The Need and Determination of Safety Stocks in a Deterministic Universe due to Lot-Sizing Problems in a Supply chain Dedicated to Customized Mass Production

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In a supply chain designed for the production of a wide variety of goods, the production schedule creates an ordered list of all the alternate components that will be mounted on the same work station of the chain. Supplying these components may have to respect some transportation-related constraints. From there systematically follows a time-gap between orders and deliveries. This mechanism, commonly called rank-change, may be analyzed with the help of the multinomial distribution associated with these alternate components. Only a simulation approach is relevant because an analytic approach is not possible. Such an analysis allows assessing the importance of the need to form safety stocks because of lot-sizing, even though requirements are known.

Introduction

This article demonstrates the need of a new kind of safety stock and gives the method to calculate its level. We study an elementary supply chain consisting of a production unit (the customer) configured as an assembly line that allows mass production of various products (e.g., an automotive production line) and another unit (the supplier) that produces alternate components (e.g., car engines) or optional components (e.g., sunroofs) assembled on a work station of this line and contributing to the required diversity (Anderson & Pine, 1997). In the context of our study, optional components are a particular type of alternate components.

For several weeks, the customer daily production (*n*) is predetermined by the opening duration of the line (stable on this horizon) and the line's cycle time. The total daily demand of alternate components to order from the supplier is thus known. From the final demand (vehicle orders), the industrial customer determines the production schedule on a horizon of K days, which in turn determines, on this horizon, the ordered list of the alternate components to assemble on the assembly line work station where they are mounted. Beyond this horizon K, the customer has information only about the average structure of the demand. At the beginning of every *t* day, the schedule of days *t* to *t*+K-1 is kept and the new orders of day *t*+K are sequenced according to the revolving planning logic. The ordered list of all alternate components to mount determines the order of demand of these components. For the purpose of our study, it is assumed that the requirements are daily. They are designed to fulfill the sequenced demand of alternate components. In our case, the lead-time λ is shorter than K so as to stay in a context in which the sequence of the alternate components to mount is known when the daily orders are sent.

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In this context, the [illegible] that are submitted must be considered as firm. We will assume that their delivery was guaranteed. The impact of batch constraints on the supply chain control does not seem to be evoked in the literature. The lot-sizing influence on capacity due to scheduling is well known. The capacity is reduced by the set-up time of a reference, possibly depending on the sequence of references to produce (White & Wilson, 1977). The optimal sizes for the batches result from the minimization of a cost function that includes possession and set-up costs. The determination of the size of the batches to produce can be constrained by conditioning or by the limited storage near a workstation that mounts alternate components. Conditioning also plays a role in certain issues regarding transportation: it influences the transportation means capacity, but not the orders assessment to submit to the supplier. Therefore we will broach the effect of lot-sizing constraints on the management of the various logistical flows, assuming that deliveries match with the orders and were made on time.

This problem will be analyzed in the context of a supply of alternate components to an assembly line but the same problem occurs when the production schedule of an assembly line is constrained by the fact that a work station assembling alternate components is supplied [illegible]equence of pallets each pallet [illegible]ing the same component. The [illegible]ection will ex[illegible] integration of the lot-sizing constraint can create rank-changes; a second section will detail the procedure to create safety stocks in a number that will allow dealing with these rank-changes.

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Analysis of the Rank-Changes Caused by the Lot-Sizing

First, we will explain the concept of rank-changes, then we will outline the rank-change function. We will focus here on the integration of the lot-sizing constraints on supply. Further on, we will see that the same mechanism applies to production when the scheduling relies on batch deliveries of alternate components that are to be mounted on the same work station of the line because the diversity of the supply is impossible to ensure at the line side.

Overview of the rank-change mechanism

The demand of alternate components to mount at a workstation on the assembly line is expressed by the ordered list S_1, periodically updated. The customer orders M engines every θ days and receives, with the same periodicity, a shipment of M engines, ordered λ days before (λ being the supplier's lead time). It is understood that conditioning constraints force the grouping of delivered references by lots of m identical products, which leads to a [illegible]ry of .γ = M/m batches. It is assumed that deliveries are completed on time and perfecty match the demand. Lot-sizing has two consequences:

- At the arrival of each delivery one must perform a reconstruction of the sequence of M alternate components respecting the order of assembly.
- The batch of M delivered components has practically no chance to coincide with the batch that will be assembled; the number of missing components corresponds to the number of surplus components.

