Finding Disjoint Paths on Edge-Colored Graphs: A Multivariate Complexity Analysis

Riccardo Dondi[1] Florian Sikora[2]

[1]Universita degli Studi di Bergamo – Italy
[2]LAMSADE, Université Paris Dauphine, CNRS – France

COCOA 2016

Finding Disjoint Paths on Edge-Colored Graphs: A Multivariate Complexity Analysis

Riccardo Dondi[1] Florian Sikora[2]

[1]Universita degli Studi di Bergamo – Italy
[2]LAMSADE, Université Paris Dauphine, CNRS – France

COCOA 2016

Outline

Introduction

FPT Vertex Cover

Parameterized Inapproximability

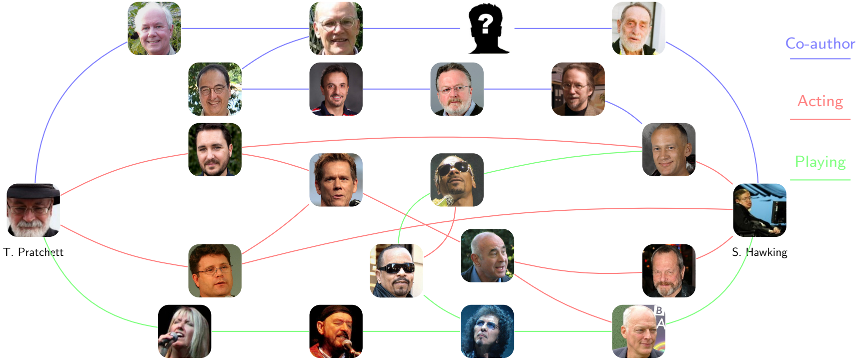
Motivations

- Originates from **Social Network Analysis**.
- Computing the **connectivity between 2 nodes** is an important problem.
  - measurement of information flow,
  - cohesion group and centrality.

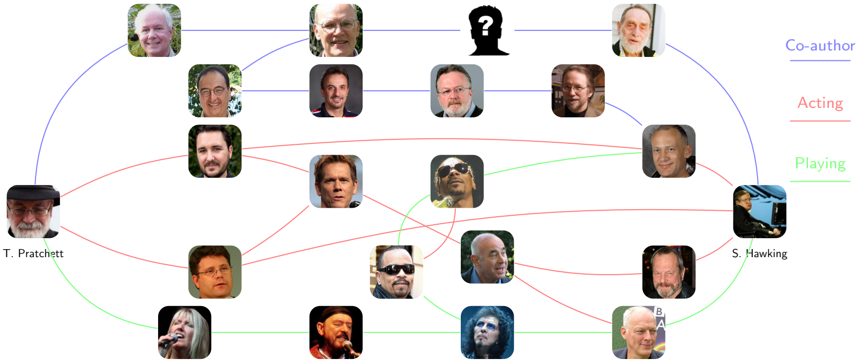
Motivations

- Originates from **Social Network Analysis**.
- Computing the **connectivity between 2 nodes** is an important problem.
  - measurement of information flow,
  - cohesion group and centrality.
- Different kind of relationship:
  - **Different colors on the edges**.
  - Integration of different type of information.
    - Different media.
    - Different protocol.
    - ...

