# Capability Theory inspired tools for aiding policy design

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**Abstract**

This paper aims at suggesting that welfare measurement could be based upon Sen's capability approach (CA). This should allow establishing a "rational" framework improving how we aide the design and assessment of public policies. We propose the use of a multi-objective mathematical program as basis for measuring individual's welfare and suggest that citizens with similar capabilities could be clustered together establishing targets for specific public policies aiming at improving or protecting the welfare on such well identified social groups.

## 1 Introduction

To a large extent aiding to design public policies consists in introducing elements of rationality (under different forms) within a public decision process. Such elements come under different forms of evidence and argumentation. A typical example of such rationalisation is "Cost Benefit Analysis"(CBA), (see HM Treasury 2020; Directorate-General for Regional and Urban Policy 2015; World Bank 2010): in reality nobody considers the result of a CBA to have a normative validity, but almost everybody is ready to accept it as a common ground for different stakeholders discussing the interest and acceptability of undergoing a certain project. Actually it is practically considered **THE** legitimating exercice for almost any institutional decision process concerning large public investments.

"Rationally speaking" designing a policy which is expected to have an impact upon the citizens' welfare implies being able to:
- observe the present situation and distribution of welfare;
- anticipate the impact of doing nothing;
- anticipate the impact of implementing a policy.

In other terms if a policy is expected to have any impact upon the citizens' welfare it makes sense to try to measure it: as it stands presently and as it could stand under different possible scenarios. However, welfare is a complex issue, implying multiple dimensions and aspects, impacting and being perceived differently among different segments of the society, being distributed unequally among the citizens. Moreover, measuring welfare is itself a policy, since any measurement will need to make choices about what and how to measure. Under such a perspective it is unlike that a single figure welfare measurement can be of any utility for effective policy design purposes.

Further on, welfare appears to be very much related on how citizens perceive themselves as being appropriately endowed and how much they feel free to use their commodities in order to realise their own aspirations. From that point of view, welfare appears to have a strong subjective dimension: as a consequence any attempt of "measuring" welfare needs to be able to capture this special feature.

Besides the above two remarks, we need to consider a third difficulty: part of the "endowments", allowing citizens to live as they do, are "commons", goods which are shared with other citizens, but consumed individually. How such "commons" affect each citizen's welfare and how much this welfare could be reduced in case any of these "commons" gets lost (totally or partially)? The contribution of "fresh water availability" to a community's citizen's welfare is far beyond the price for each liter of water consumed by each citizen.

Let's resume. We need to be able to "measure" welfare and the impact policies can have upon "welfare", but such measurement needs to consider:

- the multidimensional nature of welfare;
- the subjective dimension of welfare;
- the impact of the commons upon welfare;
- the different impact a policy or an event can have upon different groups of citizens.

The above constraints represent a challenge for decision analysts: if our tools are aimed to help (among others) policy makers to design policies and to improve how policies are designed we need to provide appropriate methods taking into consideration the above discussion.

Our attempt is to propose a framework that supports public decisions processes occurring in a *policy cycle*, within a *Policy Analytics* framework (Tsoukias et al., 2013; De Marchi et al., 2016; Daniell et al., 2016). This concept aims at supporting policy makers in a way that is meaningful, operational and legitimate, by developing, in particular, methods that take into account values of different stakeholders. However, we need a theory about how to consider welfare. For this purpose, in this paper we explore the Capability Approach (CA) (Sen, 1980, 1993, 1985, 1999, 2009) as a framework allowing to develop appropriate decision aiding for public policy design. Our proposal is to show that the CA could offer a common ground to different stakeholders assessing the impact of a policy to the citizens' welfare, offering a certain number of advantages with respect to other approaches aiming at measuring welfare (although technically more complicated and certainly more expensive to conduct in terms of analysis). The reader should not expect a detailed protocol on how to achieve that. For this paper we introduce a conceptual innovation and we show how and why it could be useful.

The first section of this paper explains our motivations: on the one hand introduces the notion of rationalisation through decision aiding for policy design purposes and on the other hand introduces the four principal approaches concerned by welfare measurement. In the following section we critically analyse the capability approach: despite being interesting as a view of welfare it is far less operational and even less suitable (as it stands) for design purposes. We then introduce in Section 4 our vision on how welfare could be measured and we present a multi-objective mathematical programming model through which we can achieve a meaningful measure of an individual's capability set. For this purpose we propose in Section 5 a small example.

# 2 Motivations

A typical activity of decision aiding consists in advising stakeholders involved in public policy making processes. For the purpose of this paper we will consider policy making within a policy cycle (see Howlett and Ramesh 1995). Part of such cycle is the policy design activity where stakeholders are expected to contribute, designing bundles of actions supposed having an impact upon a set of objectives (see Bobrow 2006). Certainly, this is a rough definition, but for the time being it is sufficient for the purpose of this paper. Notwithstanding this simple definition, it is necessary to clarify a number of concepts including: what does it mean aiding a decision process, more specifically what does it mean aiding designing and why this should be specific for public policy making purposes.

## 2.1 Aiding to decide and to design

Part of the presentation is inspired from Tsoukiàs (2007). Aiding to decide is viewed as a multi-stakeholder decision process (there are at least two of them: the client and the analyst) constructing cognitive artefacts which should enable the "client" to improve the way s.he handles a decision process for which s.he asked an advice to the analyst (see also Meinard and Tsoukiàs 2019). Such a process is guided constructing mutual convictions about and ownership of such artifacts, based upon three requirements:
- meaningfulness (for the analyst) of any information manipulation;
- usefulness (for the client) of the recommendations constructed by the process;
- legitimacy of such recommendations with respect to the decision process for which the advice has been requested.

Let's focus upon this last dimension characterising decision aiding. An essential aspect of legitimacy is the pretention of the recommendation constructed to be "rational". We are not going to expand the term rationality here: for the purpose of this paper it is sufficient the claim that within public decision processes "rationality" of any suggested action is considered to be an essential feature (although there might be no agreement what such rationality means or implies). Such rationality needs to be "arguable" (stakeholders should be able to discuss it and contrast it) and "convincing" (when no more argumentation can hold). In other terms such rationality is a common ground among the stakeholders of a public decision process upon which to construct any decisions.

A usual way to establish such a common ground is to impose it "normatively". As soon as we convince the stakeholders that respecting a certain "norm" it is rational, the decision process boils down to check how to remain coherent to the norm. A typical example in many public decision processes is the use of Cost-Benefit Analysis (CBA: see Dasgupta and Pearce 1972). We are not aiming to discuss this tool here, but to mention the fact that CBA is de-facto **THE** standard of rationality when public investments are discussed (HM Treasury, 2020; Directorate-General for Regional and Urban Policy, 2015; World Bank, 2010). The result is that the acceptability of many decisions depends upon going through such an analysis (the reader should note that the same type of reasoning applies for many environmental impact analysis tools which are established as norms of rationality[1]).

However, choosing a tool or a model as a norm of rationality comes at a price. Our concern is about two types of problems we need to pay attention to. The first concerns the axioms and hypotheses such a choice needs to impose in order to be used meaningfully. A first aim of our paper is to discuss the fundamentals behind different approaches to welfare measurement in case we consider using this as a rationality norm for public policy design and making. The second concerns the space allowed for being creative as far as the potential decisions are concerned. The basic idea here is that before assessing any solutions we need to design them and for this purpose we need to be appropriately endowed in terms of modeling. The topic is already discussed in Colorni and Tsoukiàs (2020), Ferretti et al. (2019), Howlett (2011), Pluchinotta et al. (2019) where the reader can have an overview of the existing literature (about designing actions and

[1] https://ec.europa.eu/environment/eia/eia-legalcontext.htm

policies). What we should remember is the fact that in order to be able to design alternative actions we need an explicit multidimensional representation of the solutions space. A second aim of the paper is to present simple models that nevertheless allow an explicit representation of the multidimensional nature of welfare, thus allowing alternative designs of policies.

## 2.2 Public Policies

Why aiding public policy making is different from other decision aiding activities? We will essentially use the approach developed in Tsoukias et al. (2013) which we summarise in the following.

**1**. Public policies allocate, redistribute and modify, among others, public ressources and impact the access and use of the Commons (Ostrom, 1990).

**2**. Policy cycles are participative decision processes with multiple stakeholders being involved, carrying multiple possibly independent concerns.

**3**. Policy cycles have long (possibly very long) time horizons, up to intergenerational span. Such time horizon is far beyond the political horizon of the policy makers.

**4**. Policies are deliberated in public decision processes and are normally disputed and argued not always for their content.

**5**. Policy makers are essentially driven by legitimation (long term) and accountability (short term) quests.

The result of the above features is that there is no "obvious" rationality ground for what should be considered "rational" in policy making. It looks reasonable to consider as rational the majority will (principle of democracy), but is far from clear both the notion of majority (several different definitions are possible: see Reynolds et al. 2005) and how the wills of the majority are constructed. It is also reasonable to consider as rational any action which has a positive impact among the majority of citizens, but once again it is not clear how such an impact should be assessed and which majority of citizens should we consider.

In any case we are going to focus upon this last idea: it is "rational" to suggest policies which will positively impact the citizens (or that will protect them in case they are threaten). However, this apparently simple idea needs to be better specified. The concept to use here is the one of "welfare" (see Sen 1991), but we need to understand which are the operational difficulties we may have in order to establish and measure the welfare of either a single citizen or of a group of them.

A first difficulty is what exactly welfare should represent. There exist several different dimensions to consider: health, education, shelter, work, culture, mobility, leisure ... Should these dimensions be considered separately or should we merge then in a single figure?

A second difficulty concerns the fact that each citizen may have a different appreciation of what is important for her/his well being (welfare) and how each of the possible welfare dimensions contributes to her/his overall welfare.

A third difficulty is related to the fact that even citizens sharing the same values (appreciation of the different welfare dimensions) may still have different cultural and social backgrounds and/or live in very different environments ending in having totally different realisation options.

Last, but not least, while welfare certainly depends upon the private endowments of each citizen, it also depends upon the access of each citizen to "commons" whose consumption does not depend from the will of that precise private citizen.

*Summarising: we are considering the use of a rationality norm as far as the appreciation of* *a public policy is concerned, but we need this norm to fulfill some requirements. On the one hand it should be sufficiently flexible to allow argumentation and alternative designs and on the other hand it should be able to account for the multidimensional and subjective nature of welfare for the objective diversities among citizens and for the specific impact of the commons upon welfare*. In the following we propose a brief survey of what Welfare Economics consider reasonable concepts and definitions of Welfare inspired by Sen (1980, 1985, 1997, 1999). The survey do not cover all definitions of Welfare and the controversies presented are not exhaustive, but it shows the interest of using the capability approach as a base for a rational tool of welfare measurement.

## 2.3 Welfare Measurement

**Welfare as Utility**. It can be said that the roots of welfare economics is utilitarianism (Bentham, 1789; Hicks, 1939). It is a consequentialist ethical theory (Sinnott-Armstrong, 2019), meaning it considers an action being good or bad, based on its consequences. For utilitarians, the utility is the representation of welfare, or the happiness of an individual. The principle of utility should be considered as the foundation of our moral judgement and political decision. Nothing is moral on its own, we don't have to act to maximize liberty or justice, but we have to judge actions based on their consequences over the utility. An action is considered as good, if it implies *the greatest happiness of the greatest number* (Bentham, 1789).

However, as we noted in the introduction it is necessary to understand which are the implicit hypotheses we do when we accept such a principle. There are two which are very important:
- Utility is *cardinal* (it can be represented by a quantity upon an interval scale).
- Utility allows *interpersonal comparisons* (we can compare utility of different individuals).
Such hypotheses imply that we are able to measure the difference of value between any two states, and this measurement is commensurable among individuals, further implying that we are able to compare the difference among values of a first individual with the difference among values of a second one.