The existence of a rank-change mechanism is known in the case of the handling of a quality issue on a vehicle production line, taking effect through the removal of the vehicles to reprocess for a variable time before their reinsertion on the line (Giard, Danjou, & Boctor, 2001a, 2001b; Giard, 2003). They

Figure 1
Origin of Rank-Changes

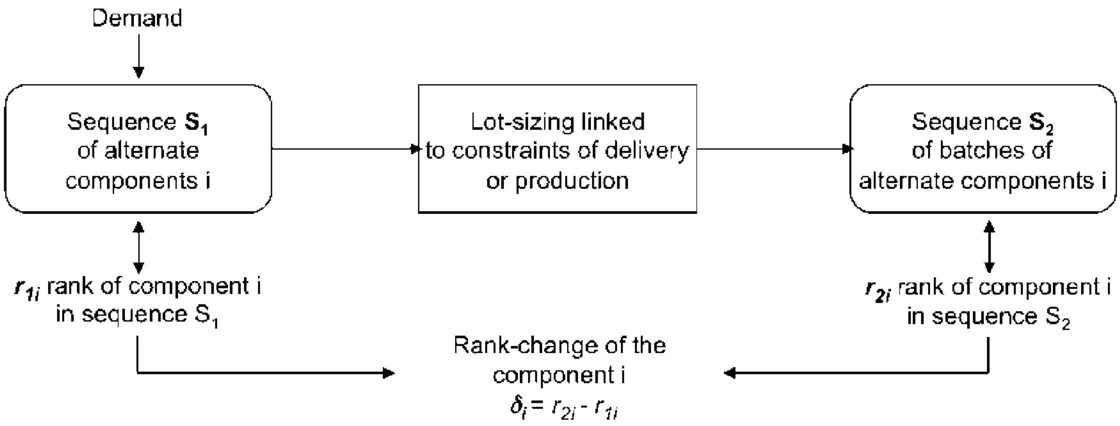

Table 1
Distribution of the Probability by Product

Product	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
Product	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	

refer to rank-changes as the “difference, positive, negative or null, between the final rank of a unit at the exit of the work station and its rank prior to entering this station (a negative rank-change thus corresponds to a situation of advance)” (Giard, Danjou, & Boctor, 2001a, p.260). Figure 1 shows the rank-change mechanism in our example.

To define the sequence S_2, we need to start from sequence S_1 and take into account the evolution of the projected stock level affected by the following demands of S_1. If the consideration of the demand of rank *i* into sequence S_1 leads to a negative stock level, we need to avoid a stock-out with a batch delivery of the concerned reference. Two main methods can be adopted:

- Deciding to launch a batch of *m* components when their projected stock level equals *–m*.
- Deciding to launch a batch of *m* components as soon as their projected stock level becomes negative (then amounting *m-1*).

Following the first method, restricted to firm demands and with a focus on product i that was not yet launched and whose projected stock level just turned negative, the probability that a batch of m units of the same completed emands of # ned by the ribution NB (*m-1*, #, p_i). This probability can be significant and lead to a stock-out. For instance, the risk that a batch of six products, with a probability of demand p = 0.9% and # = 1,000 unaddressed requirements, should not be completed amounts to 11.3%. In a context in which a stock-out is hardly acceptable because of high-line stoppage cost, the second lot-sizing method is necessarily privileged. Now that the concept of rank-change is defined, we shall study its function.

