\left[\sum_i \gamma_i \sum_j x_{ij}\right] \quad (5)$$

3.2 Reverse blending

The RB problem is an extension of traditional blending where the inputs are not available. We do not know their composition but we know that they are characterized by the components *c* (N, P, K ...) contained in the outputs. In this approach, we are actually supposed to start from output demand to define the characteristics α_{ci} of N inputs (hence the name of reverse blending given to it). The characteristics α_{ci}, therefore, become decision variables and relation (1), which was an integrity constraint for the set of parameters used in the blending problem, and become a constraint to be respected by the set of new variables that, in RB, replace this parameter set.
The constraint related to the respect of required quantity for each output remains the same (2). On the other hand, the formulation of the constraint related to respect of compositional structures remains the same (3) but the fact that x_{ij} and α_{ci} are decision variables makes the problem quadratic.

As we seek to keep the number N of inputs down to a minimum, RB becomes a ***parametric quadratic problem*** where one looks for the solution of the lowest possible value of N, starting with 3 and adding to it until a solution is found.
Among the possible solutions with the lowest N, the best ones are those where the weight of a very limited number of inputs represent the highest percentage of total inputs needed for output production. To this end, criterion (4) is replaced by (7) which maximizes use of input *i*=1.

$$Max\left[\sum_j x_{1j}\right] \quad (6)$$

3.3 Efficiency limits of the fertilizers blending solution

As mentioned before, RB aims to minimize the number of inputs whose optimal composition is to be found. To demonstrate the usefulness of this new approach versus the current one of trying to obtaining a fertilizer by blending existing fertilizers, we decided to develop a blending model, whose results are shown in (§4.2), that, rather than being guided by cost-minimization, aims to determine the minimum number of existing inputs to be mixed to produce the desired variety of outputs. This model uses the above formulation and complements it with two binary variables:

w_j which is equal to 1 if output *j* is produced,

v_i which is equal to 1 if input *i* is used.

Relations (2), (3) and (4) are therefore respectively replaced by relations (2'), (3') and (4'):

$$\sum_i x_{ij} = \mathrm{D}_j . w_j, \forall j \quad (2')$$

$$\beta_{cj}^{\mathrm{Min}} . w_j \le \sum_i \alpha_{ci} \cdot x_{ij} / \mathrm{D}_j \le \beta_{cj}^{\mathrm{Max}} . w_j, \forall i, j \quad (3')$$

$$\sum_j x_{ij} \le \mathrm{A}_i \cdot v_i, \forall i \quad (4')$$

Knowing that S is equal to the number of inputs used (6) and that the objective function of this model is to maximize the number of produced outputs (7), we seek to find the minimum number that will take S if we want to produce the full variety of outputs. To do this, we start with S = J and we decrement S until finding the minimum necessary number of inputs so that all the J outputs are produced.

$$\mathrm{S} = \sum_i v_i \quad (6)$$

$$Max(\sum_j w_j) \quad (7)$$

These problems are linear. Since the problem variables are continuous, there is an infinite number of possible solutions or none, if the problem is insoluble.

4. Case study

We set forth below a simplified case study based on actual data. We begin by defining the demand to be satisfied (§4.1) before presenting the results obtained after optimizing our RB model (§4.2).

4.1 Characteristics of the custom-made fertilizer demand

To help Moroccan farmers identify their exact needs in N, P and K, OCP, one of the world phosphate market leaders, in collaboration with the Moroccan Ministry of Agriculture and Maritime Fisheries, has developed a solution "*Fertimap*". This tool was designed by leading Moroccan agronomy experts. This software recommends the appropriate quantities, in kg / hectare, to be applied to a pre-defined parcel of land, according to the relevant soil fertility indicators, the desired crop and the yield objective. We used this tool to deduce the calculation formulas of nutrient needs so that we could use them for the determination of custom fertilizers. Rather than aiming to serve actual farmers and achieve specific yields, our purpose is to use these formulas to identify nutrient needs for optimal yields of crops whose ecological requirements are compatible with the pre-selected soil area. These formulas were deduced from linear regression using reverse engineering of the *Fertimap* approach. This regression was based on a sample of about 30 records taken from the *Fertimap* platform for each of the three nutrients. This allowed, for a particular crop, to extrapolate the functional relationships linking the explained variables (N, P and K needs) to the explanatory variables (soil fertility indicators and target yield). The optimal doses of these nutrients are determined independently since in addition to the target yield the need for N, P and K depends on organic material, available phosphorus (P_2O_5) and exchangeable potassium (K_2O). Figure 1 shows the N and K requirement curves for three different yields relevant for wheat.