*Cost-Benefit Analysis*, (HM Treasury, 2020; Directorate-General for Regional and Urban Policy, 2015; World Bank, 2010; Johansson, 1991; Boadway, 1974; Papadimitriou and Yannakakis, 1994; Adler and Posner, 1999; Frank, 2000) is a typical example of how utilitarianism is practically used in terms of decision aiding. Consider a project which is expected to have an impact upon the life and welfare of the citizens of a given territory. Given the expected consequences of the project and considering the citizens as individual *consumers*, the utility of implementing the project is the sum of utilities of each single citizen/consumer. In order to compute such utility we consider that each potential consequence is measurable at a real/proxy market revealing the citizens/consumers preferences (NB: each citizen has exactly the same preferences!). In other terms the "prices" of such consequences, observable directly or indirectly through the markets, represent the utilities lost or perceived by each citizen/consumer. Distributional weights can be applied to increase the utility of benefits (or costs) perceived by lower income citizens, to take into account the fact that one additional monetary unit has more value for low-income than for high-income citizens. A project is seen as a *change in the net supplies of commodities* (Drèze and Stern, 1987) and should be selected if its benefit exceed its cost over the period of time to consider *e.i. if the social Welfare increases over the select period*. Sensitivity analysis can be performed and as the utility of income is supposed being greater today than tomorrow, a discount rate is applied.

Utilitarian approaches have been criticised for several aspects. The first one concerns the *subjective differences* (applicable to the classic utilitarian approach); deprived people can be easily satisfied because they may have easier to meet expectations, but this is in contradiction with an intuitive representation of welfare. The fact that utility are subjective lead Robbins (1932, 1938) to conclude that utilities cannot be objectively compared because it is derived from mental states. One cannot measure the utility of someone else, which makes interpersonal comparisons impossible.

As an example, CBA suffers from the same problem; citizens are considered to have the same preferences; two people with access to the same bundle of goods are supposed obtaining the same utility, but because of their heterogeneity there is no reason that the same bundle would result to the same level of welfare.

The second difficulty concerns *distribution effects*. The classical utilitarian approach will typically disadvantage people deriving less from resources than others (for instance disable people) because they are equalising the marginal utility rate, which can be in contradiction with our moral intuition. Remaining with the CBA example, some distributional aspects can be taken into account through the use of distribution rates, but the determination of those rates can be hard to be established (HM Treasury, 2020; European Commission, 2013).

Utilitarian approach (as well as social choice approach) can be in contradiction with *Society Motivations*. For instance, Sen (1970) shows that the utilitarian and social choice approach are in conflict with the respect of personal liberties (that is being 'decisive' over some personal matter).

Finally, there is the *individual motivations* issue, utility and CBA do not take into account that individuals may act for different purposes than their own interest. As argued in Rawls (1971), individuals "have a sense for justice" and "a conception of good", that can lead them to act against their own utility.

**Welfare as Social choice**. Welfarists considered that if utilities cannot be compared among individuals, then we can only compare preferences stated by citizens upon bundles of goods or alternatives. However, when alternatives need to be compared the only information we get are their profiles within the society of citizens. The only immediate result we can get is whether there exist Pareto "optimal" social states: states which cannot improve for an individual without worsening for another.

Improvement by Pareto comparison is in practice very unrealistic if the purpose is to design public policies, because these will typically benefit to some individuals at the cost for others. The reduction of informational basis, by removing interpersonal comparison, leads to the incomparability of most of the alternatives. Then, some new criterion has to be taken into account; for this purpose Bergson (1938) developed the concept of Social welfare function (SWF) (Kaushik and López-Calva, 2011).

The result has been creating a whole field of research aiming at establishing appropriate "social welfare functions": social choice theory; a theory about how the society should make decisions only using the preferences of its members. As Arrow has shown (Arrow 1951) this is impossible under very simple conditions (unrestricted domain, Pareto principle, independence of irrelevant alternatives, no dictatorship).

This result may be seen as surprising, but Vincke (1982) shows that in fact it is not. The reason is that any SWF is a preference aggregation procedure and as such will provide a result which is poorer (from an information content point of view) from the aggregated preferences. In other terms if we aggregate complete orders (such as weak orders), any SWF satisfying Arrow's conditions cannot yield a result with the same type of information content (a complete order): there is no reason for which a society of rational decision makers will be equally rational. Not surprisingly Condorcet's procedure (which satisfies Arrow's conditions) does not guarantee the existence of a social complete order.

For Sen (1977), Arrow's impossibility result can be interpreted as a proof of Arrowian SWF *informational limitations*. Indeed, only considering preference ordering and imposing strict no-interpersonal comparison is not sufficient to be able to take decisions over social states. For this reason, some welfare economists have developed SWF allowing interpersonal comparisons (for instance Adler 2012, 2019).

**Welfare as social justice**. In *A Theory of Justice*, John Rawls (1971) was concerned with the problem of defining a fair society under the difficult question of distributive justice. For him, the concept of justice is linked, and must be achieved by fairness, which can be seen as a demand for impartiality. He wanted to define some basic structures, that would allow to have the greatest freedom and equality. Using an abstract reasoning, he defined the best basic structures (which are the political, economical and social institutions), to distribute rights and advantages that are the result of the social cooperation. To do so, Rawls got his inspiration from the social contract theory that emerge in the Enlightenment. He concluded that a fair society should maximize *primary goods* of the least well-off. The idea of primary goods is a very important concept for Rawls. It is a list of what all citizens desire, no matter what else they desire. We are not going to further develop Rawls' theory (see Rawls 1982), but notice that one important difference between the welfarism and Rawl's theory is that the first focuses on *ex post* outcomes (it evaluate the possible outcomes) whereas the later focuses on *ex ante* opportunities (it gives bases for a fair society).

A first criticism made in Sen (1980) is that Rawls focuses only on primary goods. He argues that they are means and not ends (that are freedom), and that what we have to look at, are the ends. Moreover, Sen argues that focusing on primary goods in a society where people are different can lead to unfair situations. Indeed, Rawls focuses on primary goods and not on what people can do with them. For example, a person that has a handicap may need more goods than someone else in order to achieve the same level of welfare. Having the same primary goods does not guaranty having the same welfare.

Another criticism in Sen (2009), is the fact that Rawls' theory is part of the contractarian tradition (Hobbes, 1651; Rousseau, 1762) described also as "transcendental nstitutionalism" approaches. This term is composed in two parts (Thomas, 2013); first, the *transcendental* refers to the idea of finding a set of perfect principles of justice, secondly, *institutionalism* underly the fact that the scope of distributive justice is limited to institutions only. He criticises the fact that Rawls is only concerned about *just institutions* and not *just societies*. The justice in a society can depend of non-institutional features, such as social interactions, or behaviours of people.

**Welfare as capabilities**. According to Sen (1997) and Sen (1985), we should do a distinction between owing a good, using it and obtaining utility from its use. Welfarists, mainly focus upon income and/or commodities, but for Sen, while goods and income are important, we also need to know what we can do with them. First, a commodity has to be distinguished from its characteristics. Moreover, individuals are different and can use the characteristics of a commodity differently. Such differences can be explained by different *conversion factors* that an individual possess. According to Robeyns (2005) these factors can be divided into three categories: *Personal conversion factors*, *Social conversion factors* and *Environmental conversion factors*. Since looking at an individuals' commodities in order to establish her welfare is not sufficient, we have to look at their *functionings*, which is *what a person is actually able to do*, given his/her commodities. The person's *capability* is all the possible combinations of functionings that an individual can reach. It has to be distinguished from the *achieved functioning*, that is the set of "doing" effectively chosen by the individual. A capability is the ability to achieve, whereas a functioning is an achievement. Capabilities represent the effective freedom of an individual to choose between different functioning combinations, that s.he has reason to value. From a social choice theory approach, a capability can be seen as an opportunity set. Then, only the ends (possible combinations of functionings) have an intrinsic value in order to evaluate welfare.

As argued in Robeyns (2003), we can distinguish three levels in which the Capability Approach (CA) can be used: *a critique*, *a paradigm*, *a tool to make interpersonal comparison of well-beings*. Large part of the applications of the CA concerns poverty and social inequalities (see Sen 1985; Alkire et al. 2014; Zeumo et al. 2014), but also in health economics (see Al-Janabi et al. 2008; Coast et al. 2008; Flynn et al. 2011; Al-Janabi et al. 2011, 2012) or in urban planning (see Blečić et al. 2013; Fancello et al. 2020; Fancello and Tsoukiàs 2020). The CA has a strong theoretical background and presents a number of conceptual advantages (accounting for citizens' diversity), but is far from being either an established norm for welfare measurement or an operational tool. There are several reasons for this, which we analyse in the following.

# 3 Critical perspective

The Capability approach can be criticized on several aspects (both from a theoretical and empirical point of view), but our focus will be upon using it as a rationality norm for decision aiding purposes in public policies design. In the following we present the principal critiques moved to the CA, in order to establish the attributes we consider relevant for a "rationality norm" to satisfy if it is expected to be used for policy design purposes.

## 3.1 Critics on Capability approach from a decision aiding perspective

**Interpersonal comparison.** Let's begin with a remark that has been made in Robeyns (2000): Sen's CA is not completely solving the interpersonal comparison problem. Most of the capabilities allow interpersonal comparisons, especially the basic ones. Having access to safe water, shelter, and being sufficiently nourished can be considered "objective capabilities" since there is a social consensus about their importance for any citizen's well-being in any part of the world. We do not really need to assess their subjective value and they can be objectively observed. Nevertheless, if we consider some more complex functionings such as access to culture, urban quality, self-respect (just to mention some very diverse), it seems much more difficult to do "objective" comparisons. Capabilities containing subjective judgements cannot be fully compared between different individuals. Indeed, such "subjective functionings" are not only influenced by the personal, social and environmental factors (in which individuals have not complete control), but also by their values. *In other terms in order to compare such capabilities we need to know the subjective value citizens give to certain achievements and/or opportunities: we need to enhance the information basis of commensurability.* It is not the case for most of the CA applications. This is probably due to the fact that most applications have been about poverty and inequality, as in this context all functionings are considered as crucially important (Sen, 1993). The account for individuals' values is generally only integrated in the weight for the aggregation of different functionings, especially in indexing methods (for instance Alkire et al. 2014; UNDP 2020). This can be misleading since the users and decision makers might not be aware of such hypothesis. *Taking into account the citizens' subjectivity and values should be integrated explicitly in the evaluation of functionings.*

**Individualistic.** One of the most widespread comments on the theoretical side is that the CA is too individualistic (Deneulin and McGregor, 2010; Stewart and Deneulin, 2002; Stewart, 2005; Kaushik and López-Calva, 2011), mainly because individuals are atomized. It is clear that considering individuals' capabilities, the utilisation of Commons, social structures and the interactions between individuals, raises important issues as far as the real *individual* freedom measurement is concerned. Indeed, for Commons, the substractable effect implies that the "real" capability of an individual will depend on the utilisation of the Commons by other individuals. However, there is not intrinsic impossibility in the capability approach to take into account those social environments. Most of the work referring to the CA has not paid a lot of attention to groups, but some did: Kynch and Sen (1983). Besides, social aspects are theoretically taken into account as social conversion factor of resources into functioning. See (Robeyns, 2017, Chapter 4) for more discussion on the individualistic argument.

