ELECTRE TRI-C: A Multiple Criteria Sorting Method Based on Central Reference Actions

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MCDA, Data Mining, and Rough Sets

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**Sorting problematic**: the set of categories emerges naturally from the decision aiding context through an interaction process with the decision-makers.

**Nature of the categories**: each category is defined in order to assign actions which will be subject to the same treatment or analysis.

**Absolute evaluation**: the assignment of an action only takes into account the intrinsic evaluation of this action on all the criteria and does not depend on nor influence the category to which another action should be assigned.
### Example (Credit analysis)

- **Actions**: Credit demand files
- **Categories**:
  - Accepted without additional information
  - Accepted with additional information
  - Sent to a particular department for further analysis
  - Rejected under certain conditions
  - Rejected with no conditions at all

### Example (Medical diagnosis)

- **Actions**: Patients waiting for treatment
- **Categories**: Set of pathologies studied
Profile limits

- Each category is bounded by a lower and an upper profile.
- The well-known method called up to now Electre Tri based on profile limits, or boundary actions, will be designated here by Electre Tri-B.

Central reference actions

- Each category is defined by a central reference action.
- Electre Tri-C is, therefore, the designation of the procedures based on central reference actions.
Key concepts
Actions, criteria, and credibility index

- $a_1, a_2, \ldots$ are the potential actions. The set of such actions, $A$, can be partially known \textit{a priori}.

- $F = \{g_1, \ldots, g_j, \ldots, g_n\}$ is a coherent family of criteria, with $n \geq 2$.

- $\Omega_j(a, a')$ is the advantage of $a$ over $a'$ on criterion $g_j \in F$,

$$
\Omega_j(a, a') = \begin{cases} 
  g_j(a) - g_j(a') & \text{if } g_j \text{ is to be maximized} \\
  g_j(a') - g_j(a) & \text{if } g_j \text{ is to be minimized}
\end{cases}
$$

- Each criterion $g_j \in F$ will be considered as a \textit{pseudo-criterion}.

- $\sigma(a, a')$ is the credibility of the comprehensive outranking of $a$ over $a'$ when taking all the criteria from $F$ into account.
Basic assumptions

$C = \{ C_1, \ldots, C_h, \ldots, C_q \}$ is the set of pre-defined and ordered categories, where $C_1$ is the worst category, and $C_q$ the best one, with $q \geq 2$.

Each category $C_h$ is defined by a central reference action $b_h$, $h = 1, \ldots, q$.

$B = \{ b_0, b_1, \ldots, b_h, \ldots, b_q, b_{q+1} \}$ is the set of $(q + 2)$ reference actions.

$b_0$ is a particular reference action with the worst possible evaluation $g_j(b_0)$ on criterion $g_j$, for all $g_j \in F$.

$b_{q+1}$ is a particular reference action with the best possible evaluation $g_j(b_{q+1})$ on criterion $g_j$, for all $g_j \in F$. 
### Definition (Dominance)

The set of reference actions, $B$, fulfills the (strict) dominance relation if and only if

$$\forall j, \Omega_j(b_{h+1}, b_h) \geq 0 \text{ and } \exists j, \Omega_j(b_{h+1}, b_h) > 0; h = 0, \ldots, q.$$  

### Definition (Strict separability condition)

The set of reference actions, $B$, fulfills the strict separability condition if and only if

$$\Omega_j(b_{h+1}, b_h) > p_j; j = 1, \ldots, n; h = 0, \ldots, q.$$
Structural requirements

1. **Conformity**: Each central reference action, $b_h$, must be assigned to the category, $C_h$, $h = 1, \ldots, q$.

2. **Monotonicity**: If an action $a$ strictly dominates $a'$, then $a$ is assigned to a category at least as good as the category $a'$ is assigned to.

3. **Homogeneity**: Two actions must be assigned to the same category when they compare themselves in an identical manner with the reference actions.

4. **Stability**: After a modification of the set $B$ by applying either a merging or a splitting procedure, the non-adjacent categories to the modified ones will remain with the same actions as before the modification.
Stability requirement

Definition (Basic modification procedures)

1. **Merging procedure**: The distinction between two consecutive categories, $C_{h-1}$ and $C_h$, will be ignored by introducing a new central reference action, $b'_h$, such that:
   - $\Omega_j(b'_h, b_{h-1}) \geq 0$, for all $g_j \in F$
   - $\Omega_j(b_h, b'_h) \geq 0$, for all $g_j \in F$

2. **Splitting procedure**: The category $C_h$ can be split into two new consecutive categories by introducing two new central reference actions, $b'_h$ and $b''_h$, such that:
   - $b_{h+1} \Delta_F b''_h$, $b''_h \Delta_F b'_h$, and $b'_h \Delta_F b_{h-1}$
   - $\Omega_j(b''_h, b_h) \geq 0$, for all $g_j \in F$
   - $\Omega_j(b_h, b'_h) \geq 0$, for all $g_j \in F$
Slackness functions

**Definition (Slackness functions)**

Let $\lambda \in [0.5, 1]$ denote the chosen majority level:

1. **Direct slackness function:**
   \[ \xi^+_h(a, \lambda) = \sigma(a, b_h) - \lambda, \quad h = (q + 1), \ldots, 0. \]

2. **Reverse slackness function:**
   \[ \xi^-_h(a, \lambda) = \sigma(b_h, a) - \lambda, \quad h = 0, \ldots, (q + 1). \]

**Proposition**

1. $\xi^+_h(\cdot)$ does not decrease when moving from a given category to a worst one.

