Bipolar Preference Modeling and Aggregation in Decision Support

Meltem Öztürk,* Alexis Tsoukiàs†
LAMSADE—CNRS, Université Paris Dauphine

The article discusses the use of positive and negative reasons when preferences about alternative options have to be considered. Besides explaining the intuitive and formal situations where such a bipolar reasoning is used, the article shows how it is possible to generalize the concordance/discordance principle in preference aggregation and apply it to the problem of aggregating preferences expressed under intervals.

1. INTRODUCTION

Preference modeling, aggregation, and *exploitation* constitute three main steps in elaborating an evaluation model within a decision aiding process.[1] In the preference modeling step, we are interested in finding a suitable way to translate "preference statements" (of the type "I prefer x to y"), expressed by a decision maker, into formal statements enabling to establish an evaluation model for decision support purposes. We then may need to aggregate such preference models in the case they represent several criteria or opinions. The result is then used in the exploitation step where we try to establish a final recommendation for a choice or a ranking problem.

Preferences (about a set of objects) can be explicitly stated by the decision maker or implicitly obtained through other information (prices, evaluations, measures, etc.). In both cases, we may face the situation where the information is expressed in a bipolar form that distinguishes positive information from negative one: positive or negative assessments, positive or negative impacts, etc. are typical examples. Moreover, the way by which preferences are manipulated may be based on a bipolar reasoning, from voting procedures of large bodies[a] to well-known decision support methods using the example/counter-example principle, the ideal/anti-ideal solution principle, or the concordance/discordance principle. There can be different types of bipolarity[2] and a conjoint treatment of positive and negative

*Author to whom all correspondence should be addressed: e-mail: ozturk@lamsade.dauphine.fr.
†e-mail: tsoukias@lamsade.dauphine.fr.
[a]See the Nice treaty about decision procedures in the new European Union bodies.

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information may not fit the decision situation and information at hand. We therefore, need specific procedures in such cases.

In this article, we consider such bipolar information as positive and negative reasons supporting or denying a possible preference statement. We are thus interested in the use of independent positive and negative reasons where negative information is not the complement of the positive one. Such reasons should account for the representation of bipolarity in the preference modeling, aggregation, and exploitation steps, while constructing an evaluation model.

The article is organized as follows. In Section 2, we briefly review the use of "bipolar" scales in value theory and deontic logic, some first attempts to capture the possibility to use positive and negative reasons when reasoning about values and preferences. In Section 3, we present several real-world decision situations where the principle of considering positive and negative reasons independently is the current practice. In Section 4, we introduce our notation and set our problem. In Section 5, we show how this can be handled generalizing the concepts of concordance and discordance. Section 6 is dedicated to an example of aggregating preferences expressed on intervals where the use of positive and negative reasons is essential. We conclude by showing further research directions of this work.

2. BIPOLARITY IN VALUE THEORY: HISTORICAL DISCUSSION

In many fields, such as economics, social sciences, psychology, political science, and decision aiding, we need to represent values. *Value theory*, introduced in the sixties by,[3] is one of the first attempts to develop a general theory proposing an axiology and trying to establish some relations between "evaluation" and "value." Questions in which value theory is interested are: What is a value? Is it a *property of an object* (like its size)? or, Is it a *relationship* that arises out of circumstances linking the value object with the valuing subject in some special way? The last question shows that there exists a connection between "value" and "preference" because value may be determined as a result of a preference comparison.

The realization of a value can be smaller or larger in one instance of its application as compared with another. Rescher emphasizes that *Evaluation* in the strictest sense is e-value-ation: a comparative assessment or measurement of something with respect to its embodiment of a certain value and must be understood as the result of application of a valuation to certain items in a specific case. He differentiates two types of value scales: bipolar and monopolar.

- *Bipolar scale:* A bipolar value scale covers the entire range: negative, neutral, and positive. The value and the corresponding disvalue are presented on the same scale as opposite evaluations (eg, ugly - indifferent - beautiful, disloyal - lukewarm - loyal).
- *Monopolar scale:* A monopolar value scale covers only half of range covered by bipolar ones: neutral-positive. It does not permit to express a disvalue because of the lack of the negative part, only the absence of something positive (like the wealth) can be expressed in this covered spectrum (eg, unimaginative - imaginative, unintelligent - intelligent). In this case, neutral point does not have a "neutral sense" because it represents a complete lack of something positive.

