Workshop on Game Practice and Preference aggregation

Coalitional games arising from interactive situations on networks

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Outline

- Coalitional games and centrality of genes on biological networks
- Cost allocation problems arising from connection situations

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Coalitional games and centrality on biological networks

Networks and biology

- Network based methods have been found useful in biology,
  - protein interaction networks
  - gene regulatory networks
  - gene co-expression networks
  - …
- The structure of a network can formally be represented by a graph L = (N,E)
  - The vertex set contains the genes: N = {xgene, ygene, zgene,…}
  - The **edge set** contains *interactions*.

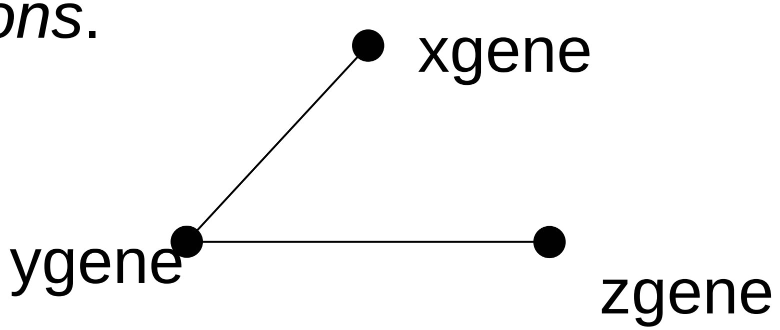
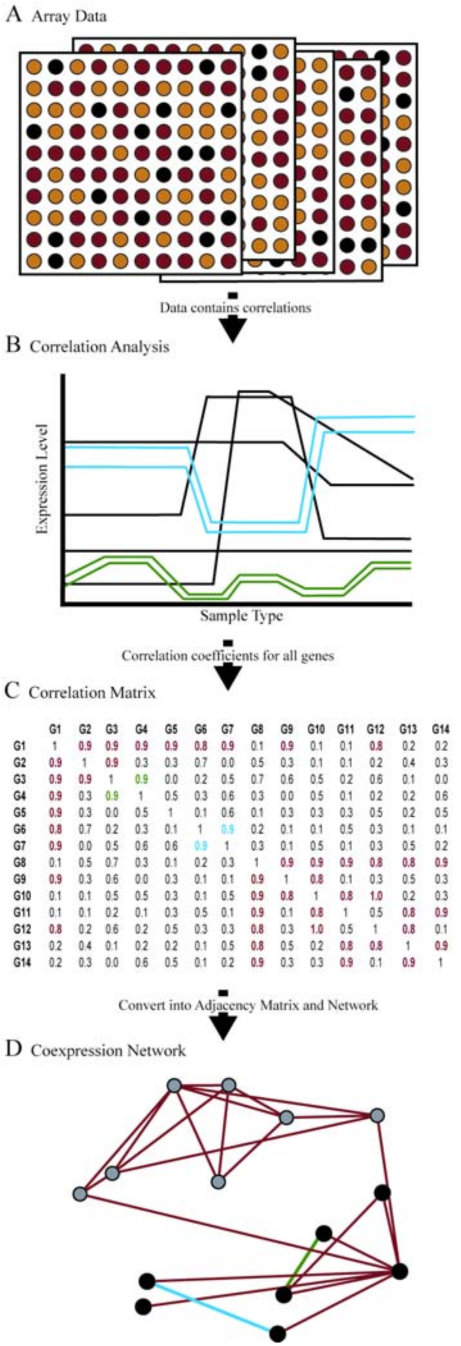


Figure 1



How to construct a biological networks?