MaxCDP

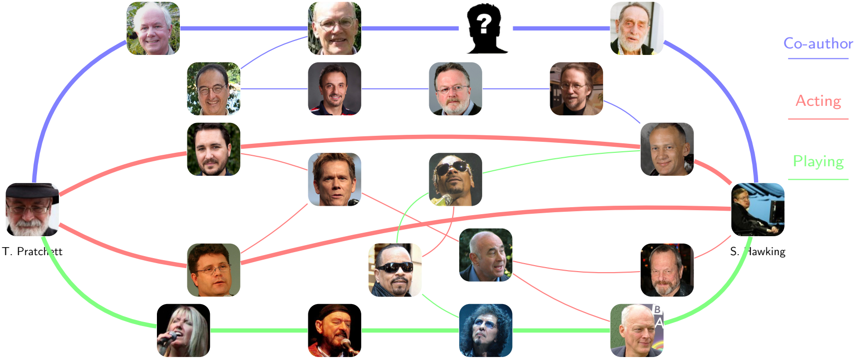


MaxCDP



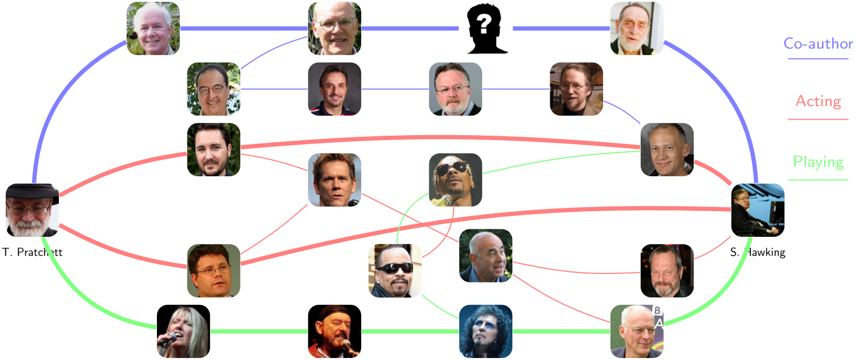
- Max nb **monochromatic disjoint paths** between 2 nodes.

MaxCDP



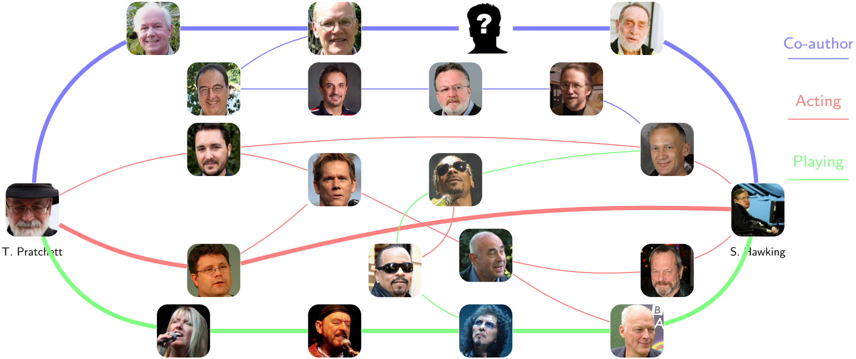
- Max nb **monochromatic disjoint paths** between 2 nodes.

MaxCDP



- Max nb **monochromatic disjoint paths** between 2 nodes.
  - Monochromatic: Info. spread among relation of the same kind.
  - Number: More connected.
  - Length: short paths are considered more significant.
  - Vertex disjoint: security, traffic congestion...

MaxCDDP



- Max nb **monochromatic disjoint paths** between 2 nodes.
  - Monochromatic: Info. spread among relation of the same kind.
  - Number: More connected.
  - Length: short paths are considered more significant.
  - Vertex disjoint: security, traffic congestion...
- Introduce vertex disjoint **and** color-disjoint version.
  - How different relations in a network connects 2 vertices

MaxCDP: Known results

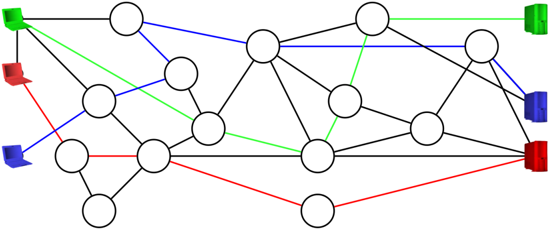
- Not approximable within with $c^{1-\varepsilon}$ [Dondi et al. 13],
  - but c-approximable [Wu 12].
- W[1]-hard w.r.t. number of paths [Dondi et al. 13].
  - Even not in XP (NP-C for 2 paths) [Gourves et al. 12].