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Study of the rank-change function

To understand the following results, we shall base our approach on the probability of the rank-change of a product, with a variable δ_i corresponding to the number of ranks that a product won or lost at delivery compared to its consumption. There is no analytic way to determine the distribution of the rank-change probability. Only an approach the Monte C drawing re in an envir precise specif distribution,

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The study of a rank-change function was conducted on the basis of a vehicle assembly line on which a workstation mounts the engine selected by the final customer, who can choose among 19 engines P_i (alternate components). The engines are produced in another plant in which there is no set-up time between references on the production line. The engines are then shipped to assembling plants in packages of six identical engines. Let us assume that the distribution of ne pi is tha

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We proceeded to the simulation of a randomly generated sequence of 3 billi the distr rom Tabl first components are excluded from the simulation because they represent the transient state coming before the steady state, which is our interest (this transient state was willingly overestimated). The packages can hold up to six identical products. Thus the size of the batches is six. Concerning the handling of the product of rank y in sequence S_1, the rule to create S_2 is as follows:

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- If the required product is in stock, it is taken out, thus, its stock level is lowered by one unit.
- Otherwise, the production of a batch of six products is launched, and the batch is included in the stock, whose level is increased by five.
- Move on to product *y+1 in* S_1.

Table 2 su results for each product P_i having the probability of use p_i, their average $\overline{\delta}_1$ of rank-changes, and the standard deviations $\sigma_{\sigma i}$ of the rank-changes for each product.

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According to Figure 1, a negative value for rank-change $\overline{\delta}_1$ means that the recently arrived product is advanced regarding its positioning in the initial demand; in the example, product 10 is systematically in advance. A positive value means that the product arrived with delay compared to its initial positioning, as product 6 illustrates. The weighted sum of average rank-changes by probability of use pi is null.

The rank-change curves of the products are not identical: the

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1. In a steady state, there is necessarily compensation between won and lost ranks. In a sample, the observed average (here $\overline{\delta}_1$*) coincides only exceptionally with the mathematical expectation (0).*

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Table 2
Rank-Change Expectations and Standard Variations of Rank-changes, per Product

Product	***P₁***	***P₂***	***P₃***	***P₄***	***P₅***	***P₆***	***P₇***	***P₈***	***P₉***	***P₁₀***
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
	-98.7	6.99	-65.2	-230	-230	39.4	29.44	-145.7	-230	-576
	134	36.8	103	257	258	9.13	16.75	178.1	259	585.7
Product	***P₁₁***	***P₁₂***	***P₁₃***	***P₁₄***	***P₁₅***	***P₁₆***	***P₁₇***	***P₁₈***	***P₁₉***	
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	
	-586	-229	-36.5	8.95	7.52	-24.64	6.40	25.20	-48.2	
	588	256	76.3	34.6	36.1	65.44	36.80	20.18	87.62	

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Figure 2
Evolution of the mathematical expectation of rank-change of products according to their probabilities p_i

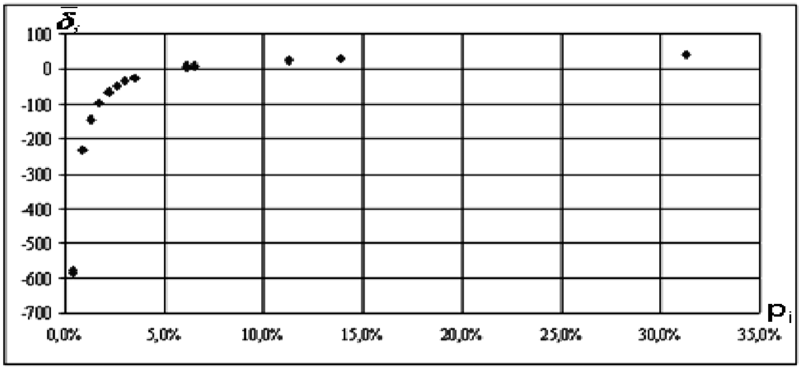
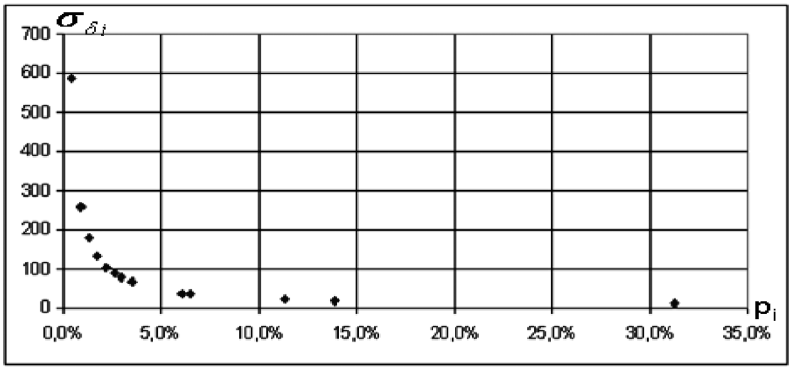


