

Preference aggregation and game theory

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Workshop on *Game Practice and Preference aggregation*

Summary

- 1 Non-strategic point of view
 - Arrow
 - Harsanyi and utilitarianism
 - Interpersonal comparison of utilities
- 2 Strategic aspects and game theory
 - Strategic voting, and G-S
 - “Small worlds”
 - Computational issues
- 3 A broader picture
 - Great confusion under the sky: excellent situation!
 - To put the cart before the horses
 - Non welfarist approaches

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Nice applications (quoted in Roemer '96):

- MacKay (1980), to pentathlon
- May (1954), to the “construction” of personal preferences on multi-attribute alternatives

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Positive results:

- Black (1958), for single-peaked preferences (needed an additional dimension)
- May (1952): characterization of simple majority voting over 2 alternatives only

Small detour: “Rawls”

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Example to see that violates IIA. Cells contain the rankings of alternatives (higher is better). Left: alternative a_2 beats a_1 . Right: the converse is true.

$N \setminus A$	a_1	a_2	a_3
1	3	2	1
2	1	2	3
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Criticism by Diamond (1967): “independence” property questionable for social preferences:

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Utilitarianism? No, see Roemer's *Theories of Distributive Justice*, 1996.

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Assuming, then, that the average happiness of human beings is a positive quantity, it seems clear that, supposing the average happiness enjoyed remains undiminished, Utilitarianism directs us to make the number enjoying it as great as possible. But if we foresee as possible that an increase in numbers will be accompanied by a decrease in average happiness or vice versa, a point arises which has not only never been formally noticed, but which seems to have been substantially overlooked by many Utilitarians

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Not to say of the difficulties in finding a common scale for measurement of “utility” (felicific, or hedonic calculus?).

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Interesting contribution: Dummett and Farquharson (1961).
Forerunners of Gibbard/Satterthwaite (and explicitly quoted by Gibbard).

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So, for voting (Corollary in Gibbard 1973):

Every voting scheme with at least 3 outcomes is either dictatorial or manipulable.

Of course simple majority is "ok", *for two outcomes* (May, 52, quoted already)

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$N \setminus A$	a_1	a_2	a_3
1	1	0	0
2	0	2	-5
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Truthful bidding is not anymore dominant. So? Well, use Vickrey-Clarke-Groves. It takes into account externalities, that enter into the definition of transfers.

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Difficulties from the fact that balance of transfers is not guaranteed.

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Rothkopf: *Thirteen Reasons Why the Vickrey-Clarke-Groves Process Is Not **Practical**.*

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On the other hand, computational complexity can be useful to prevent manipulation: see the seminal paper by Bartholdi, Tovey et al.¹ (rejected by mainstream economic journals, "of course"?). It is worst case analysis. Possibly "most" instances are easy to manipulate (relevant in **prstctice**).

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 - no expertise required (good popularization requires strong knowledge)
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Evolution of conventions (Young), of norms. The contractarian approach.

Cooter: “Three Effects of Social Norms on Law: Expression, Deterrence, and Internalization”

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We are the outcome of a long process (evolution, culture): our “preferences” are the temporary outcome of this process. Remind also Coote (previous slide).

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Don't forget that the "teleological" point of view works (if it works) at the appropriate level... Difficult to assign a goal to deoxyribonucleic acid (even if A&H say: *A gene [...] looks beyond its mortal bearer*).

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And don't forget that “preferences-goal” is an useful paradigm, allows to work at a very high level of synthesis (like the macro approach in thermodynamics).

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Other approaches, like “neuroeconomics”, can work at a more basic level, more similar to hard sciences. The “lie detector” can bypass the big problem of a mechanism designer - not to know preferences of the players (maybe there is some constitutional hurdle to be overcome).

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Contrary to Roth, Sönmez, and Ünver (2005), who prove that revealing all the set of available donors is a dominant strategy for an efficient mechanism (either deterministic or stochastic).

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Results (of the experiment [[practice?](#)]) show that suggested divisions depend on whether utilities represent *needs*, *tastes* or *beliefs*. And even a couple of cases, both dealing with needs, get different answers. But all of these cases have *the same set* of possible agreements, in utility space.

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