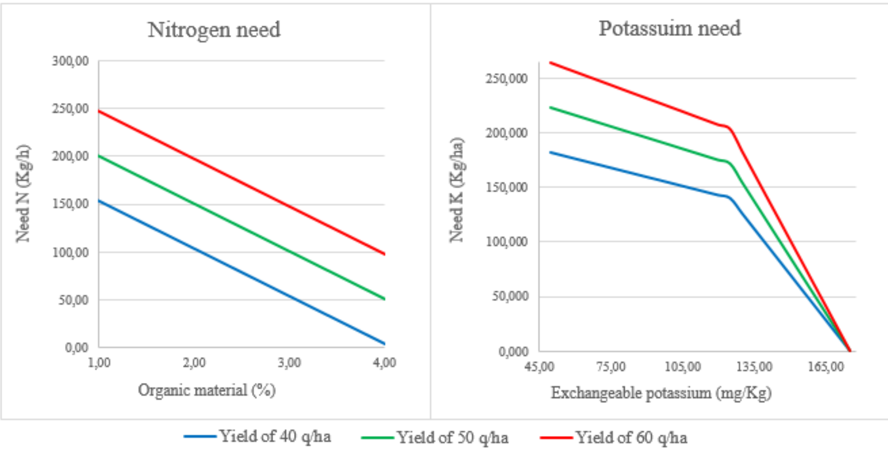

Figure 1: N and K requirement behavior versus soil indicators for three different yields

We document the customization needs calculated by these linear formulas for five different crops (olive, almond, lentil, wheat and orange citrus). The selected geographical areas are those whose soil properties are compatible with the ecological requirements for these crops. Table 1 represents the $J = 40$ fertilizer formulas inferred from *Fertimap* quantitative recommendations. The customized fertilizer formulas being the outputs *j* of the models developed in (§3).

