Multiple Criteria Decision Analysis
— Problems, Models, Methods and Applications

Professor Jian-Bo Yang
Director of Decision and Cognitive Sciences Research Centre
Manchester Business School
The University of Manchester
Room: F36 / MBS East
Tel: 0161 200 3427 (Ext: 63427)
Email: jian-bo.yang@mbs.ac.uk
Web: www.personal.mbs.ac.uk/jbyang
Main Topics of the Session

- Multiple criteria decision analysis – an introduction
- Multiple objective optimization problems in real world
- Multiple criteria assessment and decision analysis problems in real world
- Decision matrix and MCDA explained in graph
- Additive value function approach in MCDA
- Deal with uncertainties in MCDA
- Evidential reasoning MCDA – concept, model, process and tool
- A snapshot of real world MCDA applications
Decision Making at Different Levels
(Anthony’s Model, 1965)

- Super-strategic
- Strategic Planning
- Managerial Control
- Operational Control (Tactical)

Multiple Criteria Decision Analysis
Decision Issues at Different Levels

• **Strategic planning**
  – New business opportunities
  – Competition strategies
  – Technology adoption
  – Strategic partnership

• **Operational control**
  – Task scheduling
  – Production optimization
  – Coordination
  – Skill development

• **Managerial control**
  – Financial control
  – Project control
  – Quality control
  – Risk control
  – HR control

Multiple Criteria Decision Analysis
Multiple Criteria Decision Making – Typical solution procedure

1. Start investigation
2. Structure problem
3. Build model
4. Assess and analyse
5. Make decision
Main Topics of the Session

- Multiple criteria decision analysis – what is it?
- **Multiple objective optimization problems in real world**
- Multiple criteria assessment and decision analysis problems in real world
- Decision matrix and MCDA explained in graph
- Additive value function approach in MCDA
- Deal with uncertainties in MCDA
- Evidential reasoning MCDA – concept, model, process and tool
- A snapshot of real world MCDA applications
Multi-objective optimization in real world
– Production planning and scheduling

- Multiple objective optimisation for production planning in oil refinery
- Large scale optimisation methods and software
- Multiple criteria decision analysis
- Automatic model update
- Decision support systems

Multi-objective optimization in real world
– Made-to-order engineering product design

- Offshore structures
  - Construction cost
  - Layout optimisation

http://www.offshore-technology.com/contractors/pipes/project-materials/project-materials1.html
Multi-objective optimization in real world
– Made-to-order engineering product design

- Offshore structures
  - Construction cost
  - Layout optimisation

- Optimal ship design
  - Transportation cost
  - Light ship mass
  - Annual cargo

Multi-objective optimization in real world
– Made-to-order engineering product design

- Offshore structures
  - Construction cost
  - Layout optimisation
- Optimal ship design
  - Transportation cost
  - Light ship mass
  - Annual cargo
- Optimal ferry design
  - Safety measures
Multi-objective optimization in real world – Project portfolio analysis and management
Multi-objective optimization in real world
– Project portfolio analysis and management
Main Topics of the Session

• Multiple criteria decision analysis – what is it?
• Multiple objective optimization problems in real world
• **Multiple criteria assessment and decision analysis problems in real world**
• Decision matrix and MCDA explained in graph
• Additive value function approach in MCDA
• Deal with uncertainties in MCDA
• Evidential reasoning MCDA – concept, model, process and tool
• A snapshot of real world MCDA applications
Multi-Criteria Decision Analysis in real world
– Design selection of engineering products

- Offshore structures
- Container ship
- Cargo ship
- Roll-on roll-off ferry
- Aircraft
- Car
- Computer
- Motorcycle
- house
- …
Multi-Criteria Decision Analysis in real world
– Risk & safety analysis of products and systems

- Offshore structures
- Cargo ship
- Container ship
- Roll-on roll-off ferry
- Nuclear plant
- Food and drink
- Sea port
- Air port
- Hospital
- …
Multi-Criteria Decision Analysis in real world – Prioritise voices of customer via surveys (GM)

http://www.carbuyersnotebook.com/2013-chevy-cruze-pictured/

Multi-Criteria Decision Analysis in real world – *Prioritise voices of customer using surveys*