A) Microarray gene expression data

B) Measure concordance of gene expression with a Pearson correlation

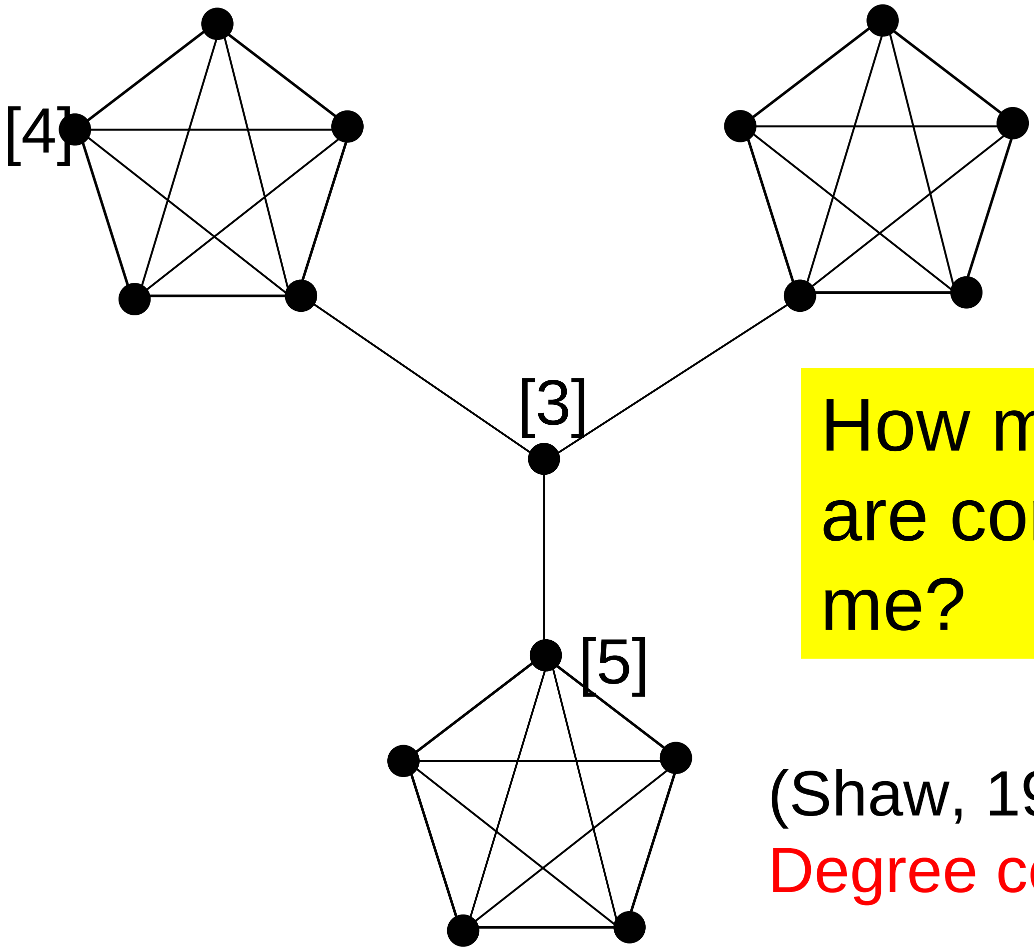
C) The Pearson correlation matrix is dichotomized to arrive at an adjacency matrix

D) → unweighted network

Lethality and centrality

- Rank genes according to their degree centrality and correlate this with the phenotypic effect of their individual removal from the yeast genome and proteome.
    - the likelihood that removal of a protein will prove lethal correlates with degree centrality in protein/gene networks (Jeong et al. *Nature*, 2001, ; Provero [arXiv:cond-mat/0207345], 2002; Carlson, BMC Genomics, 2006.)