From the empirical side, citizens' welfare depends upon a complex bundle of goods and services, some (many) of them interacting between them and/or being part of more complex systems such as cities, communities, territories etc... Designing a policy impacts the structure of such systems and therefore the welfare in ways which we do not always know. *Under such a perspective the CA, as it stands, lacks a systemic vision*: it has been mostly used to measure and identify or to find the causes of poverty/inequality under a very simple deterministic representation of the phenomenon. Another consequence of the absence of a systemic vision is the difficulty to identify the relevant Commons impacting welfare. We will turn back to the benefit of a systemic vision of welfare in section 3.2 and propose a systemic representation of CA in section 4.

**Functioning and not capabilities.** Most of the empirical works focused on present individual *functionings* rather than *capabilities* (for instance Sen, 1985; Cerioli and Zani, 1990; Qizilbash and Clark, 2005; Alkire et al., 2014). The empirical work on the CA being dominated by poverty measurement/identification, it is not a surprise. Functionings can be considered as more relevant than capabilities in this context; being undernourished is rarely a choice (Robeyns, 2006). Choosing other applications and contexts, focusing on functionings can appear as less relevant. For instance, if someone wants to *predict* the welfare of citizens of a "developed country" after implementing a public policy, measuring future *functionings* imply assumptions on what are going to be the citizens' choices and their notion of a good life. *In this context capabilities can make more sense because less prediction have to be done as far as the citizen's future actions are concerned.*

**Under-Theorisation.** Some have criticised the *Under-Theorisation* of the capability approach. The fact that Sen doesn't want to give a list of functionings (unlike Nussbaum 2003), makes the theory unclear and hard to use. This come from the fact that the capability approach is not normative. *From a decision aiding process perspective* (Tsoukiàs, 2007), *the method is the key element and the selection of particular functionings can be leaved to a discussion with different stakeholders. Not having a list of definite functioning allows a framework to be* versatile*, which is desirable from the point of view of decision makers.*

**Access to data.** The access to data is another limit to the concrete utilisation of capabilities. Indeed, in order to use the CA operationally, we need a lot of different data. First we have to find, for each individual, their means in order to achieve their functionings; their income, commodities, access to Commons etc, as well as the market mechanism. Data for this first stage are more difficult to find with respect to most "income based approaches", which deal with the net income only. Then we have to collect information about the conversion factors of individuals (*i.e. personal, social and environmental factors*). Finally, we have to construct their subjective "values". It is clear that some data are hard to collect and are both quantitative (like income, age) and qualitative (values, social group). Most of data that have been used so far in existing applications of the CA (as underlined by Robeyns, 2006) are second hand data that have not been specifically designed by CA scholars. Besides, there is no formalisation that is both operational and that fully represents the capability approach. In fact, as argued in Robeyns (2017, Chap. 4), the informational richness that is needed, is not only a data collecting problem, but also a mathematical modelling one. Social constraints and individuals different values are, among other things, difficult to model. While using a mathematical representation, we have to be careful to not lose the richness of the capability approach by over simplifying. *We think that the data quality issues need to be addressed explicitly: collecting data for an explicit use of the CA will be more expensive that collecting data for other forms of welfare measurement (such as income based approaches), but will be worth it.* Collecting such divers data is not only expensive but also hard, *new methodology on how to collect those data need to be developed.*

**No framework to compare.** Sen (2009) argues that public policies should focus on equalising the individuals' capabilities, but he did not provide a framework to compare them. How to compare someone that has the opportunity to be in good health but is not wealthy with someone that has the opportunity to be wealthy, but is not in good health? As the capability approach leads to multi-dimensional comparisons, it can be hard to find ways to compare different capabilities. Moreover, Sen claims that we should not equalise capabilities at every cost. For example, if women live longer than men all other things being equal, then we should not reduce women's access to hospital to make men's and women's life longevity equal. Then, both the difficulty to compare capability and the unanswered question of "what exactly do we have to equalise?", raise issues on the operational use of the capability approach. *However, it is not because a framework to compare solutions do not exist that any comparison can't be done, the informational base can be a sufficient ground for argumentation between different stakeholders. If it is not sufficient, some techniques have to be developed to be able to discriminates different capability set.*

The model presented in Section 4 is a proposition to answer (at least partially) those operational issues. To meet the interpersonal comparison and individualistic critics, we suggest a mathematical program model assessing something close to "capabilitie" (rather than achieved functioning), integrating explicitly the evaluation of functioning and being systemic, it is introduced in Section 4.1. The under-Theorisation is not really an issue from a decision aiding point of view, a method on how to select functionings can be find Section 4.2. Section 4.3 discuss the data collecting issue to feed our model. Finally, in Section 4.5 we will give some promising avenue for research on finding a framework to compare our "representation" of capability set.

## 3.2 Why the CA can be useful for decision aiding

Despite the CA is not really operational we consider it remains an interesting framework from a decision aiding perspective. We can identify four main steps in which a framework can be useful for decision aiding purposes, especially in the context of a public decision occurring in a policy cycle, within a policy analytics framework (Tsoukias et al., 2013; De Marchi et al., 2016; Daniell et al., 2016).

1. Help the client to have a bette *understanding* of the problem;

2. Help to *imagine and design* potential solutions;

3. *Explore* consequences of different solutions;

4. Provide some *arguments* in favour or against any selected solution.

The reader should note that our aim is to use the CA as a common ground helping to rationalise how public policies are designed, assessed and implemented. Under such a perspective our vision differs from the mainstream proposal of the CA: some scholars (as Nussbaum 2011) consider only the governments to be the actors of policy design and/or improvement and think that the CA should address "recommendations" about public policies to such governments only. Yet, as stressed by Stewart (2005) and Robeyns (2017, Chap. 4), improvement generally does not come from the only benevolence of governments. It comes with political pressure from different groups. Clients such as NGOs, trade unions or economic actors can ask help for different purposes and aims, but they all need a common ground upon which discuss, negotiate and agree (if possible). In the following we will briefly discuss why the CA can be such common rationalising ground.

We consider three main characteristics for which the CA is useful for policy design purposes.

- Introduces a *multidimensional* approach of welfare;

- Recognizes citizens' *diversity*, their different access to private or common resources, their diverse personal conversion factors and the set of various physical and social environments they can live in;

- Accounts for citizens' *subjectivity and value*; how they evaluate some set of functionings.

We also think that a fourth aspect should be developed in order for the CA to be fully efficient in a policy analytics context; a *systemic* modelling. Table 1 summarise how the advantages of the CA can help the different steps of a policy cycle.

| | Understand | Design | Explore | Argument |
|---|---|---|---|---|
| Multidimensional | ✓ | ✓ | | ✓ |
| Diversity | ✓ | ✓ | | ✓ |
| Subjective | ✓ | ✓ | | ✓ |
| Systemic | ✓ | ✓ | ✓ | |

Table 1: How the CA is useful to construct a policy analytic framework.

**Understanding.** A single figure representation of welfare is certainly easier to perceive and communicate. However, as already shown by many authors (for instance in Sen 2009) such single figures conceal the rich picture of the citizens' welfare.

Welfare is a complex social, economic and cultural reality and a single figure will not be able to represent such a complexity. Moreover, most of the times the way through which such single figure is obtained conceals both arbitrary hypotheses contained in the aggregation procedure as well as important differences among different dimensions of welfare (which could be compensated among them). A multidimensional representation of welfare offers a richer picture of the reality under observation, allows to see welfare as a distribution (and not as a figure) which on its turn allows to imagine alternative distributions in case this is considered necessary.

Recognising the diversity of well-being levels is also necessary to have a good understanding of the issue at hand. It is important to be able to measure well-being at the individual level rather than using indices that represent the "general well-being" of citizens. This allows to understand a complex situation with potentially large disparities in terms of access to resources and conversion factors. Measuring the well-being of a population as if it were a homogeneous group could lead to ignoring minority groups as well as missing what particular resources or conversion factors are limiting a specific (group of) citizens.

It is important to realise that welfare is perceived *subjectively* and also assessed subjectively. Generally policies are expected to be a reply to a problem situation, but the extent to which the present distribution of welfare is a problem and for whom is a matter of subjective appreciation by each single citizen and the same idea applies as far as the impacts of any policy are concerned. Taking into account explicitly such subjective dimension allows to have a more realistic picture and to anticipate the different reactions of groups of citizens sharing common perceptions and values.

Last, but not least, adopting a *systemic* approach as far as the representation of welfare is concerned, we allow taking into account the multiple interactions between access to private and public goods, access to the commons, private attitudes, individual and collective behaviours. Once again we obtain a richer picture upon which build a policy design.

**Designing.** Innovative policy design means:

- being able to target specific categories of citizens in order to increase and improve policy legitimacy;
- explore a space of solutions "out-of-the-box", avoiding dominant designs and creating new ideas and concepts;
- anticipate the drawbacks and negative reactions improving efficiency and long-term acceptability.

A *multidimensional* representation of welfare allows to expand the space of potential solutions including options apparently inconceivable, but potentially feasible. It also allows to explore deep "what-if" questions: what is needed in order to transform infeasible options to feasible ones or to make inconceivable actions realistic? Such an analysis is essentially possible only when the *systemic* nature of welfare's definition and structure is explicitly considered as we suggest in Section 4. It is through the explicit representation of the multiple interactions between resources, actions and values that different designs become visible.

At the same time analysing the *subjective* values driving the citizens' behaviour allows to identify different policy targets, to expand the inclusiveness of policies, while recognising the citizens' *diversity* and their expectations helps in anticipating policy legitimacy and long-term acceptability.

**Exploring.** Rational policy design is possible only if we are able to offer a common ground where the consequences of different policies can be anticipated and measured. Policies should be simulated, projected upon the citizens and their impacts studied, including possible drawbacks and unforeseeable outcomes. A *systemic* representation of welfare allows conducting such type of exercises and it is exactly for this purpose that we suggest a mathematical programming representation of the CA in Section 4. Such a model allows for quantitative analysis, to conduct simulations, to visualise impacts, offering to the stakeholders a common ground upon which discuss and negotiate (in case this results useful). A formal model is a necessary condition for any effective participative policy design process.

**Arguing.** Arguing for or against a policy (design, targets, objectives, consequences, measures, feedback etc.) is an essential feature for its legitimacy and effectiveness. Considering explicitly the *multidimensional* nature of welfare as well as the *subjectivity* and *diversity* of the citizens (with respect to the distribution and use of welfare) allows to construct the necessary basis for arguing effectively (this being a necessary condition for an effective participation to the policy design process). It also allows to identify those groups of citizens excluded by the policy designed. On the other hand it allows to construct recommendations based upon awareness, consciousness and argued convictions.

# 4 Model

In this section we introduce a Mathematical Programming framework aiming to support public policy "rationally" by measuring welfare. We consider that citizens are positively or negatively impacted by a public policy in case their welfare is "augmented" or "reduced". The proposed model is strongly inspired by the capability approach. The basic idea is that welfare is represented by a measure close to the citizens' capability sets and thus, a modification of their welfare state should become visible through the modification of such "capability sets". In our framework, citizens can chose to achieve different sets of actions that will be constrained by their access to resources (private or Commons) and conversion factors. A public policy will be seen as a modification of citizens' access to resources and conversion factors. Each set of actions is evaluated (trough the citizens' value functions and their conversion factors) in a multi-dimensional welfare space. "Solving" this MP problem for a given citizen results (at a first step) in computing the Pareto frontier: the actions s.he can undertake (which are compatible with her/his endowments) and are not dominated on some of the welfare dimensions who matter for that citizen. Since this is very near to what we call a capability set we will call such a set of efficient solutions a "capability set" (or "welfare representation"); the Pareto frontier of our mathematical program in the "welfare space". The reader should consider that the term "capability set" will be defined hereafter with that sense.

