Social Choice Inspired
Multiple Criteria Decision Analysis

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Abstract

This document is a preliminary version of a much longer survey aimed at presenting Multiple Criteria Decision Analysis methods inspired to Social Choice Theory. The paper presents how this idea evolved in the last 40 years, the basic differences between MCDA and Social Choice Theory and the methodological frameworks under which it is possible to establish common theoretical grounds and further generalisations.

1 Introduction

When multiple criteria have to be considered in order to assess alternatives for some decision purpose an idea that may appear “natural” is to use a “social choice” procedure: the “best” alternative might be considered the one indicated as such by the “majority” of criteria or an alternative x may be considered as better than alternative y if it is such on a majority of criteria. Such an intuitive reasoning needs of course to be formalised, thus borrowing concepts from Social Choice Theory, but we may also need to develop some specific original concepts in order to do so. This paper presents both how Social Choice Theory concepts are borrowed for Multiple Criteria Decision Analysis purposes and which new concepts need to be studied.

Already in the 60s Bernard Roy ([3],[28]) introduced a class of methods (known today as methods ELECTRE) aiming at aggregating preferences expressed on multiple criteria, opening the field of the so called “outranking methods” (for some early presentations see [21], [29] and for some more general discussion see also [2], [6], [8]). Arrow and Raynaud ([1] presented a first general exploration of the links between Social Choice Theory and MCDM, while more recently Marchant ([19],[20], but see also [7]) introduced a more foundational discussion on this subject.

The aim of this paper is to continue the discussion in this area with some emphasis on the general ideas inspiring the use of Social Choice Theory concepts in MCDA. The paper is organised as follows: Section 2 introduces general notation and the setting we are going to follow. Section 3 introduces the useful concepts from Social Choice Theory and emphasises the differences with Multiple Criteria Decision Analysis. Section 4 discusses how Social Choice Theory concepts influences MCDA, while Section 5 presents some examples of well known MCDA methods in the literature which are inspired from Social Choice Theory. Section 6 presents some generalisations of the ideas presented in the paper and how these can be inserted in a common methodological framework. Further research directions conclude the paper.

2 Notation and Setting

We consider the reader being acquainted with ordered sets, general properties of binary relations and the nomenclature of ordering relations (for a general presentation of such concepts in the area of preference modeling the reader can see [27]). In the following we consider a set A of alternatives (actions, candidates) which, for computation purposes, is given and finite. Possibly each element of A will be described as a vector of a multi-attribute space D^n. Each attribute D_i will be considered as a function mapping the set A to some set of numerical or nominal values E_i. In the case such a set is equipped with some specific ordering properties we can talk about a measurement scale (see [26]). The set of vectors describing A is called the “performance table” of A.

We assume the existence, for each attribute, of a preference relation $\succeq_i \subseteq A \times A$ and we call the set of such relations the set H of criteria. Generally speaking we consider $\succeq_i$ being general reflexive binary relations to be read "at least as good as". We call the asymmetric part of $\succeq_i$ strict preference and we denote it as $\succ_i$, while we distinguish the symmetric part (denoted as $\approx_i$) in an indifferent part (denoted $\sim_i$) and an incomparability one (denoted $\bowtie_i$). Usually such preference relations will be also ordering relations of different types:
- partial orders (transitive relations);
- total orders (asymmetric and transitive relations);
- weak orders (complete and transitive relations);
- interval orders (complete and Ferrers relations);

unless differently specified. Most of the times the relations $\succeq_i$ are complete although this is a restriction to be further discussed in the future.

The general setting of a MCDA problem is to establish a global relation $\succeq$ on the set A with some specific properties. Usually such a relation will be a weak order on A (a total order of equivalence classes). Occasionally we may accept as an outcome a partial order allowing incomparability among some equivalence classes. We call this preference statement as "ranking". A specific case will be the one where the equivalence classes are only two (one the complement of the other: the accepted and the rejected elements of A) and we call that a "choice" problem statement. On a more general setting we can consider also other problem statements such as rating, clustering and assignment, but for the purpose of this paper we are going to limit ourselves only to ranking and choice. For a more general discussion about different problem statements in Decision Aiding settings see [34].

