# Some results on more flexible versions of Graph Motif

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# Outline

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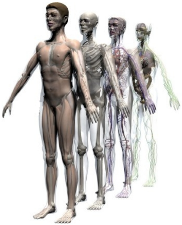
Introduction

Graph Motif : Querying motifs without topology

Approximate motifs

# Motivations



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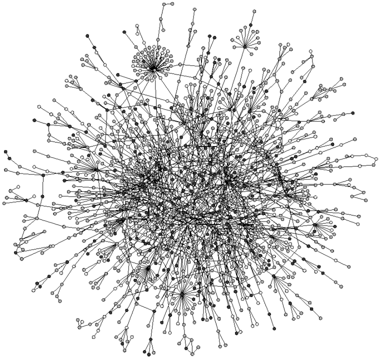
- Human complexity $\not\Leftrightarrow$ # of genes ?

# Proteins network

- Proteins can (physically) interact with other proteins (PPI).
- Biologically obtained... with noise !



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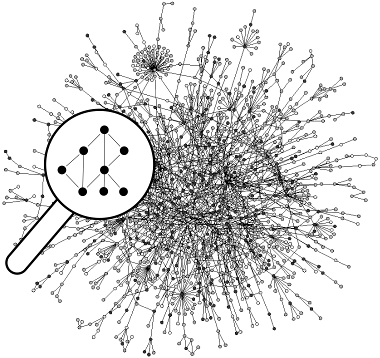

- Use a **graph** representation:
  - Proteins are nodes.
  - Interactions are edges.

# Issues

- New techniques: increase the available data [Sharan et Ideker 2006].
  - 2001: some hundreds.
  - 2006: thousands.
- Lot of databases:
  - BIND,
  - DIP,
  - KEGG,
  - MINT,
  - ...

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- Lot of databases:
  - BIND,
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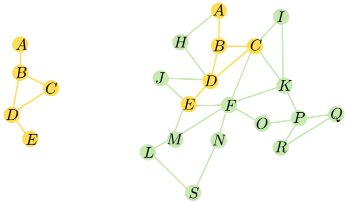
  - MINT,
  - ...
- Computational solutions are needed.

# Motif search

- Goal: find a subnetwork with both labels and topology of a given motif.



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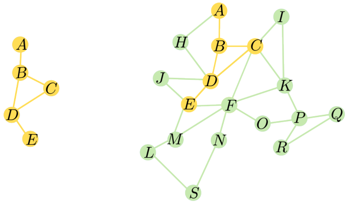


- Look for motifs to retrieve some known functions.
- **Deduce** information from well known species to less known species.

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  - Missing informations (false negatives). About 50%.
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- Topology of the motif can be unknown *a priori*.
- Different functions can have a same topology.

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- Different functions can have a same topology.
- Topology can be irrelevant.

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- Topology is only the connectivity of the solution.

# Graph Motif – A toy example

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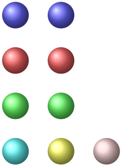

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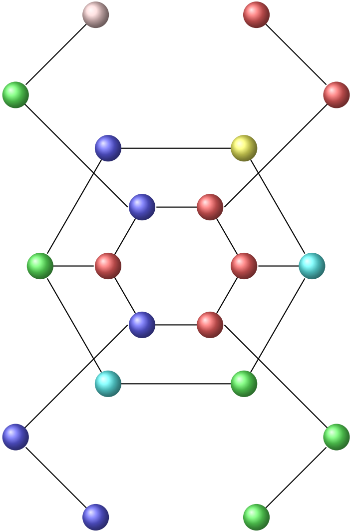

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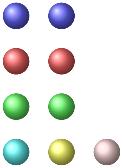

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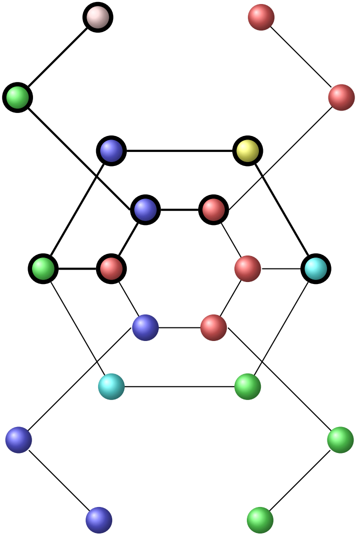

