About coordination mechanisms for selfish scheduling with multiple tasks

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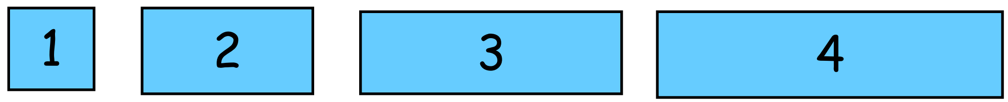
Outline

1. Problem
2. Properties of coordination mechanisms
3. Stability of classical mechanisms
4. Conclusion and future work

A scheduling problem

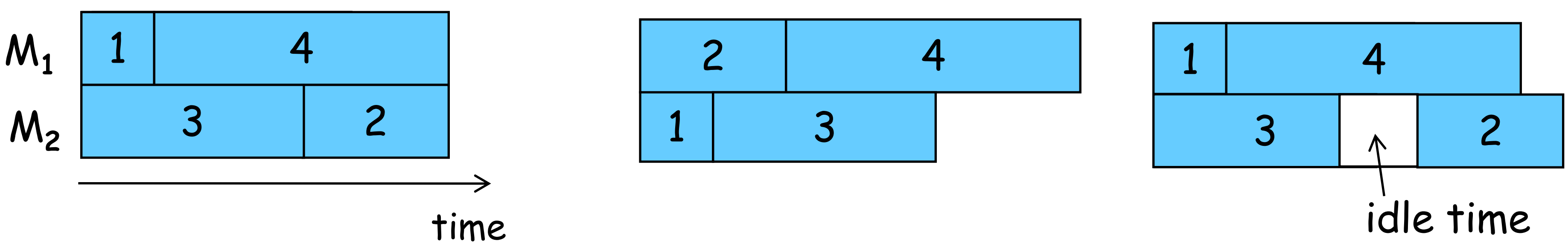
- Data : m machines $M_1, \ldots, M_m$, a set of n tasks.
  - An instance :

Tasks :



2 identical parallel machines M_1 and M_2 .

- Possible schedules :



$\sum Cj = 1+5+3+5 = 14$ $\sum Cj = 2+6+1+4 = 13$ $\sum Cj = 1+5+3+6 = 15$

- An objective function
  - Example : Min. average completion time

Context

- Algorithmic game theory (AGT).
  Shared ressources: agents with conflicting interests interact. Centralized protocols are not always possible.

  Machine : processor, printer, link in a network …

  ⇒ Scheduling problems are building blocks and have been studied extensively in AGT.

- Each agent has one objective, and a set of possible strategies. We focus on (pure) Nash equilibrium: no agent can improve its objective function by unilateraly changing its strategy.

Example

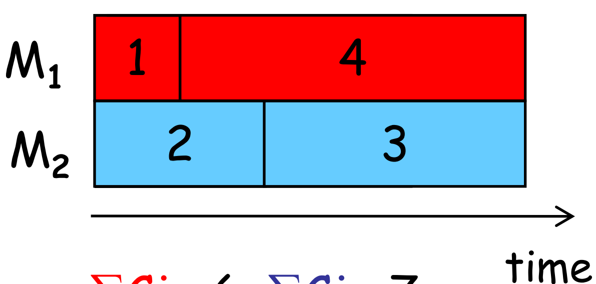
- 2 agents *A* et B

  A has 2 tasks: 2 3

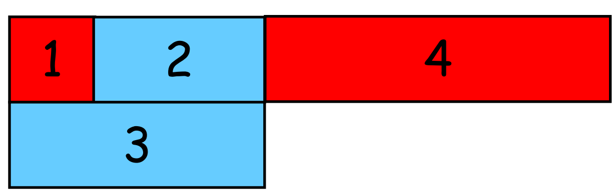
  and B has 2 tasks: 1 4

Strategy : choose on which machine to schedule each task.