3 Social Choice Theory

Given a set N of individuals we consider that each of them is endowed with a preference relation $\succeq_i \subseteq A \times A$. In Social Choice Theory we also look for constructing a global (let's call it social) preference relation $\succeq$ which should represent the whole society (the set N) and which should turn either a ranking of A or a "choice set" (a subset of A containing the most preferred elements). Once again we consider the reader acquainted with the fundamentals of Social Choice Theory, including Arrow's theorem, voting rules and their axiomatisations. Different procedures have been suggested in the literature (see [17]), but for the purposes of this paper we will introduce two archetypes of social choice procedures: the Borda rule and the Condorcet rule.

1. The Borda rule states that (denoting $\succeq_B$ the resulting binary relation):
   $x \succeq_B y \quad \Leftrightarrow \quad B(x) \geq B(y)$ where:
   $B(x) = \sum_i^N r_i(x)$, $r_i(x)$ being the rank of x in the ordering relation $\succeq_i$.
   In other terms x is ranked not worst than y iff the sum of the ranks of x is not inferior to the sum of the ranks of y.

2. The Condorcet rule states that (denoting $\succeq_C$ the resulting binary relation):
   $x \succeq_C y \quad \Leftrightarrow \quad |\{i : x \succeq_i y\}| \geq |\{i : y \succeq_i x\}|$.
   In other terms x is ranked not worst than y iff the number of individuals preferring x to y are not less than the number of individuals preferring y to x (majority principle).

It is well known that the Borda rule always turn an ordering relation, while violating Arrow's independence condition. On the other hand it is also known that the Condorcet rule does not turn an ordering relation (transitivity can fail), thus violating Arrow's transitivity condition. For a discussion on Arrow's impossibility theorem see [16].

The similarities with Multiple Criteria Decision Analysis settings are strong. We have as input a set of ordering relations and we are looking for some procedure which should turn a global (possibly) ordering relation. Intuitively we may consider the different criteria as ”voters” and the rest follows. However, there are some strong differences which should be considered before we continue the discussion.

- Social Choice Theory considers that the preferences of each individual are independent (preferentially independent). While most of the MCDA methods will make such an hypothesis this cannot be considered as being the norm. Criteria might be preferentially dependent and the literature specifically considers methods aiming at handling such a situation (see for example [4] or [14]).
- Individuals in Social Choice Theory are anonymous, while criteria in MCDA have a specific meaning and may contribute differently in establishing the global preference. In other terms criteria may have different importance, while this is not the case in Social Choice. While it makes sense to compare criteria in order to establish that one in “more important” from another this is not allowed in Social Choice Theory.
- Criteria may carry some quantitative information such as the difference of performance on the underlying attribute or some measure of intensity of preference (if meaningful), while this is not the case in Social Choice Theory.
- Last, but not least Social Choice procedures are usually “decision making” procedures aiming at producing a deliberation. It is the case with all voting procedures which are used to deliberate the winner(s) of a ballot. On the other hand usually MCDA methods are “decision aiding” procedures aiming at helping some decision maker to understand, shape, elaborate a decision problem: they are used as tools which construct reasons, arguments supporting (or adversing) some potential conclusion.

At this point does it make sense to consider Social Choice Theory as an inspiring scientific area for MCDA? Despite the differences above mentioned our reply is affirmative. As mentioned in [7] the abundant literature in Social Choice Theory about properties of ranking and choice procedures as well as the many (im)possibility theorems allow to expand our knowledge on how many MCDA methods work and how these can be improved or generalised. In any case there exist MCDA methods clearly inspired to Social Choice Theory and in the following we are going to show why.

4 Multiple Criteria Decision Analysis

Following the two social choice archetype procedures previously described we can identify two “paths” in order to create the final global ordering relation in the case of Multiple Criteria Decision Analysis (for some similar reasoning see also [40] and [39].

4.1 The Borda path

The intuitive idea is to associate to each element of A on each attribute of D a numerical value v such that $v_i(x) \geq v_i(y) \quad \Leftrightarrow \quad x \succeq_i y$. In reality since the elements of A are vectors of the performance table the value $v_i(x)$ should be read as $v_i(d_i(x))$. In other terms we do not use directly the preference relation associated to each criterion, but a numerical representation of this. The simplest way of course to compute such values is to use the rank of each element on the ordering relation $\succeq_i$ (exactly as suggested in the original Borda method).

The straightforward idea at this point will be to compute a global value for x under form of a sum: $V(x) = \sum_i v_i(h_i(x))$. The global ordering (ranking or choice) will result ranking such values. However, we can make a number of remarks.