Figure 3
Evolution of the standard variation of rank-change of products according to their probabilities p_i



mathematical expectation of the rank-change of products varies in the same way as their probability of use (Figure 2), whereas the standard deviations of product rank-change vary in an inverse order of their use probability (Figure 3). Therefore, those products in low demand are delivered somewhat ahead of schedule, whereas the delivery of products in high demand is quite delayed.

The rank-change function of a product *i* depends on both the number *I* of products (*I* = *1,..,I*) and on the distribution of probabilities p_i . Figure 4 details the occurrences of rank-change for a product with low demand, and Figure 5 shows a product with the highest demand; each is outlined with different scales. In the case of a product with low demand, the rank-change is essentially negative, with widely spread negative values (up until 1,000 won ranks, even 2,000 at certain points) and a very heterogeneous occurrence of values: when there is a peak for a small amount of lost ranks, then the occurrence of rank-changes decreases as the number of won ranks increases. In the case of a product with high demand, rank-changes are positive, but the number of lost ranks remains quite low (approximately no more than 75). The occurrence of rank-changes is symmetrical to an approximate value of 40 lost ranks, with no high difference between the occurrences.

Lot-sizing creates rank-changes, which in turn can create stock-outs. The customer must thus form safety stocks (at the vehicle assembling plant) to avoid lot-sizing on supply.

Need of a Safety Stock to Counter the Side Effects of Rank-Changes Induced by Lot-Sizing

We shall first explain the need of a safety stock to face the lot-sizing in indice i (et non 1)

Figure 4
Curve of rank-changes for product P_{12} (p_{12} = 0.9%; $\overline{\delta}_{12}$= -229)

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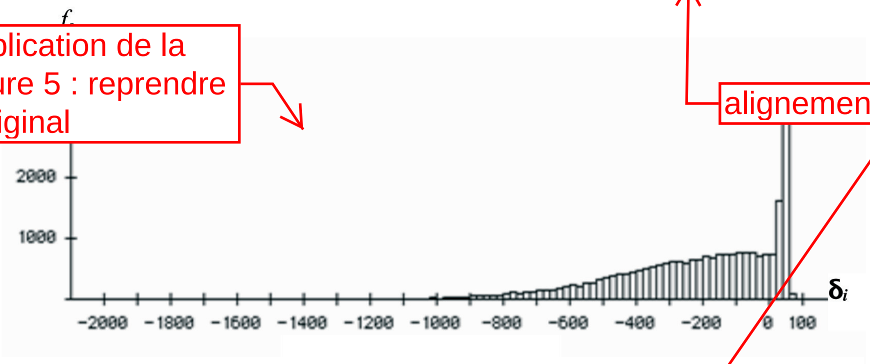
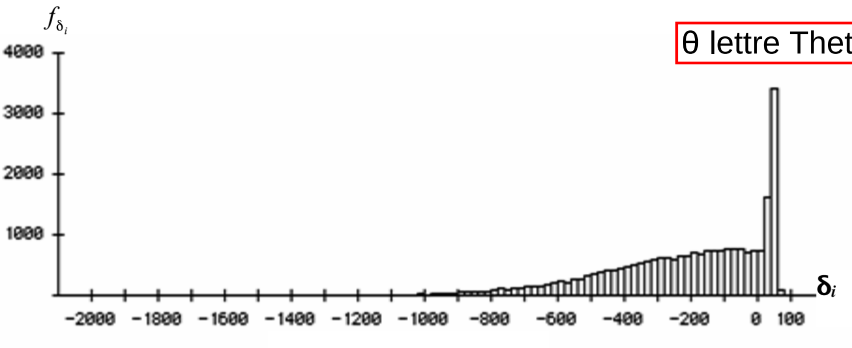


Figure 5
Curve of rank-changes for product P_6 (p_6 = 31,3%; $\overline{\delta}_6$= 39,4)



a deterministic universe, and then its determining factors will be analyzed.