Rescher points out that such a difference between scales may be necessary to differentiate two notions: *worth* and *value*. The notion of *value*, which is broader than the one of *worth*, requires the expressivity of disvalue, whereas the one of *worth* does not admit the negative pole because something having negative worth is not meaningful.

Another domain, related to value theory and preference, considering comparative evaluation of items and covering some bipolarity, is "the logic of preference." Different researchers have been interested in this subject; we undertake here the approach of Von Wright,[4,5] who was the first to introduce (to our knowledge) basic notions of such a logic. Von Wright defined five principles of a logic of preference that are related to the asymmetry and transitivity of preference relation, the connection between preference and the change of the state of affairs, the definition of preference between a disjunction of two objects and another one, and the holistic nature of preference. The central and most interesting principle of his approach concerns the connection between *change* and *preference*. He defines the preference between two objects from four situations covering all the possibilities concerning their states: $p \wedge q$, $p \wedge \neg q$, $\neg p \wedge q$, and $\neg p \wedge \neg q$. Object p is preferred to object q if and only if the world with $p \wedge \neg q$ is preferred to the one with $\neg p \wedge q$ as *end-state of contemplated possible changes in this present situation*.[b] The indifference between two different objects is interpreted as follows: *under some circumstances* the state $(p \wedge \neg q)$ is *not* preferred to the state $(\neg p \wedge q)$ *and under some circumstances* the state $(\neg p \wedge q)$ is *not* preferred to the state $(p \wedge \neg q)$. It should be observed that the two occurrences of "some" need not refer to the *same* circumstances. Indifference as defined above is not unconditional. As a consequence, the indifference of two states between themselves does not entail that the two states have the same value relative to any other state: $(pIq) \wedge (pPr) \longrightarrow (qPr)$ is not a tautology (here P represents a strict preference and I an indifference).

Such an approach has some bipolar properties:

- Concerning indifference relation, two poles can be interpreted as: one pole related to the circumstances for an affirmation and the other one against the same affirmation.
- Concerning preference relation, we can see two poles: one pole related to the states of p and the other related to the states of q.

To define the preference in such a way, it can also be useful to interpret some notions such as goodness, badness, or indifference:

A state p is *good* if it is unconditionally preferred to its negation $\neg p$.
A state p is *bad* if its negation $\neg p$ is unconditionally preferred to it.

As it can be remarked, bipolarity has been interpreted since the sixties by researchers who are interested in the representation of values and who proposed some tools (such as bipolar scales) for this purpose.

[b]This means that a change from the second one to the first one is preferable.

Bipolarity can be also found in real-life decision problems. In the following section, we present different decision problem situations (with a group of decision makers, one decision maker, with multiple criteria or one criterion, with or without veto, etc.) where bipolarity can be found under different aspects.

3. BIPOLARITY IN INTUITIVE DECISION MAKING AND IN PRACTICE: EXAMPLES

Consider the very common situation where the faculty has to deliberate on the admission of candidates to a course (let us say a management course). Consider two candidates: the first, x, having quite good grades, systematically better than the second, y, but with a very bad grade in management science; and then candidate y, who is systematically worse than x, but has an excellent grade in management science. Several faculty members will claim that, although candidate y is not better than candidate x, it is also difficult to consider x better than y because of their inverse quality concerning the key class of the course, management science. The same faculty members will also claim that the two candidates cannot be considered indifferent because they are completely different. These members are intuitively adopting the same decision rule: *candidate x is better than candidate y iff she or he has a majority of grades in her or his favor and is not worse in a number of key classes.* For an extensive discussion on the question of grades in decision support see Bouyssou et al.[6]

If we consider a class grade of a candidate as her or his value on a criterion, the reader will observe that in the above decision rule there exist criteria having a "negative power." Such a "negative power" is not compensated by the "positive power" of the majority of criteria. It acts independently and only in a negative sense.