Output	Province	Crop	N	P	K	Filler	Demand (tons)	Output	Province	Crop	N	P	K	Filler	Demand (tons)
j=1	El Hajeb	Wheat	56.00%	11.00%	0.00%	33.00%	2123	j=21	K.sraghna	Orange	55.54%	11.46%	0.00%	33.00%	33
j=2	Meknès	Wheat	67.00%	0.00%	0.00%	33.00%	1443	j=22	Berkane	Orange	47.07%	19.93%	0.00%	33.00%	1626
j=3	S.Bennour	Wheat	54.35%	13.00%	0.00%	33.00%	12421	j=23	Khénifra	Almond	37.47%	29.53%	0.00%	33.00%	34
j=4	Khémisset	Lentil	33.00%	34.00%	0.00%	33.00%	637	j=24	Khémisset	Almond	35.88%	31.12%	0.00%	33.00%	188
j=5	Sidi Kacem	Lentil	18.00%	49.00%	0.00%	33.00%	9	j=25	Meknès	Almond	44.02%	22.98%	0.00%	33.00%	746
j=6	Sidi Slimane	Lentil	20.00%	47.00%	0.00%	33.00%	73	j=26	Taza	Olive	32.95%	20.57%	13.48%	33.00%	26397
j=7	Larache	Orange	34.00%	11.00%	22.00%	33.00%	244	j=27	Taounat	Olive	45.92%	21.08%	0.00%	33.00%	36966
j=8	Fes	Wheat	34.00%	13.00%	20.00%	33.00%	3669	j=28	Meknès	Olive	59.14%	7.86%	0.00%	33.00%	7350
j=9	El Hajeb	Almond	34.00%	33.00%	0.00%	33.00%	1092	j=29	Khémisset	Olive	26.05%	11.40%	29.55%	33.00%	8480
j=10	Ifrane	Almond	54.00%	13.00%	0.00%	33.00%	9	j=30	F. Ben salah	Olive	43.63%	23.37%	0.00%	33.00%	1907
j=11	M.Yacoub	Almond	41.00%	26.00%	0.00%	33.00%	115	j=31	Béni Mellal	Wheat	37.00%	20.00%	10.00%	33.00%	500
j=12	Béni Mellal	Olive	55.00%	12.00%	0.00%	33.00%	1981	j=32	Berrechid	Wheat	37.00%	30.00%	0.00%	33.00%	670
j=13	Benslimane	Wheat	38.00%	29.00%	0.00%	33.00%	6791	j=33	El Hajeb	Lentil	30.00%	21.00%	16.00%	33.00%	230
j=14	Taounat	Wheat	53.00%	14.00%	0.00%	33.00%	13689	j=34	Berrechid	Olive	49.00%	18.00%	0.00%	33.00%	1560
j=15	Meknès	Lentil	48.00%	19.00%	0.00%	33.00%	48	j=35	Benslimane	Lentil	17.00%	22.00%	28.00%	33.00%	1000
j=16	Berrechid	Lentil	37.00%	30.00%	0.00%	33.00%	250	j=36	Taourirt	Orange	58.00%	9.00%	0.00%	33.00%	286
j=17	Settat	Lentil	0.00%	67.00%	0.00%	33.00%	1120	j=37	Taourirt	Olive	26.00%	41.00%	0.00%	33.00%	2156
j=18	F.Ben salah	Orange	23.00%	44.00%	0.00%	33.00%	804	j=38	Figuig	Almond	31.00%	0.00%	36.00%	33.00%	3021
j=19	Fes	Orange	50.00%	17.00%	0.00%	33.00%	226	j=39	Nador	Orange	29.00%	19.00%	19.00%	33.00%	800
j=20	Béni Mellal	Orange	44.77%	22.23%	0.00%	33.00%	1378	j=40	Sale	Almond	16.00%	18.00%	33.00%	33.00%	420

Table 1: 40 custom-made fertilizer formulas and demand types

Fertilizers are sold in the form of fertilizer formulas for which the percentage of N, P and K is known. However, besides these nutrients, a fertilizer has an additional component that generally has no impact on yield, called the filler. The filler is used for chemical (stabilization of granule composition) and practical reasons (excess fertilizer concentration would burn the soil). In our example, when converting recommended quantities (Kg/ha) into percentages, we opted for 33% filler in fertilizer composition.

The optimal formulas are characterized by a tolerance resulting in ranges of values for these percentages. We do not know this tolerance in advance, but in our model it is translated by a parameterized variable and we set it at ± 1% in the following calculations. Table 2 shows the target demand structure.

Globally, Moroccan soils are rich in potassium which is why recommended K levels are almost always of zero (see table 1). In fact, there is no level at which potassium becomes toxic to plants. Nevertheless, when plants get too much potassium, the absorption of some other nutrients (Nitrogen, Magnesium and Manganese) is inhibited. Therefore, even though the tolerance was fixed at 1%, if the percentage of K in the fertilizer formula is zero, it is best to keep it that way, as a first step.

