A survey on Multidimensional Poverty Measurement: a Decision Aiding Perspective

[1,2]**Vivien Kana,** [2]**Blaise Somé** and [1]**Alexis Tsoukiàs**

[1]LAMSADE, Université Paris Dauphine, France.
[2]LANIBIO, Université de Ouagadougou, Burkina Faso.

vivien.kana-zeumo@dauphine.fr
tsoukias@lamsade.dauphine.fr
some@univ-ouaga.bf

Abstract

Poverty reduction policies has often underestimated the need to define poverty as a multidimensional concept and the necessity to use multidimensional approaches to measure it. Under such a perspective research has been directed towards finding solutions to the main problems that we face when considering poverty as a multidimensional concept. This paper provides a review of literature on the recent developments in the framework of multidimensional poverty analysis. It also highlights the various approaches of multidimensional poverty measurement that can be applied to provide more accurate descriptions of poverty trends to the typical users of such statistics (such as policy analysts and policymakers) and their limitations. At the end of this paper, we consider factors other than evidence that influence policy making and policy implementation. We also introduce the concept of decision aiding and meaningful measurement. The issue of meaningfulness is thus analysed both from a theoretical point of view (measurement theory) and from an operational one (policy effectiveness).

1 Introduction

Measuring poverty is expected to be a fundamental tool in fighting against poverty, considered to be a plague of humanity. The aim of this paper is to survey how the concept of "poverty" and "poverty measurement" evolved in time and which are the different approaches established under a quest for scientific support in fighting poverty. The general perspective adopted in presenting this survey has been the one of "decision aiding": if poverty has to be measured this is because such measures are expected to be used in deciding appropriate poverty reduction policies. Under such a perspective we are going to analyse both the meaningfulness and effectiveness of how poverty can be measured.

A common method used to measure poverty is based on incomes or consumption levels: which means that an individual or household is considered as poor if his income or consumption level falls below some minimum level (poverty threshold or poverty line) necessary to meet his basic needs. However, the diversity of the reasons of poverty and the complexity of its consequences oppose a logical resistance to such an approach. The concept of poverty can be explained in sociology, but the traditional mathematical tools cannot apprehend the real level of this plague. Indeed, its multidimensional aspect, its fuzzy character and the possible substitutions between the different forms it can take, make the research and the monitoring of the explanatory factors more complex. Multidimensional poverty measurement enables us to explore which deprivations individual experience at the same time. It allows to identify such different situation of poverty.

The paper is organised as follows: Origins of poverty with its conceptual foundations and foundations of poverty measurement are briefly explored in Section 2. In Section 3 the questions "what is poverty?" and "why to measure poverty?" are discussed, while Section 4 presents our notation, some formal definitions of measurement of poverty and various approaches to multidimensional poverty measurement. The concepts of decision aiding and their contributions in poverty measurement are introduced in Section 5 while concluding comments are given in Section 6.

2 Some history

2.1 Origins and Conceptual Foundations of Poverty

Poverty is not a modern-day phenomenon. It has existed for centuries and continues nowadays to rage in many countries of the world. The concept of poverty takes its roots in social ethics, which can be regarded as a central part of political philosophy. Facets of poverty throughout the story are well known and visible from the athenian society from which Socrates, Plato, Aristotle and Xenophon (Sanderson 2004) have argued about poverty and wealth, to our era where parliamentarians and international organisations deal with portfolios of actions against poverty. Since the late Middle Ages, the fight against poverty followed different approaches until the dawn of the industrial revolution without the appearance of a *real* solution to this plague.

From 1500 to 1700, poverty is not stripped of its religious dimension. However, this theological vision of the poor is influenced by changing times. In Europe, it is perceived as a *civil disorder*, created by begging; *moral disorder*, caused by laziness and idleness beggars. In England, the English

Parliament and Queen Elizabeth I (1533–1603) worked together to enact laws (see Marshall 1985 and Slack 1990) designed to address poverty to reduce the civil and moral disorder. From that time poor (also called unfortunate) are divided into two groups: "deserving" sick, disabled, widows, orphans and thrifty old; and "undeserving" offenders, unmarried mothers, vagrants, unemployed and the old without savings. Thus, beginning in 1597, a series of laws addressing poverty in England are passed, collectively called the "Poor Law", providing government assistance to those who could not provide for themselves including the sick, the elderly and the young children. This law becomes the first major legislation allowing to deal with the poor and disadvantaged for over 200 years. It also became the basis of other legislation for dealing with the poor relief at the colonial level which helps the government to establish apprentice programs for poor children, to develop *workhouses* for those who were able to work ("good poor") and *houses of correction* for vagabonds ("bad poor"). In 1662, the recrudescence of "bad poor" among those considered as poor led to the advent of a new law in England named "Law of Settlement and Removal". Authorities aimed through this law to help only poor local citizens (good poor) and to expel from their jurisdictions anyone else who might become dependant for assistance (bad poor). The law caused authorities to codify the causes of poverty by evaluating people as to the likelihood of becoming poor and also provided harsh and punitive penalties for the bad poor. This represents until now one of the world's first "residency requirements" in determining eligibility conditions to receive help. This law remained in place long after Elizabeth's reign, providing the basis for government policies for the poor until the nineteenth century (Prothero 1934). As a matter of fact, such policies have been conceived more in terms of reducing the visibility impact of the poor for the wealthy societies of that time, rather than handling the reasons for which people were becoming poor.

During the early 17th century, while some governments established centres for the relief of the poor in their territories, others, such as Nova Scotia, choose to fully adopt the English Poor Laws in their countries. In 1795, the Speemhamland system established the earliest "poverty line" based on the price of bread and the number of dependents in a workers family; subsidisation provided when wages dipped below the poverty line. The Speenhamland system (see Marshall 1985 and Slack 1990) was a form of outdoor relief intended to mitigate rural poverty at the end of the 18th century and during the early 19th century. However, the seventeenth and eighteenth centuries are marked by the first philosophical reflections on poverty, and poverty measurement at the source of welfare economics. Bentham (1789), the forerunner of philosophical reflections on poverty in origin of the utilitarianism

doctrine in 1781, argued that individuals interpret their interests in point of *pleasure* and *suffering*. From this point of view, each individual seeks to maximise the net difference between the values of the pleasures and values of sentences for each action he intends to undertake. It is in this context that the concept of *cardinal utility* [see Ellsberg 1954 and Stigler 1950] was introduced and defined in terms of happiness or pleasure (Bentham 1789) and satisfaction of desires (Sidgwick 1874). The idea was *to maximise the pleasure and satisfaction of many (aggregate welfare), while minimising the sorrow and suffering of people* living in a given community. On the other side, the Scottish preacher and mathematician Thomas Chalmers (see Young and Ashton 1956) recommends *to help the poor to help themselves* in 1819. He developed private philanthropies to help meet the economic needs of the poor and organised a system of volunteers to meet individually and regularly with disadvantaged people to give them encouragement and training. Thus, the first social programs based on a system of volunteers to provide emergency economic and "spiritual" assistance to the poor emerged.

The nineteenth century, marked by industrialisation, will be characterised by a negative image of poverty. In 1834 the Poor Law Amendment Act (PLAA) was enacted in England to replace the 1601 Elizabethan Poor Law. The underlying emphasis of the new law was on self-reliance and its main feature was the establishment of deterrent workhouses. Public assistance is not considered a right, and government is not seen as responsible for the unemployed. The principle of "less eligibility" (a recipient of aid can never receive as much as does the lowest-paid worked) is enforced. Anyone claiming relief would have to enter the workhouse. If a man claimed relief, his entire family would also have to enter the workhouse and it would be split up in accordance with the rules of separation. In 1847 the Poor Law Commission was abolished due to the abuses which occurred, especially at the "Andover Workhouse Scandal"[1] exposed by journalists and politicians in 1845. However, poverty is still perceived as a mass phenomenon of which Karl Marx (1850) describes as proletariat (social class of proletarians also known as working poor). The proletariat has interests adverse to those of the bourgeois (or capitalists). A conflict arises between them, known as the *"class struggle"* (Marx 1850 and 1887). These are, indeed, the various phases of the 1848 revolution in France (see Marx 1850) who best reflects the newly character of the class struggle as it appears in the middle of the nineteenth century. In 1889, Bismarck, Chancellor of a newly united Germany, introduced the first national health insurance system. Issues of equality between individuals become a social concern and occupy an important place in the

[1]See http://en.wikipedia.org/wiki/Andover_workhouse_scandal, August 2011.

political discourse. Mill (1848) sets the problem of the distribution of property rights and proposes (Mill 1871) to minimise only the suffering (physical and moral) so as to take in account the well-being of individuals only when it is negative (i.e. without an evil-being). Mill's idea has been enriched to lead to readjustment of the distribution of wealth. However, the utilitarianism doctrine [see also Bentham (1829 and 1831); Sidgwick 1874; Mill 1871] is remained dominant until the late 19th century.

The twentieth century was marked by the continuation of the industrialisation process, the technological progress and the general increase in living standards which have gradually improved the situation of "working poor" until the postwar period known as the "Thirty Glorious Years" (1945-1975). This century is mainly marked by a contestation movement against the utilitarian doctrine and the principle of maximisation in different countries. The criticism derives from Bentham and Mill's assertions arguing that the utility is a psychological notion expressed by one and only one individual. It then becomes difficult to classify psychological states which reveal the own conceptions of satisfaction and to express the utility on a common scale of values. To resolve this problem, Pigou (1920) proposes to monetise the utility function. He then measures satisfaction by basing on the amount of money a person is willing to offer to obtain a certain bundle of goods. From this standpoint, it became possible to compare utility levels between individuals. The results obtained by Pigou (1920) are the origin of the *"Economic Welfare"* and of the concept of *ordinal utility* based on the ranking of preferences of individuals [see Robbins 1938 and Pareto 1927]. They mainly recommend to maximise social welfare (collective welfare) by maximising an aggregate function of individual utilities, i.e. to ensure that people be willing to pay more to consume.

The work of Pareto (1896) raising the issue of optimal allocation of endowments raises two key issues: How to improve the lot of an individual without damaging somebody else's? Is it possible to increase the satisfaction of all individuals at a time? It is about finding the Pareto's Optimum (*Equilibrium State*) i.e., an efficient allocation of resources to maximise social utility whatever the distribution of wealth between individuals. Partials answers to these two questions will be provided by the "Social Choice Theory" and will highlight some current economic theories who study especially poverty issues. The problem then consist to propose a collective preference relation (economic social welfare) from individual preferences i.e. to construct a rule for aggregating individual utility functions into a function of social welfare [see Arrow 1951 and 1963; Bergson 1938; Sen 1970]. With the publication of Rawls (1971), the policy choices of social justice are shaped gradually into society. Rawls (1971) challenges utilitarianism and proposes

an *"egalitarian"* postulate that advocates an equitable and fair sharing of resources allowing to an individual responsible to conduct all rational plans it has set. Another school proposes to exceed both *welfarism*[2] and *egalitarianism* to turn towards equal opportunities, which aims to provide citizens with equal opportunities to improve their quality of life (Sen 1979c).

2.2 Foundations of Poverty Measurement

Early scientific work[3], dealing with the issue of poverty and its manifestations dates back to Charles Booth (1892 and 1894) and Seebohm Rowntree (1901) on the situation of the poorest households in London and United Kingdom. The notion that income or consumption levels is the only reliable variable for measuring poverty was strongly entrenched among economists. Booth (1892) developed a purely exploratory methodology without providing concrete proposals to eradicate poverty. Rowntree (1941) used a more scientific approach based on assessing the resources available to households. This leads to achieve statistical results allowing him to offer avenues of economic reforms for a financial support to the poorest, through the establishment of a minimum wage and social benefits for numerous households.

The 50s were marked by the growth of theories stating that all economies should aim eventually to a level of development where economic growth should benefit to everyone (Aghion and Bolton (1997)). At this time the concept of macroeconomic poverty has been established: a situation in which the growth of the *Gross Domestic Product* (GDP) is not sufficient for all classes of people to enjoy its fruits. Therefore, the fight against poverty is simply another way to build mechanisms for *maximising* economic growth.

The 60s marked a progression in the understanding of poverty and the *monetary paradigm* is beginning to be widely criticised. Although economic thinking is influenced by the liberal model that promotes an efficient allocation of resources when market mechanisms are facilitated, sociologists [Abel-Smith and Townsend (1965), Runciman (1966)] challenge the monetary paradigm from a relative view of poverty. The publications of the late 60s show the evolution of the economic thought based on monetary paradigm towards more social considerations of poverty [Bauer (1966), Van Praag (1968), Seers (1969)] and the introduction of the *multidimensional paradigm* [Abel-Smith and Townsend (1965), Runciman (1966)].

[2]Welfarism refers to welfare economics, where the theory of welfare based on the idea that the quality of a situation can be judged entirely by the quality of utility prevails in this situation (Sen 1979b and 1979c).

[3]For further details and other references on this topic, see Fusco (2005) and Bertin (2007).

Between the late 60s and the early 70s, the economic growth still plays a central role. However, the 70s were marked by the advent of the debt crisis in developing countries, the oil shocks and the end of the system of fixed exchange rates of the Bretton Woods Institutions. This combination of negative and destabilising events brought inflation and unemployment. The Structural Adjustment Programmes were introduced for macroeconomic stabilisation and implementation of the conditions for repayment of external debts owed by the developing countries. Thus, structures such as the International Monetary Fund (IMF) and the World Bank were solicited to improve the economics of developing countries.

The 80s were marked by the failure of the structural adjustment programs which contributed to increase poverty in many countries subject to such programs. Poverty, especially in African countries, took unprecedented proportions, both in extension and intensity. Several authors (Cornia and al. 1987, Commonwealth 1989, Watkins 1995) challenged the exclusively economic approach of the structural adjustment programs and insisted on an adjustment with "a human face". In response to such criticisms, the United Nations Development Programme (UNDP), the African Development Bank (ADB) and the World Bank (World Bank 1990) launched, in 1989, the social dimension of adjustment programs which incorporated a component into its programs aimed at mitigating the consequences of such adjustments to the poorest categories of the population. Nevertheless, this willingness to consider all the socio-economic effects of macroeconomic programs of adjustment remained deeply marked by economic liberalism and the social safety nets (Morduch and Sharma 2002).

