Boolean Hedonic Games

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Abstract

We study hedonic games with dichotomous preferences. Hedonic games are cooperative games in which players desire to form coalitions, but only care about the makeup of the coalitions of which they are members; they are indifferent about the makeup of other coalitions. The assumption of dichotomous preferences means that, additionally, each player's preference relation partitions the set of coalitions of which that player is a member into just two equivalence classes: satisfactory and unsatisfactory. A player is indifferent between satisfactory coalitions, and is indifferent between unsatisfactory coalitions, but strictly prefers any satisfactory coalition over any unsatisfactory coalition. We develop a succinct representation for such games, in which each player's preference relation is represented by a propositional formula. We show how solution concepts for hedonic games with dichotomous preferences are characterised by propositional formulas.

Introduction

Hedonic games are cooperative games in which players desire to form coalitions, but only care about the makeup of the coalitions of which they are members; they are indifferent about the makeup of other coalitions. Drèze and Greenberg (1980) suggested that coalition formation involves a hedonic aspect, i.e., that apart from the yield of a coalition, players may also be interested in its composition. Bogomolnaia and Jackson (2002) and Banerjee, Konishi, and Sonmez (2001) then defined hedonic games in their present form as a simple but very versatile model of coalition formation. Hedonic games capture many social, political, and economic group formation scenarios, and can be seen as a generalisation of the stable marriage setting (Aziz and Savani 2016).

As the specification of a hedonic game requires the expression of each player's ranking over all sets of players including him, in general, such a specification requires exponential space and, when used by a centralised mechanism, exponential elicitation time. Such an exponential blow-up severely limits the practical applicability of hedonic games, and for this reason researchers have investigated compactly represented hedonic games. One approach to this problem has been to consider possible restrictions on the possible preferences that players have. For example, one may assume that each player specifies only a ranking over single players, and that her preferences over coalitions are defined according to the identity of the best (respectively, worst) element of the coalition (Cechlárová and Hajduková 2004; Cechlárová 2008). One may also assume that each player's preferences depend only on the number of players in her coalition (Bogomolnaia and Jackson 2002). These representations come with a domain restriction, i.e., a loss of expressivity. Elkind and Wooldridge (2009) consider a fully expressive representation for hedonic games, based on weighted logical formulas; in the worst case, their representation requires space exponential in the number of players, but in many cases the space requirement is much smaller.

In this paper, we consider another natural restriction on player preferences. We consider hedonic games with dichotomous preferences. The assumption of dichotomous preferences means that each player's preference relation partitions the set of coalitions of which that player is a member into just two equivalence classes: satisfactory and unsatisfactory. A player is indifferent between satisfactory coalitions, and is indifferent between unsatisfactory coalitions, but strictly prefers any satisfactory coalition over any unsatisfactory coalition.


When the space of all possible alternatives has a combinatorial structure, propositional formulas are a very natural representation of dichotomous preferences. In such a representation, variables correspond to goods (in fair division), outcome variables (Boolean games), state variables (belief merging), or players (coalition formation). In the latter case, which we will be concerned with in the present paper, each player $i$ can express her preferences over coalitions containing her by using propositional atoms of the form $ij$ ($j \neq i$), meaning that $i$ and $j$ are in the same coalition. Thus, for example, player 1 can express by the formula $(12 \lor 13) \land \neg 14$ that he wants to be in a coalition with player 2.
or with player 3, but not with player 4. Our primary aim is to present such a propositional framework for specifying hedonic games and computing various solution concepts. We will first define a propositional logic using atoms of the form \(ij\), together with domain axioms expressing that the output of the game should be a partition of the set of players. Then we consider a range of solution concepts, and show that they can be characterised by specific classes of formulas, and solved using propositional satisfiability solvers. The result is a simple, natural, and compact representation scheme for expressing preferences, and, as our characterisations are model preserving, a machinery based on satisfiability for computing partitions satisfying some specific stability criteria such as Nash stability or core stability.

Preliminaries

In this section, we recall some definitions relating to coalitions, coalition structures (or partitions), and hedonic games. See, e.g., (Chalkiadakis, Elkind, and Wooldridge 2011) for an in-depth discussion of these and related concepts.

Coalitions and Partitions We consider a setting in which there is a set \(N \subseteq \{1, 2, \ldots, n\}\) of players. We denote by \(P(T)\) the power set of \(T\). A coalition structure is an in-depth discussion of these and related concepts. See, e.g., (Chalkiadakis, Elkind, and Wooldridge 2011) for an in-depth discussion of these and related concepts. In this section, we recall some definitions relating to coalition structures (or partitions), and hedonic games. In this section, we recall some definitions relating to coalitions, coalition structures (or partitions), and hedonic games. See, e.g., (Chalkiadakis, Elkind, and Wooldridge 2011) for an in-depth discussion of these and related concepts.

Consider the following hedonic game with four players, 1, 2, 3, and 4, whose (dichotomous) preferences are as follows. (Indifferences are indicated by commas.)

1: \(\{123\}, \{124\}, \{134\}, \{1234\} \succ 1 (\{1\}, \{12\}, \{13\}, \{14\})
2: \{213\}, \{214\}, \{234\} \succ 2 \{2\}, \{21\}, \{23\}, \{24\}, \{234\}
3: \{31\}, \{32\}, \{312\} \succ 3 \{3\}, \{34\}, \{314\}, \{324\}, \{3124\}
4: \{41\}, \{42\}, \{43\}, \{412\}, \{431\}, \{4\} \succ 4 \{423\}, \{4123\}

Thus, player 1 (resp. 2) wants to be in a coalition of at least (resp. exactly) three. Player 3 wants to be in the same coalition as 1 or as 2, but not together with 4. Player 4 does not want to be with players 2 and 3 together. There is exactly one partition that is satisfactory for all four players, namely \(\{123\}\). For players 1, 2, 3, all coalitions are acceptable. For player 4, \{423\} and \{1234\} are unacceptable.

Solution Concepts for Hedonic Games A solution concept associates with every hedonic game \((N, \succeq_1, \ldots, \succeq_n)\) a (possibly empty) set of partitions of \(N\). Here we review some of the most common solution concepts.
Individual rationality captures the idea that every player prefers the coalition he is in to being on his own, i.e., that coalitions are acceptable to its members. Thus, formally, π is individually rational if, for all players i in N, \( \pi(i) \succeq_i \{i\} \).