Explanation of the need

We have seen that the probability of the delivered products matching those that need assembling is very low because of the rank-changes induced by the lot-sizing. To avoid an interruption of supply, one needs to maintain a safety stock for every complete product. These safety stocks are fed by early products, which consequently lower the amount of products left to create the safety stock. Late products refill the safety stocks from which products were taken to ensure punctual assembly. We have seen that products with low demand are ordered (and thus delivered) somewhat in advance, but the high dispersal force the creation of safety stocks. On the contrary, the strongly demanded engines are delivered with average delay, but their dispersal minimizes the need for safety stocks.

It is important to accurately gauge so that every rank-change does not systematically create an interruption of supply and, consequently, the creation of a safety stock: the supplier's deliveries concern γ delivered batches with a lead time θ. Thus it is not sequence S_2 that determines interruptions of supply but the constitution of a group o

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In this t possibl s from the rank-change function of a reference on the basis of the multinomial distribution. Only a simulation can be performed. This procedure allows highlighting tendencies of the impact of certain factors, a is likely impact of

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Every simulation was performed on a demand of 6 billion products in order to empirically obtain safety stocks SS_1 with an insignificant stock-out probability. At the beginning of the simulation, the amount of available products was set to a value W. The first delivery is made immediately before the first product is taken. This first delivery corresponds to the θ first products of sequence S_2 and the first withdrawn product corresponds to the first product of sequence S_1. Each safety stock level is calculated as the difference between W and the lowest inventory level during the simulation because W must be high enough to avoid an empty stock.

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In a deterministic universe, safety stocks depend on the rank-changes and on the lead time ?. The rank-changes depend in turn on the range I of products and on their probabilities pi. We will evaluate the safety stock level by a simulatory approach of the steady state. We shall study the impact of various factors on the safety stock in a context of a lot-sizing caused by transportation constraints. First of all, we shall analyze the impact of probabilities on the safety stock. On the basis of the data in Table 1, we obtain by those simulations the following safety stock amounts (Table 3).

The safety stock of products varies in the same way as their probabilities (Figure 6), which was priori. In this industrial example, an approximately linear relation can be noticed (ρ^2= 0.983 ; SS_i = 0.95889 p_i + 3.008).

Afterward, we shall study the impact of the dispersal of fety stock rage. The various products can have equal probabilities: this is a case of equal probabilities of demand. They can also be extremely different; all situations are possible. We thus proceeded to new simulations, replacing the observed probabilities by distribution $p_i{=}p_i$ x $k^{i-1}{+}b$ with a constraint of $\sum_i p_i{=}1$. We shall outline various

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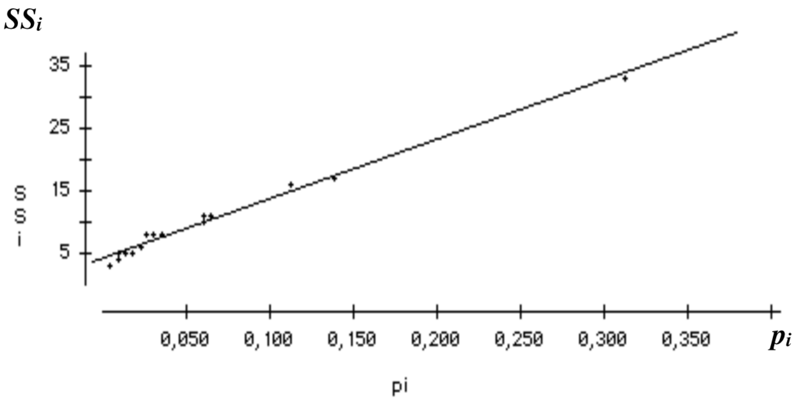
Table 3
Safety stocks related to rank-change

Product	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3%	13.9%	1.3%	0.9%	0.4%
SS_i	5	10	6	4	4	33	17	5	4	3
Product	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	
SS_i	3	5	8	11	11	8	10	16	8	