In 1990, the UNDP report (PNUD 1990) introduced the human development approach. Poverty is not only a state of deprivation, but is defined in terms of potentialities and choices available to individuals. In 1996, the development assistance committee of the Organisation for Economic Co-operation and Development (OECD) committed to eradicate extreme poverty in the world by the year 2015 while the United Nation Organisation (UNO) proclaimed the *"International Year for Poverty Eradication"* and declared at the same time the period 1997-2006 as *"International Decade for Poverty Eradication"*. During the same year, the IMF, jointly with the World Bank, took the stage under the HIPC Initiative (Highly Indebted Poor Countries) and were responsible for defining the criteria for eligibility for the HIPC initiative. In September 2000, 189 countries signed the Millennium Declaration, which led to the adoption of the Millennium Development Goals (MDGs) in reply to poverty and suffering persist.

All such declarations have encouraged many authors to develop new measurement poverty approaches for a better understanding of this plague. How-

ever, the cleavage between the advocates of monetary poverty (based on a lack of income to live decently) and the advocates of multidimensional poverty (which is expressed through a set of socio-economic indicators) remained fully throughout the 90s and continues until nowadays.

3 Poverty and Measuring

3.1 What Is Poverty?

Fighting against poverty involves measuring it and analysing the underlying causes, in order to propose appropriate policies. There is no definition of poverty winning unanimous support. Many authors have suggested different definitions of poverty over time depending on the context and their point of view of the situation. According to Watts (1968), poverty could be considered as the lack of command over commodities in general. Sen (1985) defined it as the lack of capabilities to function in a given society and the World Bank (2000) as a pronounced deprivation in well-being. Encarta World English Dictionary (Encarta 2010) defines poverty as the lack of basic human needs, such as clean and fresh water, nutrition, health care, education, clothing and shelter, because of the inability to afford them.

All such definitions show that the authors agree on the fact that the *poor* is someone who *lacks*, or is in the inability to achieve, *"something"* useful for a socially acceptable standard of living. The *"something"* lacking changes from place to place and across time, and is described in many ways. Thus, a non-poor household according to the standards of a developing country might be considered desperately poor in a developed country. Similarly, a person with a big house without vehicle may be see himself as poor while a poor pygmy rainforest in southern Cameroon covered with tree bark as a "cache-sex", living in a hut built with of tree leaves without furniture or vehicle can feel filled and "rich". Because human beings are thoroughly diverse, poverty has to be looked at through a variety of indicators. Sen (2001) argued that, *"poverty is a complex, multifaceted world that requires a clear analysis in all of its many dimensions"*. Going in the same sense, the United Nations (UN[4]) and the World Bank (World Bank 2001 and 2005) proposed an alternative definition that we have summarised as following:

[4]UN Statement, June 1998 – signed by the heads of all UN agencies.

(i) *"poverty is a denial of choices and opportunities, a violation of human dignity. It means lack of basic capacity to participate effectively in society. It means not having enough to feed and cloth a family, not having a school or clinic to go to, not having the land on which to grow one's food or a job to earn one's living, not having access to credit. It means insecurity, powerlessness and exclusion of individuals, households and communities. It means susceptibility to violence, and it often implies living on marginal or fragile environments, without access to clean water or sanitation"* (UN 1998).

(ii) *"Poverty is hunger. Poverty is lack of shelter. Poverty is being sick and not being able to see a doctor. Poverty is not having access to school and not knowing how to read. Poverty is not having a job, is fear for the future, living one day at a time. Poverty is losing a child to illness brought about by unclean water. Poverty is powerlessness, lack of representation and freedom"* (World Bank 2001 and 2005).

The definitions (i) and (ii) take into account several indicators (such as levels of income and consumption, social indicators, and indicators of vulnerability to risks and indicators of socio/political access) and seem more representative of human being situation than a purely monetary definition (unidimensional poverty). Poverty has many faces, so it has to be defined and measured in a multidimensional way.

3.2 Why do we measure Poverty?

Poverty measurement seems inevitable due to its importance in many contexts, such as fighting against poverty and social exclusion, the promotion of equality and the defence of the underprivileged, understanding vulnerability, understanding and eliminating social inequalities. In this paper, we have identified at least five main reasons[5] justifying the necessity for a rigourous measurement of poverty:

Understanding what the situation is

Ravallion (1998) argues that *"a credible measure of poverty can be a powerful instrument for focusing the attention of policy makers on the living conditions of the poor."* The first reason for measuring poverty is to understand what the situation is, how the situation is experienced over the life cycle and

[5]See Haughton and Khandker 2009 and World Bank (2005). Moreover, the list of the reasons for measuring poverty given here is not exhaustive.

how it is reproduced. Note that, understanding the causes and characteristics of poverty in a given country or geographic area is a crucial analytical step for policy makers who want to elaborate effective policies and poverty reduction strategies. In other terms, measuring poverty makes *visible* poor people, through examination of factors determining their living conditions, thus makes possible comparisons in time and space.

Targeting Domestic and Worldwide Interventions

The second reason for measuring poverty is to be able to know who the poor people are in order to design and target alleviation interventions best adapted to these. Mostly, one uses poverty profiles [Ravallion (2008), Chen and Ravallion (2007)] to achieve this goal. Then, one examines how a poverty measurement varies across subgroups of a population (for example, by geography) and compares key characteristics of poor people versus non-poor people.

Developing a social spending and growth strategy

The third reason is to be able to assess if general economic growth helped the poor to improve their living conditions. Mostly, one uses information on households and their economic status in order to understand the positive or negatives effects of general economic and financial policies. Depending on whether poverty is increasing or decreasing, policy makers could adjust reforms and evaluate how the poor are affected by such reforms. For instance, we can change the taxing policy and then evaluate how these changes influence the living conditions of poor people.

Monitoring and Evaluating Projects and Policy Interventions

The fourth reason for measuring poverty is to be able to evaluate the effectiveness of current policies and programs designed to help poor people, and then assess whether the situation is changing. In this case, poverty measurement is an instrument to judge the effects of a policy on poor people and helps to evaluate the outcomes in comparaison with a reference group. This can be helpful in order to improve policies implementation and the design of projects and programs. Measuring poverty can be used in order to simulate the impact of alternative policies on poverty and to select the most preferred alternative by ranking policy alternatives according to some poverty impact indicators.

Evaluating the Effectiveness of Institutions

The fifth reason for measuring poverty is to help evaluating institutions. To know if an institution or a government is acting correctly as far as poverty reduction is concerned, we need to assess their success in pursuing of fighting poverty. Poverty measurement is then useful in order to evaluate by how much poverty has decreased and the sustainability of results when assessing policies, projects and measurement instruments.

4 Multidimensional Poverty Measures

4.1 Notations and Definitions

Throughout this paper, we are going to adopt the following notations. $\mathfrak{P} = \{x_1, x_2, \ldots, x_n\}$, is a set of households or individuals from a given country or geographic area $\big(\text{with}|\mathfrak{P}| = \text{card}(\mathfrak{P}) = n\big)$. $\mathbb{X} = \mathbb{X}_1 \times \mathbb{X}_2 \times \cdots \times \mathbb{X}_m$, is a cartesian product of attributes $\mathbb{X}_j \subseteq \mathbb{R}^+$ (with $m \geq 2$. $\mathbb{X} \subseteq \mathbb{R}^m_+$, where $\mathbb{R}^m_+$ is the non-negative orthant of the Euclidean m-space $\mathbb{R}^m$. $\mathbb{M} = [f_j(x_i)]$ is a $n \times m$ *pattern matrix*, where $f_j(x_i)$ is the jth attribute for the ith pattern. $f_j : \mathfrak{P} \longrightarrow \mathbb{R}_+$ such that $f_j(x_i)$ is the evaluation of x_i on the jth attribute. $\xi = (\xi_1, \xi_2 \ldots, \xi_m)$ is a vector of thresholds or minimal acceptable levels on each $\mathbb{X}_J$ and $\xi \in \Xi$ (with $\Xi \subseteq \mathbb{R}^m_+ \backslash \{0\}$). $\mathcal{D}_j$ is a set of deprived individuals (or households) on the jth attribute. $f_\star(x_i) = \big(f_1(x_i), f_2(x_i), \ldots, f_m(x_i)\big)$ and $f_j(\star) = \big(f_j(x_1), f_j(x_2), \ldots, f_j(x_n)\big)^{\mathrm{T}}$ where $x_i \in \mathfrak{P}$ and T denotes vector transpose.

Definition 4.1 A Multidimensional Poverty Measurement can be defined as a non-constant function $\mathbf{P} : \mathbb{M}_{nm} \times \Xi \to \mathbb{R}$ such that:

$$\begin{aligned} \mathbf{P}(\mathbb{M}, \xi) &= \mathcal{F}(\Delta_{\mathfrak{P}}(x_1, \xi), \Delta_{\mathfrak{P}}(x_2, \xi), \ldots \Delta_{\mathfrak{P}}(x_n, \xi)) \\ &= \mathcal{F}(\pi_1, \pi_2, \ldots, \pi_n) \end{aligned} \tag{1}$$

Where $\Delta_{\mathfrak{P}}(x_i, \xi)$ is a mapping $\mathbb{X} \to \mathbb{R}_+$ which defines the aggregation procedure of multiple dimensions into a composite indicator of individual poverty π_i. $\mathcal{F}(\pi_1, \pi_2, \ldots, \pi_n)$ is a mapping of $\mathbb{R}^n_+ \to \mathbb{R}_+$ which defines the aggregation procedure of composite indicators π_i into a multidimensional poverty measurement $\mathbf{P}(\mathbb{M}, \xi)$. For all $\mathbb{M}$ and $\xi \in \Xi$, $\mathbf{P}(\mathbb{M}, \xi)$ represents the *level of poverty* associated to pattern matrix $\mathbb{M}$ according to the *minimal acceptable levels* ξ of family of attributes $\big\{\mathbb{X}_j\big\}_{j \in J}$.

The explicite forms of ξ_j, and the functions $\mathcal{F}(\cdot)$ and $\pi(\cdot)$ are chosen (or built) such that $\mathbf{P}(\mathbb{M}, \xi)$ fulfills some properties. Such properties can be algebraic properties, analytic properties, or axioms.

Let $n \geq 2$ be an integer, we denote by $\eth_n$ the permutation group of I and by $\mathcal{B} = \{e_1, e_2, \ldots, e_n\}$ the canonical basis of $\mathbb{R}^n$. For all positive integers i and j, δ_{ij} is the Kronecker symbol such that $\delta_{i,i} = 1$ and $\delta_{i,j} = 0$ when $i \neq j$).

Definition 4.2 Let $\sigma \in \eth_n$, a *transformation matrix* associated to σ is a transformation matrix Π_σ (of canonical basis $\mathbb{R}^n$) to the basis $\mathcal{B} = \left\{e_{\sigma(1)}, \ldots, e_{\sigma(n)}\right\}$. Then, if Π_σ is a transformation matrix, $\Pi_\sigma e_j = e_{\sigma(j)}$ for all integers $j \in J$:

$$\Pi_\sigma = [\delta_{i,\sigma(j)}]_{1 \leq i,j \leq n} \tag{2}$$

Definition 4.3 A matrix $A = [a_{i,j}]_{1 \leq i,j \leq n} \in \mathcal{M}_n(\mathbb{R})$ is *stochastic* if it is positive and:

$$\forall\, i \in I, \quad \sum_{i=1}^{n} a_{ij} = 1 \tag{3}$$

Definition 4.4 A *bistochastic matrix* is a stochastic matrix A such that A^T is also stochastic.

4.2 Traditional Approaches

According to Sen (1979), the poverty measurement problem can be split into two distinct exercises: (i) *identification* of the poor among the total population (who are the poor?) and (ii) *aggregation* (how are the poverty characteristics of different people to be combined into an aggregate measure?). In the traditional approach, the problem (i) has been solved by the income (or consumption) method, which needs the specification of a subsistence income cutoff (or poverty cutoff). Thus, a person is identified as poor when his or her achievements fall below this cutoff level. Many procedures have been proposed by the literature for handling problem (ii) which are based on constructing an index of poverty using the available information on the people. We present in this section a review of the most popular poverty measurements.

4.2.1 The Foster, Greer and Thorbecke Poverty Index

The *traditional approach*, known as *monetary approach*, computes the income (or consumption) of individuals as indicator of well-being. Firstly, one defines an indicator of well-being ζ_{x_i} and the poverty threshold ξ such that, $\zeta_{x_i} = \sum_{j=1}^{m} \mu_j f_j(x_i)$ (where, μ_j is the market price of the good j) and $\xi = \sum_{j=1}^{m} \mu_j \xi_j$.

Thus, the aggregation function (or Foster-Greer-Thorbecke Poverty Index) is given by the equation 4 below:

$$\mathcal{F}(\mathbb{M},\xi) = FGT_\gamma = \frac{1}{|\mathfrak{P}|}\sum_{i=1}^{|\mathfrak{P}|}\left(\mathcal{F}_{x_i}(\zeta_{x_i})\right)^\gamma \mathbf{I}(\mathcal{F}_{x_i}(\zeta_{x_i})>0) \quad (4)$$

$$= \frac{1}{|\mathfrak{P}|}\sum_{i=1}^{|\mathfrak{P}|}\left[\max\left\{\frac{\xi-\zeta_{x_i}}{\xi};0\right\}\right]^\gamma \mathbf{I}(\xi>\mathcal{F}_{x_i}(\zeta_{x_i})) \quad (5)$$

Where $\mathcal{F}_{x_i}(\zeta_{x_i}) = \max\left[\frac{\xi-\zeta_{x_i}}{\xi};0\right]$ is the function of individual poverty, $\mathbf{I}$ an indicator function and γ the sensibility parameter of distribution index among poor people. In particular, if $\gamma = 0$, FGT_0 is the "poverty head-count" (also called *"head-count ratio"*) i.e. the proportion of people with incomes less than the poverty line; if $\gamma = 1$, FGT_1 is the "poverty gap" (also called *"depth of poverty"* or *"income gap ratio"*) i.e. the gap between the poverty line and average income of the poor, expressed as a proportion of the poverty line; and if $\gamma = 2$, FGT_2 is the "poverty severity" (also called *"squared poverty gap"*) i.e. a weighted sum of poverty gaps (as a proportion of the poverty line), where the weights are the proportionate poverty gaps themselves. Let us note that, these indices belong to the family of poverty measurement named Foster-Greer-Thorbecke indices (see Foster, Greer and Thorbecke 1984).