This condition is obviously equivalent to \( \pi \succeq_i \pi[i \to \emptyset] \).

For dichotomous hedonic games, a partition π is said to be social welfare optimal if it maximises the number of players who are in a satisfactory coalition, i.e., if π maximises \( \{i \in N : \pi(i) \in N^+\} \). In a similar way, a partition π is Pareto optimal if it maximises the set of players being in a satisfactory coalition with respect to set-inclusion, i.e., if there is no partition π′ with

\[ \{i \in N : \pi(i) \in N^+\} \subset \{i \in N : \pi'(i) \in N^+\}. \]

In the extreme case in which every player is in a most preferred coalition, π is said to be perfect (Aziz, Brandt, and Harrenstein 2013). A perfect partition satisfies any other of our stability concepts.

A partition is Nash stable if no player would like to unilaterally abandon the coalition he is in and join any other existing coalition or stay on his own, i.e., if, for all \( i \in N \) and all \( S \in \pi \),

\[ \pi(i) \succeq_i S \cup \{i\}. \]

In our example, in partition \( \pi = \{1, 23, 4\} \), the group \( \{124\} \) is still weakly blocking, as \( \pi[\{124\} \to \emptyset] = \{124, 3\} \) and \( \{124, 3\} \) prefers the coalition \( \{12, 4\} \) and \( \{1, 23, 4\} \) by joining together in a separate coalition. Formally, coalitions, is said to block a partition if they would all benefit by joining together in a separate coalition. Thus, π is Nash stable: players 3 and 4 are satisfied and thus do not want to deviate, while players 1 and 2 cannot form a blocking coalition without 3 or 4. However, \( \{124\} \) is still weakly blocking, and as such \( \{14, 23\} \) is not strict core stable.

Also observe that for \( \pi = \{1, 23, 4\} \) the group \( \{124\} \) is strongly blocking, as \( \pi[\{124\} \to \emptyset] = \{124, 3\} \) and \( \{124, 3\} \) prefers the coalition \( \{1, 2, 3, 4\} \) and \( \{12, 3\} \) by joining together in a separate coalition. In the extreme case in which every player is in a most preferred coalition, \( \pi \) is Nash stable. By contrast, \( \{14, 23\} \) is core stable: players 3 and 4 are satisfied and thus do not want to deviate, while players 1 and 2 cannot form a blocking coalition without 3 or 4. However, \( \{124\} \) is still weakly blocking, and as such \( \{14, 23\} \) is not strict core stable.

For envy-freeness, consider partition \( \pi' = \{1, 24, 3\} \). Then, player 3 envy-free player 4, as \( \pi'[3 \to 4] = \{1, 23, 4\} \) and \( \{1, 23, 4\} \) prefers the coalition \( \{12, 3\} \) and \( \{1, 23, 4\} \) by joining together in a separate coalition. By contrast, player 3 does not envy player 2: we have \( \pi'[3 \to 2] = \{1, 2, 34\} \) but not \( \{1, 2, 34\} \) envy-free player 2.

### A Logic for Coalition Structures

In this section, we develop a logic for representing coalition structures. We will then use this logic as a compact specification language for dichotomous preference relations in hedonic games.

#### Syntax

Given a set \( N \) of \( n \) players, we define a propositional language \( L_N \) built from the classical connectives and containing for every (unordered) pair \( (i, j) \) of distinct players a propositional variable \( p_{(i,j)} \). The set of propositional variables we denote by \( V \). Observe that \( |V| = \binom{n}{2} \).

For notational convenience we will write \( i j \) for \( p_{(i,j)} \). Thus, \( i j \) and \( j i \) refer to the same symbol. The language is interpreted on coalition structures on \( N \) and the informal meaning of \( ij \) is “\( i \) and \( j \) are in the same coalition.” Formally, the formulas of the language \( L_N \), with typical element \( \varphi \) is given by the following grammar

\[ \varphi ::= ij \mid \lnot \varphi \mid (\varphi \lor \varphi) \]

where \( i, j \in N \) and \( i \neq j \). By \( |\varphi| \) we denote the size of \( \varphi \).

The classical connectives \( \lnot, \land, \lor, \rightarrow, \iff \) are defined in the usual way. For \( i \) a player, we write \( V_i \) for the propositional variables in which \( i \) appears, i.e.,

\[ V_i = \{ij \in V : j \in N \setminus \{i\}\}. \]

Note that for distinct players \( i \) and \( j \) we have \( V_i \cap V_j = \{ij\} \).

With a slight abuse of notation, we denote the propositional language over \( V_i \) by \( L_i \). We also make use of the following useful notational shorthand:

\[ i_1 \cdots i_m \bar{i}_{m+1} \cdots \bar{i}_p = \bigwedge_{1 \leq j \leq m} i_1 i_2 \land \bigwedge_{m+k \leq p} \lnot i_k. \]

Thus, \( i_1 \cdots i_m \bar{i}_{m+1} \cdots \bar{i}_p \) conveys that \( i_1, \ldots, i_m \) are in the same coalition and each of them in another coalition than \( i_{m+1} \cdots i_p \). Thus, where \( N = \{1, 2, 3, 4\} \), \( 1234 \lor 1324 \lor 1243 \) abbreviates \( (12 \land \lnot 13 \land \lnot 14) \lor (13 \land \lnot 12 \land \lnot 14) \lor (14 \land \lnot 12 \land \lnot 13) \) and signifies that player 1 is in a coalition of two players.

Peters (2016a; 2016b) uses a slightly different language than we, where in the goal expressed by player \( i \), \( i \) is left implicit. For instance, if the goal of agent 1 is expressed in our language by \( 12 \land \lnot 13 \) (and abbreviated into 125) then it would be expressed in his language by \( 12 \land \lnot 3 \). Obviously, both languages are almost identical, and the choice of either of them has no impact on any of the results.
Semantics We interpret the formulas of $L_N$ on partitions $\pi$ as follows.

\[
\begin{align*}
\pi \models ij & \iff \pi(i) = \pi(j) \\
\pi \models \neg \varphi & \iff \pi \not\models \varphi \\
\pi \models \varphi \land \psi & \iff \pi \models \varphi \lor \pi \models \psi \\
\end{align*}
\]

For $\Psi \subseteq L_N$, we have $\Psi \models \varphi$ if $\pi \models \psi$ for all $\psi \in \Psi$.