2. $\xi^-_h(\cdot)$ does not decrease when moving from a given category to a best one.
Assignment rules of ELECTRE TRI-C

Definition (Descending assignment rule)

- Choose a majority level \( \lambda \) (0.5 \leq \lambda \leq 1).
- Decrease \( h \) from \((q+1)\) until the first value such that \( \xi_h^+(a, \lambda) \geq 0 \).
- If \( \xi_h^+(a, \lambda) \leq |\xi_{h+1}^+(a, \lambda)| \), then assign action \( a \) to category \( C_h \). Otherwise, assign \( a \) to \( C_{h+1} \).

Definition (Ascending assignment rule)

- Choose a majority level \( \lambda \) (0.5 \leq \lambda \leq 1).
- Increase \( h \) from 0 until the first value such that \( \xi_h^-(a, \lambda) \geq 0 \).
- If \( \xi_h^-(a, \lambda) \leq |\xi_{h-1}^-(a, \lambda)| \), then assign action \( a \) to category \( C_h \). Otherwise, assign \( a \) to \( C_{h-1} \).
### Example

<table>
<thead>
<tr>
<th>Actions</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>Descending</th>
<th>Ascending</th>
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<tr>
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<td>$\succ$</td>
<td>$R^\lambda$</td>
<td>$R^\lambda$</td>
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<td>$C_4$</td>
</tr>
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<td>$R^\lambda$</td>
<td>$R^\lambda$</td>
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<td>$C_4$</td>
</tr>
<tr>
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<td>$\prec$</td>
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<tr>
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<td>$\prec$</td>
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<td>$C_3$</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$I^\lambda$</td>
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<td>$\prec$</td>
<td>$\prec$</td>
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<td>$C_1$</td>
</tr>
<tr>
<td>$a_6$</td>
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<td>$R^\lambda$</td>
<td>$R^\lambda$</td>
<td>$C_2$</td>
<td>$C_4$</td>
</tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>$C_4$</td>
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<td>$a_{10}$</td>
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<td>$\prec$</td>
<td>$\prec$</td>
<td>$C_1$</td>
<td>$C_1$</td>
</tr>
</tbody>
</table>

Note: $\lambda = 0.70$; Source: Data adapted from Merad et al., 2004
Properties of the assignment rules I

Theorem

a) The monotonicity, homogeneity, and stability requirements hold.

b) If the strict separability condition is fulfilled, then the conformity requirement holds.

Remark

The properties of uniqueness and independence of the assignments are also fulfilled.
**Definition (Dominance)**

The set of reference actions, $B$, fulfills the (strict) dominance relation if and only if

$$\forall j, \Omega_j(b_{h+1}, b_h) \geq 0 \text{ and } \exists j, \Omega_j(b_{h+1}, b_h) > 0; h = 0, \ldots, q.$$ 

**Definition (Weak separability condition)**

The set of reference actions, $B$, fulfills the weak separability condition if and only if

$$\forall j, \Omega_j(b_{h+1}, b_h) \geq 0 \text{ and } \exists j, \Omega_j(b_{h+1}, b_h) > p_j; h = 0, \ldots, q.$$
Theorem

If the weak separability condition is fulfilled, then there exists a compatible majority level, \( \lambda^c \), for which the conformity requirement holds, whenever the chosen majority level \( \lambda \geq \lambda^c \), such that

\[
\lambda^c = \frac{1}{2} + \frac{1}{2} \max_{h = 0, \ldots, q} \left\{ \sigma(b_h, b_{h+1}) \right\}
\]
### Numerical example

#### Compatible majority level

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
</tr>
</thead>
<tbody>
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<td>0.3696</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$b_2$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.0217</td>
<td>0.0217</td>
</tr>
<tr>
<td>$b_3$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.0652</td>
</tr>
<tr>
<td>$b_4$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

**Compatible majority level:**

$$\lambda^c = \frac{1}{2} + \frac{1}{2} \max_{h=0,...,4} \{\sigma(b_h, b_{h+1})\} = 0.69$$

Source: Data adapted from Merad et al., 2004
\( \hat{C} = \{ \hat{C}_1, \ldots, \hat{C}_h, \ldots, \hat{C}_q \} \) is the set of pre-defined and ordered categories, where \( \hat{C}_1 \) is the worst category and \( \hat{C}_q \) the best one, with \( q \geq 2 \).

