Parameterized Approximability of Target Set Selection

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Outline

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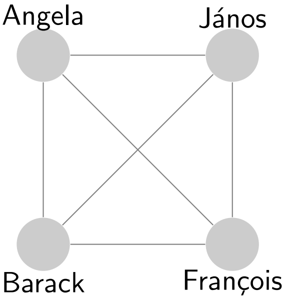
An Example from Viral Marketing



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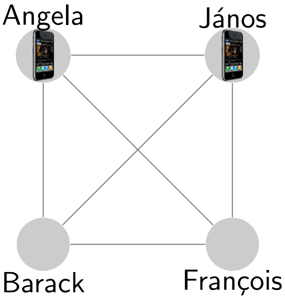

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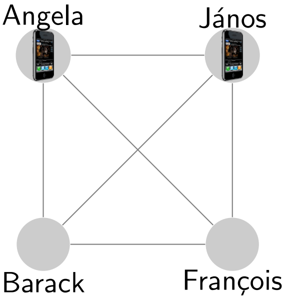

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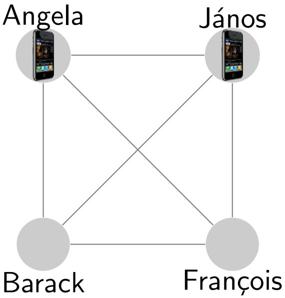

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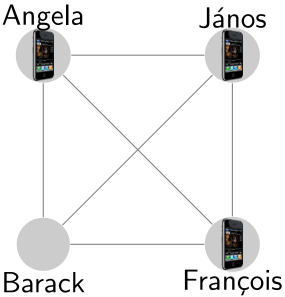

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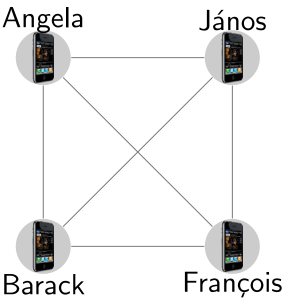

- Barack: “If **three** of my **friends** have an iPhone, I buy an iPhone too”
- François: “If **two** of my **friends** have an iPhone, I buy an iPhone too”

- Apple sold two iPhones without any advertisement!

An Example from Viral Marketing

- Goal: get the **fewest customers** with advertisement in order to **attract all customers** at the end.

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- Goal: get the **fewest customers** with advertisement in order to **attract all customers** at the end.
- Other applications:
  - Spreading of information/influence in social networks via word-of-mouth recommendations.
  - Diseases in populations.
  - Faults in distributed computing.
  - ...

Problem Definition

- **Diffusion** (threshold model):
  - A vertex of the graph is **activated** if it is in the **target set** or if at least $thr(v)$ of its **neighbors** are activated.

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- **Optimization** problem [Chen 2008]:

Min Target Set Selection:

- **Input:** A graph, a threshold for each vertex.
- **Output:** A subset of vertices of **minimum cardinality** s.t. all vertices of the graph are activated at the end of the diffusion process.

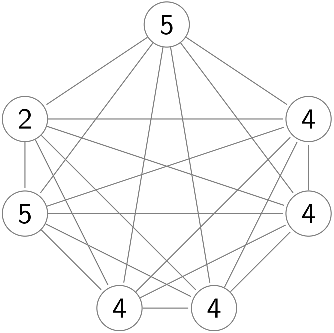
Problem Definition

- **Diffusion** (threshold model):
  - A vertex of the graph is **activated** if it is in the **target set** or if at least $thr(v)$ of its **neighbors** are activated.
- **Decision** problem [Chen 2008]:

Target Set Selection:

- **Input:** A graph, a threshold for each vertex, **an integer k**.
- **Output:** A subset of vertices of **size at most k** s.t. all vertices of the graph are activated at the end of the diffusion process.