- The idea of using a numerical value (for instance the rank as in the Borda rule) induces to consider a "distance" among the alternatives (the rank induces "equal distances" among all alternatives).
- Summing the values implies accepting that the distances on one criterion are comparable and can be exchanged with the distances on other criteria (thus introducing the concept of "commnesurability"). It is easy to see that if there is such an "exchange ratio", a trade-off, among such distances this will represent the relative importance of each criterion in forming the global value of each alternative.
- The above two remarks are at the basis of what is known as Multi-attribute Value Theory (see [15]. However, this is not the only way to interpret the Borda path which can be used as an approach in presence of valued preference relations (for more details see [18]).

4.2 The Condorcet Path

The Condorcet path is based on pairwise comparisons among the alternatives in order to establish whether one is "at least as good as" another one using some majority principle (although we may use some generalisation of the usual Condorcet rule we will keep using the $\succeq_C$ notation). Since the result will not be an ordering relation (see more details in [5]) we will need a further step transforming in some way the global relation $\succeq_C$ to an ordering relation $\succeq_i$.

The procedures transforming the relation $\succeq_C$ to an ordering relation are most of the times inspired to graph theory. In the literature the relation $\succeq_C$ is often called outranking relation and the associated graph is exploited in order to obtain a final ranking relation. Different tools are adopted in such a case such as kernels, transitive closures, different types of scores based on the out-degree and the in-degree of the nodes, covering relations etc. each of them satisfying different properties. For a discussion on which properties are satisfied and how these combine among them the reader can see [41]. For a detailed presentation see [8].

In order to handle the problem of the relative importance of each criterion we need to introduce a binary relation $\trianglerighteq \subseteq 2^H \times 2^H$ (to be read "at least as important as") among all subsets of criteria. In other terms we establish a relation of "importance" among all coalitions of criteria. We thus establish:

$$x \succeq_C y \;\Leftrightarrow\; H_{x \succeq_i y} \trianglerighteq H_{y \succeq_i x}$$

where $H_{x \succeq_i y} = \{i : x \succeq_i y\}$ is the set of criteria for which $x \succeq_i y$ is the case. In other terms x is at least as good as y is the coalitions of criteria supporting x against y is more important than the coalition of criteria supporting y against x

The relation $\trianglerighteq$ is expected to be consistent with inclusion (coalitions should be at least as important as their subsets). Generally speaking we cannot impose any further a-priori condition. However, it might be the case that such a relation could have a numerical representation (for instance if we can show that there is a weak order among all coalitions of criteria). In such a case we could talk about the "relative importance" of each coalition and thus of each criterion (since a single criterion is also a coalition). The aware reader will note that such a "relative importance" can be seen as a power index (for more details see [12], [13], [32]) of the type discussed in game theory. The introduction of such "measure of importance" allows to talk about "majority thresholds" and "winning coalitions" (the ones whose importance is above the majority threshold).

Last, but not least we may be interested to introduce a "negative coalition power" in order to represent the cases where some specific coalitions of criteria have a negative power such as a veto: such coalitions (possibly a single criterion) should be able to contract the will of any winning coalition (which cannot be the whole set of criteria since if there is a veto unanimity does not hold).

5 Some Methods

In the following we present two methods implementing the Condorcet path for different purposes. The methods present are all based to the same variant of the Condorcet rule. They first compute whether "x is at least as good as y" taking all criteria into account and in order to do so they use a "weighted majority" rule with a qualified majority threshold as well as veto conditions. Then the resulting global preference relation is manipulated in order to obtain the ordering requested by the problem statement. Roy ([30] named this general principle as concordance/discordance principle. The intuitive idea is that x is at least as good as y iff there is a strong (weighted) majority in favour and there is no strong opposition.

ELECTRE I The purpose of this method (officially appeared in 1968, see [28]), the problem statement associated to it, is to identify the "best" subset of alternatives (a choice set). In order to do so the global preference relation, called outranking relation is defined as follows:

$$x \succeq_C y \Leftrightarrow \frac{\sum_i^n w_{j\pm}}{\sum_i^n w_j} \geq \gamma \ \wedge \quad \neg \ \exists j : h_j(y) - h_j(x) \geq \delta_j$$

where:
- w_j are constants representing the relative importance of each criterion;
- $J^{\pm} = \{j : x \succeq_j y\}$ is the set of criteria for which x is at least as good as y;
- γ is a majority threshold;
- $h_j(x)$ is the score of alternative x on criterion h_j;
- δ_j is a threshold representing the difference beyond which there is a veto on criterion h_j.