Each category \( \hat{C}_h \) is defined by a lower profile limit, \( \hat{b}_{h-1} \), and an upper profile limit, \( \hat{b}_h \), such that \( \hat{b}_h \Delta_F \hat{b}_{h-1}, h = 1, \ldots, q \).

\( \hat{B} = \{ \hat{b}_0, \hat{b}_1, \ldots, \hat{b}_h, \ldots, \hat{b}_{q-1}, \hat{b}_q \} \) is the set of the \( (q + 1) \) profile limits.

\( \hat{b}_0 \) and \( \hat{b}_q \) play a similar role as \( b_0 \) and \( b_{q+1} \) when using central reference actions.
An overview of ELECTRE TRI-B
The most well-known assignment rules

Definition (Pseudo-conjunctive assignment rule)

- Choose a majority level $\lambda$ (0.5 $\leq \lambda \leq$ 1).
- Decrease $h$ from $q$ until the first value such that $\xi_{h-1}^+(a, \lambda) \geq 0$.
- Assign action $a$ to category $\hat{C}_h$.

Definition (Pseudo-disjunctive assignment rule)

- Choose a majority level $\lambda$ (0.5 $\leq \lambda \leq$ 1).
- Increase $h$ from 0 until the first value such that $\xi_h^-(a, \lambda) \geq 0$ and $\xi_h^+(a, \lambda) < 0$.
- Assign action $a$ to category $\hat{C}_h$.
Theorem

Consider \((q + 2)\) reference actions defined to apply ELECTRE TRI-C with \(q\) categories. When such reference actions are used as profile limits of the \((q + 1)\) categories in ELECTRE TRI-B,

a) if an action \(a\) is assigned to \(C_h\) by the ELECTRE TRI-C descending rule, then \(a\) is assigned to \(\hat{C}_h\) or \(\hat{C}_{h+1}\) by the ELECTRE TRI-B pseudo-conjunctive rule.

b) if an action \(a\) is assigned to \(C_t\) by the ELECTRE TRI-C ascending rule, then \(a\) is assigned to \(\hat{C}_k\), with \(k \geq t\), by the ELECTRE TRI-B pseudo-disjunctive rule.
Numerical example
Comparing assignment results

### Example

<table>
<thead>
<tr>
<th>Actions</th>
<th>Electre Tri-C</th>
<th>Electre Tri-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Descending</td>
<td>Ascending</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$C_2$</td>
<td>$C_4$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$C_1$</td>
<td>$C_4$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$C_3$</td>
<td>$C_3$</td>
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<tr>
<td>$a_4$</td>
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<td>$C_3$</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$C_2$</td>
<td>$C_1$</td>
</tr>
<tr>
<td>$a_6$</td>
<td>$C_2$</td>
<td>$C_4$</td>
</tr>
<tr>
<td>$a_7$</td>
<td>$C_1$</td>
<td>$C_2$</td>
</tr>
<tr>
<td>$a_8$</td>
<td>$C_1$</td>
<td>$C_2$</td>
</tr>
<tr>
<td>$a_9$</td>
<td>$C_1$</td>
<td>$C_4$</td>
</tr>
<tr>
<td>$a_{10}$</td>
<td>$C_1$</td>
<td>$C_1$</td>
</tr>
</tbody>
</table>

Note: $\lambda = 0.70$; Source: Data adapted from Merad et al., 2004
Conclusions

- In **Electre Tri-C** the categories are defined through central reference actions instead of profile limits.

- Central reference actions and profile limits are **two alternative ways** of defining ordered categories.

- **Electre Tri-C** fulfills the properties of uniqueness, independence, conformity, monotonicity, homogeneity, and stability.

- When the set of reference actions **does not fulfill the strict separability condition, but only a weak separability condition**, then a **compatible majority level** is required.

- A comparison with **Electre Tri-B** shows the main similarities of the two methods.
References


Appendix: Additional results

- Pseudo-criterion model
- Credibility index
- Binary relations
- Comparing ELECTRE TRI-C assignment rules
- Comparing ELECTRE TRI-B assignment rules
- Comparing results II
- ELECTRE TRI-C particular results
## Pseudo-criterion model

### Definition (Pseudo-criterion)

A **pseudo-criterion** is a function $g_j$ associated with two threshold functions, $q_j(\cdot)$ and $p_j(\cdot)$, satisfying the following condition: for all actions $a$ in the sets of actions, $g_j(a) + p_j$ and $g_j(a) + q_j$ are non-decreasing monotone functions of $g_j(a)$. (Roy, 1996)

### Key concepts

### Remark

Consider an ordered pair of actions $(a, a')$, and the two thresholds associated to the pseudo-criterion model,