Activation Process - Toy Example (K_7)



Numbers in vertex = threshold of the vertex

$k = 3.$

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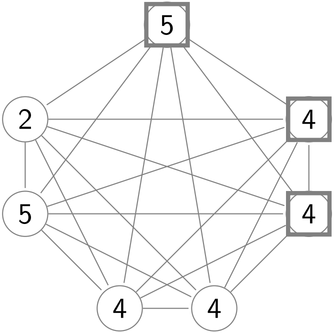

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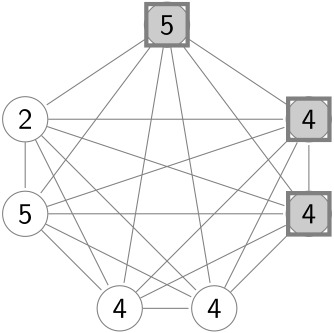

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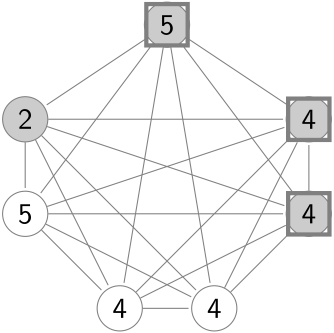

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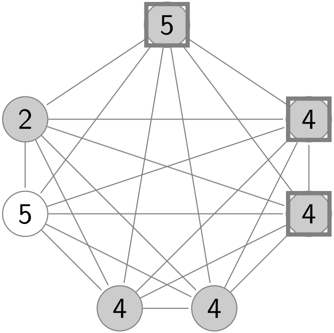

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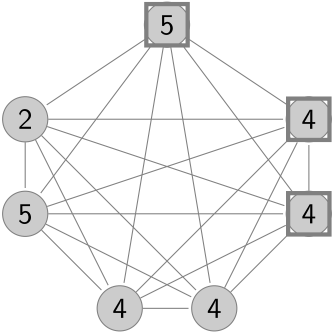

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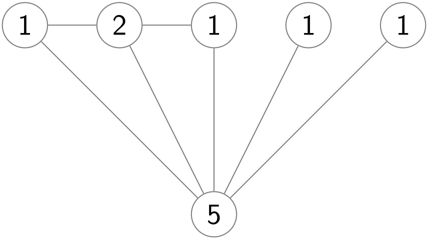
Types of thresholds

- General. $(1 \leqslant thr(v) \leqslant deg(v))$



Types of thresholds

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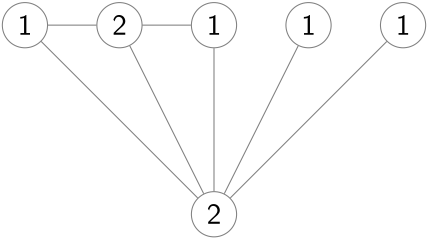
- Constant. $(1 \leqslant thr(v) \leqslant c)$



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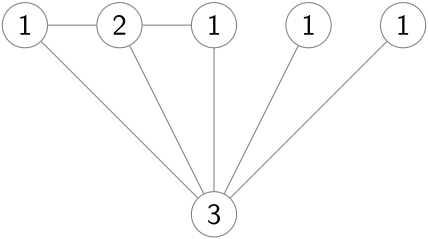

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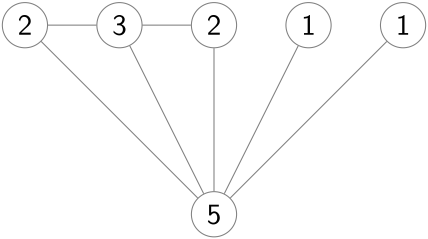
- Majority. $(thr(v) = \lceil deg(v)/2 \rceil)$.
- Unanimity. $(thr(v) = deg(v))$.



Outline

Introduction

Parameterized Approximation

Hardness

Fixed-Parameter Tractability

"*Measuring complexity only in terms of the input size means ignoring any structural information about the instances*"

J. Flum and M. Grohe

"*Question : When will the input of a problem coming from "real life" have no more structure than its size?*
Answer : Never!"

R. Downey and M. Fellows

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Answer : Never!"