Our approach consists in three distinct steps:

1. at the first step we establish a generic model aiming at computing the capability set of a given citizen (a measure of his/her welfare as it stands presently): we consider that such a set results as the solution of a multi-objective optimisation problem representing the constraints and the aspirations of the citizen;
2. at the second step we use the same model in order to simulate the impact of a public policy on a given citizen (and thus, shifting the present distribution of welfare). A public policy is considered to modify some parameters of the mathematical program. Computing the new welfare representation, we simulate the new capability set of citizens given the new public policy. The *impact* of such a policy is seen as the "modification" of the welfare representation (from the present state to the simulated new policy state);
3. at the third step we cluster the population along a number of characteristics, but essentially using the similarity of their capability sets (welfare distribution).

For the rest of the paper we will focus essentially in presenting the first step in details.

### 4.1 Mathematical programming model

We note $\mathcal{X}_i$ the set of all relevant data representing a citizen's $\mathcal{X}$ *present state* $i$, at the time of the observation (step 1). We consider $\mathcal{W}(\mathcal{X}_i)$, the welfare representation of the present state (the Pareto frontier before any new policy). Then, we need to be able to anticipate the impact of doing nothing and implementing new policies (step 2), each policy can be describe as a scenario. The welfare representation of citizen $i$ in the case of scenario $s$ is noted $\mathcal{W}(\mathcal{X}_i|scenario_s)$ or $\mathcal{W}({}^s\mathcal{X}_i)$.

**Variables.** Our decisions variables $X = \begin{bmatrix} x_1 & x_2 & \cdots & x_j \end{bmatrix}$ represent *Doings*. These are actions or activities that citizens can do, in order to "achieve", "obtain" or "reach" one or more goals. More precisely, we consider a *Doings* $\overline{X}^z$ as a combination of actions/activities that an individual can do in a set period of time. Functionings are seen as constructions of doings. The *Doings set* is all the possible Doings that a citizen can decide to achieve; all the different combinations of actions that a citizen can perform in a given interval of time.

Our mathematical optimisation program is a mix-variable problem. We use binary variables ($x_j \in \{0,1\}$) when the issue is whether a certain resource or a common is used (can be used) or not. We use real valued variables ($x_{j'} \in \mathbb{R}$) when resources or commons are consumed (at different possible levels).

**Constraints over private resources.** The consumption of a *private resource* $l$ by a citizen $i$ to achieve his/her Doings should be inferior to $R^i_l = \begin{bmatrix} r^i_1 & r^i_2 & \cdots & r^i_l \end{bmatrix}$, the set of different private resources owns or that s.he has access to. Note that private resources are to be taken in a broad sense, including for instance "abstract" items such as time or knowledge. The amount of private resources consumed/earned is obtained by the set of Doings and the conversion matrix $A^i$. The elements $a^i_{j,l} \in A^i$ are determined through *personal conversion factors*, *social conversion factors* and *environment conversion factors*.

The constraints over the quantity of the private resources are not necessarily linear and are modelled through the function $\Phi^i_l(X, A^i_l = \begin{bmatrix} a^i_{1,1} & \cdots & a^i_{j,l} \end{bmatrix})$. The quantity of private resources consumed by the citizen should be inferior to the quantity of resources that s.he possesses:
$\Phi^i_l(X, A^i_l) \quad \leq \quad R^i_l \quad$ for all $l$.
Supposing the problem being linear we get: $\quad X \cdot A^i_l \quad \leq \quad R^i_l \quad$ for all $l$.

Public policies can have a impact both on private resources ($R^i$) and the conversion matrix ($A^i$). Then, the constraints over private resources in a scenario $s$ are of the form:

$${}^s\Phi^i{}_l(X, {}^sA^i{}_l) \quad \leq \quad {}^sR^i{}_l \quad \forall l.$$

**Constraints over Commons.** We consider a vector of *Commons* $C = \begin{bmatrix} c_1 & c_2 & \cdots & c_k \end{bmatrix}$ as defined by Ostrom (1990). As for private resources, the "quantity" of a Commons used/consumed by a citizen $i$ is influenced by elements $a^i_{j,l+k} \in A^i$ from the conversion matrix. We will distinguish two types of Commons; those that are utilised and those that are consumed. In the first case, the "quantity" of Commons is not reduced when a citizen is using it, but it reduces the utility derived for other citizens. For instance, a road can be considered as an "utilised" Common. More people on a road will not consume it, but it will create traffic jam, decreasing the utility of the road. For those sort of Commons, we can use a binary representation; $C_k \in \{0,1\}$, the Common either being available or not. All actions $j$ either use this Common $k$, ($a^i_{j,l+k} = \epsilon$ with epsilon a very small value), or don't use it ($a^i_{j,l+k} = 0$). If the Common becomes not available ($C_k = 0$), being damaged or inaccessible, all actions using $k$ ($\exists a^i_{j,l+k} = \epsilon$) are then infeasible and set to 0. For such type of Commons, the consumption of other citizens have a great impact on a particular citizen conversion matrix. For instance, closing a particular road can influence the traffic on the entire road network. The use of different Commons can vary regarding different scenarios, for which we have the following constraints:

$${}^{s}\Phi^{i}{}_{k}(X, {}^{s}A^{i}{}_{l+k}) \quad \leq \quad {}^{s}C_{k} \quad \text{for some } k.$$

In the second case Commons are consumed. For instance, if a citizen consumes a quantity of water from a groundwater, this can not be consumed by someone else. In this situation, the Common pool resources which can be consumed by a particular citizen is what is left by the other citizens. The general consumption of a good $k$ in a scenario $s$ is noted ${}^{s}\Delta_k$. Consumable Commons use continuous representations; $C_k, \Delta_k \in \mathbb{R}$. Given different Commons, the quantity left can also affect the conversions parameters; it is more difficult to catch a fish if there is little left in a lake than if there are many. For consumable Commons, we have the following constraints:

$${}^{s}\Phi^{i}{}_{k}(X, {}^{s}A^{i}{}_{l+k}) \quad \leq \quad {}^{s}C_{k} - {}^{s}\Delta_{k} \quad \text{for some } k.$$

For each scenario the levels of use/consumption of each Common by other citizens must be predicted and fixed for the calculation of a citizen's welfare representation.

**Objective Function.** The *Beings vector (noted $B^z$):* is the image of our variables (*Doings*) in the welfare dimension space. A *being* $b^z_h$ is the "quantity" of how "well" a citizen is on a given *welfare dimension* $h$ and Doings $X^z$. As for Doings, a Beings $B^z$ is a combination of beings of a citizen in a certain functioning $X^z$. The *Beings set* is all the possible Beings that a person can decide to be. The welfare representation noted $\mathcal{W}(\cdot)$ is the set of Beings that are not Pareto dominated in the Beings set. They are considered as the "interesting" Doings that an individual can achieve. If we consider all dimensions of welfare that could affect the citizens' choice, we assume that all individuals would/should choose a solution that is Pareto efficient. For instance, considering Sen's example of a fasting person for political reason, focusing only on two functionings such as *being able to obtain an adequate amount of food* and *being able to be in good heath* is not sufficient to fully understand the citizens' welfare and act. Indeed, fasting is not a solution in the Pareto frontier, because by eating, the citizen should be able to be better in both dimensions. But considering more dimensions of welfare such as *the expression of his/her political opinion*, this person is choosing a solution in the Pareto frontier. In the context of decision aiding, and broadly from an operational point of view, it is clearly impossible to consider all welfare dimensions that could affect a citizens' choice. "Simply" computing the Pareto frontier is therefore an approximation.

The transformation matrix $W^i$ (with $w^i_{j,h} \in W^i$) determines how a Doings $\overline{X}^z$ will be transformed into a Beings $B^z$. These are influenced by conversion factors (personal, social and environmental) and values.

The transformation of actions in different beings are not necessarily independent. For example, going to swim, going cycling or going running may have a good impact on citizen's health, but for a non-athletic person, doing the 3 activities the same day may have a negative impact on his/her health (as he may be injured). Then, the "quantity" of the beings $h$ is converted through the non necessarily linear function $F_h(X, W^i_h)$.

For one Welfare representation, we get: $\forall h \max F_h(X, W^i_h)$.

As for other parameters, the transformation matrix can change regarding the scenario. Then, for a scenario $s$, we have: $\max_{\forall h} \quad \langle {}^{s}f_h(X, {}^{s}W^{i}{}_{h})\rangle$

The schematic representation in Figure 1 summarizes the model showing how the different items interact. Values (personal psychology and history) are interacting with the conversion factors. The personal utilisation functions are derived from the conversion factors, and given the

citizen private resources and access to Commons we can find the Doings set. Then the values and utilisation functions convert Doings into Beings. The transformation of the Doings set gives the Beings set.
From a more MP perspective: private resources and Commons are consumed to achieved some Beings level through the realisation of some Doings. For a given citizen $i$ and scenario $s$, our generic model for the welfare representation is:

$$\begin{array}{llll} \max_{\forall h} & \langle {}^s f_h (X, {}^s W^i{}_h) \rangle & & \\ s.t. & & & \\ & {}^s\Phi^i{}_l(X, {}^s A^i{}_l) & \leq & {}^s R^i{}_l & \forall l \\ & {}^s\Phi^i{}_k(X, {}^s A^i{}_{l+k}) & \leq & {}^s C_k & \text{for some } k \\ & {}^s\Phi^i{}_k(X, {}^s A^i{}_{l+k}) & \leq & {}^s C_k - {}^s\Delta_k & \text{for all other } k \end{array}$$

Finally, note that we have the present state described by $\mathcal{X}_i = (A^i, C, R^i, W^i)$ and a scenario described by $\mathcal{X}_i | senario_s = ({}^sA^i, {}^sC, {}^sR^i, {}^sW^i)$ meaning that "states" are composed of all the relevant data to find citizens' welfare representation, *e.i.* conversion matrix, access to Commons, private resources and transformation matrix.

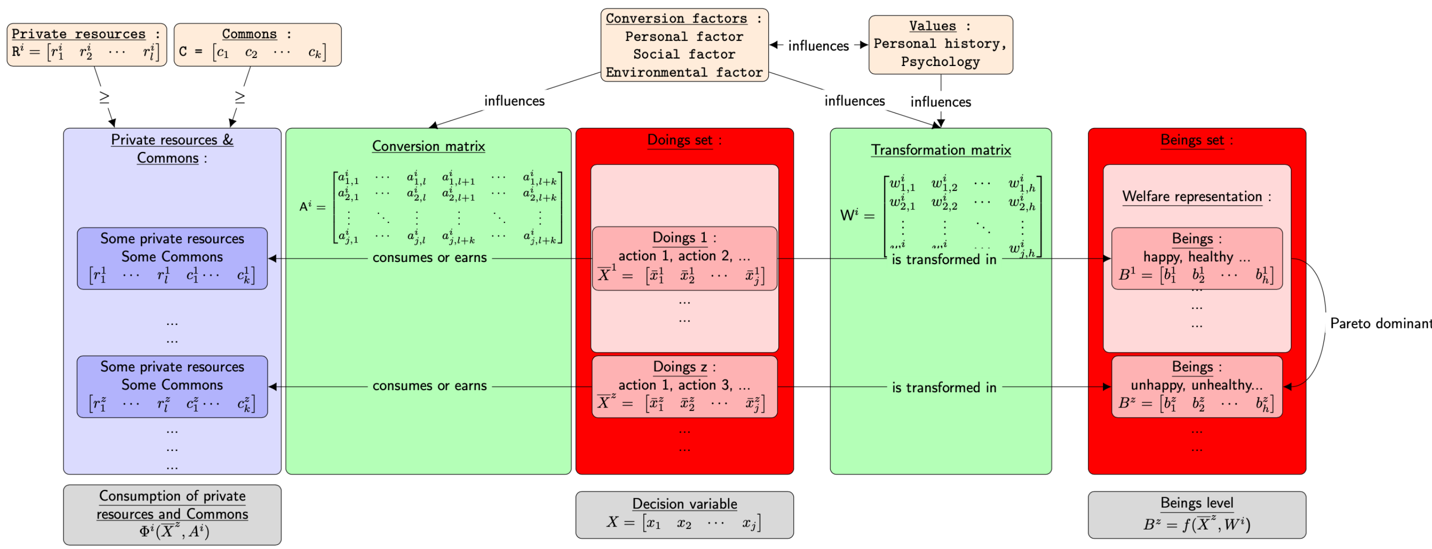


Figure 1: A schematic representation of the model

## 4.2 Construction of the model

A public policy has direct consequences on citizens' welfare by impacting :
- Their *private resources*, e.g. through taxes, flood risk management, help with energy renovation etc.;
- Their access to *Commons*, e.g. through the management of urbanism, water, forests, schools, hospitals, or the creation of laws and rights etc..