MaxCDP: Known results

- Not approximable within with $c^{1-\varepsilon}$ [Dondi et al. 13],
  - but c-approximable [Wu 12].
- W[1]-hard w.r.t. number of paths [Dondi et al. 13].
  - Even not in XP (NP-C for 2 paths) [Gourves et al. 12].
- When the length of the paths are bounded by ℓ:
  - Polynomial if $\ell < 4$, NP-C otherwise [Wu 12].
  - FPT w.r.t. number of paths $+ \ell$ [Dondi et al. 13].
    - But no polynomial kernel [Golovach Thilikos 11]

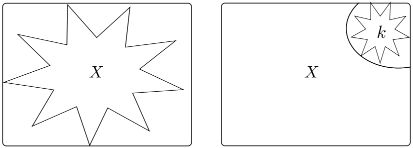
Related problems

- 1 color:
  - 1 source-target: Polynomial (flow).
  - k sources-targets: NP-C but FPT for k. [Robertson Seymour]



Fixed-Parameter Tractability

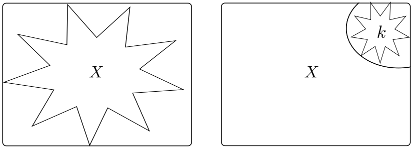
- Problem in FPT: any instance (I, k) solved in $f(k) \cdot |I|^c$.



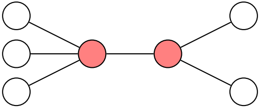
- Examples:
  - Solution of size k in a n-vertices graph.
  - n voters for k candidates.
  - Requests of size k in a n-sized database.
  - ...

Fixed-Parameter Tractability

- Problem in FPT: any instance (I, k) solved in $f(k) \cdot |I|^c$.

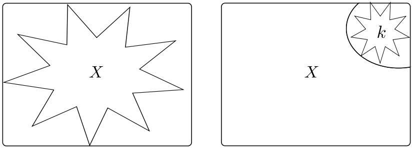


- Examples:
  - Solution of size k in a n-vertices graph.
  - n voters for k candidates.
  - Requests of size k in a n-sized database.
  - ...
- Many way to parameterize.
  - Solution size.



Fixed-Parameter Tractability

- Problem in FPT: any instance (I, k) solved in $f(k) \cdot |I|^c$.



- Examples:
  - Solution of size k in a n-vertices graph.
  - n voters for k candidates.
  - Requests of size k in a n-sized database.
  - ...

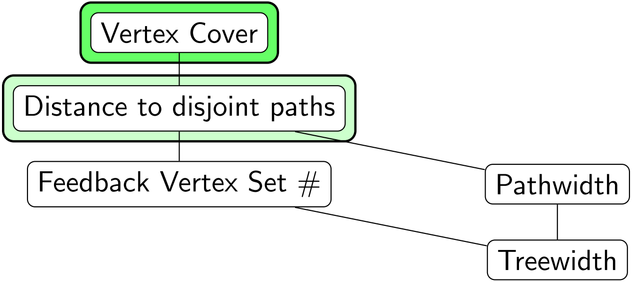
- Many way to parameterize.
  - Solution size.
  - Structure of the input.
  - ...

Structural results MaxCDP

- Real-data is **not random** (e.g. small world phenomenon).
- Information on **the structure**.
- Use it in parameterized complexity.

Structural results MaxCDP

- Real-data is **not random** (e.g. small world phenomenon).
- Information on **the structure**.
- Use it in parameterized complexity.



Outline

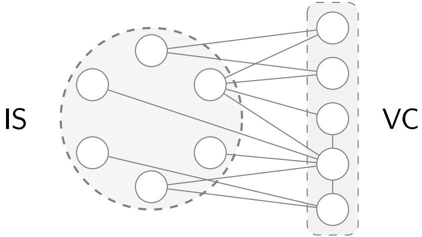
Introduction

FPT Vertex Cover

Parameterized Inapproximability

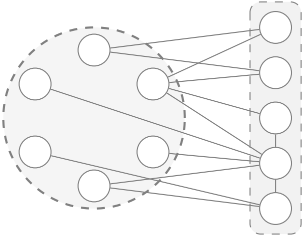
MaxCDP w.r.t. Vertex Cover number

- Aim: $f(\tau)n^{O(1)}$ exact algorithm.
- τ computed in FPT time.