Figure 6
Evolution of the safety stock SS_i according to the probabilities p_i



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Dependent variable is:    SSi
No Selector
R squared = 98,4%      R squared (adjusted) = 98,3%
s =  0,9364  with  19 - 2 = 17  degrees of freedom

Source       Sum of Squares   df    Mean Square   F-ratio
Regression   891.094           1        891.094    1.02e3
Residual      14.9061         17        0.87683

Variable    Coefficient    s.e. of Coeff    t-ratio    prob
Constant    3.95323        0.2669           14.8       ≤ 0,0001
pi          95.8887        3.008            31.9       ≤ 0,0001
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possible curves, descending and of exponent [illegible] = 0.005; [illegible] the low[illegible] product. [illegible] controlled decrease. The closer it is to 1 *k*, the more equal probabilities of component demands are. Figure 7 illustrates the distribution of these probabilities.

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The following factor of variation corresponds to the supply periodicity *θ*. We proceeded to an analysis of its impact by repeating the same simulation with different values for *θ*. This analysis was performed using three structures of demand, with 5, 10, and 19 possible products, plus an equal probability of demand between those products (yielding the worst case for safety stock). We note that when the lead time increases, the average safety stock decreases in a fairly constant way (Figure 9).

The use of three different structures of demand allows highlighting two phenomenons. The first impression, on which we will further elaborate, is that when the variety of products increases, the average safety stock decreases, which seems logical because when the variety is lower, lot-sizing induces rank-changes of necessarily lesser importance. The second impression is that of a similarity between the variation profiles of the safety stock compared to the lead time for various values of *I*: the series of points for what various values of *I* really look alike.

We proceed to a more in-depth analysis of the impact of variety *I* on the average safety stock for equal distributions, as illustrated in Figure 10. The more products, the greater the average safety stock needs to be in order to counter rank-change side effects. However, this increase is not proportional to the increase of variety: the increase is strong for values of *I* under 8 included, then slightly less pronounced for I between 8 and 12, and the average safety stock closing in to an asymptote for *I* greater than 12. This means that an increase of the variety of products has a strong impact on the average safety stock for *I* ≤ 12 in our example, but beyond those values I, adding one or several alternate components bears no consequence on the average safety stock lever (if properly calculated).

Looking more closely at the distribution of demand in our industrial case, we notice that four products have very few orders: engines 10 and 11 have a probability of 0.4% and engines 9 and 12 of 0.9%. There are neither delay nor set-up costs related to a reference change on the production

