

Arguing about voting rules

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Outline

- 1 Context
- 2 Language
- 3 Arguing for Borda
- 4 Goal: Build argumentative and adaptative recommender systems

Introduction

Context

- Voting rule: a systematic way of aggregating different opinions and decide
- Multiple reasonable ways of doing this
- Different voting rules have different interesting properties
- None satisfy all desirable properties

Our goal

We want to easily communicate about strength and weaknesses of voting rules.

Voting rule

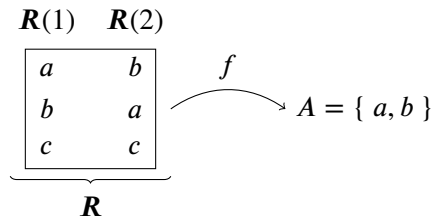
Alternatives $\mathcal{A} = \{ a, b, c, d, \dots \}$

Possible voters $\mathcal{N} = \{ 1, 2, \dots \}$

Voters $\emptyset \subset N \subseteq \mathcal{N}$

Profile partial function \mathbf{R} from \mathcal{N} to linear orders on \mathcal{A} .

Voting rule function f mapping each \mathbf{R} to winners $\emptyset \subset A \subseteq \mathcal{A}$.



Borda

- Jean-Charles de Borda, 1733–1799

Given a profile \mathbf{R} :

- count the score of each alternative;
- the highest scores win.
- Score of $a \in \mathcal{A}$ is the number of alternatives it beats.

$$\mathbf{R} = \begin{array}{ccc} a & b & b \\ d & c & a \\ c & a & c \\ b & d & d \end{array} .$$

- score a is...?

Borda

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$$\mathbf{R} = \begin{array}{ccc} a & b & b \\ d & c & a \\ c & a & c \\ b & d & d \end{array} .$$

- score a is...? $3 + 1 + 2 = 6$
- score b is $0 + 3 + 3 = 6$
- score c is $1 + 2 + 1 = 4$
- score d is $2 + 0 + 0 = 2$

Winners are $\{ a, b \}$.

Condorcet's principle

An idea from Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet (1743–1794).

Condorcet's principle

We ought to take the Condorcet winner as sole winner if it exists.

- a *beats* b iff more than half the voters prefer a to b .
- a is a *Condorcet winner* iff a beats every other alternatives.

$$\mathbf{R} = \begin{array}{ccc} a & b & b \\ d & c & a \\ c & a & c \\ b & d & d \end{array} . \text{ Who wins?}$$

How are voting rules analyzed?

- Examples featuring counter-intuitive results for some voting rules.
- Properties of voting rules, e.g. Borda does not satisfy Condorcet's principle.
- Axiomatization of a voting rule: accepting such principles lead to a unique voting rule.

Our objective

- Different voting rules
- Arguments in favor or against rules
- Dispersed in the literature
- Using mathematical formalism

We propose

- Common language
- Instantiate arguments on concrete examples

Goal: help understand strengths and weaknesses of given rules.

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Example of axiom

- Dominance: if a dominates b in \mathbf{R} , then b may not win.
- We want a language to express this kind of axioms.

Language

We use propositional logic (with connectives $\neg, \vee, \wedge, \rightarrow$).

Atoms

- One atom for each (\mathbf{R}, A) , $\emptyset \subset A \subseteq \mathcal{A}$.
- An atom talks about assigning winners A to \mathbf{R} .
- Written $[\mathbf{R} \mapsto A]$.

Semantics

Semantics v_f , given a voting rule f :

$$v_f([\mathbf{R} \mapsto A]) = \text{T iff } f(\mathbf{R}) = A.$$

L-axioms

- Now: “translate” axioms into language-axioms.
- An *l-axiom* is a set of formulæ.