4.2.2 Sen's Poverty Index

Sen (1976) used an axiomatic approach to derive another poverty index. Sen's index, $\mathcal{S}(\mathbb{M},\xi)$, is given by equation 6:

$$\mathcal{S}(\mathbb{M},\xi) = \left(\frac{2}{(q+1)n\xi}\right)\sum_{i=1}^{q}\left(\xi-\zeta_{x_i}\right)\left(q+1-i\right) \quad (6)$$

Equation 6 can also be written as

$$\mathcal{S}(\mathbb{M},\xi) = FGT_0\left(1-\left(1-FGT_1\right)\left(1-\mathbf{G}\left(\frac{q}{q+1}\right)\right)\right) \quad (7)$$

Where $\mathbf{G}$ refers to the Gini coefficient of the Lorenz distribution of incomes of the poor [see Gini (1912) and Theil (1967)] and q is the number of poor. Let us note that, if q is sufficiently high, one can easily show that Equation 7 is *equivalent* to equation 8 below:

$$\mathcal{S}(\mathbb{M},\xi) = FGT_0\left(1-\left(1-FGT_1\right)\left(1-\mathbf{G}\right)\right) \quad (8)$$

Unlike FGT_0 (proportion of persons with incomes less than the poverty line), $\mathcal{S}(\mathbb{M}, \xi)$ is sensitive to the extent of the short-fall of income of the poor from the poverty line. Unlike FGT_1 (the gap between the poverty line and average income of the poor, expressed as a proportion of the poverty line), $\mathcal{S}(\mathbb{M}, \xi)$ is sensitive to the number below the poverty line. Moreover, $\mathcal{S}(\mathbb{M}, \xi)$ is not insensitive to the redistribution of income among the poor (see Sen (1976) for further details). Thus, Sen's index $\mathcal{S}(\mathbb{M}, \xi)$ contrasts sharply with two crude measures of poverty (FGT_0 and FGT_1) used in the statistical literature and in policy discussions.

4.2.3 Watts Poverty Index

The Watts poverty index is defined as follows:

$$\mathcal{W}(\mathbb{M}, \xi) = \frac{1}{|\mathfrak{P}|} \sum_{i=1}^{|\mathfrak{P}|} \ln \left(\frac{\xi}{\zeta_{x_i}} \right) \mathbf{I}(\zeta_{x_i} \leq \xi) \tag{9}$$

In this case, ζ_{x_i} is considered as the (positive) income of individual x_i. Watts (1968) index has the particularity to satisfy focus axiom, monotonicity axiom, transfert axiom and decomposability axiom (see section 4.4.2). The logarithm plays an important role at the level of sensitivity in the sense that it ensures that a pure transfer of income from the poorest poor to those who are better off will change the Watts index.

4.2.4 Discussion

Although the most popular poverty measurements analysed in this section all have the advantage to be simple to construct and to reflect what they are supposed to capture, they nevertheless suffer of some drawbacks. FGT_0 is *"completely insensitive to the redistribution of income among the poor. A pure transfer of income from the poorest poor to those who are better off will either keep FGT_0 unchanged, or make it go down–surely a perverse response."* (see Sen 1976). FGT_1 does not capture differences in the severity of poverty amongst the poor and ignore "inequality among the poor". Therefore, FGT_0 violates the monotonicity axiom and the transfert axiom, while FGT_1 violates only the transfer axiom. FGT_2 is very difficult to understand and interpret. Sen's Poverty Index depends on the Gini coefficient and thus shares its main drawbacks. Indeed, the Gini and thus the Sen's Index cannot be used to decompose poverty into distributions from different subgroups. The Watts index is distributionally sensitive by virtue of using logarithms.

Traditional approaches rely on income or consumption expenditures per capita to compute different indices. Therefore, they explore only the monetary dimension of poverty. Poverty is multidimensional. Then we also have to take into account the non-monetary information of the concept of poverty. We need to discuss what we mean by multidimensional and take it into account when measuring. In practice, monetary measurements are mostly disconnected from the values, from the reality and from the way that people perceive themselves. One notes here a high risk of ethnocentric bias due to fact that the list of basis needs tends to reflect the views of a politician or an analyst, not those of the people themselves.

Our vision of poverty and measurement is based on the combination of three issues:

- there is around empirical evidence about the living standards of households which need to be used in order to understand both what the current situation is and in order to foresee possible ways to overcome it;
- in considering poverty we need to take into account both the subjective perception of being poor (captured by the concept of being limited to achieve aspirations) and the religious, artistic or philosophic, in short ideological, forms in which men become conscious of this plague and try to fight it out;
- poverty measurement should be seen as an instrument to design, implement and assess poverty reduction policies and not as an activity for itself. Our claim is that there is no general purpose poverty measurements, but only policy oriented ones.

We are going to further discuss these issues, while presenting Sens's capabilities approach (in Section 4.5) and in our general discussion (in Section 5).

4.3 Multidimensional Approaches

The approaches described in this section are supposed to measure the total deprivation of society in terms of each attribute separately and then aggregate the different indices for a one-dimensional index of multidimensional poverty.

4.3.1 Approaches based on the possession on each attribute

These approaches take into account dimensional deprivations and allow to evaluate how much individuals are deprived (or non-deprived) on each at-

tribute taken individually.

A) The Cut-off Method

Townsend (1979 and 1987) was the first to introduce the concept of *"deprivation"* after criticizing the *monetary paradigm* of poverty measurement. The Cut-off method consists in evaluating the deprivation on each attribute through a binary ranking of deprivation (or non-deprivation) type. Thus, an individual will be considered as poor (or deprived) on one attribute when its realisation on this attribute is lower than the poverty line or poverty threshold. Formally, one constructs an increasing function $\Psi^j : \mathbb{X}_j \to \{0; 1\}$ on each attribute which is then used to assign individuals to $\mathcal{D}_j$, the set of *"deprived"* on attribut $\mathbb{X}_j$. The function Ψ^j is given by the equation 10 as follows:

$$\Psi^j(f_j(x_i)) = \begin{cases} 1 & \text{si } f_j(x_i) \geq \xi_j \ \Rightarrow \ x_i \in \mathcal{D}_j. \\ 0 & \text{si } f_j(x_i) < \xi_j \ \Rightarrow \ x_i \in \overline{\mathcal{D}}_j. \end{cases} \tag{10}$$

where $\mathcal{D}_j \subseteq \mathfrak{P}$ and $\overline{\mathcal{D}}_j = \{x_i \ : \ x_i \in \mathfrak{P} \ \wedge \ x_i \notin \mathcal{D}_j\}$. Many applications of this approach can be found in [Townsend (1979), Mark and Lansley (1985)].

B) The "Fuzzy Sets" Approach

The fuzzy sets theory was introduced by Zadeh (1965) as an extension of the classical notion of *set*. Unlike the classical *set theory* based on a classic logic, *fuzzy sets theory* allows the gradual assessment of the membership of objects in a set. Zadeh (1965) himself defined a fuzzy set as *"a class of objects with a continuum of graded membership"*. He argues that *"such a set is characterised by a membership function which assigns to each object a membership ranging between zero and one"*.

Let $\mathcal{P}$ be a subset of poor and $\mathcal{P} \subset \mathfrak{P}$. A fuzzy set $\widetilde{\mathcal{P}}$ of $\mathcal{P}$ is defined by equation 11 below:

$$\widetilde{\mathcal{P}} = \{(x_i, \Delta_{\mathfrak{P}}(x_i, \xi))\}, \qquad x_i \in \mathcal{P}. \tag{11}$$

where $\Delta_{\mathfrak{P}}(x_i, \xi)$ is the degree of membership x_i to the subset $\mathcal{P}$. Thus, we have:

$$\begin{cases} \Delta_{\mathfrak{P}}(x_i) = 0 & \Longrightarrow x_i \in \mathfrak{P} - \mathcal{P}, \quad x_i \textit{ does not} \text{ belongs to the subset } \mathcal{P}; \\ 0 < \Delta_{\mathfrak{P}}(x_i) < 1 & \Longrightarrow x_i \widetilde{\in} \mathcal{P}, \quad x_i \textit{ partially} \text{ belongs to the subset } \mathcal{P}; \\ \Delta_{\mathfrak{P}}(x_i) = 1 & \Longrightarrow x_i \in \mathcal{P}, \quad x_i \textit{ completely} \text{ belongs to the subset } \mathcal{P}. \end{cases}$$

where $\mathfrak{P} - \mathcal{P} = \{x_i \ : \ x_i \in \mathfrak{P} \wedge x_i \notin \mathcal{P}\}$ and $\widetilde{\in}$ the partial membership.
Formally, $\Delta_{\mathfrak{P}}$ is an application of the set $\mathfrak{P} \times \Xi$ to the closed interval [0; 1]

given by equation 12:

$$\Delta_{\mathfrak{P}}(x_i,\xi) = \mathcal{F}_\gamma\big[\lambda_1\Psi^1(f_1(x_i)),\lambda_2\Psi^2(f_2(x_i))\ldots,\lambda_m\Psi^m(f_m(x_i))\big] \tag{12}$$

where Ψ^j is an increase function of the set $\mathbb{X}_j$ to $\{0;1\}$; γ represents the sensitivity parameter; λ_j the weighting systems associated to each attribute f_j (with $j \in J$) and $\mathcal{F}_\gamma$ is a real-valued function on $\prod_{j=1}^{m}\Psi^j(\mathbb{X}_j)$ that may have several additional properties, e.g., being one-to-one, nondecreasing or increasing in all its arguments.

In practice, one usually chooses $\mathcal{F}_\gamma$ as the weighted average of order γ given by the following equation:

$$\mathcal{F}_\gamma\big[\lambda_1\Psi^1(f_1(x_i)),\ldots,\lambda_m\Psi^m(f_m(x_i))\big] = \left[\sum_{j=1}^{m}\lambda_j\big[\Psi^j(f_j(x_i))\big]^\gamma\right]^{1/\gamma} \tag{13}$$

Thus, $\Delta_{\mathfrak{P}}(x_i,\xi)$ refers to aggregated index of membership degrees $\Psi^j(f_j(x_i))$ associated to the indicators of deprivation $\mathbb{X}_j$ of each x_i. In the following we give more details about two instances of this approach.

⋆ The Totally Fuzzy Approach (TFA)

The totally fuzzy approach was introduced by Cerioli and Zani (1990) to apply the concept of fuzzy sets to the measurement of poverty. Cerioli and Zani (1990) specified a membership function Ψ^j according to the fact that variables are dichotomous, polytomous or continuous.

In the case of *dichotomous variables* (variable with two modalities), the membership function Ψ^j is given by the equation 14:

$$\Psi^j(f_j(x_i)) = \begin{cases} 1 & \text{si } f_j(x_i)=1 \;\Rightarrow\; x_i \in \mathcal{D}_j. \\ 0 & \text{si } f_j(x_i)=0 \;\Rightarrow\; x_i \in \overline{\mathcal{D}}_j. \end{cases} \tag{14}$$

where $\mathcal{D}_j \subseteq \mathfrak{P}$ and $\overline{\mathcal{D}}_j = \{x_i \,:\, x_i \in \mathfrak{P} \wedge x_i \notin \mathcal{D}_j\}$.

In the case of *categorical variables* (variable with k modalities, $k > 2$), the membership function Ψ^j is given by the equation 15:

$$\Psi^j(f_j(x_i)) = \begin{cases} 1 & \text{if } f_j(x_i) \geq f_j(x_{\max}) \;\Rightarrow\; x_i \in \mathcal{D}_j. \\ \dfrac{f_j(x_i)-f_j(x_{\min})}{f_j(x_{\max})-f_j(x_{\min})} & \text{if } f_j(x_i) \in \big[f_j(x_{\min});\, f_j(x_{\max})\big[\;\Rightarrow\; x_i \widetilde{\in} \mathcal{D}_j. \\ 0 & \text{if } f_j(x_i) < f_j(x_{\min}) \;\Rightarrow\; x_i \in \overline{\mathcal{D}}_j. \end{cases} \tag{15}$$

where $f_j(x_{\min})$ is the lowest value associated to the assessing of reference individual $x_{\min}$ and $f_j(x_{\max})$ is the highest value associated to the assessing of reference individual $x_{\max}$. An individual between $x_{\min}$ and $x_{\max}$ is considered as *partially deprived*. Let us note that if $f_j(x_{\min}) = f_j(x_{\max})$, equation 15 becomes equivalent to equation 10.

In the case of *continuous variables* (as income or consumption expenditures), Cerioli and Zani (1990) have defined two reference values $f_j(x_{\min})$ and $f_j(x_{\max})$ such that if the value $f_j(x_i)$ taken by the continuous indicator for a given individual is smaller than $f_j(x_{\min})$ this person would undoubtedly be defined as poor whereas if it is higher than $f_j(x_{\max})$ he certainly should be considered as not being poor. Thus, the membership function Ψ^j is given by the equation 16:

$$\Psi^j(f_j(x_i)) = \begin{cases} 0 & \text{if } f_j(x_i) \geq f_j(x_{\max}) \ \Rightarrow \ x_i \in \overline{\mathcal{D}}_j. \\ \dfrac{f_j(x_{\max}) - f_j(x_i)}{f_j(x_{\max}) - f_j(x_{\min})} & \text{if } f_j(x_i) \in \big[f_j(x_{\min});\, f_j(x_{\max})\big[\ \Rightarrow \ x_i \widetilde{\in} \mathcal{D}_j. \\ 1 & \text{if } 0 \leq f_j(x_i) < f_j(x_{\min}) \ \Rightarrow \ x_i \in \mathcal{D}_j. \end{cases} \tag{16}$$

The membership function Ψ^j obtained is a linear increasing function with the increasing risk of poverty between $f_j(x_{\min})$ et $f_j(x_{\max})$. This approach introduces an arbitrariness because the two threshold[6] values $f_j(x_{\min})$ and $f_j(x_{\max})$ are nowhere defined.