- **a)** \( a P_j a' \iff \Omega_j(a, a') > p_j(\cdot) \)
- **b)** \( a Q_j a' \iff q_j(\cdot) < \Omega_j(a, a') \leq p_j(\cdot) \)
- **c)** \( a I_j a' \iff -q_j(\cdot) \leq \Omega_j(a, a') \leq q_j(\cdot) \)
Key concepts
Credibility index of ELECTRE methods

Definition (Credibility index)

The credibility of the comprehensive outranking of an action $a$ over $a'$, which means that $a$ may be judged at least as good as $a'$ when taking all the criteria from $F$ into account, is defined by aggregating the comprehensive concordance index, $c(a, a')$, and the partial discordance indices, $d_j(a, a')$, as follows:

$$
\sigma(a, a') = c(a, a') \prod_{j=1}^{n} T_j(a, a')
$$

where,

$$
T_j(a, a') = \begin{cases} 
1 - \frac{d_j(a, a')}{1 - c(a, a')} & \text{if } d_j(a, a') > c(a, a') \\
1 & \text{otherwise}
\end{cases}
$$
## Binary relations

### Definition ($\lambda$-binary relations)

1. **$\lambda$-outranking:** $a S^\lambda a' \iff \sigma(a, a') - \lambda \geq 0$

2. **$\lambda$-indifference:** $a I^\lambda a' \iff \sigma(a, a') - \lambda \geq 0 \land \sigma(a', a) - \lambda \geq 0$

3. **$\lambda$-incomparability:** $a R^\lambda a' \iff \sigma(a, a') - \lambda < 0 \land \sigma(a', a) - \lambda < 0$

4. **$\lambda$-preference:** $a > a' \iff \sigma(a, a') - \lambda \geq 0 \land \sigma(a', a) - \lambda < 0$
### Theorem

**a)** If an action \( a \) is \( \lambda \)-indifferent to at least one reference action, then \( a \) is assigned by the descending rule to a category at least as good as the one \( a \) is assigned to when using the ascending rule.

**b)** If an action \( a \) is \( \lambda \)-incomparable to at least one reference action, then \( a \) is assigned by the descending rule to a category at most as good as the one \( a \) is assigned to when using the ascending rule.

**c)** Otherwise, both rules assign the action \( a \) to the same category or to two different but consecutive categories.
Roy and Bouyssou, 1993, p. 395

- If an action $a$ is assigned to category $\hat{C}_k$ by the pseudo-conjunctive rule and to $\hat{C}_h$ by the pseudo-disjunctive rule, then $k \leq h$.

- Furthermore, the two assignment rules provide the same results if and only if there is no $t$ such that $\xi^+_t(a, \lambda) < 0$ and $\xi^-_t(a, \lambda) < 0$ or there is at most one $t$ such that $\xi^+_t(a, \lambda) \geq 0$ and $\xi^-_t(a, \lambda) \geq 0$. 

Electre Tri-B rules
Theorem

Consider $(q + 1)$ profile limits defined to apply ELECTRE TRI-B with $q$ categories. When such profile limits are used as reference actions of the $(q - 1)$ categories in ELECTRE TRI-C,

a) if an action $a$ is assigned to $\hat{C}_h$ by the ELECTRE TRI-B pseudo-conjunctive rule, then $a$ is assigned to $C_h$ or $C_{h-1}$ by the ELECTRE TRI-C descending rule.

b) if an action $a$ is assigned to $\hat{C}_t$ by the ELECTRE TRI-B pseudo-disjunctive rule, then $a$ is assigned to $C_k$, with $k \leq t$ by the ELECTRE TRI-C ascending rule.
**ELECTRE TRI-C particular results**

**Proposition**

a) If \( a \) \(\lambda\)-outranks \( b_h \), then \( a \) is assigned at least to \( C_h \) by the descending rule.

b) If \( b_h \) \(\lambda\)-outranks \( a \), then \( a \) is assigned at most to \( C_h \) by the ascending rule.

c) If \( a \) is \(\lambda\)-preferred to \( b_h \), then \( a \) is assigned at least to \( C_h \) by both rules.

d) If \( b_h \) is \(\lambda\)-preferred to \( a \), then \( a \) is assigned at most to \( C_h \) by both rules.
ELECTRE methods with interaction between criteria: An extension of the concordance index

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Cost Action 0602, Troina, Sicily
(11-16 April 2008)
This presentation is devoted to an extension of the comprehensive concordance index of ELECTRE methods.

Such an extension have been considered to take into account the interaction between criteria.

Three types of interaction effects has been considered, mutual strengthening, mutual weakening, and antagonistic.

In real-world decision-making situations is reasonable to consider the interaction between a small number of pairs of criteria.

Various conditions, boundary, monotonicity, and continuity have been imposed.
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... 