The first step of the procedure constructing the model is to identify all the private resources (vector $R^i$) and Commons (vector $C$, $\Delta$) that will be directly affected (created, modified or damaged) by a public policy or by an external event. The modification of resources will directly impact citizens by allowing them to do more/less actions that consume/earn those resources. The analyst must identify all the relevant actions that are directly impacted by the policy/event i.e. actions that consume/earn resources that are modified by the policy. For example, if a city council seeks to study the impact of transforming a brownfield site into a park, citizens will be able to choose to do a number of actions in this park that should be considered in the model, such as running, walking, playing, meeting friends, reading a book, sitting on a bench, breathing good air... The choice of welfare dimensions should be done taking into account both the preferences of the client and the welfare dimensions that seem to be associated with the directly impacted actions. Considering the above example, we can identify two main welfare dimensions; leisure (with running, walking, playing, meeting friends, reading a book, sitting on a bench) and health (with running, walking, playing, breathing good air). The first step of the construction of the model is represented in Figure 2, with the continuous arrows.

The second step of the model is to identify the indirect effects of the public policy. We need to consider a set of activities that have a great effect on the selected welfare dimensions. Continuing our example, if the welfare dimensions are leisure and health, we will have to consider other actions such as; similar activities but in another park, sports activities in a local sport facilities and other leisure activities in places of entertainment. These new actions will consume/earn new private and Common resources, that need to be integrated into the model. In our example; the other park, sport facilities, places of entertainment, money (to be used in places of entertainment) etc.. The second step of the construction of the model is represented by dashed arrows in Figure 2.

Of course, we can add all the activities that use these new resources until one has all the possible activities and resources that affect the well-being of a citizen, (see the dotted arrows of Figure 2). Because it would require infinite time, cognitive and financial resources, the analyst has to stop the procedure when she considers that there is sufficient resources and actions to approximate the problem to be modelled.

## 4.3 Data collection

The values of elements of ${}^sR^i$, ${}^sC^i$, ${}^s\Delta^i$ that have been determined using Section 4.2 and elements of ${}^sA^i{}_l$ (that are the result of $X \times (R+C)$) can be learned using common sense, observation, surveys and experts. Some values of consumption parameters are easy to obtain and the analyst can find parameters by himself. For instance, the cost of the action of "going to the cinema" is the same for people satisfying the same conditions; standard, student, over 60s, job seekers, large families, disabled people... In other cases, the values of consumption parameters and private resources have to be learned using surveys; for instance, it should be explicitly asked whether the citizen has a car, or how long it would take to get to work by bicycle. Finally, experts can also help to find values of consumption parameters, especially when they are impacted by policy-induced

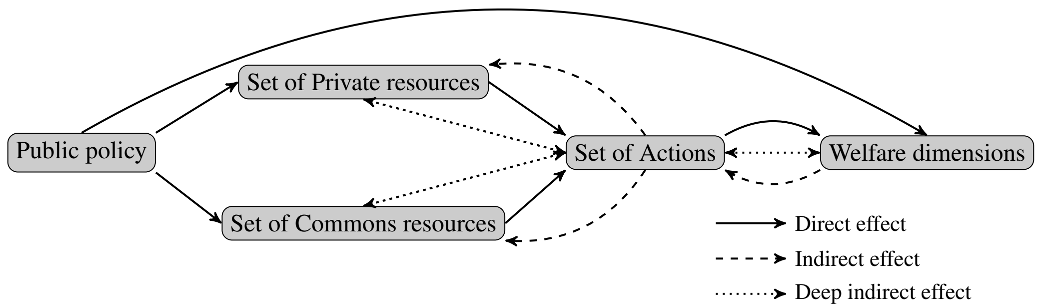


Figure 2: Schematic representation of the model construction.

change. There are many experts that can help predicting values of parameters on different area of public policies; risk analysts, urban planners, biologists, economists, sociologists, mathematicians...

The values of transformation parameters $W^i$ (that is the result of $X \times (R + C)$) needs to be learned for every citizens, the goal being to captures citizens' values and factor of conversions. One way to learn $W^i$ is to use surveys. For instance, if we consider welfare dimensions as independent, a way to learn $W^i$ is to ask questions of the type; *On a scale from -5 to 5 (-5 being I fully disagree, 0 it has no impact and 5 I fully agree)*, this *level of action would have a great positive impact on* that *dimension of welfare ?*

We can also learn values of $W^i$ using multi-attribute value theory (MAVT) (Keeney and Raiffa, 1993; Von Winterfeldt and Edwards, 1986; Jansen, 2011; Dyer, 2016). Using concepts of the MAUT (Jansen, 2011, p. 150) for each welfare dimension, we can use a multi-attribute value model such that: the alternatives will be the Doings, attributes will be the actions, attributes level will be the level of actions and single-attribute utility will be the vector $W^i_h$.

In any case, data collection methods must be developed through the learning received from different real-world applications, good data acquisition can only be achieved through practice guidelines drawn from practitioners' experiences.

## 4.4 Clustering

To be able to use the framework in a process of decision aiding, we need to be able to find clusters of citizens sharing the same values and welfare representation. We also need to be able to find a way to compare different policies from the point of view of (cluster of) individuals. The clustering problem (Zeumo et al., 2014; Fancello and Tsoukiàs, 2020; Fancello et al., 2020) are not considered in this work, but will be investigated in the future.

Essentially there are two ways to cluster citizens: one comparing their Pareto frontiers (their capability sets), the other comparing the value functions through which the Pareto frontiers have been constructed. Once citizens are clustered (and thus, become targets of policies) we can consider the design of ad-hoc policies taking into account the specific characteristics of these clusters.

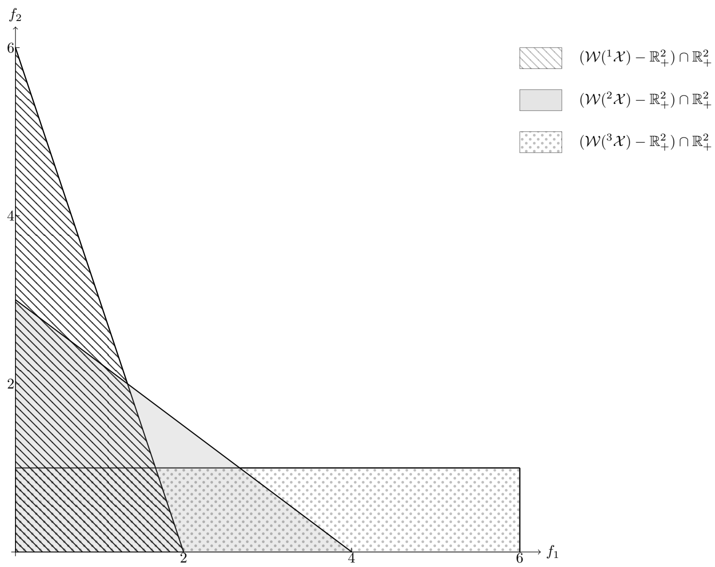


Figure 3: Three different Welfare representations with $|(\mathcal{W}(^i\mathcal{X}) - \mathbb{R}^h_+) \cap \mathbb{R}^h_+| = 6$

## 4.5 Comparison of welfare representations

In order to use this framework, we need to be able to compare welfare representations (*i.e.* Pareto frontiers). This should allow to construct an order on the possible public policies according to their "benefit" on every (cluster of) citizen(s). Pareto frontiers can be easily represented with a graph if only two welfare dimensions are considered. In this case, the client can compare different welfare representations without necessarily using a formal rule, as shown in section 5.

If the number of welfare dimensions is greater than two, the portrayal of welfare representation will by much more difficult and we will need to use a formal comparison procedure. It is clear that one Pareto frontier is preferred to another if for every solution of the latter, there is a solution of the former that is preferred to it, in other word, if the first is "above" the second. Using this simple rule can be too restrictive and can lead to a partial ordering with potentially many incomparable solutions. Another solution comparing two welfare representations consists in comparing the size of the sets they dominate in the positive part of the welfare space (above the axes).

$$\Delta(\mathcal{W}(^a\mathcal{X}), \mathcal{W}(^b\mathcal{X})) = |(\mathcal{W}(^a\mathcal{X}) - \mathbb{R}^h_+) \cap \mathbb{R}^h_+| - |(\mathcal{W}(^b\mathcal{X}) - \mathbb{R}^h_+) \cap \mathbb{R}^h_+|$$

With $\Delta(\mathcal{W}(^a\mathcal{X}), \mathcal{W}(^b\mathcal{X}))$ is $\begin{cases} \text{greater than } 0 \leftrightarrow \mathcal{W}(^a\mathcal{X}) \succ \mathcal{W}(^b\mathcal{X}) \\ \text{equal that } 0 \leftrightarrow \mathcal{W}(^a\mathcal{X}) \sim \mathcal{W}(^b\mathcal{X}) \\ \text{lower that } 0 \leftrightarrow \mathcal{W}(^a\mathcal{X}) \prec \mathcal{W}(^b\mathcal{X}) \end{cases}$

This solution is consistent with the *preferred if "above" rule* but it doesn't really capture the distribution of Welfare representations. Figure 3 shows three different welfare representations that will be ordered as indifferent because they dominate the same "amount of space", without taking into account that it is not the same areas of the welfare space.

New comparison methods need to be developed to compare Pareto frontier, there are two main aspects to pay attention to when comparing welfare representations; the general level of welfare and its distribution over the different dimensions. Intuitively, we can think to use the stochastic and Lorenz dominance approach. The First Degree Stochastic Dominance (FSD) (Levy, 2016, Chapter 3) correspond to our previous simple rule where a Pareto frontier dominates another one if it is "above" it. The Second Degree Stochastic Dominance (SSD) deals both with the mean and "dispersion" of the scope. In the same way, the general Lorenz curve (Shorrocks, 1983) dominance capture both mean and distribution of curves to order them. Those concepts cannot be directly applied to our problem because they are dealing with *cumulative* distribution functions, but their similarities had to be noticed and can be helpful to our problem.