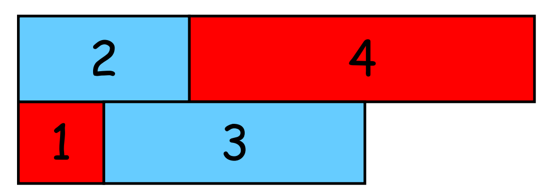
- The machines schedule the tasks by increasing order of lengths.
- Aim of the agents : Min average completion time (=Min sum of completion times)



$\sum C_j$= 6, $\sum C_j$= 7



$\sum C_j$= 8, $\sum C_j$= 6



$\sum C_j$= 7, $\sum C_j$= 6

Nash equilibrium

Price of anarchy

- Global objective function (social cost)
  - Example : Min. sum of completion times

- Price of anarchy = $\dfrac{\text{Social cost (Worst Nash equilibrium)}}{\text{Social cost (optimal solution)}}$

$\Rightarrow$ Measures the loss of efficiency du to the lack of cooperation between the selfish agents.

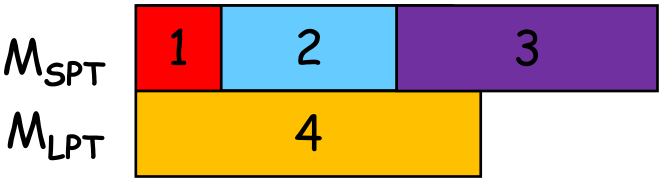
Coordination mechanisms

- Introduced by Christodoulou et al. in Icalp'04.
- Coordination mechanism = set of scheduling policies, one for each machine.

  Each policy :
  - Gives the order of the tasks on the machine, and may introduce idle times.
  - Is local : it depends on the tasks scheduled on the machine only.
  - Does not distinguish the tasks of the different agents. Each task is identified by its length and its identification number.

- Classical policies :
  - SPT (LPT): tasks are scheduled in increasing order of their lengths (resp. in decreasing order of their lengths).
  - Random : tasks are scheduled in a random order.

- Example of a coordination mechanism :



- Christodoulou et al. [ICALP'04], introduced the coordination mechanisms when 1 agent = 1 task.
- Immorlica et al. [TCS 09] : study of the convergence and the price of anarchy of the schedules induced by the classical policies.

Our problem

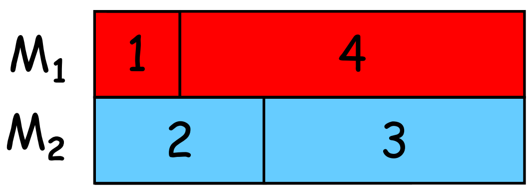
- m machines shared between 2 agents A et B having each one a set of tasks.
- Aim of each agent : Minimize the sum of the completion times of its tasks.

- Does there exist a coordination mechanism which always induce Nash equilibria ?

- What is the stability of the solutions obtained with the classical policies SPT, LPT, and Random ?

Stability of a schedule

α-approximate Nash equilibrium : no agent can improve its objective (its sum of completion times) by a ratio larger than α by changing its strategy (by moving its tasks).



Agent A: $\sum Cj$= 7; could obtain 6

-> improvement ratio =7/6 .

Agent B : $\sum Cj$= 6; could obtain 5

-> improvement ratio =6/5 .

=> 6/5-approximate Nash equilibrium.

Outline

1. Problem

2. Properties of coordination mechanisms
   - Stability
   - Price of anarchy

3. Stability of classical mechanisms

4. Conclusion and future work

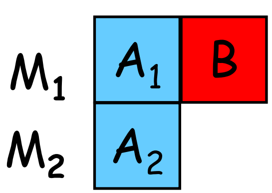
Stability

- Property : If all the machines use the same deterministic policy which doesn't use idle times, then there does not always exist a Nash equilibrium.

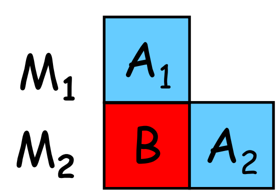
- Proof: (m=2) Let 3 tasks A_1 B A_2 s.t.
  - If A_1 and B are alone on a same machine : A_1 < B
  - If B and A_2 are alone on a same machine : B < A_2

  There is no Nash equilibrium.