### SCALE INCOMPATIBILITY IN SURVEYS

<table>
<thead>
<tr>
<th>Survey 1</th>
<th>Survey 2</th>
<th>Survey 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Disagree Strongly</td>
<td>1: Not Good</td>
<td>1: Unacceptable</td>
</tr>
<tr>
<td>2: Disagree</td>
<td>2: Good</td>
<td>3: Below Average</td>
</tr>
<tr>
<td>3: Neutral</td>
<td>3: Very Good</td>
<td>5: Average</td>
</tr>
<tr>
<td>4: Agree</td>
<td>4: Excellent</td>
<td>7: Good</td>
</tr>
<tr>
<td>5: Agree Strongly</td>
<td>5: Truly Outstanding</td>
<td>10: Outstanding</td>
</tr>
</tbody>
</table>

- Surveys use different rating scales: Limited control if not in-house
- Handling incompatibility of rating scales
  - Define common scale and create transformation functions
  - Define criteria that are independent of scales
Multi-Criteria Decision Analysis in real world
– Prioritise voices of customer using surveys
Multi-Criteria Decision Analysis in real world
– Business excellence self-assessment: EFQM
Multi-Criteria Decision Analysis in real world

- Business excellence self-assessment: EFQM

- Thoroughness
- Effort

- Knowledge Base
  (174 areas to address)

- Award Simulation
  (32 sub-criteria)

- Peer Involvement

- Workbook (9 criteria)

- Beta
- RapidScore
- Questionnaire
- Matrix Chart

EVIDENCE
PERCEPTION

GATHERING BASED
EFQM Self-Assessment Model: For total quality management in an organisation
Intelligent Decision System (IDS): Evidence Mapping Window

<table>
<thead>
<tr>
<th>Grade definitions:</th>
<th>Evidence provided:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Award Winners</td>
<td>1. Key processes and support processes have been identified and mapped.</td>
</tr>
<tr>
<td></td>
<td>The Management System manual describes links between the documentation produced and the delivery of the business goal through key performance measures.</td>
</tr>
<tr>
<td></td>
<td>2. Some departments have developed and documented their own processes and this need to be integrated within the one system.</td>
</tr>
<tr>
<td></td>
<td>- Key business processes and support processes are defined and documented to deliver policy and strategy. Flowcharts are used.</td>
</tr>
<tr>
<td></td>
<td>- A well defined and developed process exists to identify key business processes and support processes.</td>
</tr>
<tr>
<td></td>
<td>- relationships identified between individual products/services and processes.</td>
</tr>
</tbody>
</table>

Provide comments as follows:

1. Clear evidence shows that the key processes have been identified to deliver policy and strategy which matches grade B.

2. However, the system is not fully integrated and the approach still need time to be mature. Therefore C would be graded to this consideration.

To my degree of belief, B(0.6) C(0.4) would be a balanced score.
## Multi-Criteria Decision Analysis in real world – Supplier assessment and selection

**Supplier Assessment**

- Quality
- Supply Chain Evaluation
- Technical Competence evaluation
- Total Cost Evaluation
- General Factors Evaluation
- After Sales Evaluation
- Enviroethical
- Leadership and Strategy
- Project Management
- Customer Needs
- E - Readiness

---

http://www.electricalequipment.co/siemens-process-instrumentation/

6. After Sales Evaluation

6.1 Product Support

6.1.6 What is your response time?

Answers:

1> 1 – 2 hours
2> 3 – 4 hours
3> 5 – 6 hours
4> 7 – 8 hours
Supplier Assessment Model (Siemens UK)

Question & multiple choice answers

1. Quality

1.5 Quality Performance of Supplier

1.5.4 Are quality costs measured, monitored and published?

Answers:

1> No
2> Yes, occasionally
3> Yes, with improvement plans prioritised
4> Yes, with management review done regularly
2. Supply Chain Evaluation