The relation $\succeq_C$ not being an ordering relation, the search of the choice set is done using the concept of kernel of the outranking graph. In order to do so the eventual circuits within the graph are reduced to single nodes (equivalence classes) and the kernel is identified on the resulting graph. The resulting ordering relation is simple: the alternatives within the kernel are all better than the ones without it.

ELECTRE II The method (first published in [31]) is aimed at producing a ranking of the set of alternatives. The global outranking relation is computed essentially as in the previous case (ELECTRE I).

The difference consists in the procedure used in order to establish the ranking. The method first computes a "descending ranking" as follows: it identifies a first equivalence class of all alternatives which are not outranked by no other alternative, then eliminates these alternatives from the graph and computes a second equivalence class of the alternatives which now result not being outranked and so on until the whole set of alternatives is ranked. The method then computes an "ascending ranking" as follows: it identifies a last equivalence class of all alternatives not outranking any other alternative, then eliminates these alternatives and computes a second last equivalence class of the alternatives which now result not outranking any other alternative and so on until the whole set of alternatives is ranked. The two rankings do not always coincide: if the initial outranking graph contains incomparable alternatives it is likely that the two ranking will be different. The method then computes an intersection of the two rankings the result being a partial order of equivalence classes.

For methods using the "Borda path" the reader can see [11] presenting the PROMETHEE Method. In this case instead of computing a global outranking relation to be further exploited in order to establish a ranking the method computes a preference intensity c_{xy} for each pair of alternatives:

$$c_{xy} = \sum_{1}^{n} w_j F(h_j(x) - h_j(y))$$

where:
- w_j: represent the relative importance of each criterion;
- $F(h_j(x) - h_j(y))$: represents a function taking into account the differences of performance between x and y on criterion h_j.

The final ranking is then computed calculating for each alternative the out-degree and the in-degree on the valued outranking graph resulting associating to each pair of alternatives the preference intensity previously computed.

The reader will note that the methods above presented (variants of which are now present in commercial software and open source platforms, see for instance www.decision-deck.org) all make a number of hypotheses:
- criteria are expressed as functions above the performances on the underlying attributes, thus assuming the existence of a weak order among such performances;
- the relative importance of the criteria is simply given under a set of constants, thus assuming that the importance relation among coalitions of criteria can be computed in an additive way, this importance relation being a weak order among the set of coalitions of criteria;
- the veto relation is practically a semi order among the performances of each criterion, further enforcing the "quantitative" character of the information contained in each attribute.

6 Generalisations

6.1 Positive and Negative Reasons

As previously mentioned the specific way through which the concordance/discordance principle introduced by B. Roy as generalisation of the Condorcet rule has some restrictive properties. We may further note that concordance is always computed as a weighted majority, that vetoes are always expressed as result of "bad performances" which may invalidate any type of majority (if any exists).

From such observations we can consider the hypothesis to further generalise the Condorcet path (as already discussed in [35]):
- considering a "positive" ordering of coalitions of criteria to which associate if possible a "positive" importance although not necessarily additive;
- considering a "negative" ordering of coalitions of criteria to which associate if possible a "negative" importance although not necessarily additive;
- allowing the two orderings to be completely independent and compute in different ways the positive and negative importance of each coalition of criteria;
- in other terms identifying ways to compute the positive and negative reasons which support or adverse a certain global preference statement;
- extending the idea of positive and negative reasons to single criterion preference modeling or to further aggregation steps in case the attributes set is structured as an hierarchy;
- considering positive and negative reasons at the same level, as two different sources of information when comparing x to y, either through specific formalisms (see [25], [38], [36], [37], [33]) or as two distinct preference relations (see also [24], [23]).

Summarising the above discussion we can consider the use of such generalised rules (which we can also call preference aggregation methods) as procedures allowing to construct arguments for or against a certain preference statement or recommendation. After all aiding somebody implied in a

Table 1: Some methods and their properties

Methods	ORD.	ANO.	INDEP.	NO VETO
Simple majority	×	×	×	×
Weighted Majority	×		×	×
Outranking methods	×		×	
Additive value model			×	×
Non linear value model				×
Oligarchies	×			×

decision process consists exactly in constructing the reasons for which that decision maker will be convinced that the proposed solution is the one to be adopted.

6.2 How to choose a method?

The division of the MCDA methods along the two social choice archetype procedures (the Borda and the Condorcet path) allows to introduce a first major distinction: whether the distance among performances carries any quantitative information in terms of preference. Methods where such distance is meaningful will be characterised as carrying some quantitative value information, while methods where such difference only allows to establish if $x \succeq_i y$ will be considered "ordinal".