Consider an individual facing a comparison problem between two objects that are evaluated by intervals. Suppose that the price of the first one is between 100 and 130 and that of the second object is between 110 and 120. Which should she or he prefer? Let us suppose that we are interested in the first one over the second one. In this case, the difference between 100 and 110 has a "positive power" because in the left end points of intervals the first one is cheaper, but the difference between 130 and 120 occurs as "negative power" because now the first one is more expensive. Positive and negative powers are independent and there can be cases where one cannot compensate the other.

Consider now a parliament. The government has the majority in the parliament, although not a very strong one. Suppose now that a bill on a very sensitive issue (such as education, religion, national defence, minority rights, etc.) is introduced for discussion by the government. Several political, social, and ethical issues are involved. Suppose finally that the opposition strongly mobilizes, considering that this bill is a major attack against "something." Massive demonstrations are organized, an aggressive media campaign is pursued, etc. It is quite reasonable that the government will try to find a compromise on some aspects of the bill to improve its "acceptability." Note, however, that such a compromise concerns aspects argued by the minority and not the majority.

Which decision rule is the government using to choose an appropriate law proposal in such a situation? *A law proposal x is considered "better" than y iff it meets the majority will and does not mobilizes the minority aversion.* It should be observed that the minority is considered here as an independent decision power source. Such a "decision rule" is a regular practice in all mature democracies. Although the minority does not have the power to impose its political will, it has the possibility of expressing a "veto," at least occasionally. Such a "negative power" may not necessarily be codified somewhere, but is accepted. Actually, it is also a guarantee for the democratic game. When the present majority becomes a minority, it will be able to use the same "negative power."

Finally, consider the Security Council of the United Nations. Here, a number of nations are officially endowed with a veto power such that resolutions taken with a majority of votes (even the highest ones) can be withdrawn if such a veto is used. We observe that in this case the decision rule "x is better than y if it is the case for the majority and no veto is used against x" is officially adopted. Again we observe that the countries having a veto power do not have a "positive power" (impose a decision), but only a "negative" one.

Within all the cited examples, there exist two different but not compensated types of power, positive and negative ones, each one representing a different pole.

As it can be noted that the use of independent positive and negative reasons within decision rules is common practice for electoral bodies, commissions, boards, etc., besides being an intuitive rule for comparing alternatives described under multiple attributes. It is therefore necessary to consider a specific model to handle them.

4. NOTATION AND PROBLEM

In the following, A represents a finite (countable) set of objects (candidates, alternatives, actions, etc.) on which preferences are expressed and from which a choice or a ranking is expected to be established.

We are going to note with $\succeq$ (possibly subscribed $\succeq_i$) preference relations on set A to be read as "x is at least as good as y" ($x \succeq y$ or $\succeq (x, y)$). We impose only reflexivity on such a relation. If necessary, we may add other specific properties. $\succ$ represents as usually the asymmetric part of $\succeq$. We also use capital letters $P, Q, I, \ldots$ to represent specific preference relations (characterized by their properties). As usual P^{-1} represents the inverse relation of P ($P^{-1}(x, y) \equiv P(y, x)$).

We are going to use $\succeq^+$ to represent preference sentences of the type "there are positive reasons for considering x at least as good as y," whereas $\succeq^-$ represents sentences of the type "there are negative reasons for which it should not be the case that x is at least as good as y." Both $\succeq^+$ and $\succeq^-$ are binary relations.

Given a set H of such preference relations (a set of criteria) and for each couple $(x, y) \in A$ we note as H^+_{xy} the subset of H for which $x \succeq^+ y$ holds (the coalition of criteria for which there are positive reasons for which x is at least as good as y: positive coalition). In the same way, we are going to note as H^-_{xy} the subset of H for which $x \succeq^- y$ holds (the coalition of criteria for which there are negative reasons for which it should not be the case that x is at least as good as y: negative coalition).

Our problem can be summarized in two steps.

(1) Establish for each couple $(x, y) \in A$ an overall preference relation ($\succeq$), possibly separating it in $\succeq^+$ and $\succeq^-$. This should correspond to a general rule to be applied recursively in the case there is a hierarchy of criteria to take into account. We call this the *preference aggregation step*.
(2) Given such an overall preference relation, establish a final recommendation under form of a choice or a ranking on the set A, whenever this is required. We call this the *preference exploitation step*.