⋆ The Totally Fuzzy and Relative Approach

Cheli and Lemmi (1995) proposed a less arbitrary formulation than the one originally proposed by Cerioli and Zani (1990). In the case of dichotomous variables, the membership function Ψ^j is defined as in equation 14. In the case of categorical variables, Cheli and Lemmi (1995) proposed to eliminate the hypothesis of equidistance between modalities observed in Cerioli et Zani (1990). Then, the membership function Ψ^j is defined as follow:

$$\Psi^j(f_j(x_i)) = \begin{cases} 0 & \text{if } f_j(x_i) = f_j(x^1). \\ \Psi^j(f_j(x^{m-1})) + \dfrac{F_j(x^m) - F_j(x^{m-1})}{1 - F_j(x^1)} & \text{if } f_j(x_i) = f_j(x^m) \end{cases} \tag{17}$$

where $f_j(x^m)$ indicates the $m = 1 \ldots k$ modalities of the jth variable. The modality $f_j(x^1)$ represents the lowest risk of poverty while $f_j(x^m)$ the highest risk of poverty. F_j is the cumulative distribution function of f_j. In the case of continuous variables, the membership function Ψ^j is a non-linear increasing

[6] Ravallion (1996, 1998 and 2008) has widely developed this question.

function. Other "Fuzzy Set" approaches to poverty analysis can be found in the literature, such as those proposed by Chiappero (1994), Vero and Werquin (1997), Qizilbash (2000, 2003 and 2005), Kojo and al.(2007). The methods based on fuzzy sets theory have some limitations, especially when the applications are based on a multidimensional composite index designed as an aggregate average. Indeed, an individual is *"poor"* with a degree equal to 1 (i.e. he is poor in all attributes), or an individual is *"non-poor"* with a degree equal to 0 (i.e. he is non-poor on all attributes), or the individual is *poor unambiguously* on certain attributes and *non-poor unambiguously* on others (i.e. he is partially poor or partially non-poor). This result is inconsistent with a standpoint according to which an individual is poor if one of its basic needs is not accomplished. In the case of the absolute approach in which all dimensions have to be take into account and are all part of a core of poverty, fuzzy set approaches might provide contradictory results [see Qizilbash 2000 and 2003]. The fuzzy sets approach distinguishes two categories rigidly (the poor category and the non-poor category). But, it is difficult to determine and justify the existence of other categories. On the other side the notions of *"partially poor"* or *"partially non-poor"* are ambiguous and very difficult to interpret.

4.3.2 Approaches based on aggregated indicators

The principle of aggregation approaches is to combine and synthesise simultaneously several numerical values into one indice, named *composite poverty index*, so that this composite index takes into account all individual values.

⋆ The Human Poverty Index (*HPI*)

The Human Poverty Index (*HPI*) is a composite index measuring deprivations in the three basic dimensions captured in the human development index (PNUD 1990-2006) HPI_1, HPI_2 et HPI_3, respectively *deficiencies in health* (percentage of individuals whose life expectancy is less than 40 years), *deficiencies in education* (proportion of the adult population that is illiterate) and *deficiencies in terms of living conditions* (proportion of the population having access to health care, safe water and proportion of children under age five suffering from malnutrition). The proposed composite poverty index *HPI* was formulated by Arnand and Sen (1997) as follows:

$$HPI = \left(\lambda_1 HPI_1^{\gamma} + \lambda_2 HPI_2^{\gamma} + \lambda_3 HPI_3^{\gamma}\right)^{1/\gamma} \tag{18}$$

with $\lambda_1 + \lambda_2 + \lambda_3 = 1$ and $\gamma \geq 1$ being a parameter. For $\gamma = 1$, the three components of *HPI* are perfect substitutes. However, when $\gamma \to \infty$, this

index will tend to $\max(HPI_1, HPI_2, HPI_3)$. In this case, the *HPI* will only decrease if its highest-valued component decreases.

⋆ The supplementary strategy

Brandolini and D'Alessio 1998 explored the possibility of a multidimensional analysis of deprivation and inequality by adopting the capability approach proposed by Amartya Sen (see section 4.5 for more details). In their paper they examined a different strategy to measure functionings and capabilities. In this context, they defined the *supplementary strategy* (Brandolini and D'Alessio 1998) which aims to complete the information from the distribution of resources (measured by income or consumption) by the indicators of living standards. In this strategy, functionings are examined item by item in order to describe the characteristic of each dimension and to study their correlation structure. The advantage of this strategy lies in its simplicity, in that it imposes no particular structure of poverty and the prerequisites for the measure are less demanding. The lack of synthesis and the difficulty of giving a unified well-defined image are the main disadvantages of this strategy. Indeed, the plurality of elementary indicators conflicting within a same modelling can lead to difficulties both in analysis and synthesis. In addition, it is likely to get only a partial order when comparing observations between them. This constitutes a major argument in favour of the construction of aggregate indexes for synthesising information.

⋆ The Global Composite Index

The composite index approach constructs a global composite indicator $\Delta_{\mathfrak{P}}(x_i, \xi)$ for each household x_i. Let $\mathbb{M}$ be a pattern matrix, the problem is to determine an aggregation function $\mathcal{F}_\gamma$ defined from $\mathbb{R}^m$ to $\mathbb{R}$ such that:

$$\Delta_{\mathfrak{P}}(x_i, \xi) = \mathcal{F}_\gamma(\Psi^1(f_1(x_i)), \ldots, \Psi^m(f_m(x_i)); \lambda_1, \ldots, \lambda_j, \ldots, \lambda_m) \qquad (19)$$

Note that, the function $\mathcal{F}_\gamma$ (and its properties) can be defined in different ways according to the authors [see Bourguignon and Chakravarty (2003); Chiappero 1994]. The best-known aggregation function is the *weighted mean of order* γ:

$$\begin{aligned} \Delta_{\mathfrak{P}}(x_i, \xi) &= \mathcal{F}_\gamma(\ldots, \Psi^j(f_j(x_i)), \ldots; \ldots, \lambda_j, \ldots) && (20) \\ &= \left[\sum_{j=1}^{m} \lambda_j \left[\Psi^j(f_j(x_i))\right]^\gamma\right]^{1/\gamma} && (21) \end{aligned}$$

where $\Delta_{\mathfrak{P}}(x_i, \xi)$ represents the aggregated index for individual x_i of normalized degrees $\Psi^j(f_j(x_i))$ associated to different elementary indicators of deprivation, λ_j the adopted weighting system for each f_j (avec $j = 1 \dots m$), γ being a sensibility parameter. Mostly, one chooses $\lambda_j \geq 0$ such that $\sum_{j=1}^{m} \lambda_j = 1$ and γ represents a parameter which determines the substitution level between attributes.

$\Delta_{\mathfrak{P}}(x_i, \xi)$ is easy to interpret. Indeed, a good (resp. bad) performance in some given attributes implies greater (resp. lower) value on the composite index. Hence, the more an individual will be poor, the more the index value will be strong. However, this kind of aggregation operators implies commensurability of attributes and compensation/substitutability which can be argued.[7] Consider the attribute f_j: monthly salary. Assume that it is a continuous scale and the following proposal is considered realist: "all things being equal, an increase in the monthly salary of μ CFA[8] francs provides an additional level of satisfaction μ times the level achieved by an increase of one minute". The scale f_j is consistent with the requirement of commensurability of differences in preferences according to its axis of significance j (with $j = 1 \dots m$), if for all $(x_i, x_k, x_u, x_v) \in [\Omega^4]_{f_j}$, the equality 22 appears realist to define the restriction of function ϕ to this set.

$$\phi\left(\frac{x_i \ominus x_k}{x_u \ominus x_v}\right) = \frac{f_j(x_i) - f_j(x_k)}{f_j(x_u) - f_j(x_v)} \tag{22}$$

Measurement theory (see Roberts 1979, Krantz et al. 1971) has established that the only codings that transform a scale $f_j(x_i)$, conform to the requirement of commensurability of differences of preference, to an other scale $\chi(f_j(x_i))$ conform to this same requirement are affine transformations given by the equation 23:

$$\chi(f_j(x_i)) = \alpha f_j(x_i) + \beta \tag{23}$$

where $\alpha \in \mathbb{R}_+$ and $\beta \in \mathbb{R}$. This standpoint aiming to guarantee the meaningfulness of the measurement is often ignored in most of multidimensional approaches of poverty measurement based on the idea of pairwise comparisons. On the other hand, the level of compensation/substitutability depends of the γ value and the set of $\{\lambda_j : j = 1 \dots m\}$. For instance, we are in presence of a *total* substitutability when $\gamma = 1$ and λ_j for all $j = 1 \dots m$ and a *partial* substitutability when $1 < \gamma < \infty$. The goods are complementary i.e.

[7] See Fusco 2005 and Atkinson 2003.

[8] The CFA franc is the name of two currencies (the West African CFA franc and the Central African CFA franc) used in Africa which are guaranteed by the French treasury. 1 euro = 655.957 CFA francs.

no substitutability if $\gamma = \infty$. The substitutability hypothesis is not always acceptable in poverty measurement because it admits that a weakness in one or several attribute(s) can be compensate by the strongness of other(s) attribute(s).

4.3.3 Approaches based on individual data

Alternative approaches of poverty measurement are generally based on individual data. We have, in particular, the distance function approach [Deutsch and Silber 2005, Silber 2007], the information theory approach (or entropy approach)[Theil 1967, Foster and al. (1984), Maasoumi 1986 and 1993, Cowell 1977, Cowell et Kuga (1981a), Cowell et Kuga (1981b), Lugo and Maasoumi (2008),] and the inertia approach [Klasen 2000, Sahn et Stifel (2000 et 2003)].

A) The Distance Function Approach

The concept of distance function has been widely used in the efficiency analysis which highlights the distinction between input and output distance functions (see Coelli et al. 1998). Lovell and al. (1994) were the first to apply this concept in the context of the analysis of households behaviour. Deutsch and Silber (2005) then developed it in the context of multidimensional poverty measurement.

Formally, one defines $L(y)$ as the input set of all input vectors x which can produce the output vector y, that is:

$$L(y) = \{x \ : \ x \text{ can produce } y\}. \tag{24}$$

The *input distance function* $D_{in}(x)$ is then defined by equation 25 below:

$$D_{in}(x, y) = \max \left\{ \tau \ : \ \left(\frac{x}{\tau}\right) \in L(y) \right\} \tag{25}$$

Coelli and al. (1998) proved that the input distance function complies to four properties:

- (i) $D_{in}(x, y)$ is increasing in x and decreasing in y.
- (ii) $D_{in}(x, y)$ is linearly homogeneous in x.
- (iii) If $x \in L(y)$ then $D_{in}(x, y) \geq 1$.
- (iv) $D_{in}(x, y) = 1$ if x belongs to the "frontier" of the input set $L(y)$ (the isoquant of y).

Deutsch and Silber (2005) proposed an estimate of the standard of living index on the basis of information on the ownership of durable goods. Let $x = (r^{(1)}, r^{(2)}, \ldots, r^{(N)}) \in \mathbb{R}_+^N$ be the resources vector and $u = (u^{(1)}, u^{(2)}, \ldots, u^{(M)}) \in \mathbb{R}_+^M$ the functionings vector. Then, each individual is represented by the pair $(r^{x_i}, u^{x_i}),\ i \in I$. A theoretical standard of living index SL can be estimated using a Malmquist input quantity index (Coelli and al. 1998) as follows:

$$SL(u, r^{x_s}, r^{x_t}) = \frac{D_{in}(u, r^{x_s})}{D_{in}(u, r^{x_t})} \tag{26}$$

where r^{x_s} and r^{x_t} are two different resource vectors and $D_{in}$is an input distance function (given by equation 25). The Malmquist index helps to compute a *reference set* against which to judge the relative magnitudes of the two resource vectors. This reference set is the isoquant r^{x_i} and the radially farther r^{x_i} is from $L(u)$ the higher the standard of living is, for r^{x_i} must shrunk more to move back onto the reference set $L(u)$.

To estimate the distance function, Deutsch et Silber (2005) define $\lambda = 1/r_N$ and a $(N-1)$–vector $z = \{z_j\} = \{r_j/r_N\}$ with $j = 1, \ldots, (N-1)$. Then:

$$D_{in}(z, e) = (\frac{1}{x_N}) \cdot D_{in}(x, e) \tag{27}$$

Since $D_{in}(x, e) \geq 1$, we obtain:

$$(\frac{1}{r_N}) \leq D_{in}(x, e) \tag{28}$$

This implies equation 29.

$$(\frac{1}{r_N}) = D_{in}(x, e) \cdot \exp(\varepsilon), \quad \varepsilon \geq 0. \tag{29}$$

By assuming that $D_{in}(z, e)$ has a *translog functional form*, we have:

$$\ln(\frac{1}{r_N}) = \alpha_{00} + \sum_{j=1}^{N-1} \alpha_{j0} \ln(z_j) + \frac{1}{2} \sum_{j=1}^{N-1} \sum_{k=1}^{N-1} \alpha_{jk} \ln(z_j) \ln(z_k) + \varepsilon. \tag{30}$$

Estimates of the coefficients α_{00}, α_{j0} and α_{jk} (with $j = 1, \ldots, (N-1)$) may be obtained using Corrected Ordinary Least Squares or Maximum Likelihood methods while the input distance function $D_{in}(z^{x_i}, e)$ for each individual x_i is provided by the transformation:

$$D_{in}(z^{x_i}, e) = \exp\{\max(\varepsilon_{x_i}) - \varepsilon_{x_i}\} \tag{31}$$

Deutsch et Silber (2005) argue that the distance $D_{in}(z^{x_i}, e)$, by definition, will be greater than or equal to one (since its logarithm will be positive) and will hence indicate by how much an individual's resources must be scaled back in order to reach the resource frontier. The standard of living for individual x_i will then be obtained by dividing $D_{in}(z^{x_i}, e)$ by the minimum observed distance value which by definition equals one.