Centrality as a measure of the ‘importance’ of a vertex

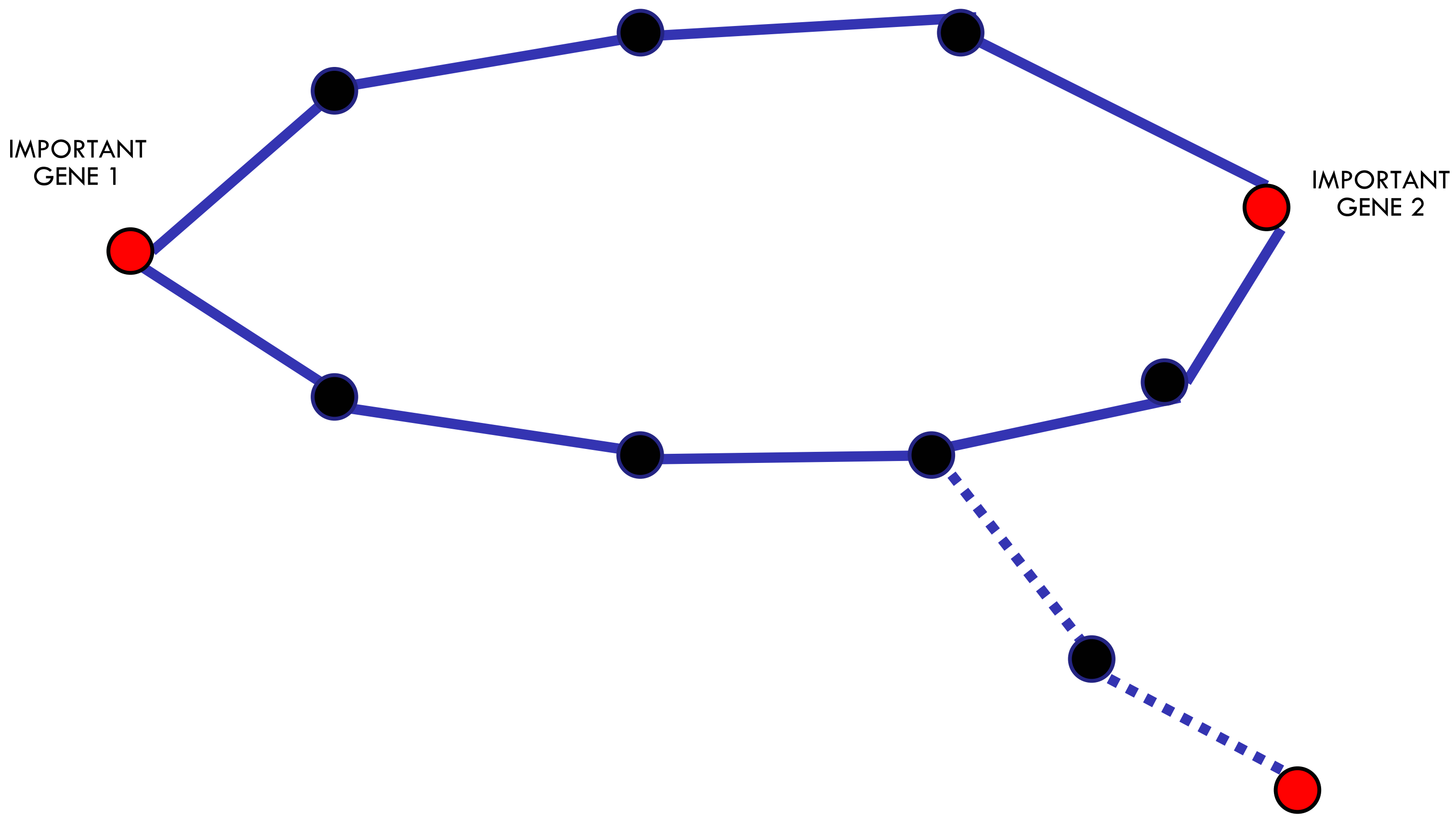


How many nodes are connected to me?

(Shaw, 1954, and Nieminen, 1974)