	Component								
	c=1			c=2			c=3		
Output	Minimum structure (≥)	Exact structure (=)	Maximum structure (≤)	Minimum structure (≥)	Exact structure (=)	Maximum structure (≤)	Minimum structure (≥)	Exact structure (=)	Maximum structure (≤)
j=1	55.00%		57.00%	10.00%		12.00%		0%	
j=2	66.00%		68.00%	0.00%		1.00%		0%	
j=3	53.35%		55.35%	12.00%		14.00%		0%	
j=4	32.00%		34.00%	33.00%		35.00%		0%	
j=5	17.00%		19.00%	48.00%		50.00%		0%	
j=6	19.00%		21.00%	46.00%		48.00%		0%	
j=7	33.00%		35.00%	10.00%		12.00%	21.00%		23.00%
j=8	33.00%		35.00%	12.00%		14.00%	19.00%		21.00%
j=9	33.00%		35.00%	32.00%		34.00%		0%	
j=10	53.00%		55.00%	12.00%		14.00%		0%	
j=11	40.00%		42.00%	25.00%		27.00%		0%	
j=12	54.00%		56.00%	11.00%		13.00%		0%	
j=13	37.00%		39.00%	28.00%		30.00%		0%	
j=14	52.00%		54.00%	13.00%		15.00%		0%	
j=15	47.00%		49.00%	18.00%		20.00%		0%	
j=16	36.00%		38.00%	29.00%		31.00%		0%	
j=17	0.00%		1.00%	66.00%		68.00%		0%	
j=18	22.00%		24.00%	43.00%		45.00%		0%	
j=19	49.00%		51.00%	16.00%		18.00%		0%	
j=20	43.77%		45.77%	21.23%		23.23%		0%	
j=21	54.54%		56.54%	10.46%		12.46%		0%	
j=22	46.07%		48.07%	18.93%		20.93%		0%	
j=23	36.47%		38.47%	28.53%		30.53%		0%	
j=24	34.88%		36.88%	30.12%		32.12%		0%	
j=25	43.02%		45.02%	21.98%		23.98%		0%	
j=26	31.95%		33.95%	19.57%		21.57%	12.48%		14.48%
j=27	44.92%		46.92%	20.08%		22.08%		0%	
j=28	58.14%		60.14%	6.86%		8.86%		0%	
j=29	25.05%		27.05%	10.40%		12.40%	28.55%		30.55%
j=30	42.63%		44.63%	22.37%		24.37%		0%	
j=31	36.00%		38.00%	19.00%		21.00%	9.00%		11.00%
j=32	36.00%		38.00%	29.00%		31.00%		0%	
j=33	29.00%		31.00%	20.00%		22.00%	15.00%		17.00%
j=34	48.00%		50.00%	17.00%		19.00%		0%	
j=35	16.00%		18.00%	21.00%		23.00%	27.00%		29.00%
j=36	57.00%		59.00%	8.00%		10.00%		0%	
j=37	25.00%		27.00%	40.00%		42.00%		0%	
j=38	30.00%		32.00%	0.00%		1.00%	35.00%		37.00%
j=39	28.00%		30.00%	18.00%		20.00%	18.00%		20.00%
j=40	15.00%		17.00%	17.00%		19.00%	32.00%		34.00%

Table 2: Constraints on the nutrient composition of fertilizer formulas

Let's now see how to satisfy these J = 40 outputs with N = 3 inputs (the lowest value found in the parametric quadratic program) whose optimal composition is to be determined.

4.2 Case Study Results

In this example, we have 129 variables and 280 constraints of which 240 are quadratic non convex. The other 40 constraints are those related to the requested quantities of fertilizers. These quantities depend on the surface of the cultivated areas as well as on the rate of fertilizer use. (The quantities shown in table 1 were found by assuming that the rate of fertilizer use is 100%). The solution of this problem is presented in Table 3.

Since the search for the optimal solution is guided by the maximization of the first input concentration in the final blends, the solver has managed to consume about 93726.30 tons from Input 1 (65.78 % of total demand). The solver (Xpress-Non Linear solver of Xpress-IVE of Fico) yields no solution by setting the number of inputs to two, and gives an infinity of solutions beyond three inputs. In this way, RB was able to find the minimum number of inputs (N=3) that allowed us to produce, under the same economic conditions, forty fertilizer formulas.
Although these formulas were established for optimal agriculture, the requested quantities are not fixed as the cultivated areas and the fertilizer use rate may change from one season to the next. This wouldn't be a problem though since these quantities will surely impact the proportion of inputs needed to meet demand but not their number and composition. To demonstrate this, we will first solve the model by taking identical demands ($D_j = 100, \forall j$) (see table 4). Subsequently, we will start from the optimal inputs proposed by this last model, but this time we take different demand types (those shown in table 1), to see how this change impacts input concentrations in total demand (see table 5).