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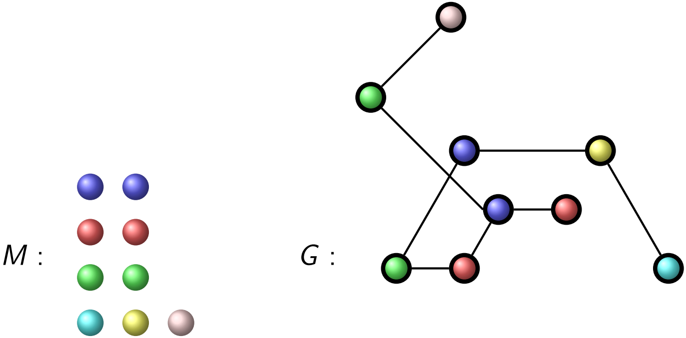

# Graph Motif

- Applied to different type of biological networks.
  - Initially for metabolic networks [Lacroix et al. 2006].
  - Useful for PPI networks [Bruckner et al. 2009].
- But also for social networks [Betzler et al. 2008, S. 2011].

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- Must cope with hardness:
  - Some FPT algorithms.
    - $k$ is the size of the solution.
    - $\mathcal{O}^*(2^k)$ for colorful motifs,
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  - Approximation.

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**Approximate motifs**

# Variants

- Experimental data, **noisy**.
- Ask for an exact occurrence is likely to fail.

- Must allow insertions, deletions...: optimisation problems!

# A variant for Graph Motif : Minimum Substitutions

- A variant: MINIMUM SUBSTITUTIONS [DONDI ET AL. 2011].
- Find an occurrence with a maximum number of colors from the motif, but with the same size.
- **Substitution** of motif colors by new colors.

# Minimum Substitutions: Toy example

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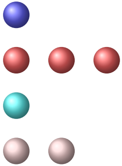

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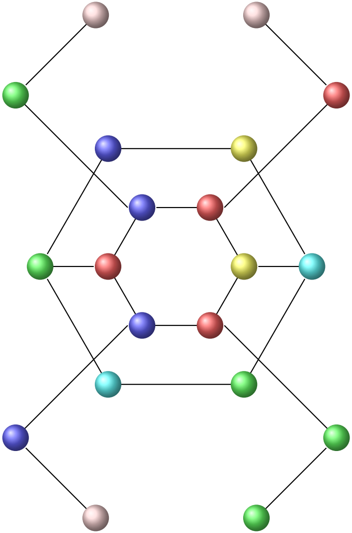

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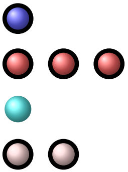

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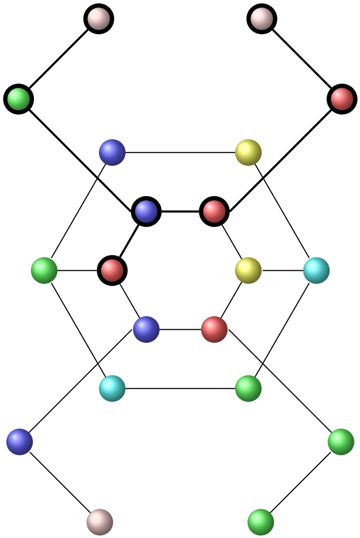

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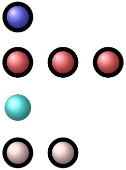

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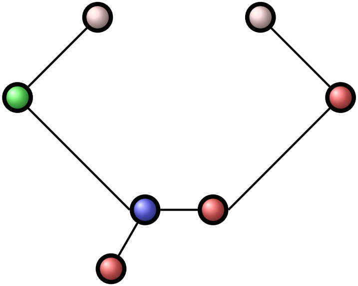

# A variant for Graph Motif : Minimum Substitutions

- NP-hard even if $G$ is a tree of maximum degree 4 where each color occurs at most twice but FPT [Dondi et al. 2011].
- Result: there is no approximation ratio within $c \log |V|$, unless $\mathrm{P} = \mathrm{NP}$, even if the motif is colorful (at most one occurrence of each color) and $G$ is a depth 2 tree.
- Reduction from Set Cover.