Notice that partitions play a dual role in our framework: both their initial role as coalition structures, and the role of models in our logic. This dual role is key to using formulas of our propositional language as a specification language for preference relations. Thus, e.g., partition $[1\{2\}345]$ satisfies the following formulas of $L_N$: $345$, $3\overline{1}$, $345\overline{2}$, $\neg 12 \land (23 \lor 34)$, and $12 \leftrightarrow 23$.

Axiomatization We have the following axiom schemes for mutually distinct players $i$, $j$, and $k$.

(A0) all propositional tautologies

(A1) $ij \land jk \rightarrow ik$ (transitivity)

as well as modus ponens as the only rule of the system:

(MP) from $\varphi$ and $\varphi \rightarrow \psi$ infer $\psi$. (modus ponens)

The resulting logic we refer to as $P$ and write $P \vdash P \varphi$ if there is a derivation of $\varphi$ from $\Psi$, (A0), and (A1), using modus ponens.

Proposition 1 (Completeness) Let $\Psi \cup \{\varphi\} \subseteq L_N$. Then,

$P \vdash P \varphi \iff \Psi \models \varphi$.

Proof sketch: Soundness is straightforward. For completeness a standard Lindenbaum construction can be used. To this end, assume $P \not\models \varphi$. Then, $\Psi \cup \{\neg \varphi\}$ is consistent and can as such be extended to a maximal consistent theory $\Psi^*$. Define a relation $\sim_{\Psi^*}$ such that for all $i$, $j \in N$,

\[i \sim_{\Psi^*} j \iff ij \in \Psi^*\]

Together with (MP), the axiom schemes (A0) and (A1) ensure that $\sim_{\Psi^*}$ is a well-defined equivalence relation. Let $[i]_{\sim_{\Psi^*}} = \{j \in N : i \sim_{\Psi^*} j\}$ be the equivalence class under $\sim_{\Psi^*}$, to which player $i$ belongs. Then define the partition $\pi_{\Psi^*} = \{[i]_{\sim_{\Psi^*}} : i \in N\}$. By a straightforward structural induction, it can then be shown that for all $\psi \in L_N$,

$P \Psi^* \models \psi \iff \psi \in \Psi^*$.

It follows that $P \Psi^* \models \Psi$ and $P \Psi^* \not\models \varphi$. Hence, $P \not\models \varphi$. \Box

As an aside, note that one can reason with coalition structures in standard propositional logic, by writing the transitivity axiom directly as a propositional logic formula. Let

\[\text{trans} = \bigwedge_{i,j,k \in N} (ij \land jk \rightarrow ik),\]

where $i$, $j$, and $k$ are assumed to be distinct. Then, for all propositional formulas $\varphi$ and $\psi$ of $L_N$,

$P \vdash P \varphi \varphi \land \text{trans} \vdash \psi$,

i.e., checking whether a formula $\varphi$ implies another formula $\psi$ in $P$ is equivalent to saying that $\varphi$ together with the transitivity constraint implies $\psi$. This means that reasoning tasks in $P$ can be done with a classical propositional theorem prover. In what follows we say that two formulas $\varphi$ and $\psi$ are $P$-equivalent whenever their equivalence can be proven in $P$, i.e., $P \varphi \leftrightarrow \psi$.

Boolean Hedonic Games

The denotation of a formula $\varphi$ of our propositional language is a set of coalition structures, and we can naturally interpret these as being the desirable or satisfactory coalition structures for a particular player. Thus, instead of writing a hedonic game with dichotomous preferences as a structure $(N, \preceq_1, \ldots, \preceq_n)$, in which we explicitly enumerate preference relations $\preceq_i$, we can instead write $(N, \gamma_1, \ldots, \gamma_n)$, where $\gamma_i$ is a formula of our propositional language that acts as a specification of the preference relation $\preceq_i$. Because every player $i$ is indifferent between any two partitions that coincide on $\pi(i)$, without loss of generality $\gamma_i$ involves only propositional variables mentioning $i$, i.e., it is a formula in the sublanguage $L_i$ of $L_N$ in which only variables in $V_i = \{ij : j \in N \backslash \{i\}\}$ occur.

Intuitively, $\gamma_i$ represents player $i$’s ‘goal’ and player $i$ is satisfied if his goal is achieved and unsatisfied if he is not. We refer to a structure $(N, \gamma_1, \ldots, \gamma_n)$ as a Boolean hedonic game. Thus, a Boolean hedonic game $(N, \gamma_1, \ldots, \gamma_n)$ represents the (standard) hedonic game $(N, \geq_1, \ldots, \geq_n)$ with for each $i$,

$P \gamma_i \iff P \gamma_i \models \gamma_i$ implies $P \models \gamma_i$.

Observe that, defined thus, the preferences of each player in a hedonic Boolean game are dichotomous.

Often, the use of propositional formulas $\gamma_i$ gives a ‘concise’ representation of the preference relation $\geq_i$, although of course in the worst case the shortest formula $\gamma_i$ representing $\geq_i$ may be of size exponential in the number of players. In what follows, we will write $(N, \gamma_1, \ldots, \gamma_n)$, understanding that we are referring to the game $(N, \geq_1, \ldots, \geq_n)$ corresponding to this specification.

Example 1 (continued) The hedonic game with dichotomous preferences in Example 1 is represented by the Boolean hedonic game $(N, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$ with $N = \{1, 2, 3, 4\}$ and the players’ goals given by:

\[\gamma_1 = 123 \lor 124 \lor 134 \quad \gamma_2 = 2134 \lor 2143 \lor 2341 \]
\[\gamma_3 = (31 \lor 32) \land \neg 34 \quad \gamma_4 = \neg 423.\]

Then, $P \models \gamma_i$ if and only if $i \in M_i^+$, for each player $i$.

Substitution and Deviation

We establish a formal link between substitution in formulas of our language and the possibility of players deviating from their respective coalition in a given partition and joining other coalitions.
Substitution  We first introduce some notation and terminology with respect to substitution of formulas for variables in our logic.

For $ij$ a propositional variable in $V_N$ and $\varphi$ and $\psi$ formulas of $L_N$, we denote by $\varphi_{ij\rightarrow \psi}$ the uniform substitution of variable $ij$ by $\psi$ in $\varphi$. If $i j = i_j, j_1, \ldots, j_k$ is a sequence of $k$ distinct variables in $V$ and $\varphi = \varphi_1, \ldots, \varphi_k$ a sequence of $k$ formulas,

$$\varphi_{ij\rightarrow \psi} = \varphi_{i_1, j_1, \ldots, i_k, j_k \rightarrow \psi_1, \ldots, \psi_k}$$

denotes the simultaneous substitution of each $i_j m_j$ by $\psi_m$ ($1 \leq m \leq k$). Thus, e.g., $(ij \rightarrow \neg kj)_{ij, jk \rightarrow jk, \neg ik}$.

