Humanitarian Logistic

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Course 1

First part

- Mathematical modelling: Linear programming (LP)
- Examples e.g. storage problem
- Excel optimisation solver

Second part

- Graph theory introduction
Course 2

Shortest path problem

- Mathematical modelling

- Graph algorithms:
  - Bellman algorithm (no cycle)
  - Dijkstra algorithm (positive weights)

- Examples
Course 3

Maximum flow problem

- Mathematical modelling

- Graph algorithms:
  - Ford & Fulkerson algorithm

- Examples
Facility location

Examples: Warehouse location (preparedness), temporary distribution center location (response), …

- Mathematical modelling
- Graph algorithms

Tasks scheduling - Planning

- Mathematical modelling
- Graph approaches: Bellman algorithm, GANTT diagram
Facility location

– Network planning –
Facility location

Data:
- $n$ locations
- $d_{ij}$: distance between locations $i$ and $j$
- $q$: number of distribution centers to open

Problem:
- Where to open the $q$ distribution centers?
- Determine location clusters
  Cluster: group of locations served by the same distribution center
- **Objective**: Minimise the total distance
MATHEMATICAL MODELLING

\[ n = 4 \text{ and } q = 2 \]

Decision variables
MATHEMATICAL MODELLING

\( n = 4 \) and \( q = 2 \)

Decision variables

For \( j = 1, \ldots, n \):

\[
y_j = \begin{cases} 
1 & \text{if a distribution center is open at location } j \\
0 & \text{otherwise}
\end{cases}
\]
Mathematical modelling

\( n = 4 \) and \( q = 2 \)

Decision variables

For \( i, j = 1, \ldots, n \):

\[
x_{ij} = \begin{cases} 
1 & \text{if } i \text{ is served by the opened distribution center } j \\
0 & \text{otherwise}
\end{cases}
\]
Mathematical modelling

$n = 4$ and $q = 2$

Constraints

- $q$ distribution centers to open:

$$
\sum_{j=1}^{n} y_j = q
$$
Mathematical modelling

\(n = 4\) and \(q = 2\)

Constraints

- Each location is served by one opened distribution centers
For \(i = 1, \ldots, n\):

\[
\sum_{j=1}^{n} x_{ij} = 1
\]
**Mathematical modelling**

$n = 4$ and $q = 2$

**Constraints**

- A location cannot be served by a non opened distribution centers

For $i, j = 1, \ldots, n$: \[ x_{ij} \leq y_j \]
Mathematical modelling

\( n = 4 \) and \( q = 2 \)

 Objective function

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}
\]
Mathematical modelling

$n = 4$ and $q = 2$

Optimal solution: total distance = 15
Mathematical modelling

\[
\begin{align*}
\text{min} \quad & \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \ x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} y_{j} = q \\
& \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, \ldots, n \\
& x_{ij} \leq y_{j} \quad i, j = 1, \ldots, n \\
& y_{j} \in \{0, 1\} \quad j = 1, \ldots, n \\
& x_{ij} \in \{0, 1\} \quad i, j = 1, \ldots, n
\end{align*}
\]
Graph approaches

- Facility location problem is hard to solve (NP-hard problem)

- Approached solution methods
  - Local search
    - Algorithm
      
      0. Choose initially any $q$ nodes (chose randomly one node, and consider the farthest node from it, and so on until selecting $q$ nodes)

      1. Swap. While there exists a swap between a current facility location and another node which improves the current objective function, execute the swap

- Approximation algorithms

- Other problems
  - Maximal covering location problem: Explosive attacks
Tasks scheduling
Tasks scheduling

Aim: Planning a project

Examples:
- Building - realisation of a warehouse
- Organization of a product in a workshop on machines
- Transport, distribution & delivery of goods

Implementing a project is decomposable into $n$ tasks $1, 2, \ldots, n$

Problem

How to best order/schedule theses tasks in order to implement the project?
Tasks scheduling

A task $i$ is perfectly described by

- Its duration (processing time) $d_i$
- Earliest start time (in French: date de début au plus tôt) $t_i$ (or $\lambda_i$)
- Latest finish time (in French: date de fin au plus tard) $t'_i$ (or $\lambda'_i$)
- The means necessary for its implementation of human, financial, material, … type

Tasks are subjects to constraints:

- Technological: a task can starts only if other tasks are achieved
- Commercial: Some tasks need to be finished before a fixed delay
- Material: a machine can treat only one task at once
- Manpower: limited staffing
- Financial: limited budget
Tasks scheduling

Central problem of scheduling

- Determine the **earliest start times** $t_i, \forall$ task $i$
- "Potential" constraints
- Minimise the total project duration
Mathematical modelling

Variables

\( t_i \): earliest start time of task \( i \), for \( i = 1, \ldots, n \)

Two fictive tasks 0 and \( n + 1 \):

\begin{itemize}
  \item \( i = 0 \): \( t_0 = 0 \) starting project
  \item \( i = n + 1 \): \( t_{n+1} \) achievement of the project
\end{itemize}