Figure 7
Example of curves of probability distribution

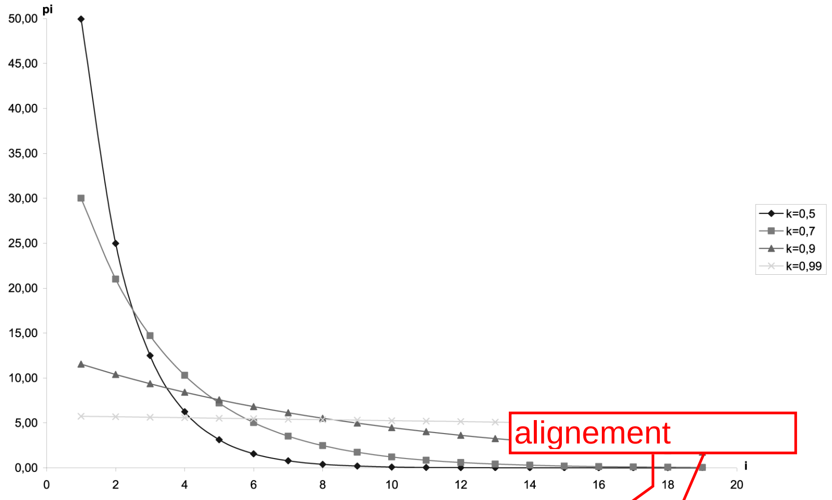
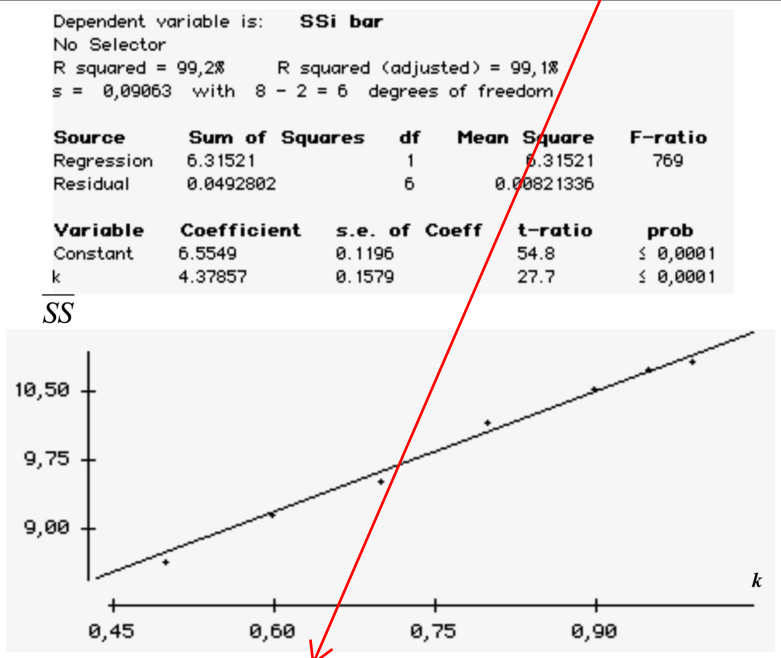