A special case, which recurs frequently in what follows, is if every $\varphi_2$ is a Boolean, i.e., if $\varphi_1, \ldots, \varphi_k \in \{\top, \bot\}$. Sequences $b = b_1, \ldots, b_k$ where $b_1, \ldots, b_k \in \{\top, \bot\}$ we will also refer to as Boolean vectors of length $k$. Thus, e.g., $\top, \bot$ is a Boolean vector of length 2 and $(ij \rightarrow jk \rightarrow kl)_{ik, jk \rightarrow \bot, \top}$.

Characterising Individual Deviations  Some of the stability concepts for Boolean hedonic games we consider, e.g., Nash stability, are based on which coalitions an individual player $i$ can join given a partition $\pi$. For instance, let partition $\pi$ be given by $[12 \mid 34 \mid 5]$. Then, player 1 can join coalition $\{3, 4\}$ but cannot form a coalition with players 4 and 5 by unilaterally deviating from $\pi$. We find that the coalitions in $\pi_{\rightarrow 1}$ can be characterised in our logic. This yields a logical characterisation of when a player $i$ can unilaterally break loose from his coalition, join another one and thereby guarantee that a given formula $\varphi$ will be satisfied. A particularly interesting case is if $\varphi$ implies the respective player’s goal. We thus gain expressive power with respect to whether a player can beneficially deviate from a given partition, a crucial concept. We make this precise in the following two lemmas, where we say, for a given player $i$, that enumeration $i j = i_j, j_1, \ldots, j_{n-1}$ of $V_i$ and Boolean vector $b = b_1, b_2, \ldots, b_{n-1}$ induce set $B \subseteq N \setminus \{i\}$ if $B = \{j_k : i j_k \in V_i \text{ and } b_k = \top\}$.

**Lemma 1** Let $\pi$ be a partition, $i$ a player and $B \subseteq N \setminus \{i\}$ induced by enumeration $i j$ of $V_i$ and Boolean vector $b$. Then, $B \in \pi_{\rightarrow 1}$ iff $\pi \models trans_{i j \rightarrow b} \varphi$.

**Proof:** As $b$ and $i j$ are fixed throughout the proof, for better readability, we write $\varphi'$ for $\varphi'_{i j \rightarrow b}$. For the “only if”-direction, assume $B \in \pi_{\rightarrow 1}$ and for contradiction that $\pi \nvdash trans'. \text{ Observe that } trans' = \bigwedge_{k,l,m} (kl' \land \varphi \rightarrow km')$. Alternatively, there are some mutually distinct $k, l, m$ such that $\pi \nvdash kl' \land \varphi \rightarrow km'$. It suffices to distinguish three cases:

(a) $i \notin \{k, l, m\}$,  
(b) $i = k$,  
(c) $i = l$.

Case (a) cannot occur as we would have $kl' = kl, \varphi \rightarrow km' = km$, and $kl \land \varphi \rightarrow km$ is a theorem of the system.

If (b), let $l = j_l$ and $m = j_m$. Then $\pi \nvdash il' \land \varphi \rightarrow im'$. It follows that $\pi \models il' \land \varphi \rightarrow km'$.

Thus, $B$ can be understood as the operation of forgetting everything about player $i$ (in the sense of (Lin and Reiter 1994)) while taking the transitivity constraint into account. Intuitively, $\exists \varphi$ signifies that given partition $\pi$ player $i$ can deviate to some coalition such that $\varphi$ is satisfied.
Proposition 2 Let \( \pi \) be a partition, \( i \) a player, and \( \varphi \) a formula of \( L_N \). Then,
\[
\pi \models \exists i \varphi \iff \pi[i \rightarrow S] \models \varphi \text{ for some } S \in \pi_{\sim i}.
\]
Proof: First assume \( \pi \models \exists i \varphi \). Then, \( \pi \models (\varphi \land \text{trans})_{\bar{b} \bar{b}} \)
for some Boolean vector \( \bar{b} = b_1, \ldots, b_{n-1} \). Define \( S \) as the set induced by \( ij \) and \( \bar{b} \). As \( (\varphi \land \text{trans})_{\bar{b} \bar{b}} = (\varphi)_{\bar{b} \bar{b}} \land \text{trans}_{\bar{b} \bar{b}} \), by Lemmas 1 and 2, we then obtain \( S \in \pi_{\sim i} \) and \( \pi[i \rightarrow S] \models \varphi \), respectively.

For the opposite direction, assume that \( \pi[i \rightarrow S] \models \varphi \) for some \( S \in \pi_{\sim i} \). Define \( \bar{b} = b_1, \ldots, b_{n-1} \) as the Boolean vector of length \( n - 1 \) such that for every \( 1 \leq k \leq n - 1 \),
\[
b_k = \begin{cases} 
\top & \text{if } j \in S \cup \{i\}, \\
\bot & \text{otherwise}.
\end{cases}
\]
Then, clearly, \( S \) is the set induced by \( ij \) and \( \bar{b} \). By Lemmas 1 and 2, it follows that \( \pi \models \text{trans}_{\bar{b} \bar{b}} \) and \( \pi \models \varphi_{\bar{b} \bar{b}} \). We may conclude that \( \pi \models \exists i \varphi \). \( \square \)

Note, however, that the number of Boolean vectors of length \( k \) is exponential in \( k \). Accordingly, \( \exists i \varphi \) abbreviates a formula whose size is exponential in the size of \( \varphi \).

Characterising Group Deviations Besides a single player deviating from its coalition and joining another, multiple players (from possibly different coalitions) could also deviate together and form a coalition of their own. This concept lies at the basis of, e.g., the core stability concept. We characterise group deviations through substitution.