Degree centrality

Alternative paths and interactions



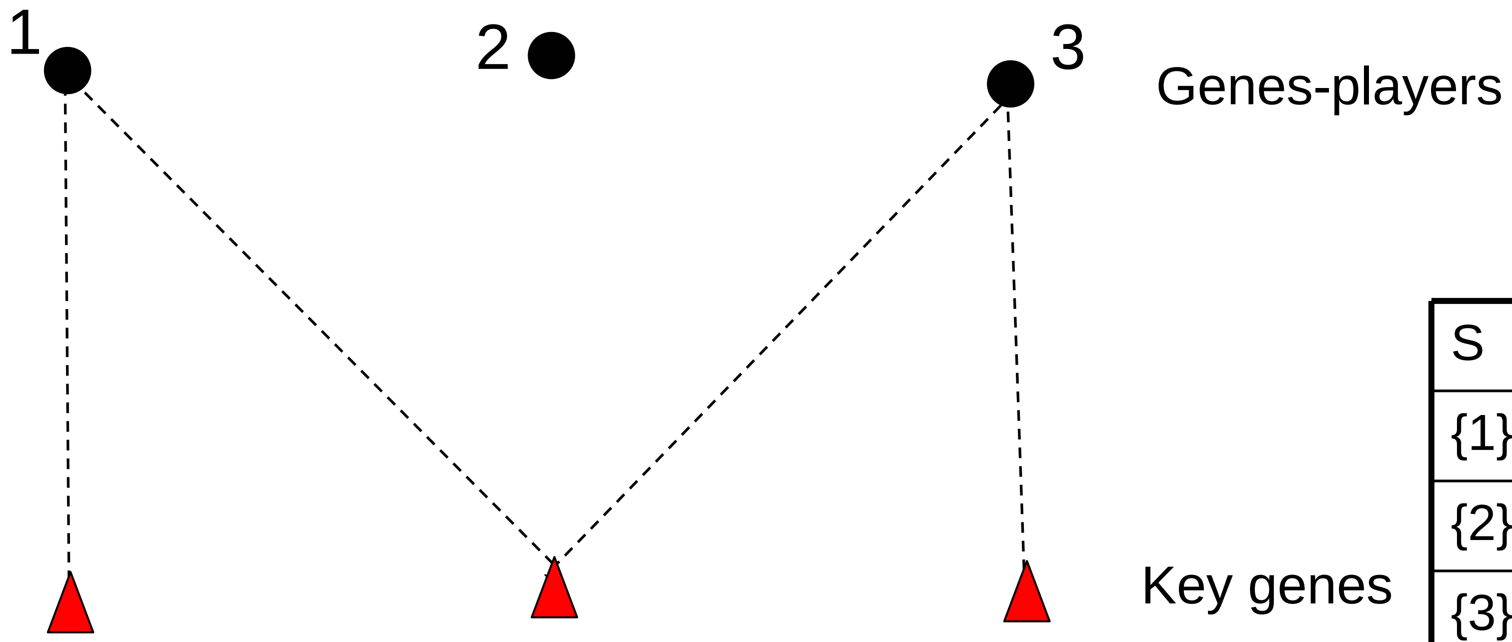
Games and Centrality

- An application of the Shapley value (Shapley (1953)), which uses both the classical one and the one by Myerson (1977), has been proposed by Gómez et al. (2003), to provide a different definition of *centrality* in social networks.
- The centrality of a node refers to the *variation* in power due to the biological situation (represented by the graph).

Centrality γ for biological networks

- (N,E) is a biological network
- An *a priori* coalitional game (N,v) is given
- The proposal is to look at the difference between:
  - $\mu(N,v,E)$: the Myerson value, that takes into account the structure of the biological network;
  - $\phi(N,v)$: the Shapley value, that disregards completely the information provided by biological networks but keeps into account the *a priori* knowledge about *key genes*.
- **Centrality** $\gamma_i(N,v,E) = \mu_i(N,v,E) - \varphi_i(N,v)$

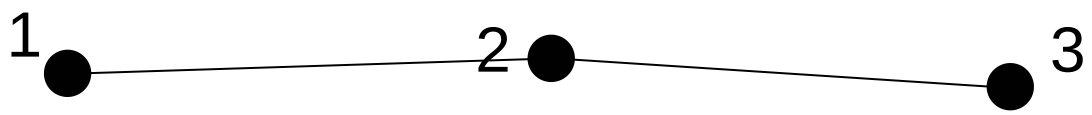
***a priori* game (N,v):** the worth v(S) of a coalition of genes i
S is the number of key genes associated only to* genes ir
S



S	v	φ(v)
{1}	1	1.5
{2}	0	0
{3}	1	1.5
{1,2}	1	
{1,3}	3	
{2,3}	1	
{1,2,3}	3	

*Means that v(S) is the number of key genes connected to S and not connected to gene-players out of S

Communication network: *a co-expression network from experimental data* ({1,2,3},E)



Genes-players

A priori game

S	v	φ(v)
{1}	1	1.5
{2}	0	0
{3}	1	1.5
{1,2}	1	
{1,3}	3	
{2,3}	1	
{1,2,3}	3	

Graph-restricted game

S	w_E	$\varphi(w_E)$
{1}	1	4/3
{2}	0	1/3
{3}	1	4/3
{1,2}	1	
{1,3}	2	
{2,3}	1	
{1,2,3}	3	

