Methods and Models for Decision Making

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Methods and Models for Decision Making (MMDM)

Aims:

• introduction to the basics of decision theory
• discussion about decision making in design (and in other fields)
• presentation of risk analysis, multicriteria, group decision, …
• definition of possible research topics (in design area)

Outline:

• (1) Introduction
• (3) Mental models
• (5) Classification
• (7) Ranking-2, multicriteria
• (9) Seminar
• (11) Group decision
• (13) Research topics
• (15) Conclusions

(2) Tools & frame

• (4) Design & decision
• (6) Ranking-1, risk analysis
• (8) A tentative case (discuss.)
• (10) Rating problems
• (12) Genetic alg. + …
• (14) Case results (if any …)
DM: an introduction
The steps of a decision

- **Alternatives**
- **Criteria**
- **Evaluation system**
- **Results**

- by elementary actions
- indicators & value functions
- what can (must) be obtained

(see in the following the different procedures)
The different (4) levels of a decision process

i. Information → Let’s go out for dinner.

ii. Feedback → Let’s go out for dinner, do you agree?

iii. Discussion → Let’s go out for dinner, where can we go?

iv. Involvement → Would you like to go out? to do what?

different actors (Decision Makers, DM’s)
a (possibly pre-defined) procedure
Decision Theories: a brief introduction

Short history:

• 40’s → Genesis (during the 2° war)
• 50-60’s → Development [*] (LP probl. & Combinatorics)
• 60-70’s → Specialization (non linear, integer, B&B, …)
• 70-80’s → Multicriteria (the importance of trade-off)
• 50-90’s → Multiple DM (the different points of view)
• 80-00’s → Decision Aiding (sw supporting the process)

[*] max f(x), s.t. x ∊ X (with X finite or infinite set)

Links & references:

• http://www.informs.org (the INFORMS site)
• http://www.euro-online.org (the EURO site)
• http://www.airo2.org (the AIRO, Italian site)
• http://corsi.metid.polimi.it (the site of Center METID)
• A. Tsoukias, From decision theory to decis. aiding method., EJOR, 2007
An “ideal” decision problem

- Someone who decides
  - with respect to one clear **objective**
  - with a set of well defined **constraints**
  - with all the suitable **information**

  in presence of a finite or infinite set of alternatives

- Two (ideal) examples
Ideal example 1

Combinatorial optimization

Your chorus is defining the storyboard of a concert and you must choose between a set of mottetti (a “mottetto” is a choral musical composition). Each mottetto \((m_1, m_2, \ldots, m_n)\) has a time of execution \(t_j\) and a level of success \(s_j\) \((j = 1, \ldots, n)\).

The total time of the exhibition is \(T\) min.

What can you do?

If you want, consider this specific instance:
\[
\begin{align*}
n &= 4; & t &= (10, 22, 37, 9); & s &= (60, 55, 100, 15); & T &= 45 \\
\end{align*}
\]

(i) What are the variables?
(ii) How many solutions?
(iii) What is the optimal choice?
Ideal example 2

Linear programming

You must define the week production of a (small) firm that has only 2 products, PA and PB.
One item of PA needs 2 units of the resource R1 and 1 unit of the resource R2.
One item of PB needs 1 unit of the resource R1 and 3 units of the resource R2.
The net revenue for each item (PA or PB) is 500 €.
You have (weekly) 400 units of R1 and 900 units of R2.
You know that the maximum possible sale for PB is 250 items.

What can you do?

(i) What are the variables?
(ii) How many solutions?
(iii) What is the optimal choice? (you can solve with Excel …)
A **real** decision problem

- **Uncertainties** (non-deterministic context, data mining)
- **Complexity** (problem dimension, non linearity, …)
- **Several stakeholders** (distributed decision power)
- **Different rationalities** (criteria and preferences)
- **Various time horizons** (often)
- **Use of simulation models**
  - what … if …
Tools

A formal decision process needs instruments for:

i. abstraction

ii. analysis

iii. synthesis

(and more …)
Tools for abstraction / 1

- 1736
- Konigsberg

- The 7 bridges
- A riddle

- Euler
- Graph theory

- The Euler model
- The answer (similar to …)
The count drunk poisoned water (from one of his 7 lovers)

All 7 lovers were in the castle the day of his death

The murderer should have come to the castle twice (one for exploring, the other for killing), while the others only one.