The reader can see an extensive discussion about the above two steps in classic Multiple Criteria Decision Analysis in Bouyssou et al.[1]

5. GENERALIZING CONCORDANCE AND DISCORDANCE

5.1. Preference Aggregation

We introduce the general rule

$$x \succeq^+ y \iff \mathcal{P}^+(H^+_{xy}) \geq \gamma \tag{1}$$

$$x \succeq^- y \iff \mathcal{P}^-(H^-_{xy}) \geq \delta \tag{2}$$

where $\mathcal{P}^+$ ($\mathcal{P}^-$) represents a measure of the importance of the "positive" (respectively negative) coalition and γ and δ represent two thresholds.

We are not going to discuss in this article how $\mathcal{P}^+$ ($\mathcal{P}^-$) is established, but without loss of generality we can assume that it is a real-valued function to the interval [0, 1]. Of course, the thresholds γ and δ are defined within the same interval.

The first rule should be read as: *when comparing x to y under all criteria, there are sufficient positive reasons to claim that x is at least as good as y* **iff** *the coalition of criteria where it is the case that x is at least as good as y is sufficiently strong*.

The second rule should be read as: *when comparing x to y under all criteria, there are sufficient negative reasons to claim that it is not the case that x is at least as good as y* **iff** *the coalition of criteria where it is not the case that x is at least as good as y is sufficiently strong*.

In principle, $\mathcal{P}^+$ and $\mathcal{P}^-$ are independently evaluated and therefore the strength of the positive and negative coalitions is not computed in the same way, nor can be considered with one being complement of the other. If we interpret the above rule within a social choice setting, we can consider $\mathcal{P}^+$ as the strength of the majority coalition, the γ threshold being the majority required to approve a bill, whereas $\mathcal{P}^-$ should be considered as the minority strength, the δ threshold representing the situation where a veto could be expressed. Consider again the UN Security Council example. Positive power for each member is $1/15$. The strength of the positive coalition is computed additively; the γ threshold being $3/5$. The negative power of each member is 0 or 1 (depending on their status: permanent or not permanent). The strength of the negative coalition is computed using the max operator; the δ threshold being 1.

The idea of using $\mathcal{P}^+$ and $\mathcal{P}^-$ has been already introduced in Multiple Criteria Decision Making methods. In the so-called "outranking methods," the global preference relation S (to be read as "at least as good as") is generally established as

$$S(x, y) \iff C(x, y) \wedge \neg D(x, y) \tag{3}$$

where $C(x, y)$ is the concordance test (is there a weighted majority of criteria in favor of x wrt to y?) and $D(x, y)$ is the discordance test (is there a veto against x wrt to y?).

Example 1. A typical application of the above rule can be seen in one of the oldest "outranking methods"[7] where:

$$C(x, y) \iff \frac{\sum_{j \in J_{xy}} w_j}{\sum_j w_j} \geq \gamma, \tag{4}$$

$$D(x, y) \iff \exists j : g_j(y) - g_j(x) > v_j \tag{5}$$

where:

- g_j is a real-valued function representing the evaluation of alternatives with respect to the criterion c_j (to be maximized);
- w_j is a nonnegative coefficient that represents the importance of the criterion c_j;
- J_{xy} represents the set of criteria for which x is at least as good as y; more precisely, $J_{xy} = \{j : g_j(y) - g_j(x) \leq q_j\}$, where q_j is the indifference threshold associated to criterion c_j; therefore $J_{xy} = H^+_{xy}$;
- γ is a majority threshold;
- v_j is a veto threshold on criterion c_j;
- consequently H^-_{xy} will be the set of criteria where a veto is expressed against $S(x, y)$.

In this case, a sufficiently strong positive coalition is any subset of criteria for which the sum of the importance coefficients is at least γ. If such a coalition exists, it means that we have positive reasons to consider that x is at least as good as y. On the other hand, we have negative reasons to consider that x is at least as good as y when y is largely better than x on at least one criterion.