Symmetric cancellation I-axiom

Definition (SYM)

For each \mathbf{R} consisting of a linear order and its inverse,

$$[\mathbf{R} \mapsto \mathcal{A}].$$

Example

$$\mathbf{R}_1 = \begin{array}{cc} a & c \\ b & b \\ c & a \end{array}, \text{ constraints? } f(\mathbf{R}_1) =$$

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$$\mathbf{R}_1 = \begin{array}{cc} a & c \\ b & b \\ c & a \end{array}, \text{ constraints? } f(\mathbf{R}_1) = \mathcal{A} = \{ a, b, c \}.$$

Example

$$\mathbf{R}_2 = \begin{array}{cc} a & b \\ b & a \\ c & c \end{array}, \text{ constraints?}$$

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$$[\mathbf{R} \mapsto \mathcal{A}].$$

Example

$$\mathbf{R}_1 = \begin{array}{cc} a & c \\ b & b \\ c & a \end{array}, \text{ constraints? } f(\mathbf{R}_1) = \mathcal{A} = \{ a, b, c \}.$$

Example

$$\mathbf{R}_2 = \begin{array}{cc} a & b \\ b & a \\ c & c \end{array}, \text{ constraints? None.}$$

Shortcut notations

$\mathcal{P}_\emptyset(\mathcal{A})$ the set of subsets of \mathcal{A} , excluding the empty set.

Let $\alpha \subseteq \mathcal{P}_\emptyset(\mathcal{A})$ be a set of possible winning alternatives.

Uni-profile clause

$[\mathbf{R} \vdash \xrightarrow{\subseteq} \alpha]$ shortcut for:

$$\bigvee_{A \in \alpha} [\mathbf{R} \vdash \rightarrow A].$$

- Intuitive content.
- Called a uni-profile clause.

Dominance I-axiom

Definition (DOM)

L-axiom DOM: for each \mathbf{R} ,

$$[\mathbf{R} \vdash_{\mathcal{L}} \mathcal{P}_{\emptyset}(U_{\mathbf{R}})],$$

with $U_{\mathbf{R}}$ the set of alternatives in \mathbf{R} that are not dominated.

Domain knowledge

- We need some formulæ encoding the voting rule concept.
- Define κ as the set of all those formulæ.

Domain knowledge κ

- 1 a voting rule can't select more than one set of winners:
for all \mathbf{R} and all $\emptyset \subset A \neq B \subseteq \mathcal{A}$,

$$[\mathbf{R} \mapsto A] \wedge [\mathbf{R} \mapsto B] \rightarrow \perp.$$

- 2 a voting rule must select at least one set of winners:
for all \mathbf{R} ,

$$[\mathbf{R} \mapsto \mathcal{P}_{\emptyset}(\mathcal{A})].$$

Reinforcement axiom

Classical reinforcement axiom: consider $\mathbf{R}_1, \mathbf{R}_2$,

- having winners A_1, A_2 ,
- with $A_1 \cap A_2 \neq \emptyset$;

then winners in $\mathbf{R}_1 + \mathbf{R}_2$ must be $A_1 \cap A_2$.

Example

$$\mathbf{R}_1 = \begin{array}{cc} a & b \\ b & a \\ c & c \end{array}, A_1 = \{a, b\},$$

Reinforcement axiom

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Example

$$\mathbf{R}_1 = \begin{array}{cc} a & b \\ b & a \\ c & c \end{array}, A_1 = \{a, b\}, \mathbf{R}_2 = \begin{array}{ccc} a & b & a \\ b & a & c \\ c & c & b \end{array}, A_2 = \{a\},$$

$$\mathbf{R} = \begin{array}{ccccc} a & b & a & b & a \\ b & a & b & a & c \\ c & c & c & c & b \end{array}. \text{Winners?}$$

Reinforcement axiom

Classical reinforcement axiom: consider \mathbf{R}_1 , \mathbf{R}_2 ,

- having winners A_1 , A_2 ,
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then winners in $\mathbf{R}_1 + \mathbf{R}_2$ must be $A_1 \cap A_2$.

Example

$$\mathbf{R}_1 = \begin{array}{cc} a & b \\ b & a \\ c & c \end{array}, A_1 = \{a, b\}, \mathbf{R}_2 = \begin{array}{ccc} a & b & a \\ b & a & c \\ c & c & b \end{array}, A_2 = \{a\},$$

$$\mathbf{R} = \begin{array}{ccccc} a & b & a & b & a \\ b & a & b & a & c \\ c & c & c & c & b \end{array}. \text{Winners? } \{a\}$$

Reinforcement I-axiom

Classical reinforcement axiom: consider $\mathbf{R}_1, \mathbf{R}_2$,

- having winners A_1, A_2 ,
- with $A_1 \cap A_2 \neq \emptyset$;

then winners in $\mathbf{R}_1 + \mathbf{R}_2$ must be $A_1 \cap A_2$.