# Construction

$$X = \{x_1, x_2, x_3\}, \mathcal{S} = \{\{x_1, x_2\}, \{x_2, x_3\}, \{x_2\}\}$$

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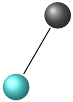

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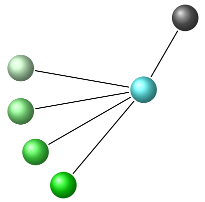

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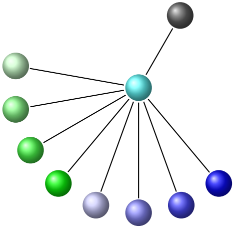

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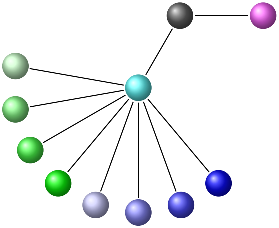

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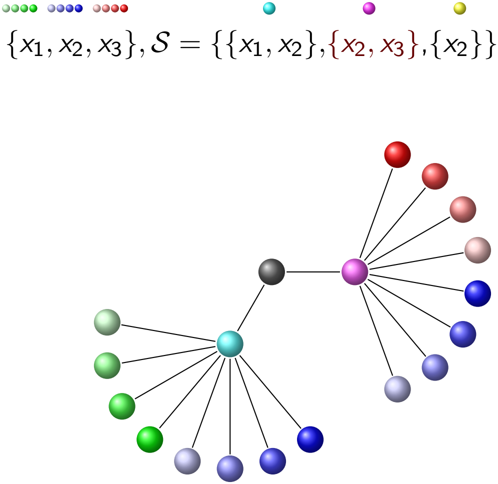

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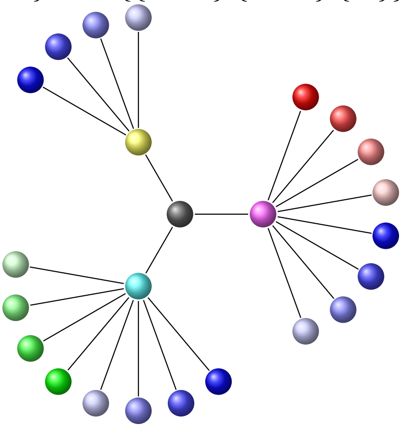

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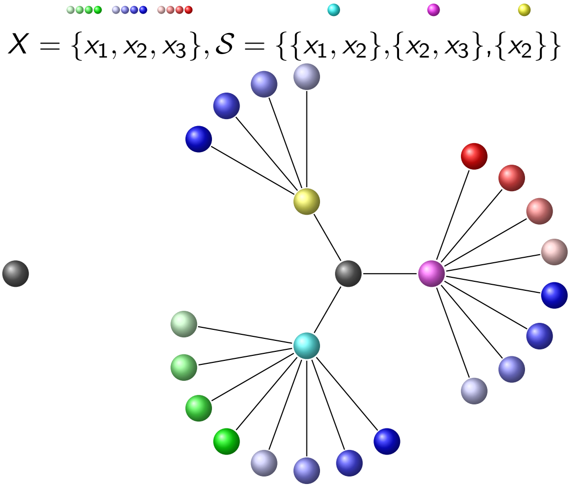

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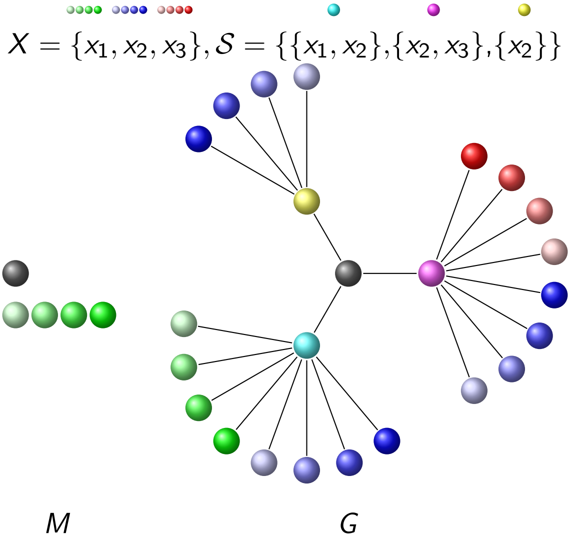