Another major distinction concerns anonymity of the different criteria. Actually, the reader will note than in case we consider the difference of performance on different criteria as comparable (thus commensurable, as in the case of additive value functions) anonymity is automatically excluded. Strictly speaking this should also contain the case where such differences are all the same (as in the case of the classic Borda rule) although we may be tempted to consider this specific case as a special one.

A third property which may characterise a method is whether it allows to take into account the existence of preferential dependencies among the criteria. The reader will note that we are not talking among the statistical correlation among the underlying attributes performance distributions, but about the possibility of having conditional preference statements of the type "if $x \succeq_k y$ then $z \succeq_l w$. In such a case we need to consider methods allowing non linear aggregations either of the values of the alternatives or of the importance of the criteria.

Finally, we may consider the case where vetoes need to be explicitly considered such that any type of majority could be overturned by any bad performance on a single criterion.

Table 1 shows some classical MCDA methods and the properties they satisfy following the above four mentioned distinctions. Our claim is that such properties are both exhaustive and useful.

1. Exhaustive in the sense that the combinations of the four properties (if consistent) define all archetypes of MCDA procedures as much as the Borda rule and the Condorcet rule establish the archetypes of Social Choice procedures. Why this happens? If we accept that all MCDA methods can be derived from the two archetype social choice procedures the only three issues distinguishing MCDA methods from such procedures are exactly anonymity, preferential dependence and possibility of veto.

2. Useful because it allows to establish a rough guideline for conducting a dialogue with a user (decision maker) in order to choose the method better fitting the problem situation. In Figure 1 we represent a concept lattice where at the top we put the (arbitrary) simplest method: simple majority (the Condorcet rule). If this is not satisfying or unfitting the preference statements of the decision maker (expressed while modeling the problem situation) then modifying properties one by one allows to check which are the ones satisfied and which not and thus choose a

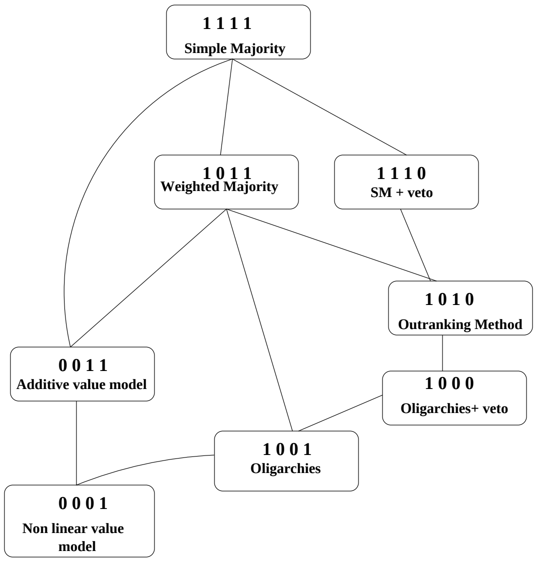

Figure 1: A lattice for navigating among methods. Source [22]

method. As has been shown in [22] this can be implemented in argument schemes allowing to construct a formal dialogue with the decision maker.

7 Conclusion

How far have we gone? We have shown that, despite crucial differences, social choice procedures can be considered as an inspiring framework for MCDA. More precisely we have shown that potentially any MCDA method can be derived from two archetype Social Choice Procedures: the Borda rule and the Condorcet rule.

MCDA methods need to take into account more complex information with respect to classic Social Choice procedures since differences of performances may be meaningful and criteria may be neither anonymous nor independent. We have shown that using the four basic properties: ordinality, anonymity, independence, presence of vetoes we can make an exhaustive classification of MCDA methods.

Is this presentation really complete? NO. There are at least two critical issues to be further discussed.

- Preference Learning. MCDA methods are characterised not only by how they model and aggregate preferences, but also on how they "learn" preferences. For the time being we have omit this aspect, but for a more complete presentation we need also to discuss what type of protocols are used in order to translate preference statements of the decision maker to preference models (for some discussion see [8]).

- Further analysis of the properties. As has been shown under a conjoint measurement theory analysis of MCDA methods (such as the ones suggested by [9], [10], [8]) both the concepts of ordinality and anonymity can be and have to be reconsidered. This should have as a result reconsidering how the MCDA methods can be classified.

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