Optimal composition of the inputs (α_{ci})

		Component			
		c=1	c=2	c=3	Filler
Input	i=1	66.0%	1.00%	0.00%	33.00%
	i=2	0.0%	0.51%	67.99%	31.50%
	i=3	0.0%	69.03%	0.00%	30.97%

Concentrations of inputs in total demand

		Total quantity	Inputs share	Total demand
Input	i=1	93726.30	65.78%	142492 Tons
	i=2	12088.28	8.48%	
	i=3	36677.44	25.74%	

Optimal quantities to be taken from the inputs (X_{ij})

		Input			Demand			Input			Demand
		i = 1	i = 2	i = 3	D (j)			i = 1	i = 2	i = 3	D (j)
Output	j=1	1833.50	0.00	289.50	2123	Output	j=21	28.25	0.00	4.75	33
	j=2	1443.00	0.00	0.00	1443		j=22	1185.01	0.00	440.99	1626
	j=3	10412.60	0.00	2008.37	12421		j=23	19.83	0.00	14.17	34
	j=4	328.15	0.00	308.85	637		j=24	105.11	0.00	82.89	188
	j=5	2.59	0.00	6.41	9		j=25	508.64	0.00	237.36	746
	j=6	23.23	0.00	49.77	73		j=26	13558.50	4853.84	7984.70	26397
	j=7	129.39	81.45	33.16	244		j=27	26268.30	0.00	10697.70	36966
	j=8	1945.68	1025.72	697.60	3669		j=28	6692.95	0.00	657.05	7350
	j=9	579.09	0.00	512.91	1092		j=29	3469.09	3567.47	1443.44	8480
	j=10	7.50	0.00	1.50	9		j=30	1288.67	0.00	618.33	1907
	j=11	73.18	0.00	41.82	115		j=31	287.88	76.94	135.18	500
	j=12	1680.85	0.00	300.15	1981		j=32	385.76	0.00	284.24	670
	j=13	4012.86	0.00	2778.14	6791		j=33	108.03	57.08	64.89	230
	j=14	11200.10	0.00	2488.91	13689		j=34	1181.82	0.00	378.18	1560
	j=15	35.64	0.00	12.36	48		j=35	272.73	426.48	300.79	1000
	j=16	143.94	0.00	106.06	250		j=36	255.67	0.00	30.33	286
	j=17	16.97	0.00	1103.03	1120		j=37	882.00	0.00	1274.00	2156
	j=18	292.36	0.00	511.64	804		j=38	1464.73	1556.25	0.03	3021
	j=19	174.64	0.00	51.36	226		j=39	363.64	233.07	203.30	800
	j=20	956.25	0.00	421.75	1378		j=40	108.18	209.97	101.85	420

Table 3: Optimal solution for potential demand types

Optimal composition of the inputs (α_{ci})

		Component			
		c=1	c=2	c=3	Filler
Input	i=1	66.00%	1.00%	0.00%	33.00%
	i=2	7.25%	1.00%	57.55%	34.20%
	i=3	0.00%	69.03%	0.00%	30.97%

Concentrations of inputs in total demand

		Total quantity	Inputs share	Total demand
Input	i=1	2355.83	58.90%	4000 Tons
	i=2	400.00	10.00%	
	i=3	1244.17	31.10%	

Optimal quantities to be taken from the inputs (X_{ij})