2.1 Performance Measures

2.1.27 Which of the following criteria are used to measure the performance?

Answers: (Yes / No)

2.1.27.1 Purchase savings
2.1.27.2 Availability of stocks
2.1.27.3 Number of purchase orders outstanding
2.1.27.4 Level of inventory
2.1.27.5 Stock turnover
2.1.27.6 Standard cost variance
Supplier Assessment Model (Siemens UK)

Overall assessment grade (TQM Concept)

Supplier Classification

World Class (ideal)
Award winners (reliable)
Improvers (potential)
Drifters (unfavourable)
Uncommitted (unqualified)
Supplier Assessment Model (Siemens UK)

Propagation of quantitative assessment

Response time ⇔ After Sales Evaluation

1 hour or less ⇔ (World Class)
3 hours ⇔ (Award winners)
5 hours ⇔ (Improvers)
7 hours ⇔ (Drifters)
8 or above ⇔ (Uncommitted)
Main Topics of the Session

- Multiple criteria decision analysis – what is it?
- Multiple objective optimization problems in real world
- Multiple criteria assessment and decision analysis problems in real world
- **Decision matrix and MCDA explained in graph**
- Additive value function approach in MCDA
- Deal with uncertainties in MCDA
- Evidential reasoning MCDA – concept, model, process and tool
- A snapshot of real world MCDA applications
Traditional Decision Matrix – Average Point Assessment

It uses average numbers to assess each alternative on all criteria.
MCDM – Graphic Interpretation for

Dominated solutions, efficient solution, efficient frontier

Dominated solutions: C
Weak efficient solution: D
Efficient frontier: A, B, D, E, F, G

Purpose of MCDM:
Find the most preferred solution from the set of efficient solutions

目的在于MCDM：
从有效解决方案的集合中找到最偏好解决方案
Distance-based Preference Modelling

Aspiration level models (minimax distance)

Reference point models: Set a reference point and find an alternative closest to the reference point in certain distance measure.
Distance-based Preference Modelling

**Ideal point models** *(minimax distance)*

**Ideal point models**: Set an ideal reference point and find an alternative closest to the ideal point in certain distance measure.
Main Topics of the Session

• Multiple criteria decision analysis – what is it?
• Multiple objective optimization problems in real world
• Multiple criteria assessment and decision analysis problems in real world
• Decision matrix and MCDA explained in graph
• Additive value function approach in MCDA
• Deal with uncertainties in MCDA
• Evidential reasoning MCDA – concept, model, process and tool
• A snapshot of real world MCDA applications
Additive Value Function Approach
Assessment of postgraduate schools – example 1

Original Decision Matrix

<table>
<thead>
<tr>
<th></th>
<th>Average book (y₁, number)</th>
<th>Student / staff (y₂, ratio)</th>
<th>Research grant (y₃, $,000)</th>
<th>Graduation delayed (y₄, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>0.1</td>
<td>5</td>
<td>5,000</td>
<td>4.7</td>
</tr>
<tr>
<td>School 2</td>
<td>0.2</td>
<td>7</td>
<td>4,000</td>
<td>2.2</td>
</tr>
<tr>
<td>School 3</td>
<td>0.6</td>
<td>10</td>
<td>1,260</td>
<td>3.0</td>
</tr>
<tr>
<td>School 4</td>
<td>0.3</td>
<td>4</td>
<td>3,000</td>
<td>3.9</td>
</tr>
<tr>
<td>School 5</td>
<td>2.8</td>
<td>2</td>
<td>284</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Multiple Criteria Decision Analysis
Assign Importance Weights by Comparisons

School performance assessment example

Comparisons: Suppose the most important criterion of the four criteria for school performance assessment is “research grant”.