Statements of the 7 women:

Alice saw B C E F
Barbara saw A C D E G
Clara saw A B D
Diana saw B C E
Elena saw A B D G
Francesca saw A G
Gloria saw B E F

Elementary, my dear Watson!
(said Sherlock H.)
The solution

S. H. & the death of count Kinskij

Women statements

\[
\begin{align*}
A & \rightarrow E \\
E & \rightarrow D \\
D & \rightarrow C \\
C & \rightarrow G \\
G & \rightarrow F \\
F & \rightarrow A
\end{align*}
\]

\[\{A, E, D, C\} \not\rightarrow \{A, E, G, F\} \not\rightarrow \{A, B, G, F\}\]

Impossible!
Graph theory & decision problems

- **General reports**
  - [http://en.wikipedia.org/wiki/Graph_theory](http://en.wikipedia.org/wiki/Graph_theory)

- **Applications**
  - [http://bla...](http://bla...)
  - [http://bla...](http://bla...)

- **A famous problem – TSP**
  - [http://www-e.uni-magdeburg.de/mertens/TSP/index.html](http://www-e.uni-magdeburg.de/mertens/TSP/index.html)
  - [http://www.tsp.gatech.edu/index.html](http://www.tsp.gatech.edu/index.html)
  - [http://www.densis.fee.unicamp.br/~moscato/TSPBIB_home.html](http://www.densis.fee.unicamp.br/~moscato/TSPBIB_home.html)
Tools for analysis / 1

- Sudoku (Corriere della Sera, 3 Sept. 2006)

- Branching rules → a tree
- A lot of (small) subproblems
Step 2

Step 6

Step 4

What number in position $X$? 2 or 9

branch (a) $\Rightarrow X = 2$

but if $X = 2$, there is no place for a 2 in the right-high block; so $X = 2 \Rightarrow NO$

branch (b) $\Rightarrow X = 9$

in this case $\neq$
**Step 8**

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**Step 9**

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*What in the position $Y$?*

5 or 9

*branch (b1) $\Rightarrow Y = 5$*

*in this case* $\notin$

*Open situations (to be explored) are (b1) with $Y = 5$, and (b2) with $Y = 9$*
### Tools for analysis / ...

#### Step 13 (of b1)

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#### Step 53 (of b1)

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Stop! (the solution is unique)

so branch (b2)
The solution (visualization)

- Branching rules
- A lot of (easier) subproblems
- Stopping rules
Tools for synthesis

Who is the all time world’s best boxeur ?

Indicators:

- strength
- speed
- n. of victories
- years of premiership
- ...

We need a common framework to compare the alternatives !
Tools & frame
Decision processes: a frame

1. Math. programming
2. Risk analysis
3. Multiple criteria
4. Social choice
5, 6, 7, 8 → Game theory, …

Information
- complete
- partial
- state identific. & risk an.

Objectives
- one
- more
- trade-off

Dec. makers
- one
- more
- conflicts
A real decision process

- **Uncertainties** (non deterministic context, …)
- Complexity (problem dimension, non linearity, …)
- Several stakeholders (distributed decision power)
- Different rationalities (criteria and preferences)
- Different time horizons (often)
- Use of simulation models
  - what … if …
- The perception of the problem:
  - differences between
  - normative approach
  - cognitive approach
Decision processes in a non-deterministic context

1. Math. programming
2. Risk analysis
3. Multi-objective (criteria)
4. Social choice
5, 6, 7, 8 → ....

Information

Objectives

Dec. makers

[∗] → non-deterministic context

perception & mental models
Two (opposite) theories

(a) Normative theory (prescriptive)
   what the DM should do

(b) Cognitive theory (descriptive)
   what the DM really does

When they are the same?
   if the (single) DM has all the information in a deterministic way and has clearly in mind the (single) criterion of evaluation
   optimization

Experimental tests
**N-1° Principle of INVARIANCE**

Equivalent (from the logical point of view) versions of the same problem **must** produce the same choice.