However, this way to interpret the concordance/discordance principle presents a number of weak points. Using the definition in Equation 3, both the concordance and the nondiscordance tests have to be verified to establish the outranking relation. Indeed, the negative reasons (discordance) are sufficient to invalid the positive ones (concordance). However, there is a big semantic difference between a situation where a majority of criteria supports that "x is at least as good as y," but there is a veto, a situation where there is neither majority nor veto, and a situation where there is a minority of criteria in favor of the outranking relation. In other words, when comparing two alternatives x and y, the use of the concordance/discordance principle introduces four different epistemic situations, but only two possible cases can occur (either the outranking relation holds or not).

Moreover, the principle does not work recursively. There is no way to consider the existence of positive and negative reasons for each single criterion that should be aggregated separately. This prevents the use of this method in a hierarchical structure of criteria and agents.

Remark 1. The two functions $\mathcal{P}^+$ and $\mathcal{P}^-$ are supposed to be the measures of strength of the positive and negative coalitions of criteria, respectively. It is reasonable to consider such functions as "fuzzy measures" or "valued binary relations" instead of using their "cuts" represented by the thresholds γ and δ. This is the approach adopted by several authors including Figueira and Greco (personal communication, 2004), Grabisch and Labreuche,[8] Fernandez and Olmedo,[9] and Öztürk and Tsoukiàs.[10] The result is a "bipolar" (positive/negative) measure of the strength of preference for each pair of objects in A.

5.2. Preference Exploitation

Aggregating preferences generally result in a binary relation that is neither necessarily complete nor transitive.[11,12] The global relations $\succeq^+$ and $\succeq^-$ obtained after aggregating preferences are not necessarily orders. Thus, it is difficult, if not impossible, to identify a best choice or a ranking of the set A just using these relations. To obtain such a result (which we may call a final recommendation), it is necessary to further elaborate the information obtained from the aggregation step.

The literature offers a large variety of procedures for this purpose when conventional preference structures are considered.[13] The interested reader can see more details in Bouyssou et al.,[1] chapter 7. However, very few, if any, procedures exist when positive and negative procedures are considered separately.[14] In this article, we present two procedures:

(1) the positive/negative net flow procedure
(2) the positive/negative dominance ranking procedure

Let us recall that the input of such procedures are the two binary relations $\succeq^+$ and $\succeq^-$ on the set A and the output is a ranking of the set A.

(1) *The positive/negative net flow*. For each element $x \in A$, we compute a score

$$\sigma(x) = |\{y \in A : x \succeq^+ y\}| + |\{y \in A : y \succeq^- x\}| \\ - |\{y \in A : y \succeq^+ x\}| - |\{y \in A : x \succeq^- y\}| \quad (6)$$

We then rank the set A by decreasing values of σ. In other terms, for each element x we count the elements for which there are positive reasons such that x should be at least as good as them plus the elements for which there are negative reasons for which they should not be at least as good as x and we subtract the number of elements for which there are positive reasons for which they should be at least as good as x and the elements for which there are negative reasons for which x should not be at least as good as them. This procedure generalizes the net flow procedure used in MCDM.[15]

(2) *The positive/negative dominance ranking*. The procedure establishes two distinct rankings, one for the positive and another for the negative reasons and works as follows:
   - consider the graph associated to the relation $\succeq^+$;
   - identify the subset A_1^+ of A such that there are no entering arcs to any of its elements (the elements of A for which there are no other elements having positive reasons for which they should be at least as good as them);
   - establish A_1^+ as an equivalence class (the best), eliminate it from A and apply the same procedure to $A \setminus A_1^+$; this will identify the second best equivalence class A_2^+;
   - proceed until the set A is totally ranked from A_1^+ (the best) to A_n^+ (the least best);
   - consider the graph associated to the relation $\succeq^-$;
   - identify the subset A_1^- of A such that there are no entering arcs to any of its elements (the elements of A for which there are no other elements having negative reasons for which they should not be at least as good as them);
   - establish A_1^- as an equivalence class (the worst), eliminate it from A, and apply the same procedure to $A \setminus A_1^-$; this will identify the second worst equivalence class A_2^-;
   - proceed until the set A is totally ranked from A_n^- (the least worst) to A_1^- (the worst);
   - the two rankings do not necessarily coincide. A partial ranking of A can be obtained from the intersection of these two rankings.