MaxCDP w.r.t. Vertex Cover number

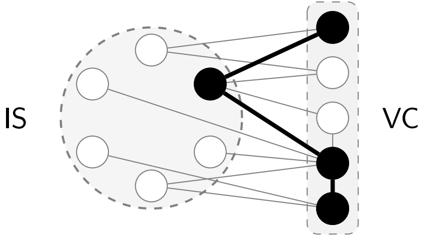
- Aim: $f(\tau)n^{O(1)}$ exact algorithm.
- τ computed in FPT time.



- For all paths (s, v, t) (length 3): remove v.
  - Only one path can use v in an optimal solution.

MaxCDP w.r.t. Vertex Cover number

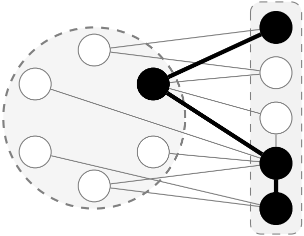
- Aim: $f(\tau)n^{O(1)}$ exact algorithm.
- τ computed in FPT time.



- For all paths (s, v, t) (length 3): remove v.
  - Only one path can use v in an optimal solution.
- In any $s - t$ path (length > 3) of the solution:
  - A vertex in IS is adjacent to one in VC or s or t.

MaxCDP w.r.t. Vertex Cover number

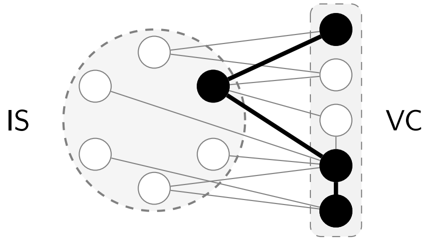
- Aim: $f(\tau)n^{O(1)}$ exact algorithm.
- τ computed in FPT time.



- For all paths (s, v, t) (length 3): remove v.
  - Only one path can use v in an optimal solution.
- In any $s - t$ path (length > 3) of the solution:
  - A vertex in IS is adjacent to one in VC or s or t.
    - Paths are of length at most 2τ.
    - At most τ different paths.

MaxCDP w.r.t. Vertex Cover number

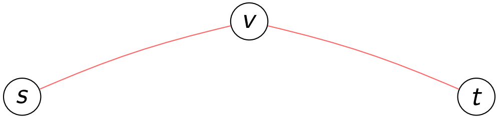
- Aim: $f(\tau)n^{O(1)}$ exact algorithm.
- τ computed in FPT time.



- For all paths (s, v, t) (length 3): remove v.
  - Only one path can use v in an optimal solution.
- In any $s - t$ path (length > 3) of the solution:
  - A vertex in IS is adjacent to one in VC or s or t.
    - Paths are of length at most 2τ.
    - At most τ different paths.
    - Known FPT [Bonizzoni et al. 13].

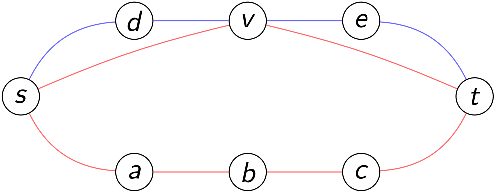
MaxCDDP and Vertex Cover

- For MaxC**D**DP: **cannot remove length 3 paths**.



MaxCDDP and Vertex Cover

- For MaxC**D**DP: **cannot remove length 3 paths**.



- $(s, d, v, e, t) \cup (s, a, b, c, t)$ better than (s, v, t).