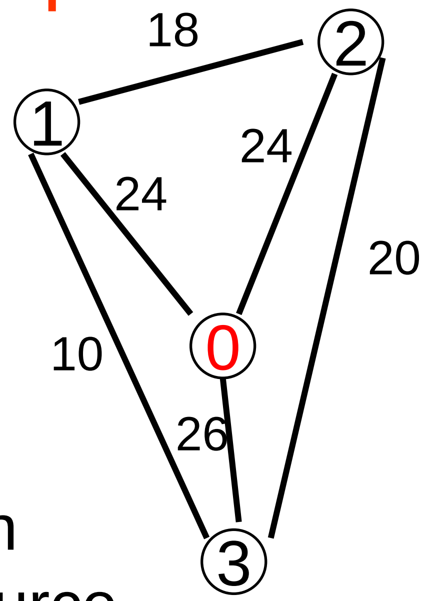
γ
-1/6
1/3
-1/6

2

Cost allocation problems arising from connection situations

Minimum Cost Spanning Tree Situation

Consider a complete weighted graph



– whose vertices represent agents

– vertex 0 is the source

– edges represent connections between agents or between an agent and the source

– numbers close to edges are connection costs

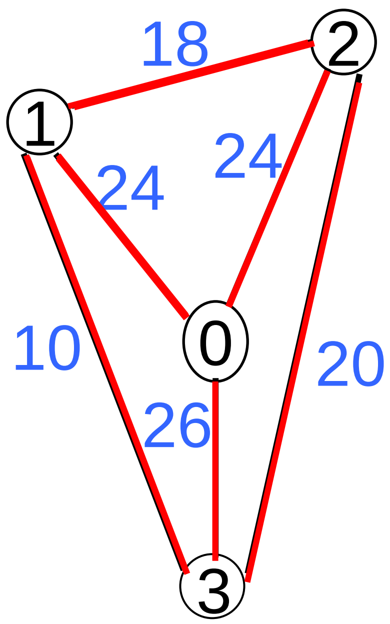
Minimum cost spanning tree (mcst) problem

Optimization problem:

How to connect each node to the source 0 in such a way that the cost of construction of a spanning network (which connects every node directly or indirectly to the source 0) is minimum?

The problem of finding a mcst may be easily solved thanks to different algorithms proposed in literature (Boruvka (1926), Kruskal (1956), Prim (1957), Dijkstra (1959))

Example: The cost game ({1,2,3},c) is defined on the following connection situation:



c(1)=24
c(2)=24
c(3)=26
c(1,3)=34
c(1,2)=42
c(2,3)=44
c(1,2,3)=52

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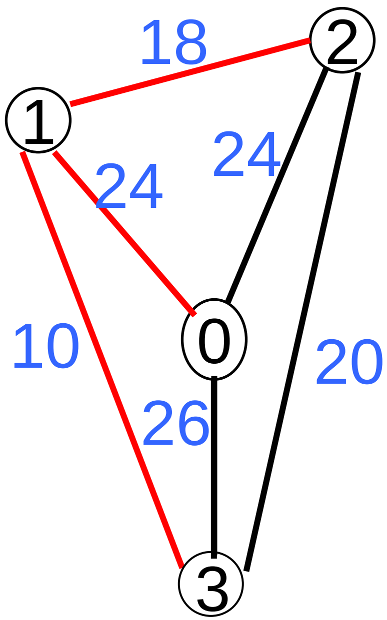
c(2,3)=44
c(1,2,3)=52

The game ({1,2,3}, c) is said mcst game (Bird (1976))

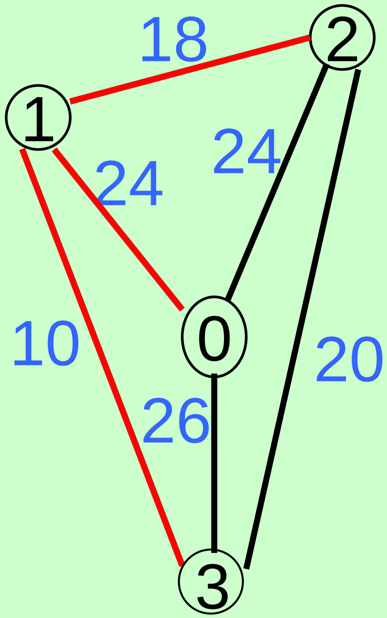
How to divide the total cost? (Bird 1976)