An interesting line of research is the literature on ranking sets of objects (See Foster 2011; Barberà et al. 2004; Pattanaik and Xu 1990, 2000a,b; Gaertner 2012). Authors think that the most relevant method to assess Pareto frontiers has been developed by Gaertner and Xu (2006, 2008, 2011). First, the idea is to chose a point $x^0$ in $\mathbb{R}^h_+$, that will help us to give a score for each Pareto frontier. Then, we need to define a function of distance between two points $d(x, y)$, for instance we can use the Euclidean distance $d(x, y) = \sqrt{\sum_{i=1}^{h}(x_i - y_i)^2}$. Let define $\succeq$ such that $x, y \in \mathbb{R}^h$, $x \succeq y \leftrightarrow x_i \geq y_i \forall i = \{1, \dots h\}$, we can adapt[2] the Gaertner-Xu method to determine the "score" of a welfare representation as;

$$r(\mathcal{W}(^s\mathcal{X}_i), x^0) =$$

$$\begin{cases} -min\{d(b-\epsilon, x^0) \mid b \in \mathcal{W}(^s\mathcal{X}_i),\ \epsilon \in \mathbb{R}^h_+\} \text{ if } \{\nexists b \in \mathcal{W}(^s\mathcal{X}_i) \mid b \succeq x^0\} \\ max\{t \mid \forall\, b \in \mathcal{W}(^s\mathcal{X}_i)\ \cap \forall \epsilon \in \mathbb{R}^h_+\ s.t.\ b - \epsilon \succeq x^0; d(b-\epsilon, x^0) \leq t]\} \text{ if } \{\exists b \in \mathcal{W}(^s\mathcal{X}_i) \mid b \succeq x^0\}. \end{cases}$$

Figure 4 shows an example of the determination of the score of four welfare representations using the Gaertner-Xu like method with $x^0 = (4, 4)$. We would obtain the order $\mathcal{W}(^1\mathcal{X}_i) \prec \mathcal{W}(^2\mathcal{X}_i) \prec \mathcal{W}(^3\mathcal{X}_i) \prec \mathcal{W}(^4\mathcal{X}_i)$ with $\mathcal{W}(^1\mathcal{X}_i) = -2.69$, $\mathcal{W}(^2\mathcal{X}_i) = -1.5$, $\mathcal{W}(^3\mathcal{X}_i) = 1$, $\mathcal{W}(^4\mathcal{X}_i) = 2$

A generalized version using cones instead of points is developed in Gaertner and Xu (2008), but the simple unique point formulation is sufficient to understand the main idea of the method and following remarks are also true for the cone-method.

First, $x^0$ can be hard to define/find. Gaertner and Xu (2006, 2008) deals with poverty identification and $x^0$ is supposed to be the individual's standard of living judged as "poor", we are not only concerned with poverty identification, and we could define $x^0$ as an average point or an objective point.

Secondly, it can be seen as a pessimistic procedure if $x^0$ is "bellow" the welfare representation and an optimistic one if it is "above". Indeed, in the case where $x^0$ is "above" the Pareto frontier, its score is determined by the closest point that is weakly dominated by $\mathcal{W}(^s\mathcal{X}_i)$ from $x^0$, all other points that are weakly dominated by $\mathcal{W}(^s\mathcal{X}_i)$ and $x^0$ would have a worse score. On the other hand, if $x^0$ is below some part of $\mathcal{W}(^s\mathcal{X}_i)$, the score is equal to the closest point on the frontier of weakly dominated solutions, but solutions that dominates $x^0$ can exist with a higher score. We can formulate other rules, for instance an optimistic version of the procedure can be define as following:

$$r(\mathcal{W}(^s\mathcal{X}_i), x^0) = \begin{cases} -min\{d(b-\epsilon, x^0) \mid b \in \mathcal{W}(^s\mathcal{X}_i),\ \epsilon \in \mathbb{R}^h_+\} \text{ if } \{\nexists b \in \mathcal{W}(^s\mathcal{X}_i) \mid b \succeq x^0\}, \\ max\{d(b, x^0) \mid b \in \mathcal{W}(^s\mathcal{X}_i)\ \cap\ b \succeq x^0\} \text{ if } \{\exists b \in \mathcal{W}(^s\mathcal{X}_i) \mid b \succeq x^0\}. \end{cases}$$

and a pessimist approach:

$$r(\mathcal{W}(^s\mathcal{X}_i), x^0) =$$

[2] The Gaertner-Xu method deals with sets that are compact, convex or star-sharped. We have adapted the procedure so that it works with our sets. We do not guarantee that our adapted model satisfies all the axiom of Gaertner and Xu (2008).

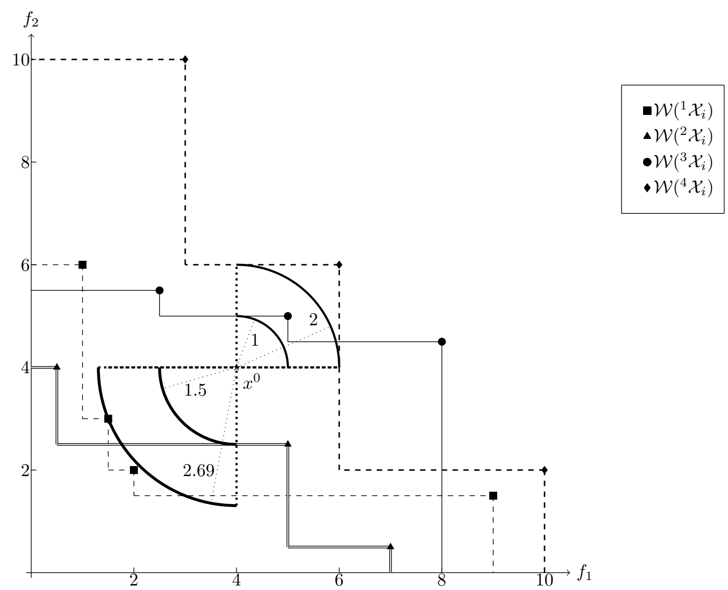


Figure 4: An example of the determination of scores from welfare representation using Gaertner-Xu like method.

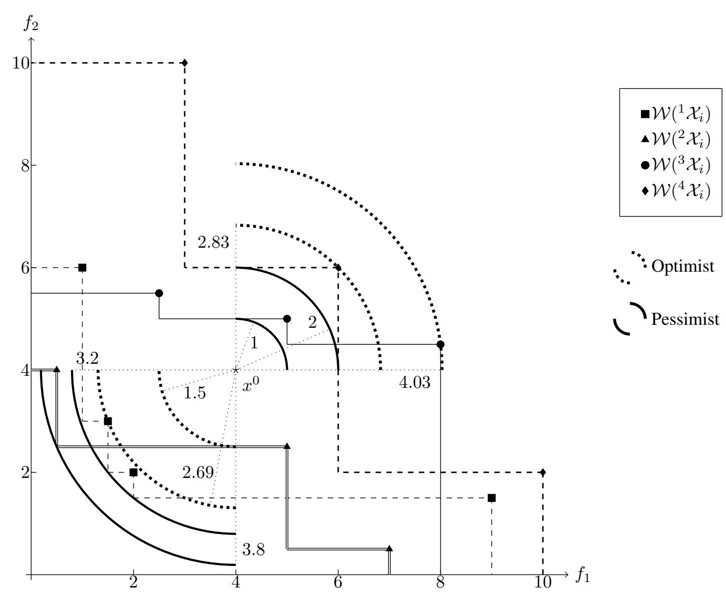


Figure 5: An example of the determination of score from welfare representation using Gaertner-Xu like method.

$$\begin{cases} -min\{t \mid \forall\, b \in \mathcal{W}(^s\mathcal{X}_i)\ \cap \epsilon \in \mathbb{R}^h_+,\ s.t.\, b - \epsilon \preceq x^0; d(b-\epsilon, x^0) \geq t\} \text{if } \{\exists b \in \mathcal{W}(^s\mathcal{X}_i) \mid b \succeq x^0\}. \\ \ max\{t \mid \forall\, b \in \mathcal{W}(^s\mathcal{X}_i)\ \cap \forall \epsilon \in \mathbb{R}^h_+\ s.t.\, b - \epsilon \succeq x^0; d(b-\epsilon, x^0) \leq t]\} \text{ if } \{\exists b \in \mathcal{W}(^s\mathcal{X}_i) \mid b \succeq x^0\}. \end{cases}$$

Using the same Pareto frontier and $x^0$ than in the previous example, we obtain different orders and scores, as illustrated in Figure 5, the order in the case of the optimist procedure it is: $\mathcal{W}(^3\mathcal{X}_i) \succ \mathcal{W}(^4\mathcal{X}_i) \succ \mathcal{W}(^2\mathcal{X}_i) \succ \mathcal{W}(^1\mathcal{X}_i)$ (with $\mathcal{W}(^1\mathcal{X}_i) = -2.69$, $\mathcal{W}(^2\mathcal{X}_i) = -1.5$, $\mathcal{W}(^3\mathcal{X}_i) = 4.03$, $\mathcal{W}(^4\mathcal{X}_i) = 2.83$) and in the pessimist procedure is: $\mathcal{W}(^4\mathcal{X}_i) \succ \mathcal{W}(^3\mathcal{X}_i) \succ \mathcal{W}(^1\mathcal{X}_i) \succ \mathcal{W}(^2\mathcal{X}_i)$ (with $\mathcal{W}(^1\mathcal{X}_i) = -3.2$, $\mathcal{W}(^2\mathcal{X}_i) = -3.8$, $\mathcal{W}(^3\mathcal{X}_i) = 1$, $\mathcal{W}(^4\mathcal{X}_i) = 2$). In all cases, the distribution of the welfare representation is not really taken into account. We only look at the extreme solutions; those closest or furthest from $x^0$. We need further investigation to develop a framework that improves how the "general" distribution of the scope is taken into account.

Finally, there is a risk that the clients only use the score. One of the goal of our model is to give a rich picture of the present and possible state of the world. Using a tool that reduce this information to a single figure can lead the client to jump to that score, ignoring the complexity of the problem.

We stop here with the question of welfare representation comparisons in this paper. This issue is of significant interest and requires further work (in particular axiomatic work), but we believe we have shown that comparison procedures can exist beyond the case where one border is "above" the other, in particular in the context of research on the comparison of set objects. In the following section we present an example on how to use our model.

# 5 Example

This example is constructed in order to demonstrate how to use our framework in the context of welfare measurement. More precisely, the example is constructed in order to show how to model the influence of the impact of the Commons upon the citizens' welfare. Considering public policies aiming at protecting Commons, being able to identify crucial Commons and measure their impact (on citizens' welfare) is a valuable step into the design and rationalisation of a public policy decision. Measuring the impact of a Common's damage can represent the impact of public policies that consist to "do not maintain", "remove" or "shut down" a Common. We want to represent the welfare of two individuals: Alice and Bob, that are in the current states $\mathcal{X}_A$ and $\mathcal{X}_B$. They are living in the same city, and have the same representation of it, through the graph $G(V, E)$ Figure 6:

- V: The set of vertices, that represent points of opportunities (*i.e. 1: Home, 2: Park, 3: City center, 4: Work place*). The two citizens are living in the same neighbourhood 1, and are both working in 4. On each vertex, citizens can perform actions, that can be derived in beings through the transformation matrix $W^i$ (see table 3 and 4 in Annex 1). Those transformations parameters are personal for each citizen, and depend both on the citizens' conversion factors and values. An action consumes private resources and Commons $A^i$ (see table 3 and 4 in Annex 1), these consumptions are different regarding only the conversion factors.

- E: The set of road $e_{vwr}$ and public transport (PT) $e_{vwp}$ that links two vertices $vw$. Both are considered as Commons. To make the example simpler, we consider that the transformation into beings of the utilisation of a road (resp. PT) and the consumption of private resources

| Citizen | Money | Time in hours | Car in hours |
|---|---|---|---|
| Alice | 0 | 24 | 24 |
| Bob | 0 | 24 | 0 |

Table 2: Vectors of private resources

are the same regardless of the road (resp. PT) the citizen uses (see table 3 and 4 in Annex 1). The utilisation of these edges are considered as actions.