B) The Information Theory Approach

Information theory (IT) was firstly developed by Claude Shannon as a discipline within the mathematical theory of communication and it aims to determine how much data can be transmitted through a channel without significant losses or errors (see Shannon 1948). Theil (1967) was probably the first one to have the idea to apply this theory to economics.

Maasoumi (1993) exposed the basic principle and defined entropy as a measurement of the uncertainty, the disorder or the volatility associated with a given *random variable* or stochastic variable. Cowell et Kuga (Cowell 1977, Cowell et Kuga 1981a, Cowell et Kuga 1981b) extended the family of IT-based inequality indices to the Generalised Entropy measures given by equation 32:

$$GE_\gamma = \frac{1}{\gamma(1-\gamma)} \frac{1}{|\mathfrak{P}|} \sum_{i=1}^{|\mathfrak{P}|} \left[1 - \left(\frac{\nu_{x_i}}{\overline{\nu}} \right)^\gamma \right] \tag{32}$$

where $\gamma \in]-\infty; +\infty[$ is a parameter capturing the sensitivity of a particular GE index to different parts of the distribution, while $\overline{\nu} = \frac{1}{|\mathfrak{P}|} \sum_{i=1}^{|\mathfrak{P}|} \nu_{x_i}$ is the mean income. The smaller the γ, the higher the measure's sensitivity to the lower tail, that is, the poor. The first index of Theil T_1 and the second index of Theil T_2 are given as follows:

- For $\gamma = 1$, $GE_1 = T_1 = \dfrac{1}{|\mathfrak{P}|} \displaystyle\sum_{i=1}^{|\mathfrak{P}|} \frac{\nu_{x_i}}{\overline{\nu}} \ln \left(\frac{\nu_{x_i}}{\overline{\nu}} \right)$

- For $\gamma = 0$, $GE_0 = T_2 = \dfrac{1}{|\mathfrak{P}|} \displaystyle\sum_{i=1}^{|\mathfrak{P}|} \ln \left(\frac{\overline{\nu}}{\nu_{x_i}} \right)$

Maasoumi (1986) applied information theory in the context of multidimensional measurement of inequality. He used information theory both in the aggregation across attributes (to obtain a well-being index for each individual) and in the aggregation across individuals (to obtain the inequality measure). The distance function $\mathcal{D}_\gamma(\cdot)$ is defined as the weighted mean of the

relative entropy divergences between $\Delta_{\mathfrak{P}} = \{\Delta_{\mathfrak{P}}(x_1), \Delta_{\mathfrak{P}}(x_2), \ldots, \Delta_{\mathfrak{P}}(x_n)\}$ and $\Psi^j = \{\Psi^j(f_j(x_1)), \Psi^j(f_j(x_2)), \ldots, \Psi^j(f_j(x_n))\}$ formulated as follows:

$$\mathcal{D}_\gamma(\Delta_{\mathfrak{P}} \parallel \mathbb{X}; \lambda) = \sum_{j=1}^{m} \lambda_j \frac{1}{\gamma(1-\gamma)} \sum_{i=1}^{|\mathfrak{P}|} \Delta_{\mathfrak{P}}(x_i) \left[1 - \left[\frac{\Delta_{\mathfrak{P}}(x_i)}{\Psi^j(f_j(x_i))}\right]^\gamma\right] \tag{33}$$

where λ_j is the weight associated to the generalized entropy distance of each attribute. The minimisation of $\mathcal{D}_\gamma(\Delta_{\mathfrak{P}} \parallel \mathbb{X}; \lambda)$ with respect to $\Delta_{\mathfrak{P}}(x_i)$, subject to $\sum_{i=1}^{|\mathfrak{P}|} \Delta_{\mathfrak{P}}(x_i)$, provides the *optimal aggregation functions* given by equation 34 and equation 35:

$$\Delta_{\mathfrak{P}}(x_i) = \left[\sum_{j=1}^{m} \lambda_j \left[\Psi^j(f_j(x_i))\right]^\gamma\right]^{1/\gamma} \text{ when } \gamma \neq 0 \tag{34}$$

$$\Delta_{\mathfrak{P}}(x_i) = \prod_{j=1}^{m} \left[\Psi^j(f_j(x_i))\right]^{\lambda_j} \text{ when } \gamma = 0 \tag{35}$$

where γ is a parameter who determines the level of substitution between attributes in the aggregate function. Maasoumi proposed to use a generalised entropy measure on the resulting well-being indices:

$$GEM_\gamma = \frac{1}{\gamma(1-\gamma)} \frac{1}{|\mathfrak{P}|} \sum_{i=1}^{|\mathfrak{P}|} \left[1 - \left[\frac{\Delta_{\mathfrak{P}}(x_i)}{\overline{\Delta}_{\mathfrak{P}}(x_i)}\right]^\gamma\right] \tag{36}$$

This measure is argued failing to satisfy the multidimensional version of the Pigou-Dalton principle of transfer, known as Uniform Majorisation (see Lugo and Maasoumi 2008). Lugo et Maasoumi (2008) proposed to overcome this criticism using a similar approach to Maasoumi (1986) in order to derive a multidimensional poverty index. Thus, they suggested two alternative approaches to the derivation of multidimensional poverty indices using instruments from information theory: the *aggregate poverty line approach* and the *component poverty line approach*.

The *aggregate poverty line* $\Delta_{\mathfrak{P}}(\xi)$, consistent with the IT aggregator functions $\Delta_{\mathfrak{P}}(x_i)$ derived above and given by the equations 34 et 35 is:

$$\Delta_{\mathfrak{P}}(\xi) = \left[\sum_{j=1}^{m} \lambda_j \left[\xi_j\right]^\gamma\right]^{1/\gamma} \text{ when } \gamma \neq 0 \tag{37}$$

$$\Delta_{\mathfrak{P}}(\xi) = \prod_{j=1}^{m} \left[\xi_j\right]^{\lambda_j} \text{ when } \gamma = 0 \tag{38}$$

Each attribute ξ^j of the poverty line plays a role in defining a multi-attribute poverty line $\Delta_{\mathfrak{P}}(\xi)$, which incorporates the same weights for the attributes as considered for each individual. The Aggregate Poverty Line (APL) multidimensional poverty measure $\mathcal{F}_\gamma$ is thus defined by equation 39:

$$\mathcal{F}_\gamma(\Delta_{\mathfrak{P}};\xi) = \frac{1}{|\mathfrak{P}|}\sum_{i=1}^{|\mathfrak{P}|}\left[\max\left\{\frac{\Delta_{\mathfrak{P}}(\xi)-\Delta_{\mathfrak{P}}(x_i)}{\Delta_{\mathfrak{P}}(\xi)};0\right\}\right]^\gamma \mathbf{I}\big(\Delta_{\mathfrak{P}}(\xi)>\Delta_{\mathfrak{P}}(x_i)\big) \tag{39}$$

In the second approach, the component poverty line, we seek to obtain the synthesis functions

$$\delta^j_{x_i} = \frac{\xi_j - \Psi^j(f_j(x_i))}{\xi_j} \tag{40}$$

instead of $\Psi^j(f_j(x_i))$. All $\delta^j_{x_i}$ can be interpreted as the *shortfalls to threshold* with $0 \leq \delta^j_{x_i} \leq 1$ for all *poor* and $\delta^j_{x_i} \leq 0$ for all non-poor (Bourguignon and Chakravarty 2003). The shortfall indicator is then given by the equations 41 and 42:

$$\Delta_{\mathfrak{P}}(\delta_{x_i}) = \left[\sum_{j=1}^{m}\lambda_j\left[\frac{\xi_j-\Psi^j(f_j(x_i))}{\xi_j}\right]^\gamma\right]^{1/\gamma} \text{ when } \gamma \neq 0 \tag{41}$$

$$\Delta_{\mathfrak{P}}(\delta_{x_i}) = \prod_{j=1}^{m}\left[\frac{\xi_j-\Psi^j(f_j(x_i))}{\xi_j}\right]^{\lambda_j} \text{ when } \gamma = 0 \tag{42}$$

The λ_j are the positive weights associated to each jth *shortfall* and γ represents the level of substitutability between shortfalls. The higher the γ, the lower the degree of substitutability between them. When $\gamma \to \infty$, the relative deprivations are non-substitutes and when $\gamma = 1$ shortfalls are perfect substitutes. The individual poverty function is then given by the equation 43 and the multidimensional poverty measurement by the equation 44.

$$\mathcal{F}_{\mathfrak{P}}(x_i) = \left[\sum_{j=1}^{m}\lambda_j \max\left\{\delta^j_{x_i};0\right\}^\gamma\right]^{1/\gamma} \tag{43}$$

$$\mathcal{M}_{\mathfrak{P}}(\Delta_{\mathfrak{P}}(\delta);\xi;\alpha) = \frac{1}{|\mathfrak{P}|}\sum_{i=1}^{|\mathfrak{P}|}\mathcal{F}^\alpha_{\mathfrak{P}}(x_i)\ \mathbf{I}\big(\mathcal{F}_{\mathfrak{P}}(x_i)>0\big) \tag{44}$$

which is the αth moment of the distribution of $\Delta_{\mathfrak{P}}(\delta) = \big\{\Delta_{\mathfrak{P}}(\delta_{x_1}),\ldots,\Delta_{\mathfrak{P}}(\delta_{x_n})\big\}$. The component poverty line (CPL) multidimensional poverty measure is thus given by the equation 46:

$$\mathcal{M}^{\gamma}_{\mathfrak{P}}(CPL_{strong}) = \frac{1}{|\mathfrak{P}|}\sum_{i=1}^{|\mathfrak{P}|}\left[\sum_{j=1}^{m}\lambda_j \max\left\{\frac{\xi_j - \Psi^j(f_j(x_i))}{\xi_j}; 0\right\}^{\gamma}\right]^{\alpha/\gamma}\mathbf{I}\big(\mathcal{F}_{\mathfrak{P}}(x_i) > 0\big) \tag{45}$$

and

$$\mathcal{M}^{\gamma}_{\mathfrak{P}}(CPL_{strong}) = \frac{1}{|\mathfrak{P}|}\sum_{i=1}^{|\mathfrak{P}|}\left[\prod_{j=1}^{m}\max\left\{\frac{\xi_j - \Psi^j(f_j(x_i))}{\xi_j}; 0\right\}^{\lambda_j}\right]^{\alpha}\mathbf{I}\big(\mathcal{F}_{\mathfrak{P}}(x_i) > 0\big) \tag{46}$$

This procedure produces an aggregate of relative deprivations that associates weights to each deprivation, and allows trade-offs between these relative deprivations in various attributes. Note that, the substitution is allowed only among attributes that are below the poverty threshold.

For more details about these approaches, we invite readers to refer to Lugo et Maasoumi (2008).

C) The Inertia Approach

The inertia approach (see Asselin 2009) is a parametric approach to the composite poverty indicator who is mainly based on multivariate analysis techniques. A brief review of literature on inertia approaches for multidimensional measurement of poverty allowed us to summarise the following techniques.

Principle Components Analysis (PCA)

Filmer and Pritchett (2001) built a linear index of wealth based on asset ownership indicator variables using data from India. They used PCA to derive the standardised first principal component of the variance-covariance matrix of the observed household assets (also called *weights*) for the asset indicators. They concluded that in the absence of data on consumption expenditures, applying PCA to compute asset indices is a coherent and stable alternative. For further details and applications, see Klasen 2000, Filmer and Pritchett 1997, Asselin 2009, Kabubo-Mariara and al. 2010.

Factor Analysis (FA)

Sahn and Stifel (2003) used factor analysis to evaluate the potential of an asset-based index as an indicator of household economic welfare. In the case

of a capability approach, Silber (2007) argued that the factor analysis would provide a theoretical framework for explaining the (observed) functionings by means of capabilities represented by the latent factors but such a model will not explain the latent variables. For further details and applications, see Hirschberg and al. (1991), Nolan and Whelan 1996, Asselin 2009.

Multiple Correspondence Analysis (MCA)

Booysen et al. (2007) used MCA to build asset-based composite poverty indicators. MCA is preferable when we are in presence of categorical data and it can easily combine quantitative variables and categorical variables. MCA is also helpful for visualisation data and variables on the same graph. For more details and application, see Asselin 2009.

Cluster Analysis (CA)

Ferro Luzzi et al. (2006) used cluster analysis to aggregate individuals according to how similar they are with regard to their various scores of multiple deprivations. Hirschberg et al. (1991) proposed statistical cluster analysis methods to explore different ways and levels for clustering of 23 diverse attributes such as political rights, civil liberties, life expectancy, literacy, real domestic product, etc. Then, they use cluster analysis to see how different welfare units (countries) cluster together on the basis of the attributes considered, in a manner (metric) consistent with the one used in order to cluster the attributes.

4.3.4 Intersection and Union Approaches

The measurement based on the intersection[9] considers that an individual x_i is poor if he is not able to satisfy the minimum requirements on all basic needs. Then, an individual x_i is poor if he has a bad score on all attributes $\mathbb{X}_j$ $(j \in J)$. According to this point of view, the function $\Delta_{\mathfrak{P}}(x_i, \xi)$ is given by the following equation:

$$\Delta_{\mathfrak{P}}(x_i, \xi) = \begin{cases} \tau > 0, & \text{if } f_j(x_i) < \xi_j, \ \forall\, j \in J; \\ 0, & \text{eitherwise} \end{cases} \tag{47}$$

The measurement based on the union considers that an individual x_i is poor if he is not able to satisfy the minimum requirements for one basic need. Then, an individual x_i is poor if he is for *at least* on of attributes $\mathbb{X}_j$

[9]See Duclos et al. 2006), for more information.