1. Compare its importance with each of the other criteria:
   “Research grant” is twice as important as “books”, \( \frac{\omega_3}{\omega_1} = 2 \)
   “Research grant” is 1.5 times as important as “ratio”, \( \frac{\omega_3}{\omega_2} = 1.5 \)
   “Research grant” is 3 times as important as “graduation”, \( \frac{\omega_3}{\omega_4} = 3 \)

Solve the four linear equations:

\[
\omega_3 - 2\omega_1 = 0, \quad \omega_3 - 1.5\omega_2 = 0, \quad \omega_3 - 3\omega_4 = 0, \quad \omega_1 + \omega_2 + \omega_3 + \omega_4 = 1
\]

So, the weights of the four criteria are given by

\( \omega_1 = 0.2, \quad \omega_2 = 0.2667, \quad \omega_3 = 0.4, \quad \omega_4 = 0.1333 \)
Definition of A Partial Value Function

*Direct assessment via visual aid – $v_3$*
Look up Partial Value Function

To get values for research grant – \( v_3 \)
Pre-processing Data Collected

Transformation of data with optimal interval

**Concept:** For some criteria neither larger nor smaller is desirable, such as student and staff ratio. A high ratio may lead to the compromise of quality, but a low ratio means low workload for staff. A desirable ratio may be shown in the following diagram.
Additive Value Function Approach

*Performance assessment for postgraduate schools*

### Variously-Transformed Decision Matrix with Weights

<table>
<thead>
<tr>
<th></th>
<th><strong>Average book</strong> ($\omega_1 = 0.2$)</th>
<th><strong>Student / staff</strong> ($\omega_2 = 0.2667$)</th>
<th><strong>Research grant</strong> ($\omega_3 = 0.4$)</th>
<th><strong>Graduation delayed</strong> ($\omega_4 = 0.1333$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>0.5950</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>School 2</td>
<td>0.6100</td>
<td>0.8333</td>
<td>0.9166</td>
<td>0.7142</td>
</tr>
<tr>
<td>School 3</td>
<td>0.6700</td>
<td>0.3333</td>
<td>0.5650</td>
<td>0.4857</td>
</tr>
<tr>
<td>School 4</td>
<td>0.6250</td>
<td>0.6666</td>
<td>0.8333</td>
<td>0.2286</td>
</tr>
<tr>
<td>School 5</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.1420</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Multiple Criteria Decision Analysis
Multiple Attribute Value Theory

Additive value function and conditions required

General form of an additive value function is given by:

\[ v = \sum_{i=1}^{m} \omega_i v_i(y_i) = \omega_1 v_1(y_1) + \omega_2 v_2(y_2) + \cdots + \omega_m v_m(y_m) \]

Conditions for use of Additive MAVF:

1. Satisfaction of preferential independence among any groups of attributes. This is only a necessary condition.
2. Satisfaction of the corresponding trade-off, or Thomsen condition.
3. Interval scale property for constructing marginal value function.
4. Weights of attributes need to be assessed as scaling constants (trade-offs), or swing weights, not necessarily relative importance.
5. Linear & complete compensation among criteria without any limit.
Additive Value Function Approach

Performance assessment for postgraduate schools

Ranking Using Variously-Transformed Decision Matrix

<table>
<thead>
<tr>
<th>School</th>
<th>$z^h_1$ ($\omega_1=0.2$)</th>
<th>$z^f_2$ ($\omega_2=0.2667$)</th>
<th>$v_3$ ($\omega_3=0.4$)</th>
<th>$z^e_4$ ($\omega_4=0.1333$)</th>
<th>$\sum \omega_i v_i$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>0.5950</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.7857</td>
<td>2</td>
</tr>
<tr>
<td>School 2</td>
<td>0.6100</td>
<td>0.8333</td>
<td>0.9166</td>
<td>0.7142</td>
<td>0.8061</td>
<td>1</td>
</tr>
<tr>
<td>School 3</td>
<td>0.6700</td>
<td>0.3333</td>
<td>0.5650</td>
<td>0.4857</td>
<td>0.5136</td>
<td>4</td>
</tr>
<tr>
<td>School 4</td>
<td>0.6250</td>
<td>0.6666</td>
<td>0.8333</td>
<td>0.2286</td>
<td>0.6666</td>
<td>3</td>
</tr>
<tr>
<td>School 5</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.1420</td>
<td>1.0000</td>
<td>0.3901</td>
<td>5</td>
</tr>
</tbody>
</table>

It is useful to conduct sensitivity analysis by changing weights, using different normalisation methods or changing value functions.