		Input			Demand		Input			Demand
		i = 1	i = 2	i = 3	D (j)		i = 1	i = 2	i = 3	D (j)
Output	j=1	86.36	0.00	13.64	100	j=21	85.61	0.00	14.39	100
	j=2	100.00	0.00	0.00	100	j=22	72.88	0.00	27.12	100
	j=3	83.83	0.00	16.17	100	j=23	58.33	0.00	41.67	100
	j=4	51.52	0.00	48.48	100	j=24	55.91	0.00	44.09	100
	j=5	28.79	0.00	71.21	100	j=25	68.18	0.00	31.82	100
	j=6	31.82	0.00	68.18	100	j=26	48.13	24.53	27.34	100
	j=7	47.95	38.82	13.23	100	j=27	71.06	0.00	28.94	100
	j=8	48.36	35.47	16.17	100	j=28	91.06	0.00	8.94	100
	j=9	53.03	0.00	46.97	100	j=29	34.35	51.84	13.82	100
	j=10	83.33	0.00	16.67	100	j=30	67.58	0.00	32.42	100
	j=11	63.64	0.00	36.36	100	j=31	55.14	18.41	26.46	100
	j=12	84.85	0.00	15.15	100	j=32	57.58	0.00	42.42	100
	j=13	59.09	0.00	40.91	100	j=33	43.23	28.84	27.93	100
	j=14	81.82	0.00	18.18	100	j=34	75.76	0.00	24.24	100
	j=15	74.24	0.00	25.76	100	j=35	21.19	49.41	29.40	100
	j=16	57.58	0.00	42.42	100	j=36	89.39	0.00	10.61	100
	j=17	1.52	0.00	98.48	100	j=37	40.91	0.00	59.09	100
	j=18	36.36	0.00	63.64	100	j=38	39.08	60.92	0.00	100
	j=19	77.27	0.00	22.73	100	j=39	41.09	33.92	24.99	100
	j=20	69.39	0.00	30.61	100	j=40	18.63	57.85	23.52	100

Table 4: Optimal solution for identical demand types

By setting demand at 100 tons by default, total demand for the 40 fertilizers is 4000 tons of which 58.9% is from the first input, 10% from the second and 31.10% from the third one.

Optimal composition of the inputs (α_{ci})

		Component			
		c=1	c=2	c=3	Filler
Input	i=1	66.00%	1.00%	0.00%	33.00%
	i=2	7.25%	1.00%	57.55%	34.20%
	i=3	0.00%	69.03%	0.00%	30.97%

Concentrations of inputs in total demand

		Total quantit	Inputs share	Total demand
Input	i=1	91702	64.36%	142492 Tons
	i=2	15274	10.72%	
	i=3	35516	24.92%	

The taken quantities from inputs i

		Input			Demand		Input			Demand
		i = 1	i = 2	i = 3	D (j)		i = 1	i = 2	i = 3	D (j)
Output	j=1	1833.50	0.00	289.50	2123	j=21	28.25	0.00	4.75	33
	j=2	1443.00	0.00	0.00	1443	j=22	1185.01	0.00	440.99	1626
	j=3	10412.62	0.00	2008.38	12421	j=23	19.83	0.00	14.17	34
	j=4	328.15	0.00	308.85	637	j=24	105.11	0.00	82.89	188
	j=5	2.59	0.00	6.41	9	j=25	508.64	0.00	237.36	746
	j=6	23.23	0.00	49.77	73	j=26	12705.03	6474.87	7217.10	26397
	j=7	117.01	94.71	32.28	244	j=27	26268.26	0.00	10697.74	36966
	j=8	1774.37	1301.38	593.25	3669	j=28	6692.95	0.00	657.05	7350
	j=9	579.09	0.00	512.91	1092	j=29	2912.64	4395.66	1171.71	8480
	j=10	7.50	0.00	1.50	9	j=30	1288.67	0.00	618.33	1907
	j=11	73.18	0.00	41.82	115	j=31	275.68	92.03	132.29	500
	j=12	1680.85	0.00	300.15	1981	j=32	385.76	0.00	284.24	670
	j=13	4012.86	0.00	2778.14	6791	j=33	99.44	66.33	64.24	230
	j=14	11200.09	0.00	2488.91	13689	j=34	1181.82	0.00	378.18	1560
	j=15	35.64	0.00	12.36	48	j=35	211.88	494.14	293.99	1000
	j=16	143.94	0.00	106.06	250	j=36	255.67	0.00	30.33	286
	j=17	16.97	0.00	1103.03	1120	j=37	882.00	0.00	1274.00	2156
	j=18	292.36	0.00	511.64	804	j=38	1180.69	1840.31	0.00	3021
	j=19	174.64	0.00	51.36	226	j=39	328.73	271.36	199.91	800
	j=20	956.25	0.00	421.75	1378	j=40	78.23	242.99	98.78	420