Let \( T = \{i_1, \ldots, i_t\} \) be a group of players. Observe that \( |V_T| = \binom{n}{2} - \binom{n-t}{2} \) and let \( i_j T \) be a fixed enumeration of \( V_T \). Given \( i_j T \), we define the \( T \)-separating Boolean vector \( \bar{b}_T \) as the unique Boolean vector of length \( \binom{n}{2} - \binom{n-t}{2} \) such that for all \( i \in T \) and all \( j \in N \),
\[
ij_{i_j \bar{b}_r} = \begin{cases} 
\top & \text{if } j \in T, \\
\bot & \text{otherwise}.
\end{cases}
\]
Intuitively, \( \bar{b}_T \) represents the choice of group \( T \) to form a coalition of their own. Whenever \( T \) is clear from the context we omit the subscript in \( \bar{b}_T \) and \( i_j T \). The following characterisation now holds.

Lemma 3 Let \( T \) a group of players, \( \pi \), a partition, \( i_j \) a fixed enumeration of \( V_T \), and \( \bar{b}_T \) the corresponding \( T \)-separating Boolean vector. Then, for every formula \( \varphi \in L_N \),
\[
\pi \models (\varphi)_{i_j \bar{b}_T} \iff \pi[T \rightarrow \emptyset] \models \varphi.
\]

Characterising Solutions Our task in this section is to show how the various solution concepts we introduced above can be characterised as formulas of our propositional language. Let \( f \) be a function mapping each Boolean hedonic game \( G \) for \( N \) to a formula \( f(G) \) of \( L_N \). Given a solution concept \( \theta \), we say that \( f \) is a characterisation of \( \theta \) if for every Boolean hedonic game \( G \) on \( N \) and every partition \( \pi \), we have that \( \pi \models \varphi \) is a solution according to \( \theta \) for game \( G \) if and only if \( \pi \models f(G) \). If, furthermore, there exists a polynomial \( p \) such that \( |f(G)| \leq p(|N|) \), then \( f \) is a polynomial characterisation of \( \theta \).

Once we have a characterisation of \( \theta \), we know that there is a one-to-one correspondence between the partitions of \( N \) satisfying \( \theta \) and the models of \( f(G) \), i.e., our characterisation is model-preserving.\(^1\) Therefore, given a Boolean hedonic game \( G \):

- checking whether there exists a partition satisfying \( \theta \) in \( G \) amounts to checking whether \( f(G) \) is satisfiable;
- computing a partition satisfying \( \theta \) in \( G \) amounts to finding a model of \( f(G) \);
- computing all partitions satisfying \( \theta \) in \( G \) amounts to finding all models of \( f(G) \).

Thus, once we have a characterisation of a solution concept, one may want to use a SAT solver to find (some or all) or to check the existence of partitions that satisfy it. This carries over to conjunctions of solution concepts. For instance, if individual rationality is characterised by \( f_{IR} \) and envy-freeness by \( f_{EF} \), there is a one-to-one correspondence between the individual rational envy-free partitions for \( G \) and the models of \( f_{IR}(G) \land f_{EF}(G) \). More generally, these techniques can be used for finding or checking partitions satisfying \( \theta \) that also have other properties expressible in \( L_N \).

In the remainder of the section we focus on a number of classical solution concepts, and see how they can be characterised in our logic.

Individual Rationality, Perfection, and Optimality Recall that a partition is individually rational if any player is at least as happy in her coalition as being alone, i.e., no player would prefer to leave her coalition to form a singleton coalition. Now we have the following characterisation of individual rationality in our logic.

Proposition 3 Let \( (N, \gamma_1, \ldots, \gamma_n) \) be a Boolean hedonic game, \( i \) a player, and \( \pi \) a partition. Let \( i_j \) be a fixed enumeration of \( V_i \), and let \( b = \bot, \ldots, \bot \) be the Boolean vector of length \( n - 1 \) only containing \( \bot \). Then,
\[
(i) \quad \pi \text{ is acceptable to } i \iff \pi \models (\gamma_i)_{i_j \bot} \rightarrow \gamma_i,
\]
\[
(ii) \quad \pi \text{ is individually rational iff } \pi \models \bigwedge_{i \in N} ((\gamma_i)_{i_j \bot} \rightarrow \gamma_i).
\]
Proof: We only give the proof for (i), as (ii) follows as an immediate consequence. For (i), observe that the set induced by \( i_j \) and \( \bot \) is \( \emptyset \) and therefore contained in \( \pi_{\sim i} \). Then consider the following equivalences,
\[
\pi \text{ is acceptable to } i \iff \pi \models (\gamma_i)_{i_j \bot} \rightarrow \gamma_i,
\]
\[
\pi \models (\gamma_i)_{i_j \bot} \rightarrow \gamma_i \iff \pi \models (\gamma_i)_{i_j \bot} \rightarrow \gamma_i.
\]
\( \square \)

\( ^1 \)Note that the fact that the existence of a partition satisfying some solution concept is \( \text{NP-complete} \) does not imply that there is a model-preserving translation into SAT.
of which the third one holds by virtue of Lemma 2. \hspace{1em} \square

Note that these characterisations are of polynomial size. To illustrate Proposition 3 we consider again Example 1.

**Example 1 (continued)** In the game of our example, all partitions are acceptable to player 1, whose goal is given by $\gamma_1 = 123 \lor 124 \lor 134$. Let $V_1$ be enumerated by $1\gamma_7 = 12, 13, 14$ and let $\delta = \bot, \bot, \bot$. Thus, $(\gamma_2)_{12,13,14} = \bot, \bot, \bot$ is P-equivalent to $\bot$ and, hence, $\pi = (\gamma_2)_{12,13,14} = \bot, \bot, \bot \rightarrow \gamma_1$ for all partitions $\pi$. According to Proposition 3 this signifies that to player 1 every partition is acceptable.

Now consider player 4, whose goal is given by $\neg 423$, i.e., by $\neg (42 \land 43)$. Let $V_4$ be enumerated by $41, 42, 43$ and let $\delta = \bot, \bot, \bot$. Then, $\neg (42 \land 43)_{41,42,43} = \neg (\bot \land \bot) = \neg \bot$, which is obviously P-equivalent to $\bot$. Hence,

$$\pi \models \neg (42 \land 43)_{41,42,43} = \bot, \bot, \bot \iff \pi \models \neg (42 \land 43),$$

meaning that a partition $\pi$ is acceptable to player 4 if and only if $\pi$ satisfies his goal.

Thus, we obtain the following logical characterisation.

**Proposition 4** Let $(N, \gamma_1, \ldots, \gamma_n)$ be a Boolean hedonic game. Then, a partition $\pi$ is perfect \iff $\pi \models \bigwedge_{i \in N} \gamma_i$.