Several other procedures can be conceived. We limit ourselves in this article to these two examples just in order to show how it is possible to obtain a final ranking after preferences have been aggregated using positive and negative reasons independently. Concluding this section, we can make the following remarks.

Remark 2. In the case $\mathcal{P}^+$ and $\mathcal{P}^-$ are considered as fuzzy measures the preference exploitation step will require different procedures. For examples see Öztürk and Tsoukiàs.[10]

Remark 3. The idea of final recommendation adopted in this article focusses on obtaining a choice or a ranking for the set A. However, in real decision support situations a final recommendation can be richer than that. For instance, it can identify conflicts and incomparabilities to analyze before any further decision. A clear representation of the positive and negative reasons behind such critical issues is extremely beneficial in such situations.

In the following, we are going to show how these ideas apply to a specific preference aggregation problem: the case where preferences are expressed through comparison of intervals.

6. AGGREGATING PREFERENCES ON INTERVALS

Consider a set of four objects $A = \{a, b, c, d\}$ and a set of four criteria $H = \{h_1, h_2, h_3, h_4\}$. To simplify the presentation, we consider that to all four criteria correspond to attributes where the objects in A can take numerical values on a scale from 1 to 8. However, in this particular case, we make the hypothesis that due to uncertainties on the real values of the objects these will take a value under form of an interval. Figure 1 shows how the objects in A are evaluated on the four attributes.

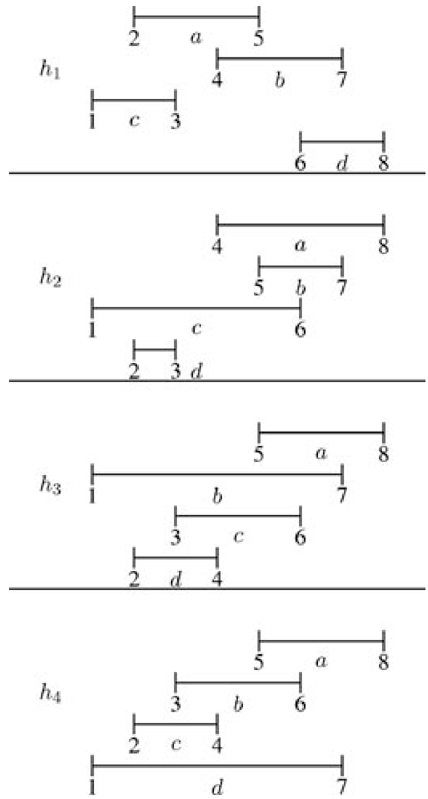

Figure 1. The values of A on the four attributes.

Where is the problem? Conventionally, comparison of intervals is based on the hypothesis that "x is preferred to y," $P(x, y)$, iff the lowest value of x is larger than the greatest value of y (the intervals thus being disjoint). In all other cases, "x is indifferent to y" ($I(x, y)$) (for more on interval orders, see Fishburn,[16] Pirlot and Vincke,[17] and Trotter.[18] If we interpret as usual the binary relation $\succeq$ as $P \cup I$, then the graphs representing this binary relation for the four criteria can be seen in Table I (here represented under form of a 0-1 matrix).

Aggregating these preferences with a simple majority rule: "x is at least as good as y iff it is the case for at least three out of four criteria," will return all four objects indifferent. This is not surprising because conventional interval orders use only positive information and are unable to differentiate between sure indifference and hesitation between indifference and preference.

To overcome this problem, we are going to introduce positive and negative reasons in comparing objects both at each criterion level and at the aggregated one. For this purpose, we are going to use a preference structure called *PQI* interval order.[19] In this structure, we consider three possibilities when comparing intervals:

- strict preference (P): when an interval is completely to the right of the other (exactly as in conventional interval orders);

Table I. The four interval orders.

h_1	a	b	c	d	h_2	a	b	c	d
a	1	1	1	0	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	0	1	0	c	1	1	1	1
d	1	1	1	1	d	0	0	1	1
h_3	a	b	c	d	h_4	a	b	c	d
a	1	1	1	1	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	1	1	1	c	0	1	1	1
d	0	1	1	1	d	1	1	1	1

- indifference (I): when one interval is completely included in the other;
- hesitation between preference and indifference or weak preference (Q): when an interval is to the right to the other, but they have a nonempty intersection.