$(j \in J)$. According to this point of view, the function $\Delta_{\mathfrak{P}}(x_i, \xi)$ is given by the following equation:

$$\Delta_{\mathfrak{P}}(x_i, \xi) = \begin{cases} 0, & \text{if } f_j(x_i) \geq \xi_j, \ \forall\, j \in J; \\ \tau > 0, & \text{eitherwise} \end{cases} \tag{48}$$

4.3.5 Stochastic Dominance Approaches

The multidimensional stochastic dominance approaches are multidimensional extensions to the stochastic approaches developed by Atkinson (1987), Foster and Shorrocks (1988a and 1988b), Zheng (1999 and 2001) for a one-dimensional framework. Yélé Maweki (2008) argued that such approaches make ordinal poverty comparisons possible over classes of procedures for aggregations across dimensions and across individuals. They also allow for robustness over areas of possible multidimensional poverty "frontiers" — analogous to the usual unidimensional poverty lines. We outline these techniques briefly below.

Let two functions of cumulative distributions ψ and ψ^* defined on $[0, \xi_1^*] \times [0, \xi_2^*]$, the poverty measurements associated to these distributions are given by the following equations 49 et 50:

$$\mathbf{P}(\psi, \xi) = \int_0^{\xi_1^*} \int_0^{\xi_2^*} \Delta_{\mathfrak{P}}(a_1, a_2; \xi_1, \xi_2)\, d\psi(a_1, a_2) \tag{49}$$

$$\mathbf{P}(\psi^*, \xi) = \int_0^{\xi_1^*} \int_0^{\xi_2^*} \Delta_{\mathfrak{P}}(a_1, a_2; \xi_1, \xi_2)\, d\psi^*(a_1, a_2) \tag{50}$$

where $\Delta_{\mathfrak{P}}(\cdot)$ is the function of individual poverty associated to a person x_i with (a_1, a_2) as attributes. The objective is then to compare the two poverty measurements obtained by the equations 49 and 50. For this, one examines the difference between the two measurements:

$$\Delta\mathbf{P}(\xi) = \mathbf{P}(\psi, \xi) - \mathbf{P}(\psi^*, \xi) = \int_0^{\xi_1^*} \int_0^{\xi_2^*} \Delta_{\mathfrak{P}}(a_1, a_2; \xi_1, \xi_2)\, d\Phi(a_1, a_2) \tag{51}$$

with $\Phi(a_1, a_2) = \psi(a_1, a_2) - \psi^*(a_1, a_2)$. Thus, the distribution ψ ***dominates*** ψ^* if $\Delta\mathbf{P}(\xi) < 0$.

Bourguignon et Chakravarty (2002) defined three classes of poverty measurements with two substitutable, complementary or independent attributes.

⋆ **Substitutability**

The first order stochastic dominance at each attribute of poverty can be applied when the two attributes are *substitutable*. According to Bourguignon and Chakravarty (2002) the first order stochastic dominance through intersection of two dimensions of poverty is given by the following equation 52:

$$\Delta \mathbf{P}(x_i) = \int_0^{a_1} \int_0^{a_2} d\Phi(b_1, b_2) \leq 0, \ \forall\, a_1 \leq \xi_1^* \text{ et } a_2 \leq \xi_2^* \tag{52}$$

⋆ **Complementarity**

The first order stochastic dominance under the condition of order robustness, through the union of two dimensions of poverty is given by the following equation 53:

$$\Delta \mathbf{P}(x_i) = \sum_{j=1;2} \int_0^{a_j} d\Phi_{b_j}(b_j) - \int_0^{a_1} \int_0^{a_2} d\Phi(b_1, b_2) \leq 0, \ \forall\, a_1 \leq \xi_1^* \text{ et } a_2 \leq \xi_2^* \tag{53}$$

⋆ **Independency**

When the two attributes are *independent*, the stochastic dominance verifies the conditions given by the equation 54:

$$\frac{\partial^2 \Delta_{\mathfrak{P}}(a_1, a_2; \xi_1, \xi_2)}{\partial a_1 \partial a_2} = 0 \text{ and } \Delta \mathbf{P}(a_j) = \int_0^{a_j} d\Phi_{b_j}(b_j) \leq 0, \ \forall\, a_j \leq \xi_j^* \tag{54}$$

For further details and applications, the readers are invited to see [Atkinson and Bourguignon 1982, Bourguignon and Chakravarty 2002, Duclos and al. (2006), Yélé Maweki 2007 and Yélé Maweki 2008, Sami and El Lahga 2008]. The main drawback of the multidimensional stochastic approach is related to the fact that its applications are in general limited to the two-dimensional context. Thus, the realm of multidimensional stochastic approaches remains an open-subject when we are in presence of more than two dimensions.

4.4 Axiomatic Characterisation

Axiomatic characterisations have been advocated in order to maintain the multidimensional conceptualisation of poverty and to aggregate such dimensions into a composite index (as in the case of monetary indices) in accordance

with the properties (or axioms) that a poverty index must verify. The first authors to adopt an axiomatic approach in the context of multidimensional poverty indices were Chakravarty and al. (1998), followed by Tsui (2002) after his previous works on the axiomatisation of multidimensional indices of inequality (Tsui 1995), then Bourguignon and Chakravarty (2002 and 2003).

4.4.1 Some axioms and their interpretation

A multidimensional measurement of poverty $\mathbf{P}(\mathbb{M}, \xi)$ is expected to verify some axioms which, mostly, are a multidimensional extension of a set of desirable properties for poverty indices. Such axioms have an impact on the aggregation functions which allows to compute the magnitude of multidimensional poverty measurement. The functions $\mathcal{F}(\cdot)$ and $\pi(\cdot)$ explicitly depend on the axioms that the poverty measurement must absolutely respect. In the following we present some axioms for multidimensional poverty measurement discussed in the literature [Zheng 1997, Chakravarty and al. 1998, Tsui 2002, Bourguignon and Chakravarty (2003), Sami 2005].

Axiom 4.1 Continuity (CN): This axiom ensures that the poverty measurement must be not too sensitive to a marginal variation in the quantity of an attribute, sometimes due to errors of observation on the quantities of attributes. Formally, for all $\xi \in \Xi$, $\mathbf{P}(\cdot)$ is continuous on $\mathbb{M}$.

Axiom 4.2 Symmetry (SM): The axiom of symmetry claims that any characteristic, other than the quantities of attributes $f_j(x_i)$ used to define the multidimensional index of poverty, does not impact on measuring poverty. Formally, for any pattern matrix $\mathbb{M}$ and for any $\xi \in \Xi$, $\mathbf{P}(\mathbb{M}, \xi) = \mathbf{P}(\Pi_\sigma \mathbb{M}, \xi)$, where Π_σ is any transformation matrix of relevant order (see definition 4.2).

Axiom 4.3 Focus (FO): The focus axiom requires that a poverty measurement $\mathbf{P}(\mathbb{M}, \xi)$ remains unchanged if the situation of an individual x_i improves on an attribute $\mathbb{X}_j$ on which it is not poor (non-lean attribute). Formally, let $\mathbb{M} = [f_j(x_i)]$ and $\widetilde{\mathbb{M}} = [g_j(x_i)]$, IF for any $x_i \in \mathfrak{P}$ s.t. $g_j(x_i) = f_j(x_i) + \epsilon$ and $f_j(x_i) \geq \xi_j$, with $\epsilon > 0$ THEN $\mathbf{P}(\widetilde{\mathbb{M}}, \xi) = \mathbf{P}(\mathbb{M}, \xi)$.

Bourguignon and Chakravarty (2003) proposed two versions of the focus axiom: *strong focus* and *weak focus*.

Axiom 4.4 Strong Focus (SF): The strong focus axiom implies non-compensation (or non-substitutability) between lean and non-lean attributes of poor people. In other terms the measurements of poverty should not

account for the improved well-being of individuals who are not poor. Formally, let $\mathbb{M} = [f_j(x_i)]$ and $\widetilde{\mathbb{M}} = [g_j(x_i)]$, IF (**a**) for any x_i s.t. $f_j(x_i) \geq \xi_j$, $g_j(x_i) = f_j(x_i) + \epsilon$ (with $\epsilon > 0$), (**b**) $f_j(x_k) = g_j(x_k)$ for all $x_k \neq x_i$, and (**c**) $f_l(x_i) = g_l(x_i)$ for all $l \neq j$ and for all x_i, THEN $\mathbf{P}(\widetilde{\mathbb{M}}, \xi) = \mathbf{P}(\mathbb{M}, \xi)$.

Axiom 4.5 Weak Focus (WF): weak focus considers that the measurement of poverty is only independent of the quantities of attributes of non-poor individuals. Thus, one should not take in account the attributes of non-poor individuals such as in strong focus. Formally, let $\mathbb{M} = [f_j(x_i)]$ and $\widetilde{\mathbb{M}} = [g_j(x_i)]$, IF for some x_i, $f_p(x_i) \geq \xi_p$ for all p and (**a**) for any $j \in J$, $g_j(x_i) = f_j(x_i) + \epsilon$ (with $\epsilon > 0$), (**b**) $f_q(x_i) = g_q(x_i)$ for all $q \neq j$, and (c) $f_l(x_k) = g_l(x_k)$ for all $x_k \neq x_i$ and for all $l \in J$, THEN $\mathbf{P}(\widetilde{\mathbb{M}}, \xi) = \mathbf{P}(\mathbb{M}, \xi)$.

Contrary to the strong focus, weak focus allows the compensation (or substitutability) between lean and non-lean attributes of poor people. The weak focus is useful when basic needs are exchangeable and are focused on the attributes of the same type. However, it poses significant analytical difficulties (because of the mixity of attributes). For this reason several authors prefer to use only the strong focus axiom in practice.

Axiom 4.6 Monotonicity (MN): The axiom of monotonicity requires that the poverty measure should not increase (or decrease) when a poor person on an attribute improves (or deteriorates) its situation in relation to this attribute. Formally, let $\mathbb{M} = [f_j(x_i)]$, $\widetilde{\mathbb{M}} = [g_j(x_i)]$, $\xi \in \Xi$ and $j \in J$, IF (**a**) for any x_i s.t. $g_j(x_i) = f_j(x_i) + \epsilon$, where $f_j(x_i) < \xi_j$ and $\epsilon > 0$, (**b**) $f_j(x_k) = g_j(x_k)$ for all $x_k \neq x_i$, and (**c**) $f_l(x_i) = g_l(x_i)$ for all $l \neq j$ and for all x_i, THEN $\mathbf{P}(\widetilde{\mathbb{M}}, \xi) \leq \mathbf{P}(\mathbb{M}, \xi)$.

The monotonicity axiom claims that a measurement of poverty should be able to consistently reflect the variation of the well-being of poor people.

Axiom 4.7 Principle of Population (PP): The principle of population requires to construct poverty measurements insensitive to the population size. Formally, let $\mathbb{M} = [f_j(x_i)]$ be a $n \times m$ pattern matrix, let $\xi \in \Xi$ and $k \in \mathbb{N}$, $\mathbf{P}(\mathbb{M}, \xi) = \mathbf{P}(\mathbb{M}^k, \xi)$, where $\mathbb{M}^k$ is the k-fold replication of $\mathbb{M}$.

The principle of population axiom is necessary for the ordinal comparisons of poverty between countries because it implies that the distributions of the basic needs corresponding to the populations of different sizes can be compared.

Axiom 4.8 Scale Invariance (SI): The axiom of scale invariance requires that the poverty measure $\mathbf{P}(\mathbb{M}, \xi)$ should be invariant with respect to transformations of the measurement scales of attributes and thresholds. Let $\mathbb{M} = [f_j(x_i)]$ be a $n \times m$ pattern matrix and let $\xi \in \Xi$, $\mathbf{P}(\mathbb{M}, \xi) = \mathbf{P}(\Lambda\mathbb{M}, \Lambda\xi)$, where $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_m)$ and $\lambda_i > 0$ for all $i \in \{1, \ldots, m\}$.

According to the axiom of scale invariance, the matrix M may involve attributes with different measurement scales (or units). Thus, the measurement of poverty in terms of the CFA franc would give the same result in terms of Euro or U.S. Dollars. The measurement of poverty should be insensitive to changes of scale. Only the relative distance between the quantities of all attributes and their respective thresholds is required.

Axiom 4.9 Subgroup Monotonicity (SGM): The axiom of subgroup monotonicity recommends that a multidimensional measure of poverty is sensitive to the level of welfare of different subgroups of the population having homogenous characteristics such as age, gender, place of residence etc.. For example if we assume a total population divided into two subgroups (a and b), we obtain the following formulation: let $\mathbb{M} = \begin{pmatrix} \mathbb{M}^a \\ \mathbb{M}^b \end{pmatrix}$ and $\mathbb{N} = \begin{pmatrix} \mathbb{N}^a \\ \mathbb{N}^b \end{pmatrix}$ with $(\mathbb{M}^a, \mathbb{N}^a) \in n^a \times m$ and $(\mathbb{M}^b, \mathbb{N}^b) \in n^b \times m$.

$$\Big[\Big(\mathbf{P}(\mathbb{M}^a, \xi) > \mathbf{P}(\mathbb{N}^a, \xi)\Big) \wedge \Big(\mathbf{P}(\mathbb{M}^b, \xi) > \mathbf{P}(\mathbb{N}^b, \xi)\Big)\Big] \Longrightarrow \mathbf{P}(\mathbb{M}, \xi) > \mathbf{P}(\mathbb{N}, \xi) \tag{55}$$

$\mathbf{P}(\mathbb{M}, \xi)$ verifies the axiom 4.9 if it is formulated as follows:

$$\mathbf{P}(\mathbb{M}, \xi) = \mathcal{F}\Big(\frac{1}{n}\Delta_{\mathfrak{P}}(x_1, \xi), \ldots, \frac{1}{n}\Delta_{\mathfrak{P}}(x_i, \xi), \ldots, \frac{1}{n}\Delta_{\mathfrak{P}}(x_n, \xi)\Big) \tag{56}$$

Axiom 4.10 Subgroup Decomposability (SD): The axiom of subgroup decomposability requires that if the population is partitioned into subgroups with respect to a homogeneous characteristic (sex, region, age, region, religion, etc..), then the multidimensional measure of poverty is a weighted average of measures of multidimensional poverty subgroups. Formally, for any $\mathbb{M}_1, \ldots, \mathbb{M}_K$ s.t. $\mathbb{M} \equiv \mathbb{M}_1 \cup, \ldots, \cup \mathbb{M}_K$ and $\xi \in \Xi$, $\mathbf{P}(\mathbb{M}_1 \cup, \ldots, \cup \mathbb{M}_K, \xi) = \sum_{i=1}^{K} \frac{n_i}{n} \mathbf{P}(\mathbb{M}_i, \xi)$, where for each i, $n_i = |\mathbb{M}_i|$ and $n = \sum n_i$.