Consequently, a perfect partition exists if and only if the formula $\gamma_1 \land \bigwedge_{i \in N} \gamma_i$ is satisfiable. Finding a social welfare maximising partition reduces thus to finding valuation satisfying a maximum number of formulas $\gamma_i \land \gamma_j$.

Note that deciding whether a perfect partition on a Boolean hedonic game exists is NP-complete (Peters 2016a), which makes this translation into satisfiability even more meaningful.

Leveraging the same idea of iteratively checking whether a perfect partition for a subset of agents exists, one can compute Pareto optimal solutions for a given game. A subset $\Psi$ of formulas is said to be maximal trans-consistent if both

(i) $\Psi \cup \{ \text{trans} \}$ is consistent, and

(ii) $\Psi' \cup \{ \text{trans} \}$ is inconsistent for all $\Psi'$ with $\Psi \subseteq \Psi'$.

We now have the following proposition.

**Proposition 5** A partition $\pi$ of a Boolean hedonic game is Pareto optimal if and only if $\{ \gamma_i : \pi \models \gamma_i \}$ is a maximal trans-consistent subset of $\{ \gamma_1, \ldots, \gamma_n \}$.

Algorithms for computing maximal consistent subsets are well-known and may be used to find Pareto optimal partitions (cf., e.g., (Ben-Eliyahu and Dechter 1996; Marquis 2000; Lian and Waaler 2008; Lifitto and Sakallah 2008)).

**Nash Stability** Recall that a partition $\pi$ is Nash stable, if no player $i$ wishes to leave his coalition $\pi(i)$ and join another coalition so as to satisfy his goal. Exploiting the results from the previous section, we obtain the following characterisation of this fundamental solution concept.

**Proposition 6** Let $(N, \gamma_1, \ldots, \gamma_n)$ be a Boolean hedonic game and $\pi$ a partition. Then,

$$\pi \text{ is Nash stable} \iff \pi \models \bigwedge_{i \in N} ((\exists i \gamma_i) \rightarrow \gamma_i).$$

**Proof:** Consider an arbitrary player $i$ and observe that following equivalences hold.

$\pi$ is Nash stable

\(\text{iff} \) for all $i$ and $S \in \pi \models i \rightarrow S$

\(\text{iff} \) for all $i$ and $S \in \pi \models i \models i \rightarrow S$

\(\text{iff} \) for all $i$, if $\pi \models i \rightarrow S$ then $\pi \models \gamma_i$

\(\text{iff} \) for all $i$, if $\pi \models \gamma_i$ then $\gamma_i$

\(\text{iff} \) for all $i$, $\pi \models ((\exists i \gamma_i) \rightarrow \gamma_i)$

$\pi \models \bigwedge_{i \in N} ((\exists i \gamma_i) \rightarrow \gamma_i).$

The fourth equivalence holds by virtue of Proposition 2. The third one is a standard law of logic; merely observe that whether $\pi \models \gamma_i$ is not dependent on $S$. \hspace{1em} \square

Our running example illustrates this result.

**Example 1 (continued)** Consider again the game of Example 1. Partition $[1234]$ satisfies each player’s goal and, consequently, is Nash stable. We also have that $[1234] \models \gamma_1 \land \gamma_2 \land \gamma_3 \land \gamma_4$ and, thus,

$$[1234] \models \bigwedge_{i \in N} ((\exists i \gamma_i) \rightarrow \gamma_i).$$

Now recall that for partition $\pi = [1234]$ player 2’s goal is not satisfied and that she cannot deviate and join another coalition to make this happen. In this case, $\pi_{-2} = \{1\}, \{3\}, \{4\}$. Moreover, $\pi_{2 \rightarrow \{1\}} = [1234]$, $\pi_{2 \rightarrow \{3\}} = [1234]$, and $\pi_{2 \rightarrow \{4\}} = [1234]$. Since, $[1234] \not\models \gamma_2$, $[1234] \not\models \gamma_2$, and $[1234] \not\models \gamma_2$, it follows that $\pi \not\models (\exists i \gamma_i) \rightarrow \gamma_i$. Hence, $\pi \models (\exists i \gamma_i) \rightarrow \gamma_i$. Player 1, however, could deviate from $\pi_2$ and join $\{2,3\}$ and thus have his goal satisfied. Thus, $\pi$ is not Nash stable. Now observe that $\{2,3\} \in \pi_{-1}$ and that $\pi_{1 \rightarrow \{2,3\}} = [1234]$. Moreover, $[1234] \models \gamma_1$. As such $\pi \models (\exists i \gamma_i)$, also $\pi \not\models (\exists i \gamma_i) \rightarrow \gamma_i$. We may conclude that

$$[1234] \not\models \bigwedge_{i \in N} ((\exists i \gamma_i) \rightarrow \gamma_i).$$

Nash stable partitions are not guaranteed to exist in Boolean hedonic games. The two-player game $\{1,2\}$, $12$, $\{1,2\}$ witnesses this fact, as can easily be appreciated. The translation into a SAT instance gives us a way to compute all Nash stable partitions of a given Boolean hedonic game. Recall, however, that the size of $\exists i \gamma_i$ is generally exponential in the size of $\gamma_i$. Of course, one may wonder why this is useful to express Nash stability as an exponentially large SAT instance, since the existence of a Nash stable partition is “only” NP-complete (Peters 2016a). However, we stress that our translation is model-preserving, which means that it is particularly useful if we want to use Nash stability in conjunction with other concepts; moreover, in many practical cases, the translation will remain of reasonable size.

**Core and Strict Core Stability** Core and strict core stability relate to group deviations much in the same way as
Nash stability relates to individual deviations. Having characterised group deviations in the previous section, we find that Lemma 3 yields a straightforward characterisation in our logic of a specific group blocking or weakly blocking a given partition.