Definition (REINF)

For each $\mathbf{R}_1, \mathbf{R}_2, A_1, A_2 \subseteq \mathcal{A}, A_1 \cap A_2 \neq \emptyset$:

$$([\mathbf{R}_1 \mapsto A_1] \wedge [\mathbf{R}_2 \mapsto A_2]) \rightarrow [\mathbf{R}_1 + \mathbf{R}_2 \mapsto A_1 \cap A_2].$$

Fishburn-against-Condorcet argument

Fishburn (1974, p. 544) argument against the Condorcet principle (see also <http://rangevoting.org/FishburnAntiC.html>).

Condorcet winner

w VS $\mu, \mu \in \{a, \dots, h\}$?

	nb voters					
	31	19	10	10	10	21
1	<i>a</i>	<i>a</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>h</i>
2	<i>b</i>	<i>b</i>	<i>w</i>	<i>w</i>	<i>w</i>	<i>g</i>
3	<i>c</i>	<i>c</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>f</i>
4	<i>d</i>	<i>d</i>	<i>h</i>	<i>h</i>	<i>f</i>	<i>w</i>
5	<i>e</i>	<i>e</i>	<i>g</i>	<i>f</i>	<i>g</i>	<i>a</i>
6	<i>w</i>	<i>f</i>	<i>e</i>	<i>e</i>	<i>e</i>	<i>e</i>
7	<i>g</i>	<i>g</i>	<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>
8	<i>h</i>	<i>h</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>
9	<i>f</i>	<i>w</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>

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2	<i>b</i>	<i>b</i>	<i>w</i>	<i>w</i>	<i>w</i>	<i>g</i>
3	<i>c</i>	<i>c</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>f</i>
4	<i>d</i>	<i>d</i>	<i>h</i>	<i>h</i>	<i>f</i>	<i>w</i>
5	<i>e</i>	<i>e</i>	<i>g</i>	<i>f</i>	<i>g</i>	<i>a</i>
6	<i>w</i>	<i>f</i>	<i>e</i>	<i>e</i>	<i>e</i>	<i>e</i>
7	<i>g</i>	<i>g</i>	<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>
8	<i>h</i>	<i>h</i>	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>
9	<i>f</i>	<i>w</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>

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	nb voters					
	31	19	10	10	10	21
1	<i>a</i>	<i>a</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>h</i>
2	<i>b</i>	<i>b</i>	<i>w</i>	<i>w</i>	<i>w</i>	<i>g</i>
3	<i>c</i>	<i>c</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>f</i>
4	<i>d</i>	<i>d</i>	<i>h</i>	<i>h</i>	<i>f</i>	<i>w</i>
5	<i>e</i>	<i>e</i>	<i>g</i>	<i>f</i>	<i>g</i>	<i>a</i>
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9	<i>f</i>	<i>w</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>

ranks

	1	≤ 2	≤ 3	≤ 4	≤ 5	≤ 6	≤ 7	≤ 8	≤ 9
<i>w</i>	0	30	30	51	51	82	82	82	101
<i>a</i>	50	50	80	80	101	101	101	101	101

Fishburn-versus-Condorcet I-axiom

Define \mathbf{R}_F the profile shown in the previous slide.

Definition (Fishburn-versus-Condorcet)

The Fishburn-versus-Condorcet I-axiom FvSC is defined as:

$$[\mathbf{R}_F \vdash_{\in} \mathcal{P}_{\emptyset}(\mathcal{A} \setminus \{w\})].$$

L-axiomatization

An l-axiomatization is a set of l-axioms.

Definition (Conforming to J)

The rule f conforms to the l-axiomatization J iff v_f assigns the value T to all formulæ in j , for all $j \in J$.

An l-axiomatization is consistent iff there exists a voting rule conformant to it.