The axiom of subgroup decomposability is especially important for the targeting of the subgroups sensitive to poverty. This is extremely useful for improving the effectiveness of policies to reduce poverty.

Definition 4.5 Pigou-Dalton Progressive Transfer (PDPT): The pattern matrix $\mathbb{M} = [f_j(x_i)]$ is said to be obtained from $\widetilde{\mathbb{M}} = [g_j(x_i)]$ by Pigou-Dalton progressive transfer of attribute j from one poor person if for some people $x_i,\ x_k$: (**a**) $g_j(x_k) < g_j(x_i) < \xi_j$, (**b**) $f_j(x_k) - g_j(x_k) = g_j(x_i) - f_j(x_i) > 0$, with $g_j(x_i) - f_j(x_i) > 0$ and $f_j(x_k) \leq f_j(x_i)$, (**c**) $f_j(x_l) = g_j(x_l)$ for all $x_l \neq x_l,\ x_k$, and (**d**) $f_p(x_l) = g_p(x_l)$ for all $p \neq j$ and all x_l.

Axiom 4.11 Multidimensional Transfer Principle (MTP): The multidimensional transfer principle is a multidimensional extension of Sen's (1976) assertion according to which the monetary measurements have to be sensitive to income inequality among the poor. It argues that the multidimensional measurement of poverty decreases (or does not increase) when the inequality between the attributes of the poor decreases formally. Thus, for all $\mathbb{M} = [f_j(x_i)]$ and $\xi \in \Xi$, IF $\widetilde{\mathbb{M}}$ is obtained from $\mathbb{M}$ such that $\widetilde{\mathbb{M}} = \Pi_\sigma \mathbb{M}$ THEN $\mathbf{P}(\widetilde{\mathbb{M}}, \xi) \leq \mathbf{P}(\mathbb{M}, \xi)$.

Note that, $\widetilde{\mathbb{M}}$ is a matrix obtained from the matrix $\mathbb{M}$ by redistributing the attributes of the poor according to a bistochastic matrix Π_σ.

Axiom 4.12 One Dimensional Transfer Principle (ODTP): The one dimensional transfer principle is a form of the multidimensional transfer principle that refers to a transfer of a single attribute $\mathbb{X}_j$ between two poor people. Thus, it states that a Pigou-Dalton progressive (regressive) transfer between two poor people should not increase (decrease) poverty. Formally, for all $\widetilde{\mathbb{M}} = [g_j(x_i)]$, IF $\mathbb{M}$ is obtained from $\widetilde{\mathbb{M}}$ by a Pigou-Dalton progressive transfer of some attribute between two poor, THEN $\mathbf{P}(\mathbb{M}, \xi) \leq \mathbf{P}(\widetilde{\mathbb{M}}, \xi)$, where $\xi \in \Xi$ is arbitrary.

Axiom 4.13 Non-Decreasing Poverty under Correlation Increasing Switch (NDCIS): This axiom is an extension of the proposal made by Atkinson and Bourguignon (1982) to highlight the situation in which rearrangements of the attributes between individuals increase the correlation between attributes. These rearrangements occur during a series of transfers of attributes within the poor population that increase the correlation between attributes without improving the situation of the poor. The fact that the situation of a poor person gets worse on the different attributes implies that poverty is not expected to decrease. Formally, let $\mathbb{M}_p$ be a sub-matrix of $\mathbb{M}$ corresponding to individuals in situation of poverty.

$$\forall\, (\mathbb{M}, \xi) \in \mathcal{M}_{nm} \times \Xi \quad \left[\widetilde{\mathbb{M}} \leftarrow \Pi_\sigma \mathbb{M}_p\right] \Longrightarrow \mathbf{P}(\widetilde{\mathbb{M}}, \xi) \geq \mathbf{P}(\mathbb{M}, \xi) \tag{57}$$

where Π_σ is a stochastic matrix such that $\Pi_\sigma \mathbb{M}_p$ be different from a transformation matrix of the lines of $\mathbb{M}_p$.

Given that the baskets of attributes of wealthy individuals remain unchanged, this axiom can be applied to substitutable attributes. Note that, when the attributes are substitutable, a rearrangement that increases their inter-correlation should not decrease the measurement of poverty because the poorest people are unable to compensate the leaner attributes by the strongest attributes. By contrast, when the attributes are complementary, a rearrangement that increases the correlation of attributes can not increase poverty.

The converse property of NDCIS will be denoted by NICIS (Non-Increasing Poverty under Correlation Increasing Switch). The detailed discussion on the axiomatic approaches may be found, for example, in the studies of Zheng (1997), Chakravarty and al. (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Fusco 2005, Sami and El Lahga 2008. However, defining an axiom on an aggregated measure is not an easy task because the desired properties may differ depending on whether we aggregate preferences or knowledge.

4.4.2 List of measurements characterised axiomatically

Sen (1976) pioneered the axiomatic approach to the measurement of poverty by explicitly setting out the axioms against which a poverty measure should be judged. The idea to use axioms is based on the observation that, despite the limitations of the head-count ratio FGT_0 (see subsection 4.2.4) which violates both of the following axioms (monotonicity axiom and the transfert axiom), it continues to be very widely used. Another common measure is the so-called "poverty gap" FGT_1 (used by the United States Social Security Administration; see Sen 1076) which is the aggregate shortfall of the income of all the poor taken together from the poverty line. This satisfies the monotonicity axiom but violates the transfer axiom. Sen elaborated a new poverty measurement by noticing the violation of these elementary conditions by the existing poverty measures. A poverty measurement may be assumed to satisfy certain postulates. The list of measurements characterised axiomatically is given by the following table: In a general way, a multidimensional poverty measurement $\mathbf{P}(\mathbb{M}, \xi)$ must satisfy **SM**, **PP**, **MN**, **FO** and **CN** (see Tsui (2002)). Note however, that the set of axioms for multidimensional poverty measurements which have been introduced in this paper are very restricted compared to the large literature on this field.

Measurements	**Axiomatic Characterisation**	
	Satisfies	Violates
Headcount Ratio FGT_0	-	**MN**, ODTP
Income Gap Ratio FGT_1	**MN**	**ODTP**
Sen's Poverty Index $\mathcal{S}(\mathbb{M}, \xi)$	**FO**, **MN** and **ODTP**	-
Watts index $\mathcal{W}(\mathbb{M}, \xi)$	**FO**, **MN**, **ODTP** and **SD**	-
Chakravarty and al. (1998)	**FO, MN, SD, SI, MTP and CN**	-
Bourguignon and al. (2003)	**MTP**, **NDCIS** or **NICIS**	-
Bourguignon and al. (2003)	**MN, CN, SD and OTP or MTP**	-
Bourguignon and al. (2003)	**SD**	-

Table 1: List of measurements characterised axiomatically

4.5 Sen's Capabilities Approach

The origins of the capabilities approach can be found in a series of papers on critiques of traditional welfare economics written by Sen in the early 1980s (see Sen (1979c, 1983 and 1985)) where he developed the concepts of *capabilities* and *functionings* for assessing the well-being of individuals. Functionings refer to an individual's achievements, i.e. what she or he manages *to do* or *to be*. Capabilities, which incorporate the idea of *freedom*, refer to an individual's *ability* and *real* opportunity to achieve a given functioning (*"doing"* or *"being"*).

Sen (1985) was the first to formalise the concept of capabilities. Let $f_\star(x_i)$ representing the vector of commodities possessed by an individual (or household) x_i (with $i \in I$). Let $\psi(\cdot)$ representing the conversion function of a commodity vector into a vector of objective characteristics. Let $g_{x_i}[\cdot]$ representing a personal utilisation function of x_i reflecting one pattern of use that x_i can actually make. Let $\mathcal{L}_{x_i}$ represent the set of utilisation function g_{x_i} among which individual x_i can make its choice and let $\mathcal{V}_{x_i}\big[\,\cdot\,\big]$ represent the valuation of the vector of functionings h_{x_i} which evaluates the level of well-being of x_i. Thus, the achieved function vector h_{x_i} can be given by equation 58 and represents the achieved functioning of individual x_i when he chooses utilisation function g_{x_i} for a vector of commodities $f_\star(x_i)$.

$$h_{x_i} = g_{x_i} \circ \psi(f_\star(x_i)) = g_{x_i}\big[\psi(f_\star(x_i))\big] \tag{58}$$

The vector h_{x_i} represents what individual x_i is *able to do* (doing) and *to be* (being). The well-being of individual x_i can then be given by equation 59.

$$\widetilde{\mathcal{V}}_{x_i} = \mathcal{V}_{x_i}[h_{x_i}] = \mathcal{V}_{x_i}\big[g_{x_i} \circ \psi(f_\star(x_i))\big] \tag{59}$$

By using $\mathcal{V}_{x_i}\big[\,\cdot\,\big]$, it is possible to characterise the valuations of well-being that individual x_i can potentially achieved as shown by equation 60 below.

$$\widetilde{\mathcal{V}}_{x_i} = \Big\{ \mathcal{V}_{x_i} \;:\; \exists\; \mathcal{V}_{x_i}[h_{x_i}],\; h_{x_i} \in \mathcal{Q}_{x_i}(f_\star(x_i)) \Big\} \tag{60}$$

From this, one can derive a set of vectors of achievable functionings $\mathcal{R}_{x_i}(f_\star(x_i))$ given by equation 61.

$$\mathcal{R}_{x_i}(f_\star(x_i)) = \Big\{ h_{x_i} \;:\; h_{x_i} = g_{x_i} \circ \psi(f_\star(x_i)),\; \forall\; g_{x_i}[\cdot] \in \mathcal{L}_{x_i} \Big\} \tag{61}$$

Therefore, the vector of achievable functionings becomes:

$$\mathcal{Q}_{x_i}(f_\star(x_i)) = \Big\{ h_{x_i} \;:\; h_{x_i} = g_{x_i} \circ \psi(f_\star(x_i)),\; \forall\; g_{x_i}[\cdot] \in \mathcal{L}_{x_i} \wedge \forall\; f_\star(x_i) \in \widetilde{\mathbb{X}}_{x_i} \Big\} \tag{62}$$

and, $\mathcal{Q}_{x_i}(f_\star(x_i))$ formally represents the set of capabilities of individual x_i, i.e. the freedom that individual x_i has to choose among all possible functionings alternatives according to its personal characteristics and its social environment.

Critics on Sen's capability approach tend to focus on the ambiguities of capabilities theory and questioned its ability to aggregate into a single index the multiple capabilities. Bénicourt (2004 and 2006) evokes the impasse of an approach on several ethical criteria. Navarro (2000) highlights the lack of analysis of power relations between international organisations and developing countries. Pogge (2002) defends the Rawlsian primary goods against the capabilities. Nussbaum (1988) criticises Sen to not have provided a clear list of capabilities, and thereafter it (Nussbaum 2000, 2003 and 2005a) isolated human capability that can be convincingly argued to be of central importance in all human life no matter what the person is or she chooses. However, the capability approach, whatever its flaws are, is more complete (in term of conceptualising poverty) than other existing approaches.

4.6 New Multidimensional Poverty Indexes

4.6.1 Alkire-Foster Approach

Alkire and Foster (2010) proposed a new methodology for multidimensional poverty measurement consisting of an identification method ρ_k that extends the traditional intersection and union approaches, and the class of poverty measures FGT_α. They used two forms of cutoff at the identification step: one within each dimension to determine whether a person is deprived in that dimension, and a second across dimensions that identifies the poor by *counting*

the dimensions in which a person is deprived. At the aggregation step they used the FGT measures, appropriately adjusted to account for multidimensionality. In their paper (Alkire and Foster 2010) they introduce a number of axioms and they show that their methodology satisfies a range of desirable properties including decomposability. The Alkire-Foster approach allows us to measure outcomes at the individual level (individual or household) against multiple criteria (dimensions and indicators). Their approach is flexible and can be used with different dimensions and indicators to create measures of incidence, intensity and depth of poverty, as well as inequality among the poor to different societies and situations according to the type of data available. For further details about the methodology and applications in the USA and in Indonesia, the reader is invited to see Alkire and Foster (2010).

4.6.2 The Multidimensional Poverty Index (MPI)

In 2010, the Oxford Poverty and Human Development Initiative (OPHI) and the United Nations Development Programme Human Development Report (see UNDP 2010) launched the Multidimensional Poverty Index (MPI) as a new measure that gives a *multidimensional* picture of people living in poverty. The MPI was created by Alkire and Santos (2010) using the Alkire-Foster approach (see Alkire and Foster 2010). According to Alkire and Santos (2010), the MPI is an index of acute multidimensional poverty which reflects deprivations in very rudimentary services and core human functionings for people across 104 developing countries. The MPI identifies deprivations across three dimensions: health, education and living standards (which are reflected in 10 indicators, each with equal importance), and shows the number of people who are multidimensionally poor and the deprivations that they face on the household level. The MPI is computed as follows:

$$MPI = H \times A \tag{63}$$

where H is the percentage of people who are MPI poor (incidence of poverty) and A the average intensity of MPI poverty across the poor (%). The MPI is most appropriate for less developed countries. It allows to capture the widespread deprivations in developing countries such as in South Asia and in Sub-Saharan Africa, and in the poorest Latin American countries. The MPI shows the magnitude of poverty beyond monetary measures—an important accomplishment. In short, the MPI allows us to compute and vividly convey overlapping deprivations— building on international consensus, captured in the Millennium Development Goals, about the dimensions of serious, and indeed unacceptable, disadvantage [see UNDP 2010; Alkire and Santos (2010)].

Note that, the MPI index uses household-level data and then aggregate them to the country level. As we mentioned above, MPI identifies three broad dimensions – health, education and living standards – which are weighted equally (one-third each) to form the composite multidimensional poverty index. This way to choose the weights and to add up fundamentally different things is arbitrary and greatly simplist. The choice of "weights" is in itself a decision problem and it is crucial to 'elicit' these parameters through robustness tests. The negative influence of compensation effects are neither studied nor considered in the MPI index.

