Théorie de la décision et théorie des jeux

Stefano Moretti

UMR 7243 CNRS
Laboratoire d'Analyse et Modélisation de Systèmes pour l'Aide à la décision (LAMSADE)
Université Paris-Dauphine

email: Stefano.MORETTI@dauphine.fr

2

Basic definitions

- **From decision theory to game theory**
- **Game form**
- **Preferences and games in strategic form**
- **Some examples**

Relevant characteristics

- Decision makers (=players) engaged in an interactive decision problem:
  - more than one decision maker (DM) (=player). [The “easy case", 1 DM, is left to Decision Theory (DT)]
  - the result is determined by the choices made by each player
  - the decision makers' preferences w.r.t. outcomes are (generally speaking) different.
- Classical assumptions about players: rational and intelligent.

Relevant parameters

- players know the relevant data of the interaction decision problem:
- available strategies
- payoffs
- rationality and intelligence of each player
- each player knows that all players know what is listed above
- each player knows that each player knows...

  COMMON KNOWLEDGE
- not available the possibility of binding agreements:

  NON COOPERATIVE GAMES

Basic model in Decision Theory

- (X, E, h,$\sqsupseteq$) where:
- X set of alternatives (choices) available to the DM
- E set of outcomes
- h:X→E maps alternatives into outcomes
- $\sqsupseteq$ total preorder on E (math object to describe the preferences on E of the DM)
- Very important remark: the rationality assumption is essentially subsumed in the transitivity condition.

Squeezing the model

- We have: (X, E, h,$\sqsupseteq$).
- Using utility functions, we get: (X, E, h, u). To which we can associate the diagram:

$$X \xrightarrow{h} E \xrightarrow{u} \mathbb{R}$$

- Composition of functions: f = u$\circ$h:

$$X \xrightarrow{f} \mathbb{R}$$

- Or, (X, f). Nice simplification.

From one to two DMs: Game form

- A game form (in strategic form), with two players, is:

$$(X, Y, E, h).$$

- New aspects w.r.t. decision theory:
- two DMs (we shall call them "players"), so two sets of available
- alternatives (choices, but here we use the word "strategies")
- $h : X \times Y \rightarrow E$ is the map that converts a couple of strategies

(one for each player) into an outcome.

- Easy to generalize to a finite set of players N:

$$(N, (X_i)_{i \in N}, E, h)$$

- with $h : \prod_{i \in N} X_i \rightarrow E$.

Preferences of the players

- To get a game we need a second ingredient, the preferences of the players.
- If we have two players (called I and II), each will have his preferences: $\sqsupseteq_I$, $\sqsupseteq_{II}$.
- Each $\sqsupseteq_I$, $\sqsupseteq_{II}$ is a total preorder on E.
- We shall represent them by utility functions: u and v.
- We shall often make the assumption that these utilty functions are vNM (von Neumann-Morgenstern).

Game in strategic form

- Patching all together (game form + preferences)...
- We use utility functions. In the 2 players case:

$$(X, Y, E, h, u, v).$$

- The corresponding diagram:

$$\begin{tikzcd} & & \mathbb{R} \\ X \times Y \arrow[r, "h"] & E \arrow[ur, "u"] \arrow[dr, "v"'] & \\ & & \mathbb{R} \end{tikzcd}$$

Game in strategic form: squeezed

- Still in the 2 players case:

$$(X, Y, f, g)$$

- where f = uʒh and g = vʒh.
- The squeezed diagram:

$$\begin{tikzcd} & \mathbb{R} \\ X \times Y \arrow[ur, "f"] \arrow[dr, "g"'] & \\ & \mathbb{R} \end{tikzcd}$$

Example 1: Prisoner's dilemma

- Consider the following game:

I \ II	L	R
T	(3,3)	(1,4)
B	(4,1)	(2,2)

- You are the row player (I).
- The left number in each cell represents the evaluation that you give to the outcome. The number on the right represents the evaluation of player (II)…
- Which row do you choose? T or B?

Prisoner's dilemma tale (not very relevant)

- Two suspects are arrested by the police.
- The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal.

  If one testifies (defects from the other) for the prosecution against the other and the other remains silent (cooperates with the other), the betrayer goes free and the silent accomplice receives the full 10-year sentence (strategies (B,L) or (T,R)).

  If both remain silent, both prisoners are sentenced to only six months in jail for a minor charge (strategies (T,L)).

  If each betrays the other, each receives a five-year sentence (strategies (B,R)).
- Each prisoner must choose to betray the other or to remain silent.
- Each one is assured that the other would not know about the betrayal before the end of the investigation.

Example 2: Coordination game

- Consider the following game:

I \ II	L	C	R
T	(0,0)	(1,1)	(0,0)
M	(0,0)	(0,0)	(1,1)
B	(1,1)	(0,0)	(0,0)

- Again you are the row player (I).
- Which row do you choose? T, M or B?