Multiple Criteria Decision Analysis
For purchase of MP3 players, suppose three attributes are taken into account: *price*, *memory*, and *sound quality*

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Memory</th>
<th>Sound Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP3-A</td>
<td>High price + Large memory</td>
<td>High sound quality</td>
<td></td>
</tr>
<tr>
<td>MP3-B</td>
<td>Low price + Small memory</td>
<td>High sound quality</td>
<td></td>
</tr>
</tbody>
</table>

Suppose MP3-A is preferred to MP3-B

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Memory</th>
<th>Sound Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP3-C</td>
<td>High price + Large memory</td>
<td>Low sound quality</td>
<td></td>
</tr>
<tr>
<td>MP3-D</td>
<td>Low price + Small memory</td>
<td>Low sound quality</td>
<td></td>
</tr>
</tbody>
</table>

Would MP3-C still be preferred to MP3-D?
Limitation or Bias of Additive VFA

Efficient frontier: A, B, D, E, F, G
Efficient convex hull: A, E, G
Additive VFA cannot find B or F as the most preferred solution

\[ \omega_s v_s + \omega_p v_p = \mathbf{v} \]
Main Topics of the Session

• Multiple criteria decision analysis – what is it?
• Multiple objective optimization problems in real world
• Multiple criteria assessment and decision analysis problems in real world
• Decision matrix and MCDA explained in graph
• Additive value function approach in MCDA
• Deal with uncertainties in MCDA
• Evidential reasoning MCDA – concept, model, process and tool
• A snapshot of real world MCDA applications
Multi-Criteria Decision Analysis

Belief distribution versus average assessment

- Frequencies of customer responses from external surveys

- The average score of GM-B is about the same as that of GM-A

- Is GM-B of the same priority to GM as GM-A in future design?
**Belief Decision Matrix – Distribution Assessment**

1. It can represent precise numbers for all criteria on each alternative
2. It can represent subjective judgements
3. It can represent ignorance explicitly

\[
A_{mn} = \{(H_1, \beta_1), (H_2, \beta_2), \ldots, (H_N, \beta_N)\}
\]
## Multi-Criteria Decision Analysis

**Belief decision matrix for problem modelling**

<table>
<thead>
<tr>
<th>House Criteria</th>
<th>House 1 in Altrincham</th>
<th>House 2 in Heaton</th>
<th>House 3 in Mercy</th>
<th>House 4 in Didsbury</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
<td>{(G, 0.5), (E, 0.5)}</td>
<td>{(G, 0.5)}</td>
<td>{(A, 0.2), (G, 0.8)}</td>
<td>{(G, 0.2), (E, 0.8)}</td>
</tr>
<tr>
<td><strong>Distance (mile)</strong></td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>Asking Price (£)</strong></td>
<td>113,000</td>
<td>110,000</td>
<td>118,000</td>
<td>150,000</td>
</tr>
<tr>
<td><strong>Attractiveness</strong></td>
<td>{(P, 0.05), (G, 0.35), (E, 0.60)}</td>
<td>{(A, 0.4), (G, 0.6)}</td>
<td>{(G, 0.3), (E, 0.7)}</td>
<td>{(G, 0.6), (E, 0.4)}</td>
</tr>
</tbody>
</table>
### Construct Qualitative Value Function

**Assess the location of houses in south Manchester**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Definition (list of indicators for collecting evidence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>excellent</td>
<td>Pleasant surrounding, Excellent neighbours, First class facilities, Very convenient transportation, Excellent schools, and Many shops around</td>
</tr>
<tr>
<td>Good</td>
<td>Good surrounding, Friendly neighbours, Good facilities, Convenient transportation, Good schools, and A number of shops around</td>
</tr>
<tr>
<td>Average</td>
<td>Normal surrounding, Ordinary neighbours, Some facilities, Some transportation, Average schools, and A few shops around</td>
</tr>
<tr>
<td>Poor</td>
<td>Noisy surrounding, Unfriendly neighbours, Poor facilities, Inconvenient transportation, Poor schools, and Few shops around</td>
</tr>
<tr>
<td>Bad</td>
<td>Unbearable surrounding, Terrible neighbours, No facilities, No transportation, No schools, and No shops around</td>
</tr>
</tbody>
</table>
Belief Decision Matrix