*in* Greco and Figueira (2003)
Application areas:

- Environmental problems
- Constructions of indices (in this case several pairs ... of interaction criteria)
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- Environmental problems
- Constructions of indices (in this case several pairs ... of interaction criteria)
Introduction

Application areas:

- Environmental problems

- Constructions of indices (in this case several pairs ... of interaction criteria)
Illustrative Example 1

CHOOSING THE SITE FOR CONSTRUCTING A NEW HOTEL

Criteria:

- $g_1$: land purchasing and construction costs (investment costs) \([\text{min}]\);
- $g_2$: annual operating costs (annual costs) \([\text{min}]\);
- $g_3$: personnel recruitment possibilities (recruitment) \([\text{max}]\);
- $g_4$: target client perceptions of the city district (image) \([\text{max}]\);
- $g_5$: facility of access for the target clients (access) \([\text{max}]\).

Mutual strengthening between criteria: investment and annual costs.

Mutual weakening between criteria: image and access.
CHOOSING THE SITE FOR CONSTRUCTING A NEW HOTEL

Criteria:

$g_1$: land purchasing and construction costs (investment costs) [min];

$g_2$: annual operating costs (annual costs) [min];

$g_3$: personnel recruitment possibilities (recruitment) [max];

$g_4$: target client perceptions of the city district (image) [max];

$g_5$: facility of access for the target clients (access) [max].

Mutual strengthening between criteria: investment and annual costs.

Mutual weakening between criteria: image and access.
Illustrative Example 1

CHOOSING THE SITE FOR CONSTRUCTING A NEW HOTEL

Criteria:

- $g_1$: land purchasing and construction costs (investment costs) [min];
- $g_2$: annual operating costs (annual costs) [min];
- $g_3$: personnel recruitment possibilities (recruitment) [max];
- $g_4$: target client perceptions of the city district (image) [max];
- $g_5$: facility of access for the target clients (access) [max].

- Mutual strengthening between criteria: investment and annual costs.
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Choosing the site for constructing a new hotel

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Illustrative Example 1

The comparison of site $a$ with sites $b$, $c$, and $d$, in terms of the two financial criteria $g_1$ and $g_2$

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According to the classic definition of the concordance index, the role that $g_1$ and $g_2$ should have for supporting the answer to the assertion “$a$ is at least as good as $b$ (or $c$ or $d$)” is characterized by the following weights,

- $k_1$ in the comparison with $b$,
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Figueira et al. (CEG-IST) Interaction Between Criteria
Illustrative Example 1

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\begin{align*}
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Figueira et al. (CEG-IST) Interaction Between Criteria 11.04.2008 8 / 41
DMR considers the weights $k_1$ and $k_2$ appropriate, when only one criterion, supports a decision that one action is better than another one.

However, he/she judges that the sum $k_1 + k_2$ is not sufficient to characterize the role of this criteria pair when both supports the decision.

Because in this case each criterion is strengthened by the other given the degree of complementarity between them.

If one action is better than another one with respect to criteria $g_1$ and $g_2$ conjointly, it would be interesting to be able to take this mutual strengthening effect into account.

This effect can be taken into account by increasing the weights $k_1$ and $k_2$ in the concordance index.
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The comparisons of site \(a\) with sites \(b, c, d'\) in terms of the two purely ordinal criteria, \(g_4\) and \(g_5\)

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<td>(a) is better than (b)</td>
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</tr>
<tr>
<td>(g_5)</td>
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The DMR judges that the sum \(k_4 + k_5\) is too high to characterize the role of this criteria pair when both supports the decision that one action is better than another one, because in this case each criterion is weakened by the other due to the degree of redundancy between them.
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The comparisons of site \( a \) with sites \( b, c, d' \) in terms of the two purely ordinal criteria, \( g_4 \) and \( g_5 \):

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Illustrative Example 2

LAUNCHING A NEW DIGITAL CAMERA MODEL

- Criteria:
  - $g_1$: purchasing costs (cost) [min];
  - $g_2$: weaknesses (fragility) [min];
  - $g_3$: user friendliness of the controls (workability) [max];
  - $g_4$: image quality (image) [max];
  - $g_5$: aesthetics [max];
  - $g_6$: volume [min];
  - $g_7$: weight [min].

- Antagonistic effect between criteria: cost and fragility.
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LAUNCHING A NEW DIGITAL CAMERA MODEL

Criteria:

\( g_1 \): purchasing costs \((\text{cost})\) [\(\min\)];

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Antagonistic effect between criteria: \(\text{cost}\) and \(\text{fragility}\).
**Illustrative Example 2**

**Comparisons** of digital camera model *a* with models *b*, *c*, and *d*, according to criteria *g*<sub>1</sub> and *g*<sub>2</sub>:

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According to the **classic definition of the concordance index** the role these criteria should play in supporting the assertion “model *a* is at least as good as model *b* (or *c* or *d*)” is characterized by the following weights,

- *k*<sub>1</sub> + *k*<sub>2</sub> in the comparison with *b*,
- *k*<sub>2</sub> in the comparison with *c*,
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Figueira et al. (CEG-IST)
DMR considers that weights $k_1$ and $k_2$ adequately characterize the role these two criteria should play when comparing $a$ with $b$ and $a$ with $c$

However, he/she considers that the same is not true when comparing $a$ with $d$

Based on a customer survey, it seems that when one model is less fragile than another, the benefit derived from the lower cost is partially masked by the fact the model is less fragile.