- The predecessor of 1 is 0: the Bird allocation gives to player 1 the cost of {0,1}.
- The predecessor of 2 is 1: the Bird allocation gives to player 2 the cost of {1,2};
- The predecessor of 3 is 1: the Bird allocation gives to player 3 the cost of {1,3}.

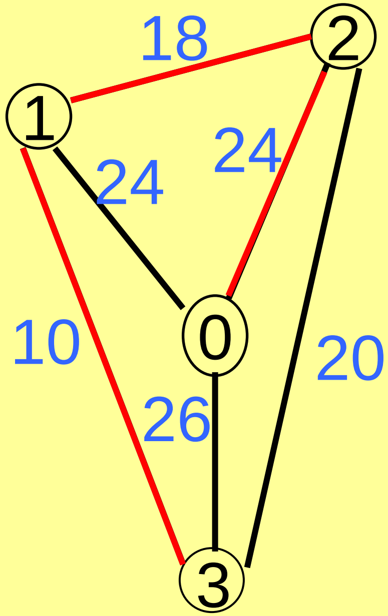
$w(\Gamma)=52$

Bird allocation w.r.t. to Γ, $(x_1, x_2, x_3)=(24, 18, 10)$ is in the core of ({1,2,3},c).



The Bird allocation w.r.t .this mcst is

$(x_1, x_2, x_3)=(24, 18, 10)$



The Bird allocation w.r.t. this mcst is

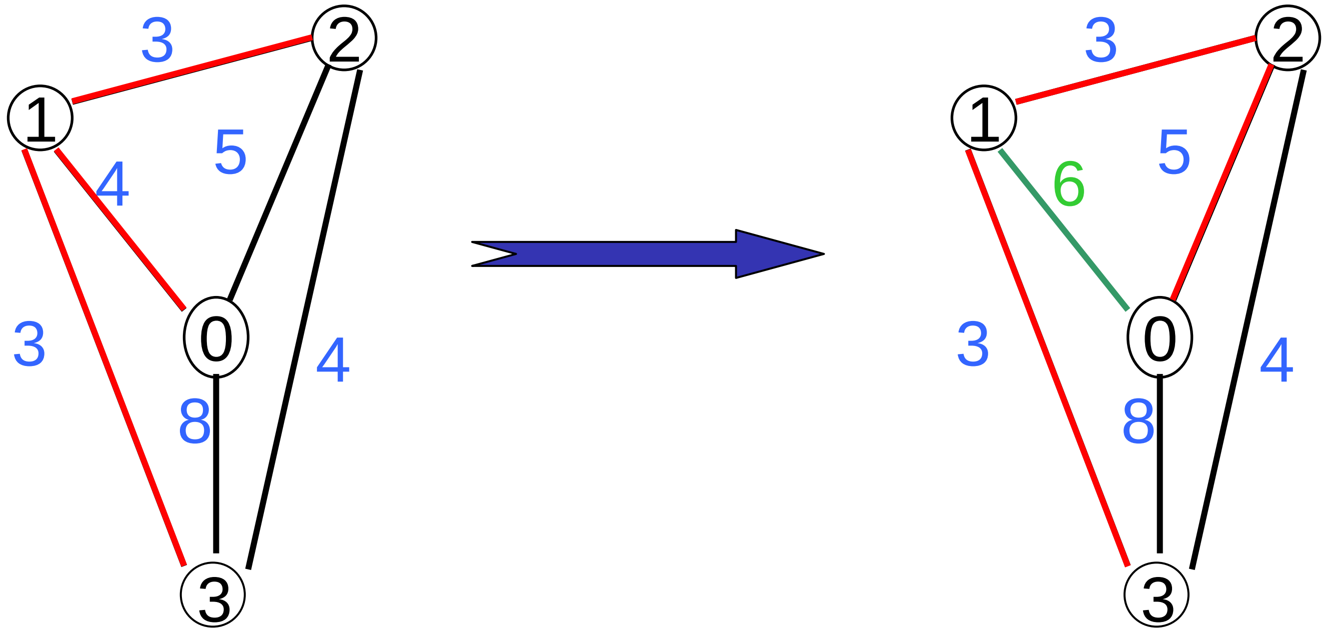
$(x_1, x_2, x_3)=(18, 24, 10)$

Both allocations belong to the core of the mcst game (and also their convex combination).