Assessment based on evidence collected

Assessing the Location of House 1 in Altrincham using the collected evidence against the agreed assessment standards
Belief Decision Matrix

Examples for uncertainty modelling

- **From comparing evidence to grading standards**
  Supplier 1’s performance on Technical Competence
  \{\text{(Excellent, 50%), (Good, 40%), (Poor, 10%)}\}

- **Group opinion distribution**
  Deep repository on health risk
  \{\text{(High, 30%), (Medium, 30%), (Low, 40%)}\}

- **Random data**
  Car fuel consumption in mpg (miles/gallon):
  \{\text{(20mpg, 30%), (22mpg, 30%), (25mpg, 40%)}\}
Belief Decision Matrix

Examples for uncertainty modelling

- **Judgments from Experience** - Personality Test:

  Do you always try to avoid the gaps on pavement?
  
  {\(\text{(Yes, 20\%), (No, 80\%)}\)}

- **From converting numerical data to grades**

  If   Excellent=100, Good=75,
  
  then   90={\(\text{(Excellent, 60\%), (Good, 40\%)}\)}
Belief Decision Matrix

Examples for uncertainty modelling

• **Data with ignorance (partial or complete)**

  _Car engine quality assessment:_

  \{(Excellent, 30\%), (Good, 50\%)\}

  with unknown 20\% — **Partial ignorance**

  \{(Excellent, 0\%), …, (Poor, 0\%)\}

  with unknown 100\% — **Complete ignorance**
Belief Decision Matrix

Examples for uncertainty modelling

- **Data with interval uncertainties**

  Belief assigned to an interval of grades:

  \{(Excellent-Good), 60\%), (Good, 40\%)}

- **Interval belief assessed to individual grades:**

  \{(Moderately Negative, 20-30\%),
  (Neutral, 30-40\%), (Positive, 40-50\%)}
Main Topics of the Session

• Multiple criteria decision analysis – what is it?
• Multiple objective optimization problems in real world
• Multiple criteria assessment and decision analysis problems in real world
• Decision matrix and MCDA explained in graph
• Additive value function approach in MCDA
• Deal with uncertainties in MCDA
• Evidential reasoning MCDA – concept, model, process and tool
• A snapshot of real world MCDA applications
## Multi-Criteria Decision Analysis

### Belief decision matrix for problem modelling

<table>
<thead>
<tr>
<th>House Criteria</th>
<th>House 1 in Altrincham</th>
<th>House 2 in Heaton</th>
<th>House 3 in Mercy</th>
<th>House 4 in Didsbury</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
<td>{(G, 0.5), (E, 0.5)}</td>
<td>{(G, 0.5)}</td>
<td>{(A, 0.2), (G, 0.8)}</td>
<td>{(G, 0.2), (E, 0.8)}</td>
</tr>
<tr>
<td><strong>Distance (mile)</strong></td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>Asking Price (£)</strong></td>
<td>113,000</td>
<td>110,000</td>
<td>118,000</td>
<td>150,000</td>
</tr>
<tr>
<td><strong>Attractiveness</strong></td>
<td>{(P, 0.05), (G, 0.35), (E, 0.60)}</td>
<td>{(A, 0.4), (G, 0.6)}</td>
<td>{(G, 0.3), (E, 0.7)}</td>
<td>{(G, 0.6), (E, 0.4)}</td>
</tr>
</tbody>
</table>
Multiple Criteria Decision Analysis