Applying this structure to the information previously presented, we obtain the preference relations in Table II.

We are now going to interpret such preference relations in terms of positive and negative reasons. For this purpose, we are going to use the $\succeq^+$ and $\succeq^-$ relations and define:

$$P(x, y) \iff \succeq^+ (x, y) \wedge \not\succeq^- (x, y) \wedge \not\succeq^+ (y, x) \wedge \succeq^- (y, x) \tag{7}$$

$$I(x, y) \iff \succeq^+ (x, y) \wedge \not\succeq^- (x, y) \wedge \succeq^+ (y, x) \wedge \not\succeq^- (y, x) \tag{8}$$

$$Q(x, y) \iff \succeq^+ (x, y) \wedge \not\succeq^- (x, y) \wedge \succeq^+ (y, x) \wedge \succeq^- (y, x) \tag{9}$$

In other words, whereas P and I represent "sure" situations of interval comparison, the relation Q represents an hesitation between them: indeed, when comparing x to y we have positive reasons claiming that x is at least as good as y and no negative

Table II. The four *PQI* interval orders.

h_1	a	b	c	d	h_2	a	b	c	d
a	I	Q^{-1}	Q	P^{-1}	a	I	I	Q	P
b	Q	I	P	Q^{-1}	b	I	I	Q	P
c	Q^{-1}	P^{-1}	I	P^{-1}	c	Q^{-1}	Q^{-1}	I	I
d	P	Q	P	I	d	P^{-1}	P^{-1}	I	I
h_3	a	b	c	d	h_4	a	b	c	d
a	I	Q	Q	P	a	I	Q	P	Q
b	Q^{-1}	I	I	I	b	Q^{-1}	I	Q	I
c	Q^{-1}	I	I	Q	c	P^{-1}	Q^{-1}	I	I
d	P^{-1}	I	Q^{-1}	I	d	Q^{-1}	I	I	I

Table III. Positive reasons.

$\succeq_1^+$	a	b	c	d	$\succeq_2^+$	a	b	c	d
a	1	1	1	0	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	0	1	0	c	1	1	1	1
d	1	1	1	1	d	0	0	1	1
$\succeq_3^+$	a	b	c	d	$\succeq_4^+$	a	b	c	d
a	1	1	1	1	a	1	1	1	1
b	1	1	1	1	b	1	1	1	1
c	1	1	1	1	c	0	1	1	1
d	0	1	1	1	d	1	1	1	1

reasons claiming the opposite; when comparing y to x, we have both positive and negative reasons (because the larger value of y is larger than the smaller value of x, but smaller than the larger value of x). For further details on such models, the reader can see Öztürk,[20] Tsoukiàs et al.,[21] and Tsoukiàs and Vincke.[22] Applying this reasoning to the information concerning the set A, we get the results in Table III (for the positive reasons) and in Table IV (for the negative reasons).

To aggregate these positive and negative reasons, let us apply now the principle introduced in Equations 1–2. In this precise case, we use the following specific rule:

$$x \succeq^+ y \iff \frac{|\{h_j : x \succeq_j^+ y\}|}{|H|} \geq \frac{3}{4} \tag{10}$$

$$x \succeq^- y \iff \frac{|\{h_j : x \succeq_j^- y\}|}{|H|} \geq \frac{1}{2} \tag{11}$$

Actually, we use a very simple aggregation rule. Both $\mathcal{P}^+$ and $\mathcal{P}^-$ are additive, and for both the positive and negative distribution of power we consider the criterion equivalent. The results of this aggregation can be seen in Table V.

Table IV. Negative reasons.

$\succeq_1^-$	a	b	c	d	$\succeq_2^-$	a	b	c	d
a	0	1	0	1	a	0	0	0	0
b	0	0	0	1	b	0	0	0	0
c	1	1	0	1	c	1	1	0	0
d	0	0	0	0	d	1	1	0	0
$\succeq_3^-$	a	b	c	d	$\succeq_4^-$	a	b	c	d
a	0	0	0	0	a	0	0	0	0
b	1	0	0	0	b	1	0	0	0
c	1	0	0	0	c	1	1	0	0
d	1	0	1	0	d	1	0	0	0

Table V. Positive and negative reasons after aggregation.