What happens when the structure of the network changes?

- Imagine to use a certain rule to allocate costs.
  - The cost of edges may increase: if the cost of an edge increases, nobody should be better off, according to such a rule (*cost monotonicity*);
  - One or more players may leave the connection situation: nobody of the remaining players should be better off (*population monotonicity*).

Cost monotonicity: Bird allocation behaviour

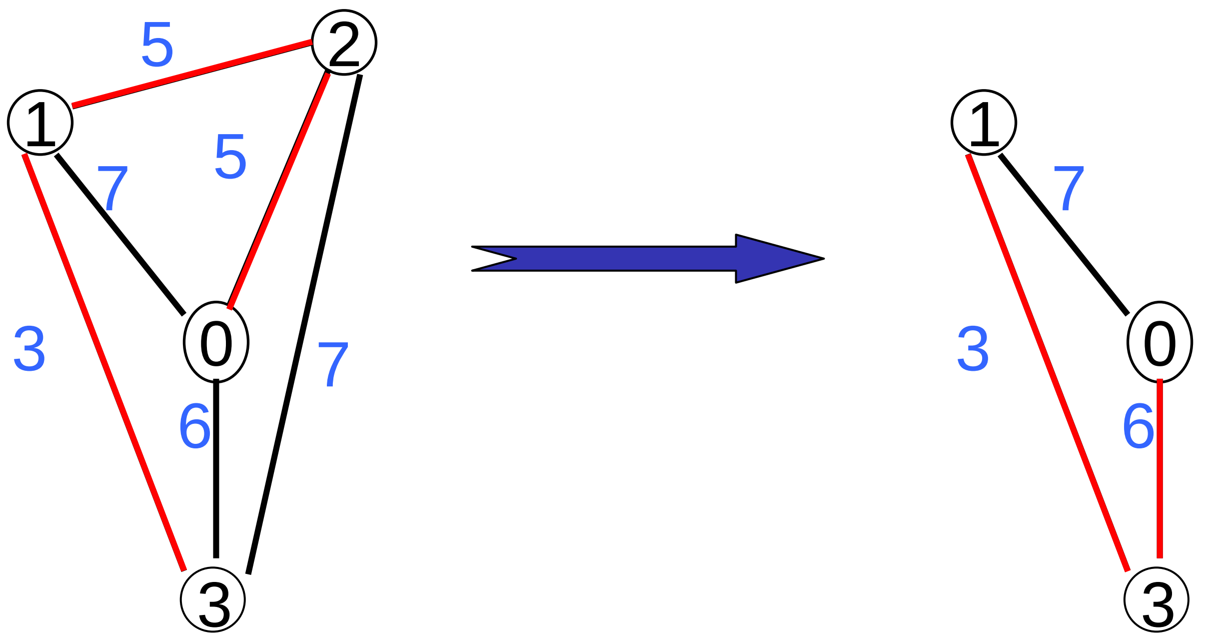


Bird allocation: (4, 3 ,3)

Bird allocation: (3, 5 ,3)

Bird rule does not satisfy cost monotonicity.

Population monotonicity: Bird allocation behaviour



Bird allocation: (5, 5 ,3)

Bird allocation: (3, * ,6)

➡ Bird rule does not satisfy population monotonicity

Conclusions

- This theoretic problem has been solved:
  - *Obligation rules* provides allocations that are in the core, tree independent, and also are monotonic wrt both costs and population. (http://arno.uvt.nl/show.cgi?fid=80868)
- What about practice?
  - Attempt to find a dynamic justification based on intuitive grounds that directs the people who are engaged with the real topic to the desired solution.