Arguments

Definition (Argument)

An argument grounded on J is a pair (*claim*, *proof*),

- J an l-axiomatization,
 - *claim* a uni-profile clause (thus of the form $[\mathbf{R} \vdash_{\in} \alpha]$),
 - *proof* a natural deduction proof of the claim grounded on J .
-
- The argument shows that for all voting rules f conformant to J , $f(\mathbf{R})$ selects a set of winners among α .
 - The argument claims that it is only reasonable to choose the winners among α for \mathbf{R} (provided J is accepted).
 - *Consistent* arguments require a consistent l-axiomatization.

A simple argument

Claim

$$\bullet \mathbf{R} = \begin{array}{cccc} & a & b & a & c \\ & b & c & b & b \\ \text{Consider:} & c & a & c & a \end{array},$$

$$\bullet \mathbf{J} = \{ \text{DOM}, \text{SYM}, \text{REINF} \}.$$

We can prove that for f compliant with \mathbf{J} :

$$[\mathbf{R} \vdash_{\mathbf{J}} \{ \{ a \}, \{ b \}, \{ a, b \} \}].$$

See how?

A simple argument

Claim

$$\bullet \mathbf{R} = \begin{array}{cccc} & a & b & a & c \\ & b & c & b & b \\ \text{Consider:} & c & a & c & a \end{array},$$

$$\bullet J = \{ \text{DOM}, \text{SYM}, \text{REINF} \}.$$

We can prove that for f compliant with J :

$$[\mathbf{R} \vdash_{\in} \{ \{a\}, \{b\}, \{a,b\} \}].$$

$$\text{See how? Consider } \mathbf{R}_D = \begin{array}{cc} a & b \\ b & c \\ c & a \end{array}, \mathbf{R}_S = \begin{array}{cc} a & c \\ b & b \\ c & a \end{array}, \mathbf{R} = \mathbf{R}_D + \mathbf{R}_S.$$

Example proof

$$\mathbf{R}_D = \begin{array}{cc} a & b \\ b & c \\ c & a \end{array}, \mathbf{R}_S = \begin{array}{cc} a & c \\ b & b \\ c & a \end{array}, \mathbf{R} = \mathbf{R}_D + \mathbf{R}_S = \begin{array}{cccc} a & b & a & c \\ b & c & b & b \\ c & a & c & a \end{array}.$$

- ① $[\mathbf{R}_D \vdash \{\{a\}, \{b\}, \{a,b\}\}]$ (DOM)
- ② $[\mathbf{R}_S \vdash \{a,b,c\}]$ (SYM)
- ③ $([\mathbf{R}_D \vdash \{a\}] \wedge [\mathbf{R}_S \vdash \{a,b,c\}]) \rightarrow [\mathbf{R} \vdash \{a\}]$ (REINF)
- ④ $([\mathbf{R}_D \vdash \{b\}] \wedge [\mathbf{R}_S \vdash \{a,b,c\}]) \rightarrow [\mathbf{R} \vdash \{b\}]$ (REINF)
- ⑤ $([\mathbf{R}_D \vdash \{a,b\}] \wedge [\mathbf{R}_S \vdash \{a,b,c\}]) \rightarrow [\mathbf{R} \vdash \{a,b\}]$ (REINF)
- ⑥ $[\mathbf{R}_D \vdash \{a\}] \rightarrow [\mathbf{R} \vdash \{a\}]$ (PR from 2 & 3)
- ⑦ $[\mathbf{R}_D \vdash \{b\}] \rightarrow [\mathbf{R} \vdash \{b\}]$ (PR from 2 & 4)
- ⑧ $[\mathbf{R}_D \vdash \{a,b\}] \rightarrow [\mathbf{R} \vdash \{a,b\}]$ (PR from 2 & 5)
- ⑨ $[\mathbf{R}_D \vdash \{a\}] \vee [\mathbf{R}_D \vdash \{b\}] \vee [\mathbf{R}_D \vdash \{a,b\}]$ (rewrite 1)
- ⑩ $[\mathbf{R} \vdash \{a\}] \vee [\mathbf{R} \vdash \{b\}] \vee [\mathbf{R} \vdash \{a,b\}]$ (PR from 6–9)
- ⑪ $[\mathbf{R} \vdash \{\{a\}, \{b\}, \{a,b\}\}]$ (rewrite 10)

Example shortened

Tweak l-axioms to skip steps which will seem intuitive to humans.