This phenomenon can be modeled by decreasing the weight of criterion $g_1$ in the concordance index of the assertion “$a$ is at least as good as $b$”.
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Concepts: Definitions and notation

- Notation
- Pseudo criterion
- The criteria weights and the concordance index
- Partial concordance index $c_i(a, b)$
- Properties of the comprehensive concordance index $c(a, b)$
Notation

Pseudo criterion

The criteria weights and the concordance index

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Properties of the comprehensive concordance index $c(a, b)$
- $F = \{g_1, g_2, \ldots, g_i, \ldots, g_n\}$ denote a coherent set or family of criteria; for the sake of simplicity we shall use also $F$ as the set of criteria indices (the same will apply later on for subsets of $F$);

- $A = \{a, b, c, \ldots\}$ denote a finite set of actions with cardinality $m$;

- $g_i(a) \in E_i$ denote the evaluation of action $a$ on criterion $g_i$, for all $a \in A$ and $i \in F$, where $E_i$ is the scale associated to criterion $g_i$ (no restriction is imposed to the scale type).

- $k_i$ is the relative importance or weight of criterion $g_i$.

- $C(aTb)$ represents the coalition of criteria in favor of the assertion “$aTb$”, where $T \in \{P, Q, S\}$ (introduced later).

- $\bar{C}(aTb)$ denote the complement of $C(aTb)$. 
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- $k_i$ is the relative importance or weight of criterion $g_i$.

- $C(a \bowtie b)$ represents the coalition of criteria in favor of the assertion “$a \bowtie b$”, where $T \in \{P, Q, S\}$ (introduced later).

- $\bar{C}(a \bowtie b)$ denote the complement of $C(a \bowtie b)$.
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A pseudo criterion is a function $g_i$ associated with the two threshold functions $q_i(g_i(a))$ and $p_i(g_i(a))$ satisfying the following condition, for all $a \in A$ (Roy, 1991, 1996): $g_i(a) + p_i(g_i(a))$ and $g_i(a) + q_i(g_i(a))$ are non-decreasing monotone functions of $g_i(a)$.

By definition, for all pairs $(a, b) \in A \times A$ with $g_i(a) \geq g_i(b)$ and $q_i(g_i(a)) \leq p_i(g_i(a))$, 

$$al_i b \iff g_i(a) \leq g_i(b) + q_i(g_i(b));$$

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The criteria weights and the concordance index

The concordance index can be defined as follows,

\[ c(a, b) = \sum_{i \in F} \frac{k_i}{K} c_i(a, b), \quad \text{with} \quad K = \sum_{i \in F} k_i \]

where,

\[ c_i(a, b) = \begin{cases} 
1, & \text{if} \quad g_i(a) + q_i(g_i(a)) \geq g_i(b), \quad (aS_i b), \\
\frac{g_i(a) + p_i(g_i(a)) - g_i(b)}{p_i(g_i(a)) - q_i(g_i(a))}, & \text{if} \quad g_i(a) + q_i(g_i(a)) < g_i(b) \leq g_i(a) + p_i(g_i(a)), \quad (bQ_i a), \\
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Partial concordance index, $c_i(a, b)$
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Properties of the comprehensive concordance index, $c(a, b)$

- **Boundary conditions:** $0 \leq c(a, b) \leq 1$.

- **Monotonicity:** $c(a, b)$ is a monotonous non-decreasing function of $\Delta_i = g_i(a) - g_i(b)$, for all $i \in F$.

- **Continuity:** if $p_i(g_i(a)) > q_i(g_i(a))$, for all $i \in F$ and $a \in A$, then $c(a, b)$ is a continuous function of both $g_i(a)$ and $g_i(b)$.

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**Positive net balance condition.**

$$\forall i \in F, \quad (k_i) - \left( \sum_{\{i,j\}: k_{ij} < 0} |k_{ij}| + \sum_h k'_{ih} \right) > 0$$
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On the antagonism effect: Remarks

- The presence of an antagonism coefficient $k'_{ih} > 0$ is compatible with both the absence of antagonism in the reverse direction ($k'_{hi} = 0$) and the presence of a reverse antagonism ($k'_{hi} > 0$).

- The antagonism effect does not double the influence of the veto effect; in fact, they are quite different. If criterion $g_h$ has a veto power, it will always be considered, regardless of whether $g_i$ belongs to the concordant coalition. The same is not true for the antagonism effect, which occurs only when the criterion $g_i$ belongs to the concordant coalition.