Constraints

\begin{itemize}
  \item Potential constraints: general form
  \[ t_j - t_i \geq a_{ij} \]
\end{itemize}
### Mathematical modelling

#### Potential constraints

- **Temporal location**
  - Task $j$ cannot start before a certain time $e_j: t_j - t_0 \geq e_j$
  - Task $j$ must be achieved before a certain limit $\ell_j > d_j: t_j + d_j - t_0 \leq \ell_j$

- **Succession**
  - **Simple**
    Task $j$ can start only if task $i$ is achieved: $t_j - t_i \geq d_i$
  - **With delay** $d'_{ij}: t_j - t_i \geq d_i + d'_{ij}$
  - **Immediate**
    Task $j$ starts immediately after task $i$: $t_j - t_i = d_i$

#### Objective function

$$\min \ t_{n+1} - t_0$$
Mathematical modelling

\[
\begin{align*}
\min & \quad t_{n+1} - t_0 \\
\text{s.t.} & \quad t_j - t_i \geq a_{ij} \\
& \quad \ldots \\
& \quad t_i \geq 0 \quad \forall i = 0, \ldots, n + 1
\end{align*}
\]
**Task scheduling - Example**

A logistic operations with 9 tasks

<table>
<thead>
<tr>
<th>Task designation</th>
<th>Condition</th>
<th>Duration (h)</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>Can starts 5h after the origin</td>
<td>16</td>
</tr>
<tr>
<td>b</td>
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<td>c</td>
<td>3h after the origin</td>
<td>20</td>
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<tr>
<td>d</td>
<td>a &amp; b completed</td>
<td>8</td>
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<tr>
<td>e</td>
<td>b completed</td>
<td>18</td>
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<tr>
<td>f</td>
<td>b &amp; c completed</td>
<td>25</td>
</tr>
<tr>
<td>g</td>
<td>d, e, f completed</td>
<td>15</td>
</tr>
<tr>
<td>h</td>
<td>e completed, c half completed</td>
<td>17</td>
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<td>i</td>
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<td>10</td>
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</table>
**Graph modelling**

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**Graph potential task**

- a
- b
- c
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**Graph potential task**

```
  a
  
  b          e
  
  c          f
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**Graph potential task**

- a
- d
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Graph modelling

Graph potential task
Graph modelling

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Graph potential task
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Graph potential task
How to compute $t_g$?
How to compute $t_g$?
Graph algorithm

How to compute $t_g$:

$$t_g = \max\{21 + 8, 14 + 18, 23 + 25\} = \max\{t_d + d_{dg}, t_e + d_{eg}, t_f + d_{fg}\} = 48$$
How to compute $t_j$:

$$t_j = \max_{i \in \Gamma^{-1}(j)} \{t_i + d_{ij}\}$$
**Graph algorithm - Bellman algorithm**

**Condition:** No cycle

**Algorithm**

Initialise $t_0 : t_0 = 0$

Mark node 0

**While** (There exist nodes not marked) **do**

Select a non marked node $j$ that has all its predecessors nodes marked

Compute $t_j$ using $t_j = \max_{i \in \Gamma^{-1}(j)} \{ t_i + d_{ij} \}$

Mark node $j$
APPLICATION OF BELLMAN ALGORITHM
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APPLICATION OF BELLMAN ALGORITHM

Critical path: 0 → c → f → g → 10
Critical tasks: 0, c, f and g
How to compute $t'_e$?
Graph algorithm - Latest finish time

How to compute $t'_e$?
**Graph Algorithm - Latest Finish Time**

How to compute $t'_e$:

\[
t'_e = \min \{ 48 - 18, 53 - 18, 46 - 18 \} = \min \{ t'_g - d_{eg}, t'_i - d_{ei}, t'_h + d_{eh} \} = 28
\]
Graph algorithm

How to compute $t'_i$:

$$t'_i = \min_{j \in \Gamma(i)} \{ t'_j - d_{ij} \}$$
Task $i$ is critical: $t'_i = t_i$
Margin $m_i = t'_i - t_i$, ∀ task $i$. If $m_i = 0$ then $i$ is critical.
GANTT DIAGRAM REPRESENTATION

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Resource

Time
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Resource

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GANTT diagram representation

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- **Resource**

- **b**
  - Start: 3
  - End: 14

- **C**
  - Start: 3
  - End: 23
Gantt Diagram Representation

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<tr>
<th>i</th>
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![Gantt Chart](image)
GANTT DIAGRAM REPRESENTATION

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Resource

- **a**
  - Start: 5
  - End: 29

- **b**
  - Start: 14
  - End: 49

- **c**
  - Start: 3
  - End: 23

- **d**
  - Start: 14
  - End: 29

- **e**
  - Start: 23
  - End: 49

- **f**
  - Start: 32
  - End: 48

- **g**
  - Start: 21
  - End: 49

- **h**
  - Start: 14
  - End: 49

- **i**
  - Start: 14
  - End: 49
Minimum resources needed = 3
Exercises