Definition (Reinforcement-sets)

For each $\mathbf{R}_1, \mathbf{R}_2, \alpha_1, \alpha_2 \subseteq \mathcal{P}_{\emptyset}(\mathcal{A})$, $\cap \emptyset \neq \emptyset, \emptyset \in \alpha_1 \times \alpha_2$:

$$([\mathbf{R}_1 \vdash \alpha_1] \wedge [\mathbf{R}_2 \vdash \alpha_2]) \rightarrow [\mathbf{R}_1 + \mathbf{R}_2 \vdash \bigcup_{A_1 \in \alpha_1, A_2 \in \alpha_2} \{ A_1 \cap A_2 \}].$$

- 1 $[\mathbf{R}_D \vdash \{ \{ a \}, \{ b \}, \{ a, b \} \}]$ (DOM)
- 2 $[\mathbf{R}_S \vdash \{ a, b, c \}]$ (SYM)
- 3 $((1) \wedge (2)) \rightarrow [\mathbf{R} \vdash \{ \{ a \}, \{ b \}, \{ a, b \} \}]$ (REINF-SETS)
- 4 $[\mathbf{R} \vdash \{ \{ a \}, \{ b \}, \{ a, b \} \}]$

Soundness and completeness

Consider an L -axiomatization J and a claim $c = [R \vdash_{\in} \alpha]$.

Theorem (Soundness)

If there exists an argument (c, proof) grounded on J , the claim holds given J .

Theorem (Completeness)

If the claim holds given J , then there exists an argument (c, proof) grounded on J .

This is easily obtained from the soundness and completeness of natural deduction in propositional logic.

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Argument building for Borda

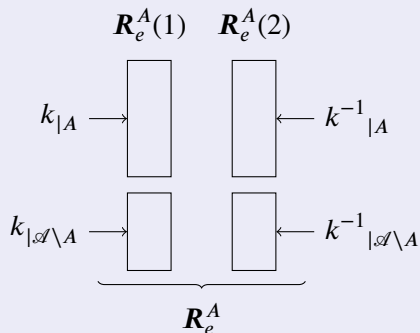
Write f_B for the Borda rule.

- We want to produce an argument justifying Borda's output.
- Given \mathbf{R} , we want an argument with claim $[\mathbf{R} \mapsto f_B(\mathbf{R})]$.
- Basis: Young (1974)'s axiomatization of the Borda rule.
- Our l-axiomatization uses three simple profile types plus REINF.

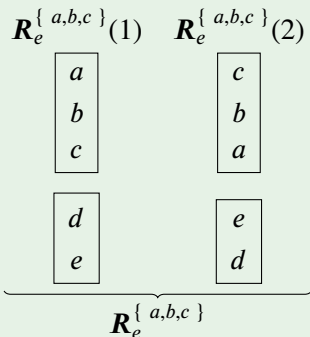
Elementary profile

Fix an arbitrary linear order k on \mathcal{A} . Given $A \subseteq \mathcal{A}$, define \mathbf{R}_e^A .

Definition (Elementary profile)



Example



Cyclic profiles

Given S a complete cycle in \mathcal{A} , define \mathbf{R}_c^S .

Definition (Cyclic profile)

\mathbf{R}_c^S is the profile composed by all $|\mathcal{A}|$ possible linearizations of S as preference orderings.

Example

$$\mathbf{R}_c^{\langle a,b,c,d \rangle} = \begin{array}{cccc} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{array} .$$

Borda L-axiomatization

ELEM for all A : [$\mathbf{R}_e^A \mapsto A$].

CYCL for all \mathcal{S} : [$\mathbf{R}_c^{\mathcal{S}} \mapsto \mathcal{A}$].

REINF as previously but generalized to any number of summed profiles.

CANC cancellation: when all pairs of alternatives (a, b) in a profile are such that a is preferred to b as many times as b to a , the set of winners must be \mathcal{A} .