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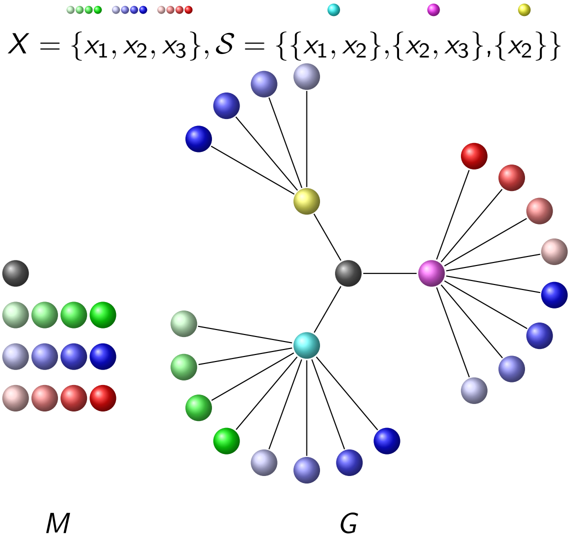

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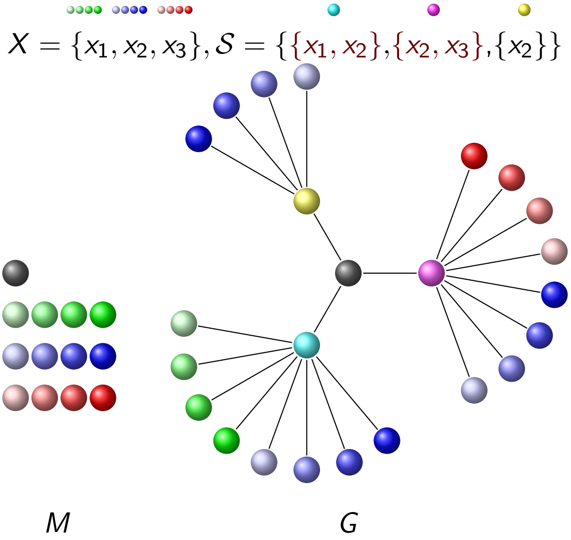


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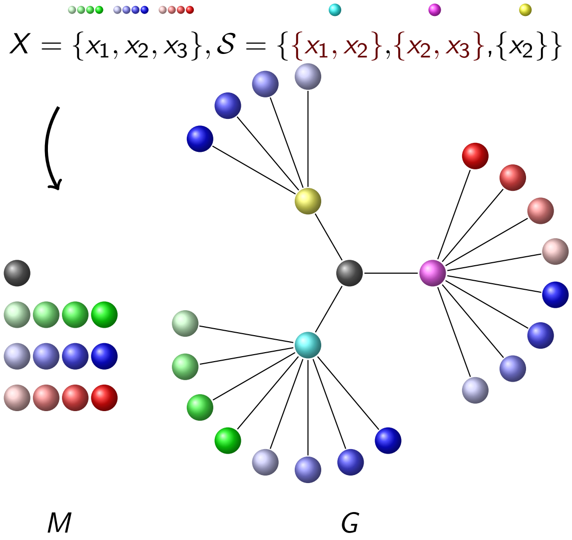


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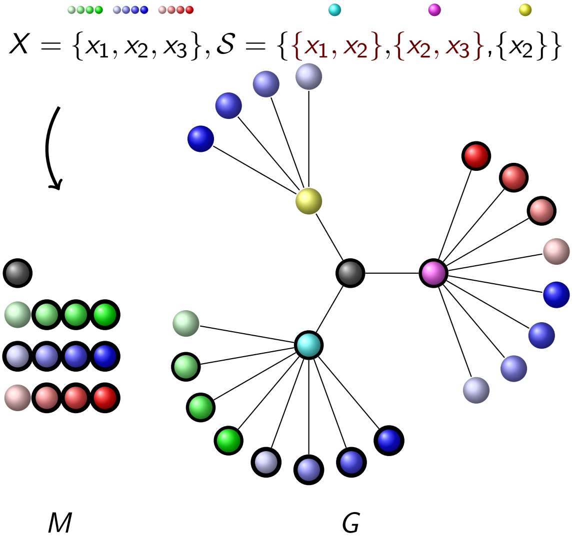