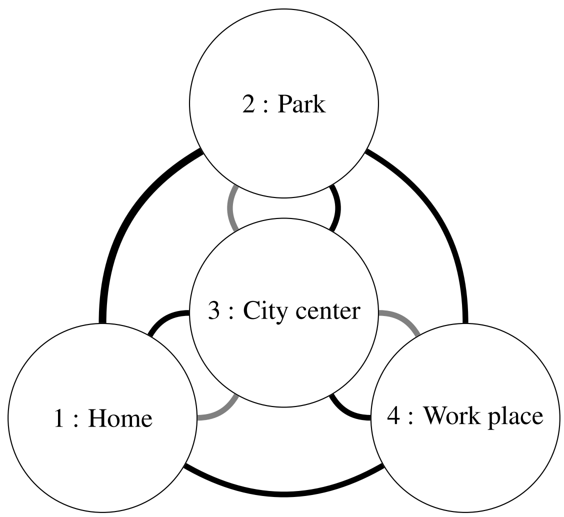


Figure 6: Graph of the city of Alice and Bob, the arcs in gray represent public transports and arcs in black represent roads.

Each citizen has a quantity of private resources $A^i = \{A_1^i = \text{Money}, A_2^i = \text{Time}, A_3^i = \text{Car}\}$, described in table 2. The goal of a citizen is to maximise his/her Beings ($f_h^i$ = Health,$f_p^i$ = Pleasure) through his/her transformation functions. The reader should note that, strictly speaking, Health and Pleasure are welfare dimensions. The measure of the citizen's "ability" to take profit of these dimensions is a "measure" of the citizen's capability. For the sake of simplicity we use the terms "Health" and "Pleasure" as if these were capabilities. The solutions obtained will establish the Pareto frontier representing the citizen $i$; the Welfare representation $\mathcal{W}(\mathcal{X}_i)$.

The Welfare representations $\mathcal{W}(\mathcal{X}_a)$ and $\mathcal{W}(\mathcal{X}_b)$ are graphically represented in Figure 7. To obtain these solutions, we solved two linear programs; Table 5 in Annex 1 displays the linear program corresponding to $\mathcal{W}(\mathcal{X}_a)$.

We can now simulate how the citizens' welfare is impacted by a damage to a Common. The associated parameter is turned to 0 (this resource being not accessible any more; for the sake of simplicity we will not change other parameters). We solve the new LP; for instance Table 6 in Annex 1 displays the LP used in order to obtain the new welfare representation of Alice after a damage to the park. Figure 8 represents the welfare representation of Alice $W(\mathcal{X}_a|Road_{14})$ and Bob $W(\mathcal{X}_b|Road_{14})$ after the closure of the road between their 1:home and their 4:work place. Figure 9 represents the welfare of Alice $W(\mathcal{X}_a|park)$ and Bob $W(\mathcal{X}_b|park)$ after a damage to the park.

These two figures allow a simple analysis of the negative impact of a Common being not available. For instance, the closure to the Road linking 1 to 4 does not have a high negative impact on citizens' Welfare representation. Indeed, it does not affect the Welfare representation of Bob (as he has no car), and the possible Beings of Alice are "decreased" but are still close to what they were. On the other hand, the damage to the park can be seen as having a high impact, specially because of the impact upon health. The Beings with a high impact upon Health are not achievable anymore.

Now let's consider the negative impact of the same Commons damage using a (simplified) utilitarian analysis (using CBA). According to Metz (2008), 80% of the benefits of a road come from its time saving (other benefit will be ignored); in our example, Alice uses $r_{1,4}$ around 8 times a week (and Bob never use it), which is a general use of the road of an average of 1.5 times a day (and 45 minutes/day) for the considered population (Alice and Bob). If the road 1 to 4 is closed it is necessary to go through the park or the city center, wasting 30 minutes compared to the direct road, which is an average waste of 45 minutes/day ($1.5 \times$ 30min). We suppose that the appraisal values for time saving (or time losing) for not working time is about 0.11 euro/minute (UK's Department for Transport (2015) considers it as 0.11 pound/minute). So the general lose is equal to 3.77 euros/day (time wasted $\times$ values for times losing).

Considering the impact of the park's damage, we can find a proxy market for its price evaluation. The main benefits of the park are linked to health, by allowing to practice physical activities and by cleaning the air. We can choose the price of gym pass as proxy for physical activities and the amount of carbon capture by the park times the carbon emissions price of the European Union Emissions Trading System (EU ETS[3]) as proxy for the air quality. We obtain a benefit of approximately 3.75 euro/day.

For the CBA we used the value of the time for the road, and health and entertainment/pleasure for the park. We can note that all such dimensions are present in our model; as welfare dimensions for health and pleasure and as constraints for time. Time is seen as a constraint because the question is not how many time (or free time) we have in a day, but rather how we use it.

In this simple example, we show that our capability based approach can be more useful to a decision maker than an utilitarian approach. Indeed, using a CBA the loss of the road and the loss of the park seem to have the same negative impact. Using the CA instead we are able to identify that :

-Bob will not be impacted by the road's closure whereas Alice will.

-The park's damage seems to have a greater negative impact on citizens welfare.

-The park's damage will mainly impact citizens on their ability to achieve better health.

The reader will note that the CBA is just an archetype of an utilitarian approach to welfare measurement: the reasons for which the CBA will result in less interesting conclusions (compensation among impacts and considering all citizens as undistinguishable consumers) apply to any other utilitarian welfare measurement approach.

# Conclusions

A common rational argument justifying several public policies or their designs is that, if and when implemented (these policies), the welfare of the citizens (at least some of them) will improve. However, little is specifically said as far as the measurement of welfare is concerned in order to

[3] https://ec.europa.eu/clima/policies/ets_en

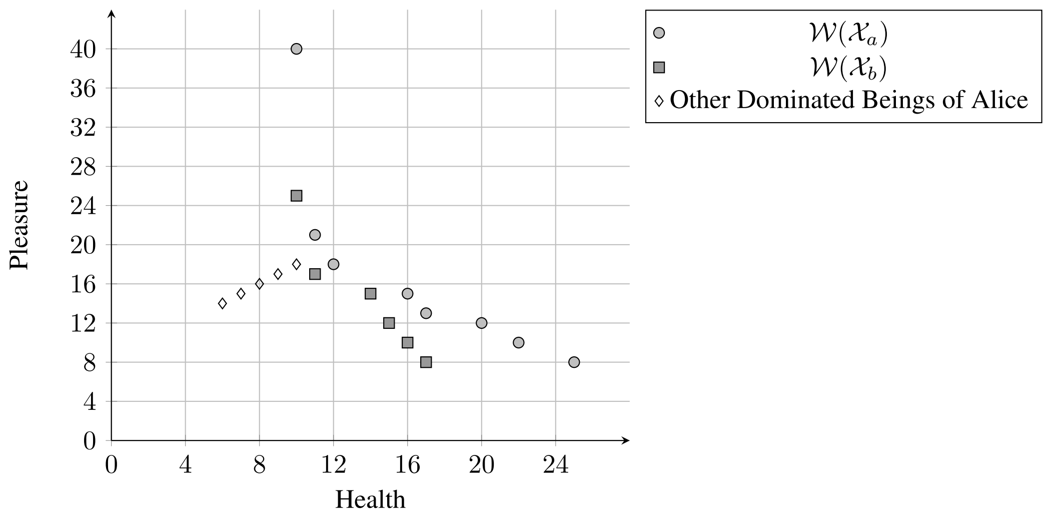


Figure 7: Citizens Welfare representation in the initial state.

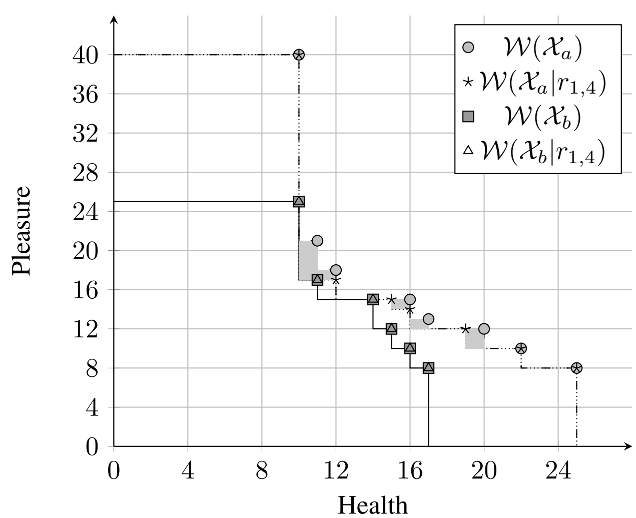


Figure 8: Impact of the $r_{1,4}$'s closure

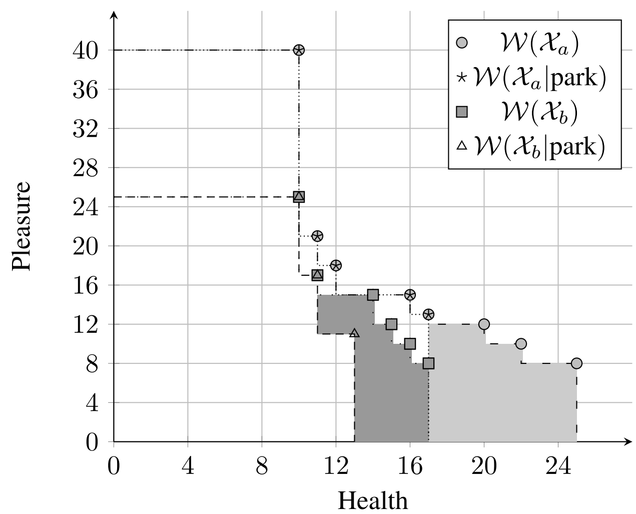


Figure 9: Impact of the park's destruction

be able to check whether such "predictions" (or promises) are well founded and sound. Welfare is measured either through simple figures (such as the GDP) or when specific projects are to be assessed welfare is practically considered as the sum of the utilities of undistinguished consumers.

Such measures allow to establish a common discussion ground for the different stakeholders and provide some rational justification, but conceal important differences among the impacts of policies to different categories of citizens. The aim of this paper is to outline an alternative framework allowing to construct sound assessments (about the modification of welfare) which should also be of more practical interest, specifically improving and expanding the design space.

For this purpose we first stress the importance to have a rational framework for decision aiding for public policy design. We do a small survey of how welfare has been considered within different approaches and we focus specifically to the so called Capability Approach introduced by A. Sen in the 80s. The advantage of this approach consists in considering explicitly the subjective difference of how welfare is constructed and perceived by the citizens through their acts and beings and not just because of their endowments. However, the capability approach, despite opening interesting theoretical opportunities and despite some fine applications it is far from being a really operational tool which can be generally used in order to model and assess welfare for some decisions making and aiding purposes.

For this reason we suggest a mathematical programming formulation of a single citizen's welfare given his/her private endowments, access to a set of "Commons" and considering a set of potential actions and different welfare dimensions relevant or interesting (for that citizen). We show through an example that our method can be more meaningful and useful than the usual utilitarian approaches (today considered the golden standard as far as public policy assessment is considered).

This suggestion opens two interesting directions. On the one hand we can cluster citizens on the basis of the similarity of their capability sets (these being the Pareto frontiers of their welfare) obtaining more interesting and efficient policy targets identification. On the other hand we can simulate different scenarios of policies where private endowments and/or the Commons can be modified. Such simulations should provide fundamental rational insight as far as the design of public policies is concerned.