  Agent A wants :

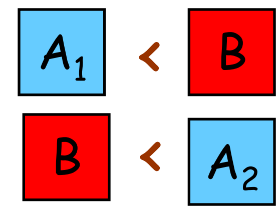


  Agent B wants :



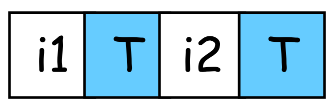
Price of anarchy

- Property : If all the machines have the same deterministic policy, then the price of anarchy is at least 2.
- Proof: m=2, 3 tasks of length 1 s.t.

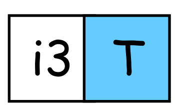


Idle times :

- If there are 2 tasks of length 1 on the same machine :

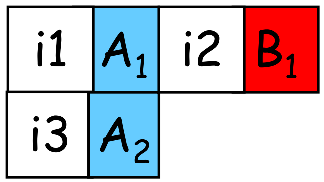


- If there is one task of length 1 alone on a machine :



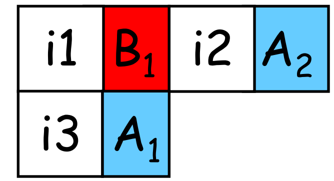
- We distinguish 4 cases:

  - Case 1:



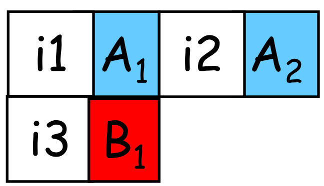
$\sum Cj$= i1+i2+2
If B_1 goes on M_2 : $\sum Cj$= i1+1
=> This not a Nash equilibrium.

  - Case 2:



$\sum Cj$= i1+i2+i3+3
Exchange $A_1 \leftrightarrow A_2$: $\sum Cj$= i1+i3+2
=> This not a Nash equilibrium.

  - Case 3:



$\sum Cj$= 2 * i1+i2+3
Nash equilibrium only if
i1+i3+2 $\geq$ 2 * i1+i2+3,
i.e. if i3 $\geq$ 1+i1+i2
If i3 $\geq$ 1 then price of anarchy $\geq$ 2.

  - Case 4: the 3 tasks are together

By conctradiction. If the price of anarchy is < 2 $\Rightarrow$ i3 < 1.
With other properties, we can deduce than there is an instance without Nash equilibrium.

=> With identical deterministic policies, the price of anarchy is $\geq$ 2.

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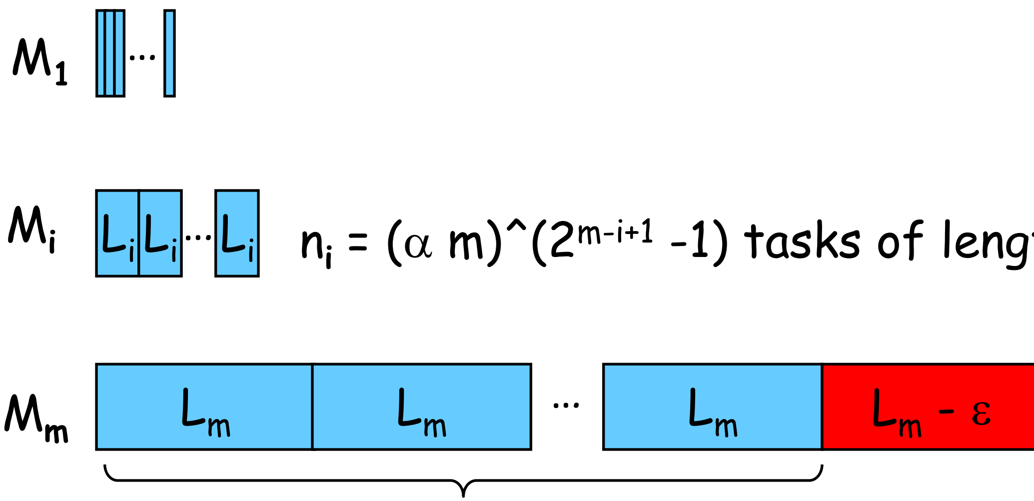
3. Stability of classical mechanisms
   - LPT and Random
   - SPT

4. Conclusion and future work

The LPT and Random policies

- Property : If the machines use the policy LPT or Random, then there does not always exist an α-approximate Nash equilibrium, for all α.
- Proof for LPT:

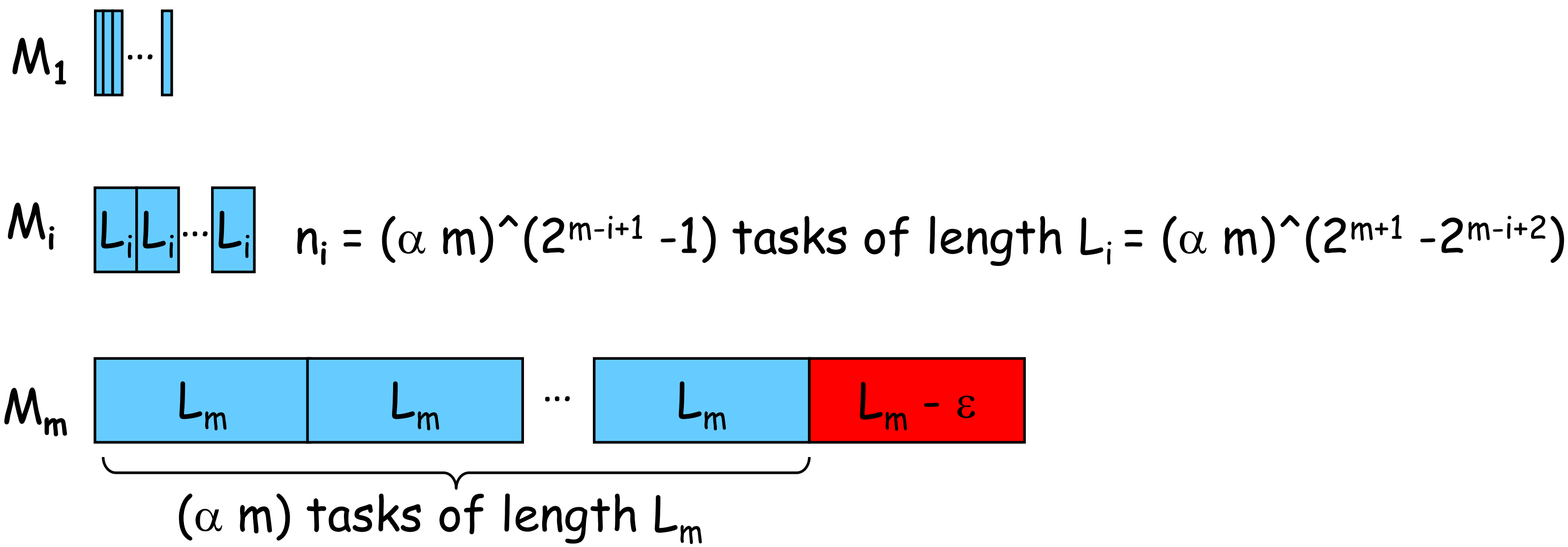


$n_i = (\alpha\ m)$^$(2^{m-i+1} - 1)$ tasks of length $L_i = (\alpha\ m)$^$(2^{m+1} - 2^{m-i+2})$

$(\alpha\ m)$ tasks of length L_m

1- In S, agent B decreases its completion time with a factor larger than α by going on another machine.

2- In any other schedule, agent *A* decreases its sum of completion times by a factor > α by moving its tasks.



- Sum of the completion times of *A* in S :