Evidential Reasoning MCDA
Modelling structure and graphic interpretation

Use ER to generate overall belief
Combine evidence

Overall Criterion $y$

Grade $H_1$

Grade $H_n$

Grade $H_N$

Sub-Criterion $y_1(\omega_1)$

Sub-Criterion $y_i(\omega_i)$

Sub-Criterion $y_m(\omega_m)$
Evidential Reasoning Approach

Framework and algorithm

Step 1: Construct a belief decision matrix

Step 2: Weight assignment and normalised

Step 3: Convert belief to basic probability mass

Step 4: Combine basic probability mass

Step 5: Generate combined distribution assessment

Step 6: Utility function based alternative ranking
Directly assigning criterion weights

The house purchase example
Assigning weights by Comparisons

The house purchase example

DS Dialog: Assign Weights Using Pairwise Comparisons

For the following father attribute

House selection

Compare the relative importance of a selected child attribute with the other child attributes in the following pairwise fashion

Attribute Selected: Location

Attribute Compared to: Attractiveness

Times as important as: 5

Provided Pairwise Comparisons:

<table>
<thead>
<tr>
<th>Attribute Selected</th>
<th>times a.i.e.</th>
<th>Attribute Compared to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Location</td>
<td>2.500000</td>
<td>2. Distance to office</td>
</tr>
<tr>
<td>1. Location</td>
<td>2.500000</td>
<td>3. Price</td>
</tr>
<tr>
<td>1. Location</td>
<td>5.000000</td>
<td>4. Attractiveness</td>
</tr>
</tbody>
</table>

Generated Weights:

<table>
<thead>
<tr>
<th>Attribute Name</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Location</td>
<td>0.500000</td>
</tr>
<tr>
<td>2. Distance to office</td>
<td>0.200000</td>
</tr>
<tr>
<td>3. Price</td>
<td>0.200000</td>
</tr>
<tr>
<td>4. Attractiveness</td>
<td>0.100000</td>
</tr>
</tbody>
</table>

Inconsistency Index: 0

OK
Evidential Reasoning MCDA

The evidential reasoning algorithm

Generation of overall belief:

\( \beta_n \) can be generated by using the following nonlinear evidential reasoning algorithm:

\[
\beta_n = k \left[ \prod_{i=1}^{m} (\omega_i \beta_{i,n} + 1 - \omega_i) - \prod_{i=1}^{m} (1 - \omega_i) \right]
\]

\[
k = \left[ \sum_{n=1}^{N} \prod_{i=1}^{m} (\omega_i \beta_{i,n} + 1 - \omega_i) - N \prod_{i=1}^{m} (1 - \omega_i) \right]^{-1}
\]

\[
S = \{(H_n, \beta_n), \ n = 1, ..., 5\}
\]
An attribute is judgementally independent of other attributes if the assessment of the former does not depend on the assessment of the latter as long as they are fixed.

For example, for purchase of MP3 players, suppose only two attributes *price* and *sound quality* are taken into account. It is then commonly accepted that

1. For any fixed price, high sound quality MP3 is judged to be better
2. For any fixed sound quality, low price MP3 is judged to be better

So, the two attributes *price* and *sound quality* are mutually judgementally independent, though they may be correlated.
Buy house – IDS Main Interface

Assessment hierarchy and alternative houses

Multiple Criteria Decision Analysis
Assess a partial value function

**Direct assessment method**

The marginal value function of the price
Assess a partial value function

Bisection assessment method

The marginal value function of the distance to office
Example 2: Buy house
Assess value functions for other attributes
# Distributed Assessments of Four Houses

## House in Altrincham
- Very Poor: 9.41%
- Bad: 14.42%
- Average: 2.71%
- Good: 46.37%
- Excellent: 27.08%

## House in Heaton Moor
- Very Poor: 0.22%
- Bad: 15.88%
- Average: 13.50%
- Good: 70.40%
- Excellent: 0.00%

## House in Heaton Mersey
- Very Poor: 2.97%
- Bad: 19.92%
- Average: 12.39%
- Good: 48.70%
- Excellent: 16.02%

## House in East Didsbury
- Very Poor: 14.86%
- Bad: 23.77%
- Average: 9.64%
- Good: 20.24%
- Excellent: 31.49%
Rank Order of the Four Houses