The paper introduces a conceptual innovation showing the interest in being inspired by Sen's Capability Approach and identifies some operational suggestions on how to use it. There are several open questions which need further investigation:
- how to learn the citizens' values?
- how to compare welfare distributions beyond the useless dominance relation?
- how to introduce scenarios and likelihoods in order to take into account the consequences of uncertain events?

These (and many other) are essential theoretical and practical questions in order to establish a solid framework, but not answered in this paper.

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# Annex 1

| Var | Activity | Transformation matrix $W^a$ | | Conversion matrix $A^a$ on private resource | | | Conversion matrix $A^a$ on Commons | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Health | Pleasure | Money | Time | Car | Road $ij$ | PT $ij$ | Park |
| $x_{ijr}$ | Take the road $ij$ | 0 | -1 | 2 | 0.5 | 0.5 | $\epsilon$ | 0 | 0 |
| $x_{ijc}$ | Take the pubic transport $ij$ | 0 | -2 | 2 | 0.5 | 0.5 | 0 | $\epsilon$ | 0 |
| $y_{11}$ | Sleep | 10 | 10 | 0 | 9 | 0 | 0 | 0 | 0 |
| $y_{12}$ | Family Time | 0 | 2 | 0 | 1 | 0 | 0 | 0 | 0 |
| $y_{21}$ | Walk | 3 | 2 | 0 | 2 | 0 | 0 | 0 | $\epsilon$ |
| $y_{22}$ | Run | 6 | -2 | 0 | 1 | 0 | 0 | 0 | $\epsilon$ |
| $y_{31}$ | Museum | 1 | 6 | 10 | 3 | 0 | 0 | 0 | 0 |
| $y_{32}$ | After work | -1 | 1 | 5 | 1 | 0 | 0 | 0 | 0 |
| $y_{33}$ | Doctor | 5 | -1 | 30 | 1 | 0 | 0 | 0 | 0 |
| $y_{41}$ | Work | 1 | 1 | -100 | 8 | 0 | 0 | 0 | 0 |

Table 3: Table of the conversion and transformation matrix of Alice

| Var | Activity | Transformation Matrix $W^b$ | | Conversion matrix $A^b$ on private resources | | | Conversion matrix $A^b$ on Commons | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Health | Pleasure | Money | Time | Car | Road $ij$ | PT $ij$ | Park |
| $x_{ijr}$ | Take the road $ij$ | 0 | 0 | 2 | 0.5 | 0.5 | $\epsilon$ | 0 | 0 |
| $x_{ijc}$ | Take the pubic transport $ij$ | 0 | -1 | 2 | 0.5 | 0.5 | 0 | $\epsilon$ | 0 |
| $y_{11}$ | Sleep | 10 | 10 | 0 | 9 | 0 | 0 | 0 | 0 |
| $y_{12}$ | Family Time | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| $y_{21}$ | Walk | 1 | 2 | 0 | 2 | 0 | 0 | 0 | $\epsilon$ |
| $y_{22}$ | Run | 3 | 4 | 0 | 1 | 0 | 0 | 0 | $\epsilon$ |
| $y_{31}$ | Museum | 0 | 2 | 10 | 3 | 0 | 0 | 0 | 0 |
| $y_{32}$ | After work | 0 | 2 | 5 | 1 | 0 | 0 | 0 | 0 |
| $y_{33}$ | Doctor | 2 | -3 | 30 | 1 | 0 | 0 | 0 | 0 |
| $y_{41}$ | Work | 1 | 1 | -100 | 8 | 0 | 0 | 0 | 0 |

Table 4: Table of the conversion and transformation matrix of Bob

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| max | ( | | $10y_{11}$ | | $+3y_{21}$ | $+6y_{22}$ | $+y_{31}$ | $-y_{32}$ | $5y_{33}$ | $+y_{41})$ | | |
| max | $(-\sum_{ijr\in E} x_{ijr}$ | $-\sum_{ijc\in E} 2x_{ijc}$ | $+10y_{11}$ | $+2y_{12}$ | $+2y_{21}$ | $-2y_{22}$ | $+6y_{31}$ | $+y_{32}$ | $-2y_{33}$ | $+y_{41})$ | | |
| S.T. | $2\sum_{ijr\in E} x_{ijr}$ | $+2\sum_{ijc\in E} x_{ijc}$ | | | | | $+10y_{31}$ | $+5y_{32}$ | $+30y_{33}$ | $-100y_{41}$ | $\leq$ | 0 |
| | $\frac{1}{2}\sum_{ijr\in E} x_{ijr}$ | $+\frac{1}{2}\sum_{ijc\in E} 2x_{ijc}$ | $+9y_{11}$ | $+y_{12}$ | $+2y_{21}$ | $+2y_{22}$ | $+3y_{31}$ | $+y_{32}$ | $+y_{33}$ | $+8y_{41}$ | $\leq$ | 24 |
| | $\frac{1}{2}\sum_{ijr\in E} x_{ijr}$ | $+\frac{1}{2}\sum_{ijc\in E} 2x_{ijc}$ | | | | | | | | | $\leq$ | 24 |
| | $\epsilon x_{ijr}$ | | | | | | | | | | $\leq$ | $1\ \forall ijr \in E$ |
| | | $\epsilon x_{ijc}$ | | | | | | | | | $\leq$ | $1\ \forall ijc \in E$ |
| | | | $y_{11}$ | $+y_{12}$ | | | | | | | $\leq$ | $M(\sum_{i1r\in E} + \sum_{i1c\in E})$ |
| | | | | | $y_{21}$ | $+y_{22}$ | | | | | $\leq$ | $M(\sum_{i2r\in E} x_{i2r} + \sum_{i2c\in E} x_{i2c})$ |
| | | | | | | | $y_{31}$ | $+y_{32}$ | $+y_{33}$ | | $\leq$ | $M(\sum_{i3r\in E} x_{i3r} + \sum_{i3c\in E} x_{i3c})$ |
| | | | | | | | | | | $+y_{41}$ | $\leq$ | $M(\sum_{i4r\in E} x_{i4r} + \sum_{i4c\in E} x_{i4c})$ |
| | $\sum_{i1r\in E} x_{i1r} - \sum_{1jr\in E} x_{1jr}$ | $+\sum_{i1c\in E} x_{i1c} - \sum_{1jr\in E} x_{1jc}$ | | | | | | | | | $=$ | 0 |
| | $\sum_{i2r\in E} x_{i2r} - \sum_{2jr\in E} x_{2jr}$ | $+\sum_{i2c\in E} x_{i2c} - \sum_{2jr\in E} x_{2jc}$ | | | | | | | | | $=$ | 0 |
| | $\sum_{i3r\in E} x_{i3r} - \sum_{3jr\in E} x_{3jr}$ | $+\sum_{i3c\in E} x_{i3c} - \sum_{3jr\in E} x_{3jc}$ | | | | | | | | | $=$ | 0 |
| | $\sum_{i4r\in E} x_{i4r} - \sum_{4jr\in E} x_{4jr}$ | $+\sum_{i4c\in E} x_{i4c} - \sum_{4jr\in E} x_{4jc}$ | | | | | | | | | $= x$ | 0 |
| | $x_{ijr}$, | $x_{ijc}$ | | | | | | | | | $\in$ | $\mathcal{N}, \forall ijr, ijc \in E$ |
| | | | $y_{11}$, | | $y_{21}$, | $y_{22}$, | $y_{31}$, | | $y_{33}$, | $y_{41}$ | $\in$ | $\{0,1\}$ |
| | | | | $y_{12}$, | | | | $y_{32}$ | | | $\in$ | $\mathcal{N}$ |

Table 5: LP for $\mathcal{X}_A$

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| max | ( | | $10y_{11}$ | | $+3y_{21}$ | $+6y_{22}$ | $+y_{31}$ | $-y_{32}$ | $5y_{33}$ | $+y_{41})$ | | |
| max | $(-\sum_{ijr\in E} x_{ijr}$ | $-\sum_{ijc\in E} 2x_{ijc}$ | $+10y_{11}$ | $+2y_{12}$ | $+2y_{21}$ | $-2y_{22}$ | $+6y_{31}$ | $+y_{32}$ | $-2y_{33}$ | $+y_{41})$ | | |
| S.T. | $2\sum_{ijr\in E} x_{ijr}$ | $+2\sum_{ijc\in E} x_{ijc}$ | | | | | $+10y_{31}$ | $+5y_{32}$ | $+30y_{33}$ | $-100y_{41}$ | $\leq$ | 0 |
| | $\frac{1}{2}\sum_{ijr\in E} x_{ijr}$ | $+\frac{1}{2}\sum_{ijc\in E} 2x_{ijc}$ | $+9y_{11}$ | $+y_{12}$ | $+2y_{21}$ | $+2y_{22}$ | $+3y_{31}$ | $+y_{32}$ | $+y_{33}$ | $+8y_{41}$ | $\leq$ | 24 |
| | $\frac{1}{2}\sum_{ijr\in E} x_{ijr}$ | $+\frac{1}{2}\sum_{ijc\in E} 2x_{ijc}$ | | | | | | | | | $\leq$ | 24 |
| | $\epsilon x_{ijr}$ | | | | | | | | | | $\leq$ | $1\ \forall ijr \in E$ |
| | | $\epsilon x_{ijc}$ | | | | | | | | | $\leq$ | $1\ \forall ijc \in E$ |
| | | | | | $\epsilon y_{21}$ | $+\epsilon y_{22}$ | | | | | $\leq$ | 0 |
| | | | $y_{11}$ | $+y_{12}$ | | | | | | | $\leq$ | $M(\sum_{i1r\in E} + \sum_{i1c\in E})$ |
| | | | | | $y_{21}$ | $+y_{22}$ | | | | | $\leq$ | $M(\sum_{i2r\in E} x_{i2r} + \sum_{i2c\in E} x_{i2c})$ |
| | | | | | | | $y_{31}$ | $+y_{32}$ | $+y_{33}$ | | $\leq$ | $M(\sum_{i3r\in E} x_{i3r} + \sum_{i3c\in E} x_{i3c})$ |
| | | | | | | | | | | $+y_{41}$ | $\leq$ | $M(\sum_{i4r\in E} x_{i4r} + \sum_{i4c\in E} x_{i4c})$ |
| | $\sum_{i1r\in E} x_{i1r} - \sum_{1jr\in E} x_{1jr}$ | $+\sum_{i1c\in E} x_{i1c} - \sum_{1jr\in E} x_{1jc}$ | | | | | | | | | $=$ | 0 |
| | $\sum_{i2r\in E} x_{i2r} - \sum_{2jr\in E} x_{2jr}$ | $+\sum_{i2c\in E} x_{i2c} - \sum_{2jr\in E} x_{2jc}$ | | | | | | | | | $=$ | 0 |
| | $\sum_{i3r\in E} x_{i3r} - \sum_{3jr\in E} x_{3jr}$ | $+\sum_{i3c\in E} x_{i3c} - \sum_{3jr\in E} x_{3jc}$ | | | | | | | | | $=$ | 0 |
| | $\sum_{i4r\in E} x_{i4r} - \sum_{4jr\in E} x_{4jr}$ | $+\sum_{i4c\in E} x_{i4c} - \sum_{4jr\in E} x_{4jc}$ | | | | | | | | | $=$ | 0 |
| | $x_{ijr}$, | $x_{ijc}$ | | | | | | | | | $\in$ | $\mathcal{N}, \forall ijr, ijc \in E$ |
| | | | $y_{11}$, | | $y_{21}$, | $y_{22}$, | $y_{31}$, | | $y_{33}$, | $y_{41}$ | $\in$ | $\{0,1\}$ |
| | | | | $y_{12}$, | | | | $y_{32}$ | | | $\in$ | $\mathcal{N}$ |

Table 6: LP for $\mathcal{X}_A|park$