Determination of the most vital elements for graph problems

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Abstract

For security or reliability problems, it is important to assess the ability of a system to resist to a destruction or a failure of a number of its entities. This amounts to identifying critical entities which can be determined with respect to a measure of performance or a cost associated to the system. In many contexts, the network can be modelled by a weighted graph where nodes, arcs or edges constitute the entities (connections of a network, assignment opportunities,...). One way to formulate the research of a subset of critical entities consists of identifying, among all subsets of nodes, arcs or edges, a subset whose removal causes the largest degradation of the measure of performance or the cost of the system. Another way is to find the maximum number of nodes, arcs or edges that can be removed and which provide some measure of performance or a certain system cost.

In my thesis, we considered the k most vital edges (nodes) and min edge (node) blocker versions for different problems of graphs. Given an optimization problem $\mathcal{P}$ defined on a weighted graph, the problem k Most Vital Edges (Nodes) $\mathcal{P}$ consists of determining a subset of k edges (nodes) whose removal from the graph causes the largest perturbation on the optimal value of $\mathcal{P}$. The complementary problem, called Min Edge (Node) Blocker $\mathcal{P}$, consists of deleting a subset of edges (nodes) of minimum cardinality such that the optimal value of $\mathcal{P}$ is, according to the nature of $\mathcal{P}$, less than or equal or greater than or equal to a specific threshold. These problems have been studied in the literature for different problems of graphs, we cite for example the shortest path [1, 2] and the minimum spanning tree [3, 4]. We studied the complexity, the approximation and exact resolution of the k most vital edges (nodes) and min edge (node) blocker versions for the following problems of graphs : minimum spanning tree, assignment, maximum weighted independent set, minimum weighted vertex cover, 1-median, 1-center, maximum flow and flow of fixed value and minimum cost. Thus, we proposed theoretical results by providing proofs of strong NP-hardness or polynomiality for particular classes of graphs, results of approximation or inapproximation. These NP-hardness, approximation and inapproximation results have been obtained either by classical polynomial reductions or by particular reductions preserving the approximation ratios (E-reduction and gap-reduction). Beside these theoretical results, we also proposed practical explicit and implicit enumeration algorithms, as well as linear programming formulation. We have implemented these algorithms in the particular case of the minimum spanning tree and compared with those existing in the literature. The implementations were done using the C language and the CPLEX 12.1 solver, showed that our implicit enumeration algorithm is much better than the linear program.

References

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