- The pair $\{g_i, g_h\}$ is antagonistic when the antagonism effect exists for one or the other criterion associated with this criteria pair.
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Practical aspects

A procedure to assign numerical values to the coefficients:

1. **Assign numerical values to the intrinsic weights.** The SRF method can be used. (the “cards” should be ranked ignoring all the possible inter-criteria interactions.)

2. **The analyst should ask the DRM about the possible interactions between criteria.** Considering criterion $g_1$ and reviewing the remaining criteria $g_2, g_3, \ldots, g_n$, it should be easy (and relatively quick), given the very nature of the criteria, to recognize if there is or not interaction between the criteria and also identify the type of the interaction involved.
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3. A numerical value is assigned to the interaction coefficient associated with each pair identified in the previous step. Example of antagonism (cf. example, criteria $g_1$ and $g_2$):

- Suppose that when using SRF the result is $k_1 = 6$ and $k_2 = 4$, and thus $k_1 + k_2 = 10$.
- Since criterion $g_2$ is antagonistic with respect to $g_1$, the weight should be lower than 6, when comparing two digital camera models $a$ and $d$ (cf. Table).
- The analyst can ask the DMR to set the value to be replaced to 6 in this comparison in order to adequately model the interaction that the DMR wants to take into account.
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- Definition of the new $c(a, b)$ for pseudo criteria
- Function $Z(\cdot, \cdot)$
- Properties of $Z(\cdot, \cdot)$
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Definition of the new $c(a, b)$ (quasi criteria)

Let,

- $L(a, b)$ denote the set of all pairs $\{i, j\}$ such that $i, j \in \bar{C}(bPa)$;

- $O(a, b)$ denote the set of all ordered pairs $(i, h)$ such that $i \in \bar{C}(bPa)$ and $h \in C(bPa)$.

$$c(a, b) = \frac{1}{K(a, b)} \left( \sum_{i \in \bar{C}(bPa)} k_i + \sum_{\{i, j\} \in L(a, b)} k_{ij} - \sum_{(i, h) \in O(a, b)} k'_{ih} \right)$$

where,

$$K(a, b) = \sum_{i \in F} k_i + \sum_{\{i, j\} \in L(a, b)} k_{ij} - \sum_{(i, h) \in O(a, b)} k'_{ih}$$
Definition of the new $c(a, b)$ (quasi criteria)

Let,

- $L(a, b)$ denote the set of all pairs $\{i, j\}$ such that $i, j \in \bar{C}(bPa)$;

- $O(a, b)$ denote the set of all ordered pairs $(i, h)$ such that $i \in \bar{C}(bPa)$ and $h \in C(bPa)$.

$$c(a, b) = \frac{1}{K(a, b)} \left( \sum_{i \in \bar{C}(bPa)} k_i + \sum_{\{i, j\} \in L(a, b)} k_{ij} - \sum_{(i, h) \in O(a, b)} k'_{ih} \right)$$

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Definition of the new $c(a, b)$ (pseudo criteria)

$$c(a, b) = \frac{1}{K(a, b)} \left( \sum_{i \in \bar{C}(bPa)} c_i(a, b)k_i + \sum_{\{i, j\} \in L(a, b)} Z(c_i(a, b), c_j(a, b))k_{ij} + 
\right.$$ 

$$- \sum_{(i, h) \in O(a, b)} Z(c_i(a, b), c_h(b, a))k'_{ih} \right)$$

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Function $Z(\cdot, \cdot)$ in the previous formula is used to capture the interaction effects in the ambiguity zone. It should be remarked that in the third summation $c_h(b, a)$ is always equal to 1.

Let $x = c_i(a, b)$ and $y = c_j(a, b)$ or $y = c_h(b, a)$. Consequently, $x, y \in [0, 1]$.

Function $Z(x, y)$ is used to get the reduction coefficients for $k_{ij}$ and $k'_{ih}$, when at least one of the arguments of $Z(x, y)$ is within the range $[0, 1]$. 
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Properties of $Z(\cdot, \cdot)$

**Extreme value conditions:** When leaving the ambiguity zones $c(a, b)$ should regain the form presented in formula. Thus, $Z(1, 1) = 1$ and $Z(x, 0) = Z(0, y) = 0$.

**Symmetry:** $Z(x, y) = Z(y, x)$.

**Monotonicity:** When the ambiguity diminishes the effect due to the interaction cannot increase. Then $Z(x, y)$ is a *non-decreasing monotone function* of both arguments $x$ and $y$. 
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Monotonicity: When the ambiguity diminishes the effect due to the interaction cannot increase. Then $Z(x, y)$ is a non-decreasing monotone function of both arguments $x$ and $y$. 
Properties of $Z(\cdot, \cdot)$ (cont)

**Marginal impact condition:** When the ambiguity diminishes we pass from $x + w$ to $x$, the relative marginal impact of the interactions is bounded from above,

$$\frac{1}{w} \left( Z(x + w, y) - Z(x, y) \right) \leq 1 \quad x, y, w, x + w \in [0, 1]$$

**Continuity:** $Z(x, y)$ is a continuous function of each argument. This permits $c(a, b)$ to be a continuous function of $g_i(a)$ and $g_i(b)$ when $p_i(g_i(a)) > q_i(g_i(a))$, for all $a \in A$ and $g_i \in F$.