Outline

Introduction

FPT Vertex Cover

Parameterized Inapproximability

Coping with the hardness

	Time	Solution Quality
FPT		
Poly. Approx.		

Coping with the hardness

	Time	Solution Quality
FPT		
Poly. Approx.		
FPT Approx.		

FPT-Approximation

- A (minimization) problem is **fpt-ρ-approximable** if for any input (I, k):
  - If $opt(I) \leqslant k$, computes a solution of value bounded by $\rho(k) \cdot k$ in time $f(k)|I|^{O(1)}$,
  - Otherwise, output can be arbitrary .

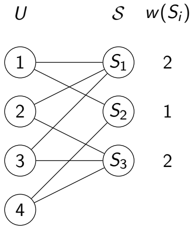
Example: computing treewidth

	Time	Ratio	
FPT	$2^{O(k^2)} \cdot n$ 	1 	[Bodlaender 96]
Poly. Approx.	$poly(n)$ 	$O(k\sqrt{\log k})$ 	[Feige et al. 05]

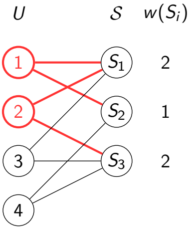
Example: computing treewidth

	Time	Ratio	
FPT	$2^{O(k^2)} \cdot n$ 	1 	[Bodlaender 96]
Poly. Approx.	$poly(n)$ 	$O(k\sqrt{\log k})$ 	[Feige et al. 05]
FPT Approx.	$2^{O(k)} \cdot n$ 	5 	[Bodlaender et al. 13]

Threshold Set

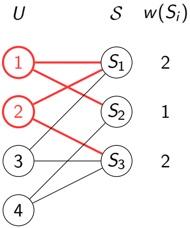


Threshold Set



A maximum solution: $T = \{1, 2\} \subseteq U$

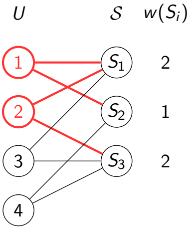
Threshold Set



A maximum solution: $T = \{1, 2\} \subseteq U$

- INDEPENDENT SET when $U = V$, $\mathcal{S} = E$, weights all 1.

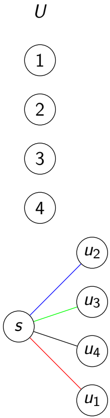
Threshold Set



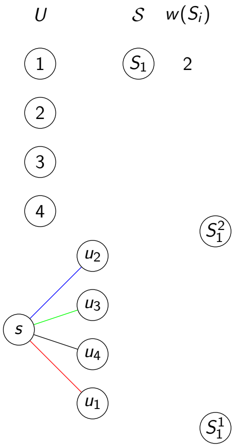
A maximum solution: $T = \{1, 2\} \subseteq U$

- Independent Set when $U = V$, $\mathcal{S} = E$, weights all 1.
- No fpt cost ρ-approximation, for any ρ function (unless FPT=W[1]) [Marx 2013].