5 Discussion

5.1 Position of Problem

Whatever the measurement designed to capture the acute poverty that people face at the same time, such as income per capita, GDP per capita, HDI or MPI, a majority of the worlds poorest countries today are in Africa, despite a wealth of natural resources. Of course some African countries like South Africa and Egypt are not quite as poor as others like Niger and Ethiopia. In 2010, 22 of the 24 nations identified as having "Low Human Development" on the United Nations' (UN) Human Development Index were located in Sub-Saharian Africa. In the other side, Africa presents the highest MPI poverty rates the same year. In 2006, 34 of the 50 nations on the UN list of Least Developed Countries (LDCs[10]) are in Africa. It is true that acute multidimensional poverty is more pronounced in African countries than others and what is more important is that the situation does not seem to improve.

However, the research of Alkire and Santos (2010) shows that multidimensional poverty is not solely an African problem, but it also affects others countries of the whole world. For instance, South Asia has the world's highest levels of poverty in term of human lives. Countries of Latin America, of Caribbean, of East Asia and Pacific countries reveal a high proportion of MPI poor. A long-term research project completed within the European Observatory on the Social Situation (see Lelkes and Zólyomi (2008)) argues that the rate of poverty varies between 10% and 23% in the countries of the European Union. Low levels of poverty characterise the Scandinavian countries, the so-called Corporatist countries (Austria, Germany), and the Czech Republic, Slovakia and Slovenia among the ex-Socialist countries. In contrast, the risk of poverty tends to be relatively high in the Mediterranean and the Baltic states. Altogether around 75 million people in the EU are at risk of

[10]See http://www.un.org/special-rep/ohrlls/ldc/statistics.htm

poverty. Countries with the highest poor population include France, Germany, Italy, Poland, Spain and the UK. According to Donna Smith (2010), the U.S. Census Bureau, approximately 43.6 million (14.3%) Americans were living in absolute poverty in 2009, up from 39.8 million (13.2%) in 2008.

This shows that poverty remains a worldwide scourge despite the variety of measurements existing in the literature to end poverty. Why this 'catastrophic failure' to eradicate poverty? Yet, poverty is known to all! We think that the problem is methodological.

Certainly there is a problem of political will. How much is the present distribution of wealth functional to maintaining the global geo-political status quo? Are the governments REALLY pursuing poverty reduction policies? Certainly there is a problem of social will. Are the wealthier societies aware and ready to transfer part of their wealth (thus becoming less wealthy) to less wealthy societies? Certainly there is a cultural problem. Are we sure that reducing poverty implies increasing consumption (of goods and services) using the standards of consumption of the wealthier societies?

All the above are serious methodological problems which we cannot discuss in this paper. But there is one more methodological problem and this concerns how science can help in fighting poverty and in designing adequate poverty reduction policies. Most of the actual poverty reduction policies are based on simplistic characterisations of who is poor. To put it the other side, most of the poverty measurement approaches are not aimed at aiding to conceive policies, but at representing an "objective situation of poverty". Unfortunately we have seen that this is misleading. Measuring the GDP increase of a country does not allow to understand if its citizens are becoming less poor. Qualitative information about the standards of life of the population are often neglected by poverty indexes if they cannot be quantified. The importance of offering chances to each individual to improve (chances to be used in very different ways) has only been seriously considered by Sen's capabilities theory, yet to become an operational approach to measure poverty. Our claim is that beyond political, social and cultural reasons for which we fail to be effective in reducing poverty we are also missing adequate measurement tools, able to help designing and implementing poverty reduction policies. We lack adequate policy analytics tools in this critical area.

Before starting a broader discussion on the issue of how we can improve poverty measurement through meaningful measurement, let us present some major features from the literature that make poverty a *complex problem*.

5.1.1 Poverty is multidimensional

Many authors [Townsend 1987; Sen (1979c, 1985, 1993 and 2001); Bourguignon and Chakravarty 2003; Fusco 2005; Bertin 2007; Kakwani and Silber (2008); Sami et El Lahga 2008; Alkire and Foster 2010; Alkire and Santos 2010] and international organisations [World Bank 2001 and 2005, UN 1998; UNDP 2010] agree that one indicator alone can not capture the multiple aspects that constitute poverty. Poverty is multidimensional. It includes several factors such as poor health, lack of education, inadequate living standard, lack of income, lack of representation and freedom, powerlessness, poor quality of work and threat from violence. Thus, when trying to build a measure of poverty, we first have to accept its multidimensional nature. Second, we have to develop a multidimensional poverty measurement able to support policy making processes: it has to provide the more policy-relevant information available. However, policies can be multi-dimensional on their turn: they can target different groups at the same time, focus on achieving multiple objectives, take into account multiple priorities. A poverty reduction policy is not just efforts to increase the income of the population. Under such a perspective, poverty measurement must be an instrument for such multi-dimensional policy making and policy implementation, and it has to allow to assess the effectiveness of current policies and to determine whether the situation is changing. The MPI (see Alkire and Santos 2010) has been developed with this in mind. The problem is then to construct an multidimensional poverty measurement (MPM) which might incorporate a range of indicators capturing the situation, allowing well informed decisions about policies, programmes or projects aiming to alleviate it.

Technically speaking the problem facing all attempts to construct a multidimensional poverty index is how to aggregate the information available on single attributes (access to health services, income, water, sanitation, education, food etc.) to a single composite index. On the one hand this has to be done meaningfully (see Roberts 1979), on the other hand it needs to be done in such a way that the index can really be used in order to support the design, implementation and assessment of poverty reduction policies. Although many of the suggested indexes try to take into account seriously the meaningfulness problem in conceiving the aggregation procedure, it is unclear whether they are really able to fulfil the quest for being operational. Most of the times this is due to the misleading idea that a general purpose poverty index could fit for all type of poverty reduction policies.

5.1.2 Poverty is evolutive

What allows us to know about the situation of people -during poverty measurement process- varies across time and societies. In addition, the manifestations of poverty are intimately related to the structures of the society and the period in which poverty is discussed. From this standpoint, a poor in developed countries (such as United States and French) must not be considered in the same way as the poor in developing countries (such as South Asia and Sub-Saharian Africa) or poorest Latin American countries. A multidimensional poverty measurement has to perform in a given period and place according to a particular context. Thus, the theoretical and empirical models, based on the capability of households to satisfy the basic needs, have to incorporate evidence from social analysis.

Under such a perspective it is clear that poverty reduction policies are contingent. Policies conceived to be applied, let's say in Africa, do not necessary apply, let's say in Latin America and surely not across time. Knowing about poverty is not only a matter of measuring a quantifiable phenomenon (how many households do not have access to clean water or aggregate country-level data), but also about the trend and the social importance of what we observe. Designing policies needs such fine knowledge as much as it needs quantified information, but this is rarely taken into account by the existing poverty indexes.

5.1.3 Poverty is a non-objective situation

Besides objective information which may characterise "poor" households (such as income, access to services or quality of housing) there is a subjective dimension of "poorness". A household without tap water in house will feel poor if all other households in the village do have tap water available. Instead if nobody in the village has access to such a facility this lack of service will not be considered by the households as "poverty", although objectively speaking (and with respect to normal standards of wealth) this is a sign of poverty. The same reasoning applies for many other types of observable situations including unexpected symptoms of feeling poor: for instance do not own a radio in Africa is a sign of extreme poverty ...

People are not only poor, they also feel more or less poor comparatively with the economic context, the culture, the history and the local traditions. Moreover, the ambitions of each individual on how to get out of poverty are different from one case to another. Some may aspire for better education for their children, while others to better access to water and sanitation facilities or to health services, all of such aspirations being legitimate (but not

necessary feasible). This subjective dimension of poverty has been captured by Sen in his capability theory: an individual is (feels) poor because he is not able to realize his aspirations, the later being subjectively defined. In conceiving a poverty reduction policy it is thus necessary to be able to take into account such a dimension.

5.1.4 Different Poverties

The previous discussion lead us to a final observation. It is misleading to talk only about "poor" and "not poor", at least when a multidimensional perspective of poverty is considered. What we observe in reality are different types of poverty. We observe people lacking to different extend, to different severity and differently perceived, access to services, goods, opportunities or security. Certainly there are individuals lacking access to (almost) everything (the extreme poor), but once again this may depend among countries, cultures and traditions. Moreover the extreme poor are far less from the vast population which is intuitively poor, but yet not extremely poor. From a policy making perspective it is this vast majority of more or less poor which becomes the target.

It is objectively difficult to construct a single class of poor. Moreover, perhaps it makes no sense to do so. After all why do we need to classify some people as poor? Positively speaking the reason is that these "poor people" will be the target of some policy aiming at helping them to get out this situation. But then why do we need a single class of poor? Perhaps it makes more sense to have different classes of people being differently poor to each of which we may tailor adequate policies. It is likely to have more effective policies if these are correctly targeted.

5.2 What can we do?

The review of the literature on poverty measurement allows us to conclude that measuring poverty is not a representation of an objective situation, it is rather an instrument for pursuing a policy. Poverty is an evolutive, multidimensional, fuzzy and non-objective situation which does not contain anything of numerical, but only the sensation of those who are suffering. However, despite being such an informal ambiguous and ill defined concept we consider important the use of formal models in measuring poverty as well as in aiding the design, implementation and assessment of poverty reduction policies. The reason for this is that the use of formal models allows to have a clear idea of the properties a certain measure has. It also helps in finding the best way to represent a certain empirical observation and more

important to understand if inferences done out of that empirical observation are grounded and meaningful. Finally it allows to choose among different methods, procedures and approaches. The key concept in all the above is meaningfulness. However, it is necessary to have a more clear idea of what meaningfulness means in our case and why it is so important?

A most complete definition of the term 'measurement' has been given by Mari (2003) who argued that *"measurement is a specific king of evaluation, i.e. it is an operation aimed at associating an information entity, the result of measurement, with the state of the system under measurement in reference to a given quantity, the measurand"*. We thing that a measurement of poverty should be considered as a set of operations allowing to build a *bridge* (realm of subjective human experiences) between the *physical world* (realm of physical things) and the *informational world* (realm of objective knowledge). The concept of meaningfulness comes from measurement theory [see Suppes 1959; Krantz et al. 1971]. Fred Roberts (1979) presented the 'meaningfulness' as an essential condition for a measurement to be well-defined in the meaning of *correctness*, *completeness* and *rationality*. The Roberts's standpoint is clearly in the same line of definition given by Stevens (1946) according to which *"measurement is the assignment of numerals to objects or events according to rule, any rule"*.

However, in the case of poverty, a measurement is not only performed in order to assign numbers to individuals or households, but it has to help decision makers make well-informed decisions about policies, programmes and projects. This latter standpoint relates to the concept of decision aiding process and introduces two conditions which are essential in the field of poverty measurements: *operationality* and *legitimacy*. A poverty measurement is *operational* if it can be used efficiently to recognise actors drawn from some universe it denotes and if it can help decision makers to make well informed decisions about policies, programmes or projects. On the other side, a poverty measurement is *legitimated* if it takes in account how a final recommendation is presented, implemented and perceived by the other actors besides its precise contents. Note that operationality and legitimacy have not been defined explicitly in this paper, so the definitions given here are based on our retrospective analysis and reconstruction (see Tsoukias 2007).

Therefore, in the realm of poverty, a measurement is *"meaningful"* if it complies to three conditions:

Theoretical soundness: poverty measurement needs to be *theoretically sound*, in the sense that the concepts used to construct it are in adequacy with measurement theory;

Operational Completeness: poverty measurement needs to be *operationally*

complete, in the sense that it is useful for policy making, policy implementation and it helps decision makers to make well informed decisions about policies, programmes or projects.

Legitimacy: poverty measurement needs to be *legitimated* in the sense that, it should reflect the perception of the society, the stakeholders and the actors.

It is important to note that our position concerning measuring poverty is in adequation with the following three positions ('Three P') and the Sen's capability approach sketched at the standpoint of its operationalization:

(P1). Measurements are inherent properties of the measured things [see Mari 2003].

(P2). Measurements are results of operations that preserve the relations observed among measured things [see Mari 2003; Roberts 1979].

(P3). Measurements are results of a decision aiding [see Bouyssou et al. 2000 and Tsoukiàs (2007)] process.

Under such a perspective meaningful measurements are a crucial instrument for providing solutions to the societal problems such as public health and education, the fighting against poverty and social exclusion, the promotion of efficient redistribution of richness, the promotion of justice and the defence of the underprivileged by efficient management of richness and ressources.

6 Conclusion

The purpose of this paper was to provide a more recent review of literature on multidimensional poverty measurements. Furthermore, we have discussed on some drawbacks about existing measurements and how we can improve multidimensional poverty measurement. Two of multidimensional poverty measurements have particularly attracted our attention: Sen's Capabilities Approach (Sen 1985 and 1993) and The MPI created by Alkire and Santos (2010). The two measurements have been developed by considering the multidimensional nature of poverty and both have drawbacks that we have outlined above. However, many authors [such as Saith 2001; Robeyns 2009 and 2010; Nussbaum 2000, 2003 and 2005a; Alkire and Santos 2010] and international organisations [such as the UNDP (2010) and the World Bank (2001)] agreed that the capabilities approach is more complete

and more appropriate in order to evaluate the wellbeing of people. Capabilities explicitly incorporate the 'well-being freedom' (or 'living standard freedom' or 'agency freedom') and implicitly promote the 'justice'. The main stumbling block of this approach is its difficulty to be operationalised due to the subjectiveness of poverty. Poverty is indeed evolutive and multidimensional and measuring poverty is not a representation of an objective situation but rather an instrument of pursuing a policy. Measuring poverty is more than identifying the people living beyond the poverty threshold ($ 1.25 a day) or averaging some indexes in order to get a single composite one. The problem is not just to know if somebody is poor, but to know what we can do in order to allow him not to be poor in the future. To our opinion this is the starting point from which to try to give some operational directions in using capabilities theory for poverty measurement. Our standpoint points out the necessity to consider the problem of poverty measurement as a decision problem and to tackle its measurement issue with that in mind.

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