**Boundary condition:** For preserving the net balance condition, it is sufficient that $Z(x, y) \leq \min\{x, y\}$. 
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Some possible forms for $Z(\cdot, \cdot)$

Among the multiple forms that can be chosen for $Z(x, y)$, we only present two of them which have an intuitive and meaningful interpretation.

$$Z(x, y) = \min\{x, y\};$$

$$Z(x, y) = xy.$$

When $x$ and $y$ are both different from 1, i.e., when the two interacting criteria belong to the ambiguity zone, then the impact of the interaction is weaker with $xy$ than with $\min\{x, y\}$.

Choosing the $\min\{x, y\}$ means that the reduction coefficient is not influenced by what happens in the other ambiguity zone. For these reasons formula $xy$ seems preferable to $\min\{x, y\}$. 
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<table>
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<th>$g_3$[max]</th>
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<td>220</td>
<td>Average</td>
<td>Average</td>
<td>Rather Good</td>
<td>Average</td>
<td>190 cm$^3$</td>
<td>155 g</td>
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<tr>
<td>b</td>
<td>300</td>
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<td>Rather Good</td>
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<td>145 g</td>
</tr>
<tr>
<td>c</td>
<td>160</td>
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<td>Average</td>
<td>Rather Bad</td>
<td>140 cm$^3$</td>
<td>130 g</td>
</tr>
<tr>
<td>d</td>
<td>280</td>
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<td>170 g</td>
</tr>
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</table>

Qualitative scale: very bad, bad, rather bad, average, rather good, good, very good.
Consider the weights obtained using SRF: $k_1 = 6$, $k_2 = 4$, $k_3 = k_4 = k_5 = 1$, $k_6 = k_7 = 2$, where $K = 6 + 4 + 1 + 1 + 1 + 2 + 2 = 17$.

The concordance index for $(a, d)$ is $c(a, d) = \frac{(6+1+1+2+1)}{17} = \frac{11}{17} = 0.647$ (criterion $g_7$ is in the ambiguity zone, and it only counts for 50% of its overall weight).

Now, consider the antagonistic effect, where $k'_{12} = 2.5$. 
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Now, consider the antagonistic effect, where $k'_{12} = 2.5$. 
The new concordance index takes the value
\[ c(a, d) = \frac{6+1+1+2+2-2.5}{17-2.5} = \frac{8.5}{14.5} = 0.586. \]

But, \( c(d, a) \) remains the same (i.e.,
\[ c(d, a) = \frac{4+3+1}{17} = \frac{8}{17} = 0.471. \]

If \( s \) is defined at \( s = 0.6 \), when taking the antagonism effect into account, the actions become incomparable, although \( a \) was preferred to \( d \) before.

This incomparability shows that this effect can imply significant changes.
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Theorem (quasi criterion). Monotonicity and boundary conditions hold for $c(a, b)$ as defined in slide 15.

Theorem (pseudo criterion). If function $Z(x, y)$ satisfies Extreme value conditions, Symmetry, Monotonicity, Marginal impact condition, and Continuity, then $c(a, b)$ satisfies Boundary conditions, Monotonicity and Continuity.
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Fundamental results: monotonicity

Consider all the possible cases. The proof is based on the fact that if the difference $g_f(a) - g_f(b)$ decreases, either $c(a, b)$ remains constant or it decreases.

1. Criterion $f$ belongs to $C(bPa)$.

2. Criterion $f$ belongs to $\bar{C}(bPa)$. Four subcases should be considered:
   - a) Criterion $f$ belongs to $C(aSb)$ and it continues in $C(aSb)$ after decreasing $\Delta_f$.
   - b) Criterion $f$ moves from $C(aSb)$ to $C(bQa)$.
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Concordance index and Choquet integral

• Choquet integral is an **aggregation operator** permitting to model interactions between criteria.

• Choquet integral is used to build a **value function** giving a complete preorder, i.e. a transitive and strongly complete binary relation, rather than simply an outranking relation, being only reflexive and not transitive and complete, like in ELECTRE methods.

• Choquet integral **is questionable especially** with respect to two main points (Roy 2007): the evaluation of each criterion is supposed to be expressed, ...

• It is interesting to investigate more in detail the **relationship between Choquet integral and the extension of the concordance index of ELECTRE methods**. (Look at concordance index from the viewpoint of the Choquet integral.)
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We showed how to take into account these types of interaction in the concordance index used within the ELECTRE methods framework.

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A re-implementation of ELECTRE methods taking account the ideas proposed in our presentation is now possible.
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