**Ranking of Houses**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Altrincham</td>
<td>0.6682</td>
</tr>
<tr>
<td>2. Heaton Moor</td>
<td>0.6352</td>
</tr>
<tr>
<td>3. Heaton Mersey</td>
<td>0.6372</td>
</tr>
<tr>
<td>4. East Didsbury</td>
<td>0.5743</td>
</tr>
</tbody>
</table>

Multiple Criteria Decision Analysis
Sensitivity of the Ranking of Houses

**Average scores of alternatives on House selection**

- Weight of 1. Location
- Weight of 2. Distance to office
- Weight of 3. Price
- Weight of 5. Attractiveness

Multiple Criteria Decision Analysis
Main Topics of the Session

• Multiple criteria decision analysis – what is it?
• Multiple objective optimization problems in real world
• Multiple criteria assessment and decision analysis problems in real world
• Decision matrix and MCDA explained in graph
• Additive value function approach in MCDA
• Deal with uncertainties in MCDA
• Evidential reasoning MCDA – concept, model, process and tool
• A snapshot of real world MCDA applications
MCDA Applications in Real World

Example 3: Motorbike performance assessment hierarchy

<table>
<thead>
<tr>
<th>Alternative Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kawasaki</td>
</tr>
<tr>
<td>Yamaha</td>
</tr>
<tr>
<td>Honda</td>
</tr>
<tr>
<td>BMW</td>
</tr>
</tbody>
</table>

- Motorcycle selection
  - Price
  - Displacement
  - Range
  - Top speed
- Engine Performance
- Operation Quality
- General finish
  - Quality of finish
  - Seat comfort
  - Headlight
  - Mirrors
  - Horn

**Example 4: Organisational quality self-assessment**


<table>
<thead>
<tr>
<th>Alternative Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>UUUSD 2002</td>
</tr>
<tr>
<td>ND 2000</td>
</tr>
<tr>
<td>Corning 2000</td>
</tr>
<tr>
<td>Vertex 2001</td>
</tr>
<tr>
<td>NWW 2000</td>
</tr>
</tbody>
</table>

**EFQM Self-assessment**

- **Enablers**
  - 1 Leadership
  - 2 Policy and Strategy
  - 3 People
  - 4 Resources and Partnerships
  - 5 Processes
    - 5a Processes are systematically designed and managed
    - 5b Processes are improved
    - 5c Products and Services are designed and developed
    - 5d Products and Services are produced, delivered and serviced
    - 5e Customer relationships are managed and enhanced

- **Results**
  - 6 Customer Results
  - 7 People Results
  - 8 Society Results
  - 9 Key Performance Results
MCDA Applications in Real World

Example 5: Performance assessment for SME

Example 6: Company innovation capability assessment

Example 7: R&D project performance assessment

MCDA Applications in Real World

Example 8: Customer satisfaction survey & assessment

- Section 1: Service
  1. What is your perception of the service you receive from Silcos?
  2. How satisfied are you with the level of service provided by Silcos personnel?
  3. Is the response to manufacturing problems or quality issues to your company?
  4. How accessible are Silcos personnel to your company?
  5. Is the communication flow between yourselves and Silcos personnel?
  6. Is the flexibility of Silcos personnel to your production demands?
  7. Is the advice and support you receive from Silcos personnel?
  8. At the start of any new product, how proactive are Silcos personnel?
  9. Is the standard of technical documentation provided by Silcos?

- Section 2: Quality

- Section 3: Cost/Sales
  1. Do you believe you receive value for money for Silcos products?
  2. Is the response to your initial enquiry?
  3. Is the costing information provided by Silcos?
  4. Is the response time for providing costing information?
  5. Do Silcos meet your costing targets?
  6. What is your opinion of the sales service provided by Silcos?
  7. What is your opinion of the after sales service provided by Silcos?

- Section 4: Delivery

For Help, press F1
MCDA Applications in Real World

Example 9: Selection of construction contractors

Example 10: Company supplier selection

Joanna Teng "Development of a supplier prequalification model for Siemens UK", MSc Dissertation, Manchester School of Management, UMIST, 2002
Example 11: Environmental impact assessment