**Proposition 7** Let \((N, \gamma_1, \ldots, \gamma_n)\) be a Boolean hedonic game and \(T\) a group of players, and \(\pi\) be a partition. Let, furthermore, \(\bar{\imath}j\) a fixed enumeration of \(V_T\) and \(\bar{b}_T\) the corresponding \(T\)-separating Boolean vector. Then,

(i) \(T\) blocks \(\pi\) iff \(\pi = \bigwedge_{i \in T} (\neg \gamma_i \land (\gamma_i)_{j \in \bar{T}_i})\),

(ii) \(T\) weakly blocks \(\pi\) iff

\[\pi = \bigwedge_{j \in T} (\gamma_j \rightarrow (\gamma_j)_{j \in \bar{T}_j}) \land \bigvee_{i \in T} (\neg \gamma_i \land (\gamma_i)_{j \in \bar{T}_j}).\]

**Proof:** We give the proof for (i), as the one for (ii) runs along analogous lines. Consider the following equivalences:

\(T\) blocks \(\pi\) iff for all \(i \in T\):

- \(\pi[T \rightarrow \emptyset] \ni \pi\)
- If for all \(i \in T\): \(\pi[T \rightarrow \emptyset] = \gamma_i \land \pi \neq \gamma_i\)
- For all \(i \in T\): \(\pi = (\gamma_i)_{j \in \bar{T}_i} \land \pi \neq \gamma_i\)
- If \(\pi = \bigwedge_{i \in T} (\neg \gamma_i \land (\gamma_i)_{j \in \bar{T}_i})\).

The third equivalence holds by virtue of Lemma 3. \(\square\)

Observe that the size of \(\bigwedge_{i \in T} (\neg \gamma_i \land (\gamma_i)_{j \in \bar{T}_i})\) is polynomial in \(\sum_{i \in T} |\gamma_i|\) and, hence, a partition \(\pi\) being blocking by a group \(T\) of players can be polynomially characterised.

As a corollary of Proposition 7 and the de Morgan laws, we obtain the following characterisations of a partition being core stable and of a partition being strict core stable. The characterisations, however, involve a conjunction over all groups of players and as such is not polynomial.

**Corollary 1** Let \((N, \gamma_1, \ldots, \gamma_n)\) be a Boolean hedonic game and \(\pi\) be a partition. Let for each coalition \(T\), \(\bar{\imath}j\) be an enumeration of \(V_T\) and \(\bar{b}_T\) the corresponding \(T\)-separating Boolean vector. Then,

(i) \(\pi\) is core stable iff \(\pi = \bigwedge_{T \subseteq N \ni i \in T} (\gamma_i)_{j \in \bar{T}_j} \rightarrow \gamma_i)\).

(ii) Then, \(\pi\) is strict core stable iff

\[\pi = \bigwedge_{T \subseteq N \ni i \in T} (\bigvee_{\gamma_j \land (\gamma_j)_{j \in \bar{T}_j} \land \bigvee_{i \in T} (\gamma_i)_{j \in \bar{T}_i} \rightarrow \gamma_i)).\]

Although core stable coalition structures are not guaranteed to exist in general hedonic games, if preferences are dichotomous we have the following positive result.

**Proposition 8** For every Boolean hedonic game, a core stable coalition structure is guaranteed to exist.

**Proof:** We initialise \(N\) to \(N\) and partition \(\pi\) to \(\emptyset\). We find a maximal subset of \(S \subseteq N\) for which all players are in an approved coalition that satisfies their formulas. We modify \(\pi\) to \(\pi \cup \{S\}\) and \(N'\) to \(N' \setminus S\). The procedure is repeated until no such maximal subset \(S\) exists. If \(N' \neq \emptyset\), then \(\pi\) is set to \(\pi \cup \{i\} : i \in N'\).

We now argue that \(\pi\) is core stable. We note that each player who was in some subset \(S\) will never be part of a blocking coalition. If \(N'\) was non-empty in the last iteration, then no subset of players in \(N'\) can form a deviating coalition among themselves. \(\square\)

By contrast, a strict core stable partition is not guaranteed to exist. To see this consider the three-player Boolean hedonic game \((\{1, 2, 3\}, 12, 21\{23, 32\})\); each of the five possible partitions is weakly blocked by either \(\{1, 2\}\) or \(\{2, 3\}\).

The characterisation of strict core stability is not of polynomial size, but it is highly unlikely that such a characterisation exists, since deciding whether there exists a strict core stable partition is \(\Sigma^P_2\)-complete (Peters 2016b).

**Envy-freeness** Recall that a partition is envy-free if no player would strictly prefer to exchange places with another player. Observe that for the trivial partitions \(\pi^0 = [1 \cdots n]\) and \(\pi^1 = [1, \ldots, n]\), we have \(\pi^0[i \rightarrow j] = \pi^0\) and \(\pi^1[i \rightarrow j] = \pi^1\) for all players \(i\) and \(j\). Accordingly \(\pi^0\) and \(\pi^1\) are envy-free. Envy-free partitions are thus guaranteed to exist in our setting. The following lemma allows us to derive a polynomial characterisation of envy-freeness.

**Lemma 4** Let \((N, \gamma_1, \ldots, \gamma_n)\) be a Boolean hedonic game and \(i\) and \(j\) players in \(N\), and \(\varphi\) a formula in \(L_N\). Fix, furthermore, an enumeration \(k_1, \ldots, k_{n-2}\) of \(N \setminus \{i, j\}\) and let \(i\bar{k} = ik_1, \ldots, ik_{n-2}\) and \(j\bar{k} = jk_1, \ldots, jk_{n-2}\) enumerate \(V_i \setminus \{ij\}\) and \(V_j \setminus \{ij\}\), respectively. Then,

\[\pi = \psi_{i\bar{k}, j\bar{k} \rightarrow j\bar{k}, i\bar{k}} \iff \pi[i \rightarrow j] = \varphi.\]

**Proof:** With \(i\bar{k}\) and \(j\bar{k}\) being fixed we write \(\varphi'\) for \(\psi_{i\bar{k}, j\bar{k} \rightarrow j\bar{k}, i\bar{k}}\). The proof is then by a structural induction on \(\varphi\), of which only the basis is interesting.

Let \(\varphi = lm\). There are three possibilities: (a) \(lm = ij\), (b) \(lm \in (V_i \cup V_j) \setminus \{ij\}\), and (c) \(lm \notin V_i \cup V_j\). If (a), we have that \(lm' = ij' = ij = lm\). Now, either \(\pi(i) = \pi(j)\) or \(\pi(i) \neq \pi(j)\). If the former, \(\pi[i \rightarrow j] = \pi\) as well as both \(\pi = \psi'\) and \(\pi[i \rightarrow j] \ni ij\). If the latter, however, it can easily be seen that both \(\pi = \psi'\) and \(\pi[i \rightarrow j] \ni ij\).