An example

Consider $\mathcal{A} = \{ a, b, c, d \}$ and a profile \mathbf{R} defined as:

$$\mathbf{R} = \begin{array}{cc} a & c \\ b & b \\ d & a \\ c & d \end{array} .$$

An example

Consider $\mathcal{A} = \{ a, b, c, d \}$ and a profile \mathbf{R} defined as:

$$\mathbf{R} = \begin{array}{cc} a & c \\ b & b \\ d & a \\ c & d \end{array} .$$

We want to justify that $f_B(\mathbf{R}) = \{ a, b \}$.

Sketch

- Consider any $\mathbf{R}' = q_1 \mathbf{R}_e^{a,b} + q_2 \mathbf{R}_e^{a,b,c} + \sum_{S \in \mathcal{S}} q_S \mathbf{R}_c^S$,
 $q_1, q_2, q_S \in \mathbb{N}$, \mathcal{S} some set of cycles.
- In \mathbf{R}' , $W = \{a, b\}$ must win.
- Find $k \in \mathbb{N}$ such that $\overline{k\mathbf{R}} + \mathbf{R}'$ cancel.
- Then $k\mathbf{R}$ has winners W . (Skipping details.)
- Then \mathbf{R} has winners W .

Our task: find \mathbf{R}' a combination of elementary and cyclic profiles such that $\overline{k\mathbf{R}} + \mathbf{R}'$ cancel.

Good news: this is always possible.

Application on the example

Define $R' = R_e^{a,b} + 2R_e^{a,b,c} + R_c^{\langle c,b,a,d \rangle} + R_c^{\langle b,d,c,a \rangle}$.

- 1 $[R_e^{a,b} \vdash \{ a, b \}]$ (ELEM)
- 2 $[R_e^{a,b,c} \vdash \{ a, b, c \}]$ (ELEM)
- 3 $[R_c^{\langle c,b,a,d \rangle} \vdash \mathcal{A}]$ (CYCL)
- 4 $[R_c^{\langle b,d,c,a \rangle} \vdash \mathcal{A}]$ (CYCL)
- 5 $[R' \vdash \{ a, b \}]$ (REINF, 1, 2, 3, 4)
- 6 $[4R + \overline{4R} \vdash \mathcal{A}]$ (CANC)
- 7 $[4R + \overline{4R} + R' \vdash \{ a, b \}]$ (REINF, 5, 6)
- 8 $[\overline{4R} + R' \vdash \mathcal{A}]$ (CANC)
- 9 $[4R \vdash \{ a, b \}]$ (REINF, 7, 8)
- 10 $[R \vdash \{ a, b \}]$ (REINF, 9)

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Counter-argument against Borda

Counter-argument against Borda?

Counter-argument against Borda

Counter-argument against Borda?

Not Condorcet-consistent!

Example

$$\mathbf{R} = \begin{array}{ccc}
 & a & b & b \\
 d & c & a & \\
 c & a & c & \cdot \\
 b & d & d &
 \end{array}$$

- Argument against Borda: use a COND I-axiom
- Then?

Counter-argument against Borda

Counter-argument against Borda?

Not Condorcet-consistent!

Example

$$\mathbf{R} = \begin{array}{ccc}
 & a & b & b \\
 d & c & a & \\
 c & a & c & . \\
 b & d & d &
 \end{array}$$

- Argument against Borda: use a COND I-axiom
- Then?
- Counter-argue with FvsC.

Building argumentative recommender systems

General goal

- Recommend complex objects
- Recommend *and* argue

Complex objects

- Voting rule
- Planning
- Strategy (game, negotiation, ...)
- Travel itinerary

Multi-level argumentation:

- NOT persuasion
- NOT predicting the natural user choice

Building adaptive recommender systems

Role of preference models

- Capture the alternatives to be recommended
- Determine the best argumentation strategy

Conclusion

- A language to express desirable properties of voting rules.
- We can then instantiate concrete arguments (example-based).
- May render some arguments in the specialized literature accessible to non experts.
- Extensions may permit to *debate* about voting rules.
- Provides a way to study appreciation of arguments.

Thank you for your attention!

Bibliography I

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