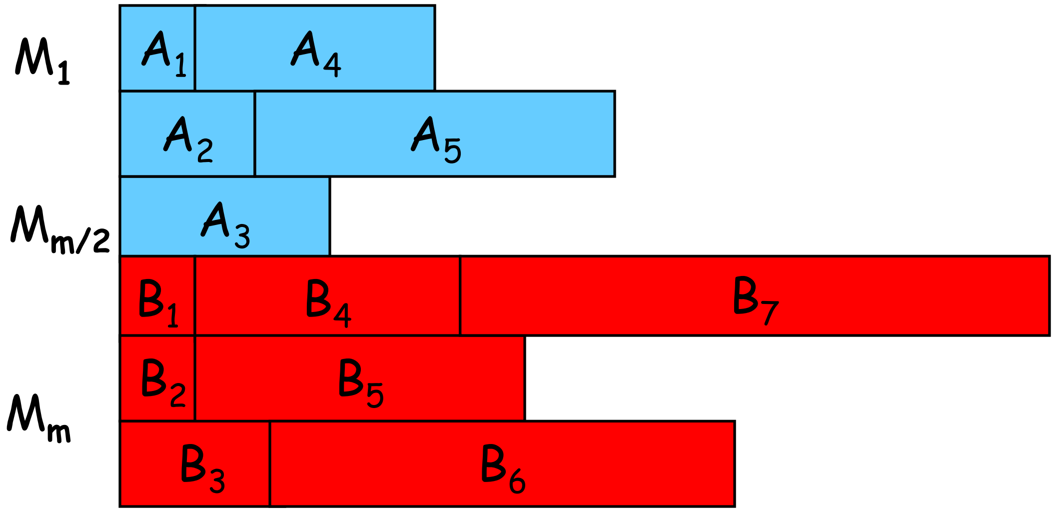
$\sum Cj(S) < m (\alpha\ m)^\wedge(2^{m+1} - 2)$

- Sum of the completion times of the tasks of length L_i with a longest task > $L_{i+1}\ n_i = (\alpha\ m)^\wedge(2^{m+1} - 1) > \alpha \sum Cj(S)$

$\Rightarrow$ There is no α-approximate Nash equilibrium in this game.

The SPT policy

- Property : If all the machines use the SPT policy, then there exist always a 3-approximate Nash equilibrium.
- Proof : (m is even)



Sum of the completion times on a set of m/2 machines $\leq$ 2 x Sum of the completion times of the same tasks on m machines.

The SPT policy : lower bound

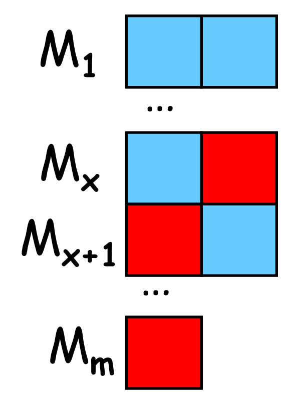
- Property : If all the machines use the SPT policy, then there does not always exist $(3/2-\varepsilon)$-approximate Nash equilibrium, for all ε.
- Proof: 2m-1 tasks of length 1 s.t.

$A_1 < B_1 < A_2 < B_2 < \dots < A_{m-1} < B_{m-1} < A_m$

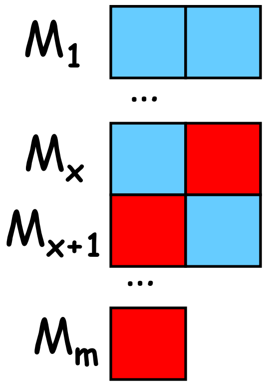
Let S be the most stable schedule.

- At most 2 tasks per machine in S.

- Each agent can, by moving its tasks, make them start at time 0.



Let x be the number of tasks of *A* in 1st position in S.



- $\sum C_j = x + 2(m-x)$.
  A could gain a factor $\frac{x+2(m-x)}{m} = 2 - \frac{x}{m}$
  by moving.

- B could gain a factor $1 + \frac{x}{m-1}$ by moving.

- S is a max($2 - \frac{x}{m}$, $1 + \frac{x}{m-1}$)-approximate Nash equilibrium.

- There is no α-approximate Nash equilibrium with
  $\alpha < \min_{x \in \{1,\dots,m\}} \max\left(2 - \frac{x}{m}, 1 + \frac{x}{m-1}\right) = \frac{3}{2}$.

Conclusion

- Machines with deterministic identical policies
  - without idle times : instances without Nash equilibrium
  - with idle times : price of anarchy at least 2 (social cost = sum of completion times).

- Classical policies :
  - LPT, Random induce schedules as instable as wanted
  - SPT induces α-approximate Nash equilibria with α between 3/2 et 3.

Future work

- Tight bound for SPT
- Complexity for an agent to compute its best response (for a given coordination mechanism) ? Convergence time to obtain a Nash equilibria?
- Does there exist a coordination mechanism which induces Nash equilibria for this problem?

  For example : one machine uses SPT, and another one LPT?