Table 5: Input consumption when combining optimal composition obtained for identical demand types and different demand types of table 1

We note that starting from the same inputs and having changed demand structure, the proportion of the inputs in total production (142492.08 tons) is no longer the same (64.36% of input 1, 10.72% of input 2 and 24.92% of input 3). The concentration flows, therefore, depend on demand structure: changing it may imply a diversity of flows. An analysis of the results of the RB model shown in Table 2, revealed that the

share of the first input in total demand increased slightly (65.78% of input 1, 8.48% of input 2 and 24.92% of input 3). This means that when defining new inputs based on a new demand structure, the structure of input flows may vary slightly.
Basically, RB is a new, valuable, approach for fertilizer producers: instead of responding to a wide variety of fertilizer requirements by blending a multitude of them, they can now do so by blending a very limited number of components that may not be ready-to-use fertilizers. To show the benefits of this new approach compared with the existing way of producing fertilizers from a blend of fertilizers using the traditional blending model (§3.1), we have tested this approach to find the minimum number S of fertilizers (inputs) that will be able to produce the 40 fertilizer formulas (outputs). With 32 inputs (from the 40 potential ones) one is able to produce the 8 other outputs (see table 6). Under S=32 inputs, some outputs are impossible to produce.

		Share of the 3 selected inputs out of 32			Demand D_j
		$i = 1$	$i = 17$	$i = 40$	
Unused inputs (N-S = 8 inputs)	$j = 3$	98.21%	1.79%	0%	100%
	$j = 12$	100%	0%	0%	100%
	$j = 14$	96.43%	3.57%	0%	100%
	$j = 15$	87.50%	12.50%	0%	100%
	$j = 18$	42.86%	57.14%	0%	100%
	$j = 30$	79.70%	20.30%	0%	100%
	$j = 34$	89.29%	10.71%	0%	100%
	$j = 35$	8.77%	9.42%	81.82%	100%

Table 6: The selected inputs required for the production of the other 8 outputs

5. Conclusion

Reverse blending is a very efficient way to deliver customized fertilizers that exactly satisfy the needs of sustainable agriculture while reducing production and transportation issues. The data used, deduced from *Fertimap* by linear regression may not be the right ones but they do not put the approach into question. The theoretical feasibility of this innovative approach has now definitely been demonstrated. For it to be operational, several complementary studies are required: first, the rules for establishing fertilizer needs, which have been determined empirically must be validated by agronomists. In addition, the census of these needs for an entire country, that depends on the “soil / crop” couple is indispensable and involves cooperating with agronomists. Similarly, chemists shall be required to assess the constraints related to chemical feasibility of the inputs. Finally, we will have to work on large instances and study the impact of this transformation of production processes on OCP’s supply chain and eventually design new distribution schemes. These will be based on several scenarios describing the impact of huge transportation flow consolidation as well as the nature, location and sizing of post-manufacturing blending facilities to be implemented in Africa, one of OCP’s largest markets.

6. Bibliography

Aldeseit, B. (2014). Linear Programming-Based Optimization of Synthetic Fertilizers Formulation. Journal of Agricultural Science *6*.
Ashayeri, J., van Eijs, A.G.M., and Nederstigt, P. (1994). Blending modelling in a process manufacturing: A case study. European Journal of Operational Research *72*, 460–468.
Babcock, Bruce, M. Edward Rister, Ronald D. Kay, et Jo Ann Wallers. (1984). « Identifying Least-Cost Sources of Required Fertilizer Nutrients ». *American Journal of Agricultural Economics* 66, nº 3: 385-91.
Barrie M. Cole and Steven Bradshaw. (2015). An optimisation methodology for a supply chain operating under any pertinent conditions of uncertainty - an application with two forms of operational uncertainty, multi-objectivity and fuzziness. International Journal of Operational Research: Vol 23, No 2.
Bown HE, Watt MS, Clinton PW, Mason EG. (2010). Influence of ammonium and nitrate supply on growth, dry matter partitioning, N uptake and photosynthetic capacity of *Pinusradiata* seedlings. Trees; 24(6):1097–1107.

