

Exercises

1

Ex.1

Consider a TU-game $(\{1,2,3,4\}, w)$ such that $w(S)=0$ if $S \in \{\{1\}, \{4\}, \{1,4\}\}$; $w(S)=1$ for each other non-empty coalition S . Calculate the Shapley value of w (without using the formula!) and say whether the Shapley value is in the core of the game.

Ex.2

Consider a TU-game (N, v) such that $v(S)=f(s)$ for each $S \in 2^N \setminus \{\emptyset\}$ where s is the cardinality of S and $f(s)$ is a real valued function. Take f as a strictly increasing function.

- 1) Is (N, v) a superadditive game?
- 2) The Shapley value of (N, v) gives $f(n)/n$ to each player?
- 3) Find an f such that the core of (N, v) is non-empty, with $n=3$.

Ex.3

Calculate the Shapley value of the TU-game (N, v) such that

$$N = \{1, 2, 3\},$$

$$v(1) = 3$$

$$v(2) = 4$$

$$v(3) = 1$$

$$v(1, 2) = 8$$

$$v(1, 3) = 4$$

$$v(2, 3) = 6$$

$$v(1, 2, 3) = 10$$

Permutation	1	2	3
1,2,3			
1,3,2			
2,1,3			
2,3,1			
3,2,1			
3,1,2			
Sum			
$\phi(v)$			