$\succeq^+$	a	b	c	d	$\succeq^-$	a	b	c	d
a	1	1	1	1	a	0	0	0	0
b	1	1	1	1	b	1	0	0	0
c	1	1	1	1	c	1	1	0	0
d	0	1	1	1	d	1	0	0	0

Table VI. The *PQI* preference structure after aggregation.

PQI	a	b	c	d
a	I	Q	Q	P
b	Q^{-1}	I	Q	I
c	Q^{-1}	Q^{-1}	I	I
d	P^{-1}	I	I	I

What do we get from these results?

First of all, we are able to reconstruct a *PQI* preference structure at the aggregated level. We can establish the type of preference relation holding for any pair of objects in the set A. More precisely, applying Equations 7–9, we get the results shown in Table VI.

We can now check whether such a *PQI* preference structure is also a *PQI* interval order and if it is the case we can try to reconstruct a numerical representation for each element of A under form of interval. Using results known in the literature,[23,24] we can prove that in this precise case this is indeed a *PQI* interval order, a numerical representation of which can be seen in Figure 2.

In this case, we definitely need a more operational result such as a ranking we can use for any of the procedures introduced in Section 4.2. More precisely, adopting the positive/negative net flow procedure (see Equation 7) we get $a > b > d > c$ ($>$ representing the ranking relation). If we use the positive/negative dominance ranking, we obtain a different result: $a > b > d, c$. This is not surprising, if we consider the nature of the aggregation procedure and the information available. Concluding this section, we may make the following remarks:

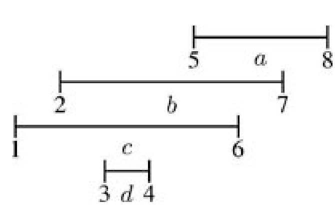

Figure 2. The global *PQI* interval order.

Remark 4. To our knowledge, this is the only way through which it is possible to aggregate preferences expressed on intervals without ending with only indifference, losing precious information.[25] The identification of positive and negative reasons in intervals comparison allows to exploit information that in conventional preference modeling is usually neglected. The specific suggestions done in this article should be considered as examples because several other possibilities can be considered depending on the problem on hand.

Remark 5. The reader can check that modifying the parameters and rules in the aggregation and exploitation steps one can obtain significantly different results. This is not surprising because these are not preferential information obtained from the decision maker and are more or less arbitrary. Care should be taken to tune them robustly.

Remark 6. Fortunately the algorithmic part of the above procedures is "easy." Indeed, as shown in Ngo The and Tsoukias[23] and Ngo The et al.,[24] checking if a *PQI* preference structure is a *PQI* interval order and finding a numerical representation are all problems in **P**.

7. CONCLUSIONS

In this article, we focus on the advantages of using independent positive and negative reasons in preference aggregation. More precisely:

- aggregating independent positive and negative reasons allows to clearly distinguish situations of sure preference from situations of hesitation as well as between incomparabilities due to conflicts (presence of both positive and negative reasons) and incomparabilities due to ignorance (absence of both positive and negative reasons);
- modeling independently positive and negative reasons allows to use the same principle for any level of preference modeling (single criterion, single agent, multiple criteria, multiple agents and their combinations), thus generalizing the concordance/discordance principle;
- the use of positive and negative reasons when objects evaluated on intervals are compared allows to solve the problem of aggregating such preferences, a situation encountered not only in decision aiding but also in several other fields.[26]

Several research problems remain unresolved in the article. Among these, we note the following:

- axiomatize preference aggregation and exploitation procedures based on the independent use of positive and negative reasons (of the type given in this article);
- study appropriate formalisms (multiple valued logics, argumentation theory, etc.), enabling elegant and compact representations besides further extending the potentialities of this approach;
- further investigate the problem of aggregating *PQI* preference structures: under what conditions the aggregation of such preference structures will result in a *PQI* interval order or any other order representable by intervals?

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