Figure 8
Evolution of the average safety stock $\overline{SS}$, depending on *k*



```
Dependent variable is:    SSi bar
No Selector
R squared = 99,2%     R squared (adjusted) = 99,1%
s =  0,09063  with  8 - 2 = 6  degrees of freedom

Source        Sum of Squares    df    Mean Square    F-ratio
Regression    6.31521            1    6.31521        769
Residual      0.0492802          6    0.00821336

Variable     Coefficient    s.e. of Coeff    t-ratio     prob
Constant     6.5549         0.1196           54.8        ≤ 0,0001
k            4.37857        0.1579           27.7        ≤ 0,0001
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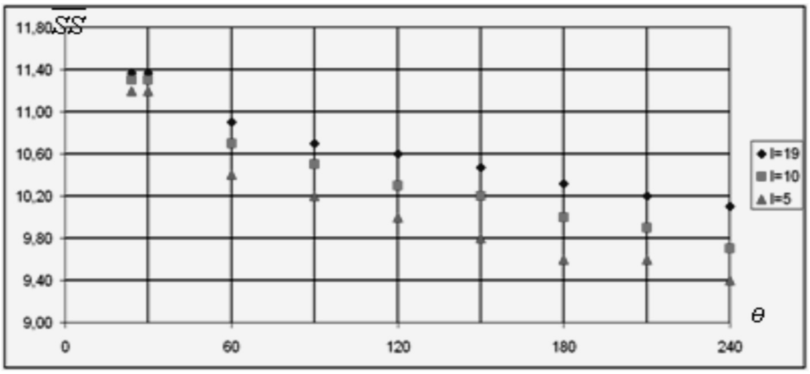
Figure 9
Average safety stock $\overline{SS}$ depending on θ for various values of *I*



line. Removing these four products makes no change on the average safety stock: it is the same whether 15 or 19 references are produced. Thus removing the offer of these four products is not advantageous to either the customer (average safety stock does not decrease) or the supplier (no reference change cost).

Safety stocks are also created to counter other hazards: quality problems in a production context, variation of transportation time, or modification of the firm sequence of requirements. These are explicitly excluded from the analysis, but it is obvious that the integration of a combination of disruptions leads to a mutualization of risks. The safety stock required to face multiple disruptions is thus less than the sum of all required safety stocks if the disruptions are considered separately.

We illustrate the impact of a combination of disruptions on the safety stock with an example of rank-changes caused by lot-sizing issues at production. The analysis of this type of lot-sizing is similar to that of delivery constraints. The submitted information concerns batches between the selected unit (engine manufacturing plant) and its provider. Several technical constraints, such as limited space at the line side of a station handling big components, can lead to production scheduling in a sequence of successive batches. The difference with the previous situation (procurement) is that these batches may not concern the same product, but products sharing a common specification, such as containing the same component (in our case, the same crankcase used by several different engines).

This lot-sizing is slightly more complex. On the one hand, it implies submitting information about batches of components to deliver to the supplier. On the other hand, it leads to the creation of a second lot-sizing of the production of the designated unit, which is not homogeneous because the batch

Figure 10
Average safety stock $\overline{SS}$ depending on variety I

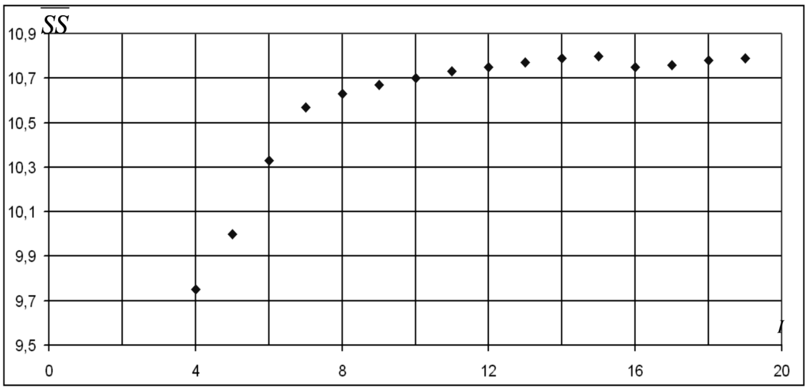


Table 4
Decrease of the safety stock induced by a mutualization of disruptions

Product	P_1	P_2	P_3	P_4	P_5	P[illegible]	[illegible]	[illegible]	P_9	P_{10}
p_i	1.7%	6.1%	2.2%	0.9%	0.9%	31.3[illegible]	[illegible]	[illegible]	0.9%	0.4%
	6	6	6	6	6	18	12	6	6	6
	1	2	1	1	1	3	2	1	1	0
	7	8	7	7	7	21	14	7	7	6
	6	8	6	6	6	18	12	6	6	6
Product	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	**Total**
p_i	0.4%	0.9%	3%	6.5%	6.1%	3.5%	6.1%	11.3%	2.6%	100%
	6	6	6	6	6	6	6	12	6	138

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thus created contains various products. Table 4 gives an illustration of this. We simulate the impact on the safety stock of a separate then simultaneous integration of the following:

- the rank-change related to production constraints: lot-sizing of the engines by batches of 24 identical crankcases
- quality issues: percentage of rejection after production amounting to 2%

Once again, the safety [illegible] by the custome[illegible] manufacturing plant) who has to own a safety stock of engines. The simultaneous integration of both disruptions allows decreasing the safety stock required to avoid stock-out by.

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Managerial implications

The supply chain is subjected to multiple risks. To avoid stock shortages, factories constitute safety stocks of the components to be assembled. When the factory can assemble on order and its supplier can produce on order, synchronous production is possible. When the differentiation of the components is made in the supplier's plant, it is often [illegible]ry to adapt the containers [illegible]hapes of the products to be [illegible]d. Batch constraints due to an efficiency search of transportation then enforces to constitute safety stocks even if the problem is in a deterministic universe. Most of the time, the need and importance of these stocks are not perceived because they were created to face various risks. From a managerial point of view, the existence of this kind of stock related to batching considerations is important to understand as it involves limits to stocks reduction. In addition, it poses the problem of the localization of the differentiation: if made at the supplier plant, it may multiply the types of containers and increase the transportation costs; made at the customer plant, the transportation cost may be reduced, due to standardization of containers, but the differenciation cost may be higher at the customer plant than at the supplier's one (due to time cycle constraints). A new trade-off has to be considered in the supply chain.

Conclusion

We have studied synchronization and decoupling of the control of the last two links of a supply chain dedicated to a customized mass production. Taking into account lot-sizing in a deterministic universe entails the creation of safety stocks. The explanatory factors of their importance were analyzed.

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