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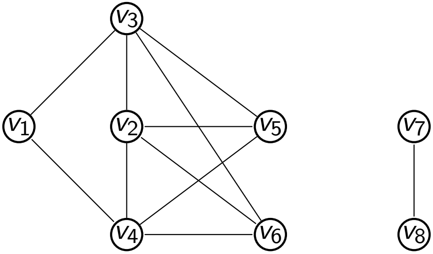
- There is no approximation ratio within $c \log |V|$ for MINIMUM SUBSTITUTIONS (unless P $=$ NP).

# Using graph modules

- Introduction of the (well-known) notion of **graph modules** into Graph Motif .



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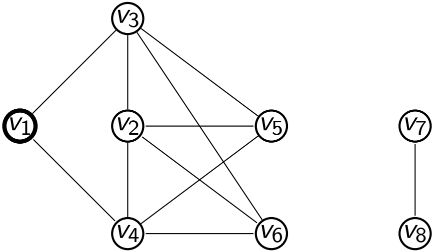

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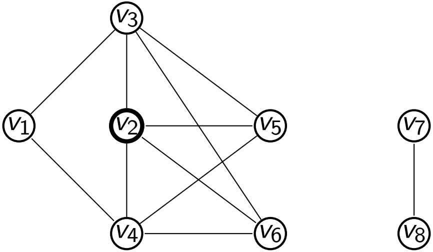

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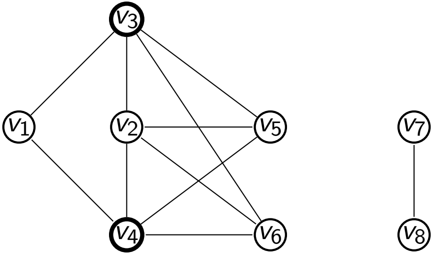

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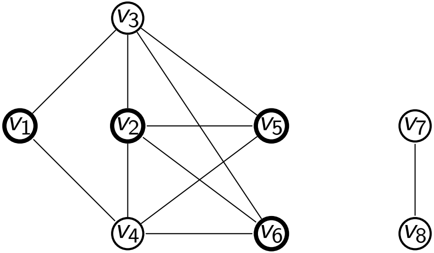

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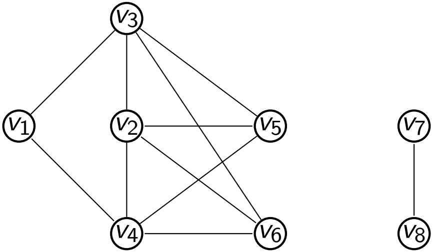


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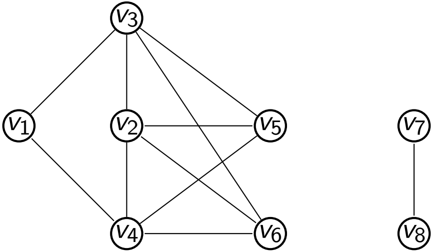
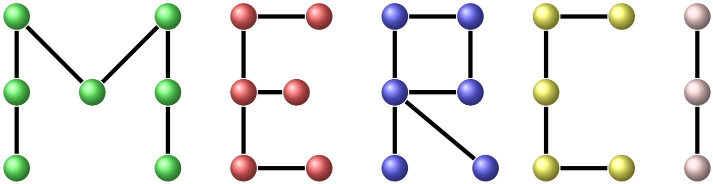



- NP-complete
- Exponential number of modules...
- ... but "generators" can be store in a linear tree.
- Using this to have FPT algorithms.

# Other result in the paper

- MAX MOTIF is hard to approximate.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