Brady, N. C. and R. R. Weil. (2004). Elements of the Nature and Properties of Soils, 2nd Edition. Atlanta, GA: Prentice Hall.
Cottenie A. (1978). Soil and plant testing as basis of fertilizer recommendations. FAO soils bulletin, 38/2.
Darryl D. Warncke, Crop & Soil Sciences Dept. (2000). Michigan State University Extension, Extension Bulletin E498.
Epstein E, Bloom AJ. (2004). Mineral nutrition of plants: Principles and perspectives (Second Edition). Sunderland, MA: Sinauer Associates, Inc.; 402p.
FAO – L'agriculture mondiale à l'horizon 2050. (2009). Comment nourrir le monde 2050.
Kalavati Prajapatil, H.A. Modi. (2012). The importance of potassium in plant growth - a review. Indian Journal of Plant Sciences, ISSN: 2319-3824
Lima, R.L.S., Severino, L.S., Sampaio, L.R., Sofiatti, V., Gomes, J.A., and Beltrão, N.E.M. (2011). Blends of castor meal and castor husks for optimized use as organic fertilizer. Industrial Crops and Products *33*, 364–368.
Loh, S.K., Cheong, K.Y., Choo, Y.M., and Salimon, J. (2015). Formulation and optimisation of spent bleaching earth-based bio organic fertiliser. Journal of Oil Palm Research *27*, 57–66.
López-Bucio J, Cruz-Ramírez A, Herrera-Estella L. (2003). The role of nutrient availability in regulating root architecture. Current Opinion in Plant Biology, 6(3):280–287.
M. Ines Minguez, Carlos Romero and Joaquin Domingo. (1988). Determining Optimum Fertilizer Combinations through Goal Programming with Penalty Functions: An Application to Sugar Beet Production in Spain. J. Opl Res. Soc. Vol. 39, No. 1, pp. 61-70.
O'Brien, J.A., Vega, A., Bouguyon, E., Krouk, G., Gojon, A., Coruzzi, G., and Gutiérrez, R.A. (2016). Nitrate Transport, Sensing, and Responses in Plants. Molecular Plant *9*, 837–856.
O. Traoré, B. Koulibaly and D. Dakuo2. (2007). Effets comparés de deux formes d'engrais sur les rendements et la nutrition minérale en zone cotonnière au Burkina Faso. TROPICULTURA, 25, 4, 200-203.
Pratt, C.J. (1965). CHEMICAL FERTILIZERS. Scientific American *212*, 62–75.
Sheridan, Richard C. (1979). « Chemical Fertilizers in Southern Agriculture ». *Agricultural History* 53, nº 1: 308-18.
Srichaipanya, Wichian, Pramin Artrit, et Anucha Sangrung. (2014). « Fertilizer quality control of a bulk-blending plant using intelligent systems ». *Suranaree Journal of Science & Technology* 21, nº 3.
Stoorvogel, J.J., Smaling, E.M.A., and Janssen, B.H. (1993). Calculating soil nutrient balances in Africa at different scales. Fertilizer Research *35*, 227–235.
SYERS, J.K., and SPRINGETT, J.A. (1984). Earthworms and soil fertility. Plant and Soil *76*, 93–104.
Watson, C. a., Atkinson, D., Gosling, P., Jackson, L. r., and Rayns, F. w. (2002). Managing soil fertility in organic farming systems. Soil Use and Management *18*, 239–247.
Watson, C. a., Stockdale E., Lois P. (2009). Soil Analysis and Management. Institute of Organic Training & Advice, Cow Hall, Newcastle, Craven Arms, Shropshire SY7 8PG.
Zingore, S., Murwira, H.K., Delve, R.J., and Giller, K.E. (2007). Influence of nutrient management strategies on variabililerty of soil fertility, crop yields and nutrient balances on smallholder farms in Zimbabwe. Agriculture, Ecosystems & Environment *119*, 112–126.