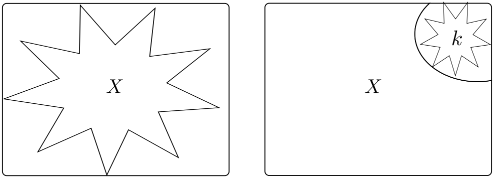
R. Downey and M. Fellows

"*The fundamental idea is to restrict the combinatorial explosion, seemingly unavoidable, that causes the exponential growth in the running time of certain problem-specific parameters...*"

R. Niedermeier

Fixed-Parameter Tractability

- Problem in FPT: any instance (I, k) solved in $f(k) \cdot |I|^c$.



- Examples:
  - Solution size k in a n-vertices graph.
  - n voters for k candidates.
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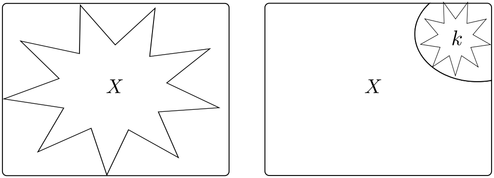

- Examples:
  - Solution size k in a n-vertices graph.
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  - Requests of size k in a n-sized database.
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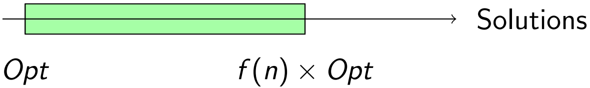
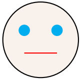
- Complexity classes: $\text{FPT} \subseteq \overbrace{\text{W[1]} \subseteq \text{W[2]} \subseteq \ldots}^{\text{presumably} \not\subset \text{FPT}}$

Polynomial Approximation (minimization)



Coping with the hardness

	Time	Solution Quality
FPT		
Poly. Approx.		

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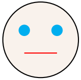
FPT		
Poly. Approx.		
FPT Approx.		

FPT-Approximation

- A (minimization) problem is **fpt-ρ-approximable** if for any input (I, k):
  - If $opt(I) \leqslant k$, computes a solution of value bounded by $\rho(k) \cdot k$ in time $f(k)|I|^{O(1)}$,
  - Otherwise, output can be arbitrary .

Example for treewidth

	Time	Ratio	
FPT	$2^{O(k^2)} \cdot n$	1	[Bodlaender 96]
Poly. Approx.	$poly(n)$	$O(k\sqrt{\log k})$	[Feige et al. 05]

Example for treewidth

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FPT	$2^{O(k^2)} \cdot n$	1	[Bodlaender 96]
Poly. Approx.	$poly(n)$	$O(k\sqrt{\log k})$	[Feige et al. 05]
FPT Approx.	$2^{O(k)} \cdot n$	5	[Bodlaender et al. 13]

FPT-inapproximability

- Not fpt-ρ-approximation for any function ρ:
  - INDEPENDENT DOMINATING SET [DOWNEY ET AL. 2008]
  - WEIGHTED CIRCUIT SAT [CHEN ET AL. 2006]

FPT-inapproximability

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- Not-monotone problems...
  - If optimum is k and every feasible solution has cost k $\rightarrow$ as hard as decision.

FPT-inapproximability - Monotone problems

- Clique or Dominating Set remain open and challenging.

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- Clique or Dominating Set remain open and challenging.
- Monotone Weighted Circuit Sat (no negation) is not fpt-approximable. [Marx 2013]

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Parameterized Approximation

Hardness

Known results for Target Set Selection

- **Hard to approximate** in poly-time within $O(2^{\log^{1-\epsilon} n})$ [Chen 2009].
- W[2]-**hard** for parameter solution size [Nichterlein et al. 2012].

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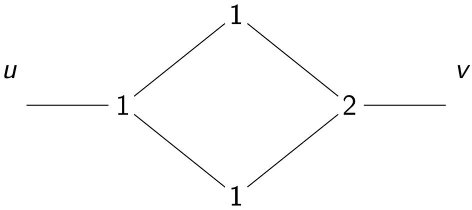
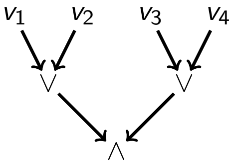
- **Hard to approximate** in poly-time within $O(2^{\log^{1-\epsilon} n})$ [Chen 2009].
- W[2]-**hard** for parameter solution size [Nichterlein et al. 2012].
- TSS is hard to approximate and W[2]-hard:
  - Can we have better approximation ratio if we allow fpt-time?