For case (b), we may assume without loss of generality that \(lm = ik\) for some \(k \neq j\). Then, \(i\bar{k}' = jk\). In case \(\pi(i) = \pi(j)\), obviously, \(\pi = \psi[i \rightarrow j] = i\bar{k}\) as well as \(\psi[i \rightarrow j] = \psi\). So, assume \(\pi(i) \neq \pi(j)\). Now, either \(i \in \pi(i)\) and \(k \notin \pi(j)\), (ii) \(k \notin \pi(k)\) and \(k \in \pi(j)\), or (iii) \(k \notin \pi(i)\) and \(k \notin \pi(j)\). If (i), \(\pi = ik'\) as well as \(\pi[i \rightarrow j] = jk\). In cases (ii) and (iii), we have \(\pi = ik'\) and \(\pi[i \rightarrow j] \ni jk\).

Finally, if (c), then \(lm' = lm\). As \(lm \notin \{i, j\}\), it is easily seen that \(\pi = \psi[i \rightarrow j] = lm\). \(\square\)

We are now in a position to state the following result.

**Proposition 9** Let \((N, \gamma_1, \ldots, \gamma_n)\) be a Boolean hedonic game. Furthermore, for every two players, \(i\) and \(j\), and enumeration \(k_1, \ldots, k_{n-2}\) of \(N \setminus \{i, j\}\), let \(i\bar{k} = ik_1, \ldots, ik_{n-2}\) and \(j\bar{k} = jk_1, \ldots, jk_{n-2}\) enumerate \(V_i \setminus \{ij\}\) and \(V_j \setminus \{ij\}\), respectively. Then,

\[\pi\] is envy-free iff \(\pi = \bigwedge_{i, j \in N} ((\gamma_i)_{i\bar{k}, j\bar{k} \rightarrow j\bar{k}, i\bar{k}} \rightarrow \gamma_i).\]
Proof: By Lemma 4, the following equivalences hold:

\[ \pi \text{ is envy-free} \]
\[ \text{iff } \forall i, j \in N: \pi_i \geq_i \pi_j \]
\[ \text{iff for all } i, j \in N: \pi_i \equiv_i \pi_j \]
\[ \text{iff for all } i, j \in N: \pi_i \Rightarrow \pi_j \text{ implies } \pi_i \Rightarrow \pi_j \]
\[ \text{iff for all } i, j \in N: \pi_i \Rightarrow (\gamma_i) \text{ holds } \]
\[ \text{iff for all } i, j \in N: \pi_i \Rightarrow (\gamma_i) \text{ implies } \pi_i \Rightarrow \gamma_i \]
\[ \text{iff for all } i, j \in N: \pi_i \Rightarrow \gamma_i \]

This concludes the proof. \( \square \)

Observe that the size of \( \bigwedge_{i,j \in N} (\gamma_i) \text{ is polynomial in } \sum_{i \in N} |\gamma_i| \), hence we get a polynomial characterization of envy-freeness. Note that an envy-free partition always exists (e.g., the partition where all agents are together is envy-free, or the partition where they are all isolated), but once again, the fact that the translation is model-preserving allows us to compute all envy-free partitions.

Example 1 (continued) Recall that \( \gamma_3 = (31 \lor 32) \land -34 \) and that player 3 envies player 4 if partition \( \pi' = [1][2][3] \) obtains. To see how this is reflected by Proposition 9, let 31, 32 and 41, 42 enumerate \( V_3 \setminus \{34\} \) and \( V_4 \setminus \{43\} \), respectively. Then,

\[ (31 \lor 32) \land -34, 31, 32, 41, 42, 31, 32, 41, 42 \land -34. \]

Now, both \( \pi' = (41 \lor 42) \land -34 \) and \( \pi' \neq (31 \lor 32) \land -34 \), and, hence, \( \pi' \not\models (\gamma_3)34, 31, 32, 41, 42, 31, 32, 41, 42 \rightarrow \gamma_3. \)

After some simplifications, the formulas expressing that players 1, 2, 3, 4 are not envious are

- for 1: 234 \( \rightarrow \) 1234
- for 2: -1342
- for 3: -1234 \( \land -413 \land -423 \)
- for 4: -423 \( \lor \) 1234.

Therefore, \( \pi \) is envy-free if and only if it satisfies

\[ 1234 \lor (-234 \land -134 \land -1234 \land -413 \land -423) \]

or, equivalently, 1234 \( \lor (-24 \land -14 \land -124) \). This formula (that we call \( EF \)) is satisfied by the partitions \([12][3][4], \[1][2][34], \[1][23][4], \[1][234] \) and \([1][2][3][4] \). Now, we may want to require envy-freeness in addition with another property. For instance, assume there should be exactly two coalitions, which is expressed by the polynomial-size formula \( \psi = \bigvee_{1 \leq i < j \leq 4} \neg ij \land \bigwedge_{1 \leq i < j < k < 4} (ijk \land jik \land kij). \)

\( EF \land \psi \) has a single model corresponding to \([4][123]\).

Related Work and Conclusions

Our motivation and approach is strongly reminiscent of the setting of Boolean games in the context of non-cooperative game theory (Harrenstein et al. 2001). A major difference with Boolean games and propositional hedonic games is that in Boolean games, players have preferences over outcomes, where an outcome is a truth assignment to outcome variables, and each outcome variable is controlled by a specific player. This control assignment function, which is a central notion in Boolean games, has no counterpart here, where the outcome is a partition of the players. However, there are technical similarities with and conceptual connections to Boolean games, especially when characterising solution concepts. For instance, the characterisation of Nash stable partitions by propositional formulas (Section 4) is similar to the characterisation of Nash equilibria by propositional formulas in Boolean games as by Bonzon et al. (2009). The basic Boolean games model of Harrenstein et al. (2001) was adapted to the setting of cooperative games by Dunne et al. (2008); there, however, the logic used to specific player’s goals was not intended for specifying desirable coalition structures, as we have done in the present paper.

Our work also shares some common ground with the work of Bonzon, Lagasse-Schiex, and Lang (2012), who study the formation of efficient coalitions in Boolean games, especially when characterising solution concepts. For instance, the characterisation of Nash stable partitions by propositional formulas (Section 4) is similar to the characterisation of Nash equilibria by propositional formulas in Boolean games as by Bonzon et al. (2009). The basic Boolean games model of Harrenstein et al. (2001) was adapted to the setting of cooperative games by Dunne et al. (2008); there, however, the logic used to specific player’s goals was not intended for specifying desirable coalition structures, as we have done in the present paper.

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would be interesting to see whether these classes can also be polynomially characterised in our logic.

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References

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