**Examples**
- Change names or positions for the options
- Change measure units
- Add a constant value for all the results

**Counterexamples**
- Lotteries (A, B, C)
- Ellsberg paradox (1961)
Lotteries (case A and case B)

Better A1 or A2?

Better B1 or B2?
Lotteries (case C)

**Better C1 or C2?**

- **C1**: 25% chance of 240, 75% chance of -760.
- **C2**: 25% chance of 250, 75% chance of -750.

**But notice that**

- **C1** can be expressed as a linear combination of **A1** and **B2**.
- **C2** can be expressed as a linear combination of **A2** and **B1**.

**C1 → lin. comb. of A1 and B2**

**C2 → lin. comb. of A2 and B1**
Ellsberg

Now you have a second chance (after the ball is re-inserted)

Better to take from A or B?

ambiguity aversion

White ball win

the same ...

Black ball win

Better to take from A or B?

ambiguity aversion?

Better to take from A or B?

better ...

better ...

50 (b) \ 50 (n)

\[100 - \alpha\] (n)

\[\alpha\] (b)
Cognitive theory: a first principle

Principle of NON NEUTRALITY

The aggregation of (decisional) options is not a neutral operation!

Given the two preferences on A1 and B2, it is not guaranteed that their aggregation (C1) is the preferred one.

- Caution: do not combine too easily the options

- Normally, the ambiguity is avoided, "even if this is not rational" (Ellsberg)
N-2° Principle of DOMINANCE

If the DM prefers A with respect to B in every scenario (or context or state of nature) the choice must be A.

Examples
- I prefer to be missionaire (with respect to engineer) in peace and prefer to be missionaire (…) in war
- I prefer chicken with respect to beef (when there is nothing else) and I prefer chicken … also when there is fish

Counterexamples (see in next lessons)
- Extraction from an urn filled with 100 balls (Tversky e Kahneman, 1986)
- The possible choices in uncertainty conditions (see “Sindaco di Utopia”)

so choice … is better then …
(leaving … out of consideration)
### Extraction (in two conditions) / 1

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<td>2 yellow</td>
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Better A or B?

- better ...

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Better C or D?

- better ...

but C ≡ A and D ≡ B
**Extraction (in two conditions) / 2**

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Better \[\text{Invest or Build}\]

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Cognitive theory: three more principles

**C-2° Principle of EVIDENCE**

The dominance among options should be obvious

**C-3° Principle of ASYMMETRY**

The possibility of losing $K$ is more important than that to win $K$

**C-4° Principle of COMPACTNESS**

An aggregated option ($A$) has an importance less than the sum of the importances of the single sub-options ($A_1.A_2$)

$$\pi(A) < \pi(A_1) + \pi(A_2)$$
**Normative theory: principles & (counter)examples / 3**

**N-3° Principle of TRANSITIVITY**

If the decision prefers A over B and B over C, then A **must** be preferred over C.

**Examples:**
- Since V. Rossi is better than Stoner, and Stoner is better than Melandri, ...
- Buying emission units (Kyoto protocol) is better than cutting the production, and cutting the production is better than not respecting the constraints on emissions, so ...

**Counterexamples:**
- A new car + accessories

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standard 10.000€
+ air cond. 1.000€
+ alloy rims 1.000€
+ ...

(but finally ...)

D > A? or rather
**the options are incomparable?**
Cognitive theory: progression vs. crash

**Principle of CRASH**

The decision-maker is (relatively) indifferent to small progressive changes, but at some point become aware of the (large) gap and ...

Cognitive theory: estimation

**Principle of OVER/UNDER-ESTIMATION**

There is an inclination to

- over-estimate events with small probability
- under-estimate events with high probability (except in case of certainty)

Asymmetry in dealing with subjective probability