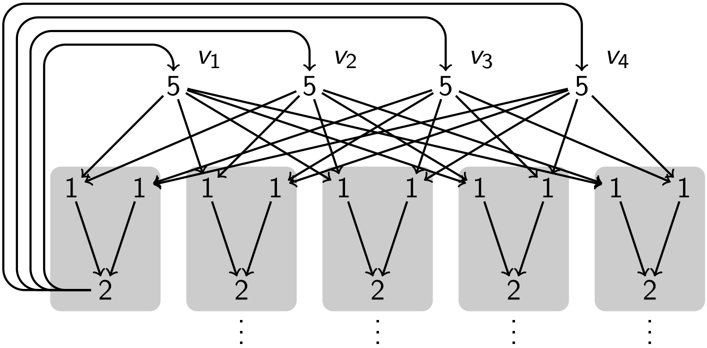
Directed edge gadget



General idea - Bipartite graphs

- From Monotone Weighted Circuit Sat.



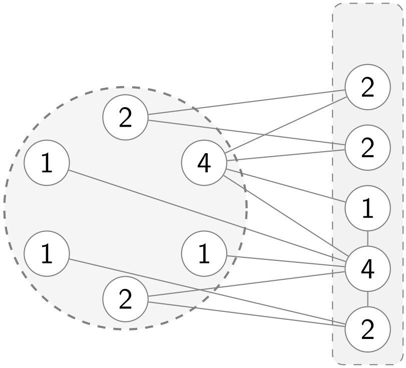


Results

- Reduction with one-to-one correspondence between solutions.
- As Monotone Circuit Sat, Target Set Selection is not fpt-ρ-approximable, for any function ρ.
- Can be extended to **majority** and **constant** threshold via additional gadgets.

Unanimity Thresholds

- All the neighbors of a vertex must be activated.
  - Only one round!



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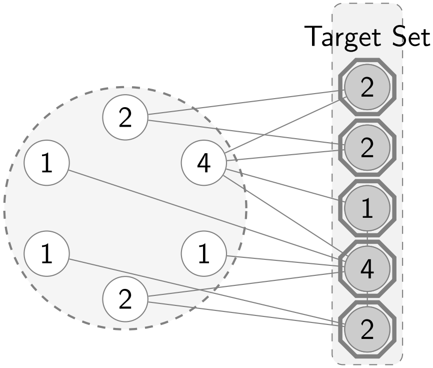

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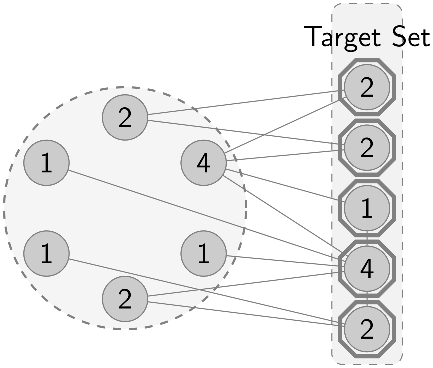

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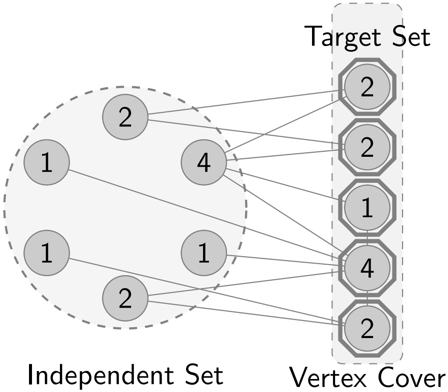

- Equivalent to Vertex Cover: in FPT.

Conclusion

- Hard hard hard hard.

Conclusion

- Hard hard hard hard.
- Dual of the problem?
  - Unanimity thresholds:
    - Equivalent to INDEPENDENT SET: W[1]-hard.
  - Majority or constant thresholds?
  - fpt-approximation?

Köszönöm!