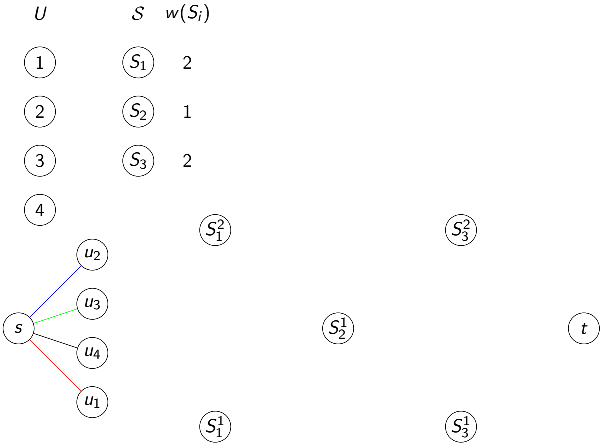
Reduction from Threshold Set



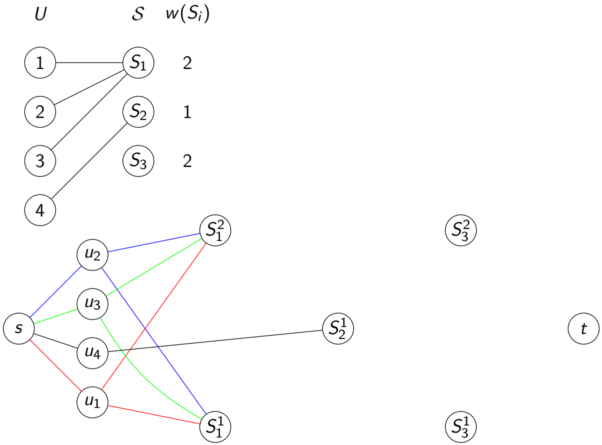
Reduction from Threshold Set



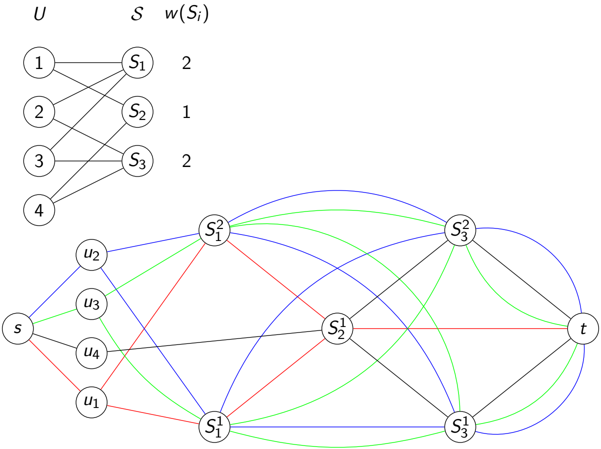
Reduction from Threshold Set



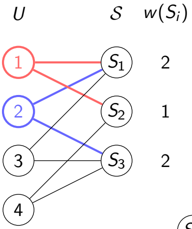
Reduction from Threshold Set



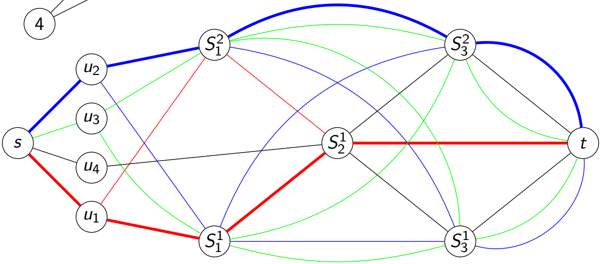
Reduction from Threshold Set



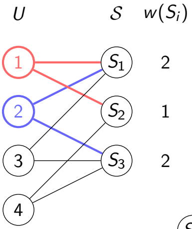
Reduction from Threshold Set



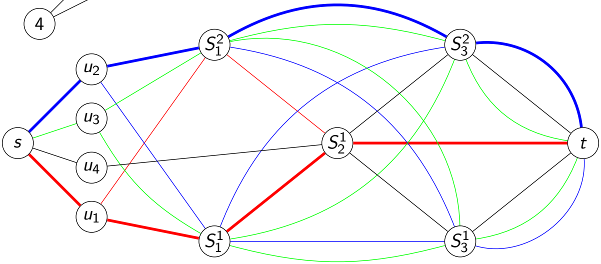
A maximum solution: $T = \{1, 2\} \subseteq U$



Reduction from Threshold Set



A maximum solution: $T = \{1, 2\} \subseteq U$



- Reduction with one-to-one correspondence between solutions.
  - MaxCDP (and MaxCDDP du to the s_i) are not fpt-ρ-approximable, for any function ρ (unless FPT=W[1]).

Open questions

- Complexity on **special class** of graphs? (planar + 2 colors ?)
- Parameterized complexity w.r.t. **feedback vertex set** ? (XP ? FPT ?)
- Fine grained complexity lower bounds?

谢谢