What is a Decision Problem?

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Outline

Problems

- Patients triage in emergency room;
- Identification of classes of similar DNA sequences;
- Star ratings of hotels;
- Waste collection vehicle routing;
- Vendor rating and bids assessment;
- Optimal mix of sausages;
- Chip-set lay out;
- Airplanes priority landing;
- Tennis tournament scheduling ...

What is a decision problem?

Consider a set A established as any among the following:

- an enumeration of objects;
- a set of combinations of binary variables (possibly the whole space of combinations);
- a set of profiles within a multi-attribute space (possibly the whole space);
- a vector space in $\mathbb{R}^n$.

Technically:

A Decision Problem is a partitioning of A under some desired properties.

What is important?

What does really matter?

In designing, choosing, applying, implementing, understanding, explaining, justifying, a method?

What are the primitives?

And what is the derived information and the expected outcomes?

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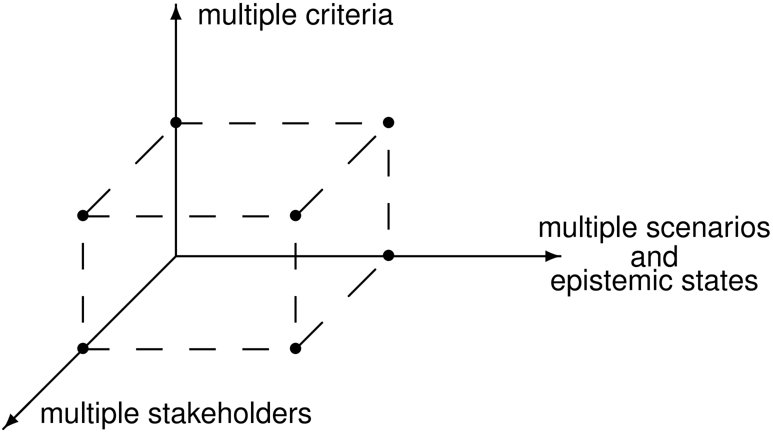
And what is the derived information and the expected outcomes?

Why is not straightforward?

- multiple opinions
- multiple values
- multiple likelihoods

+ algorithmic aspects

Partitioning? How?



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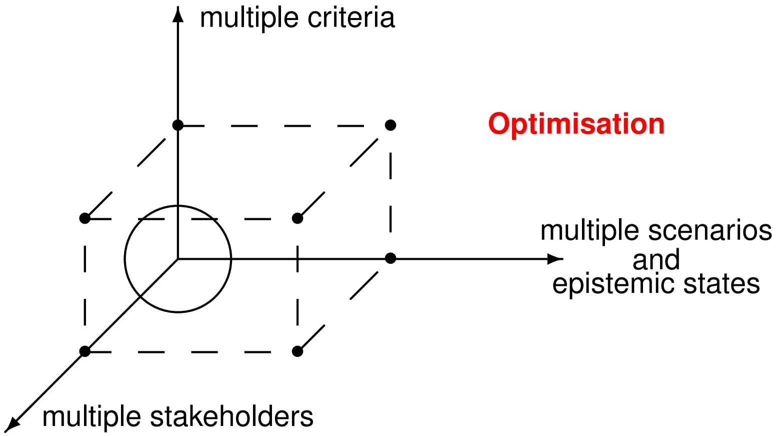

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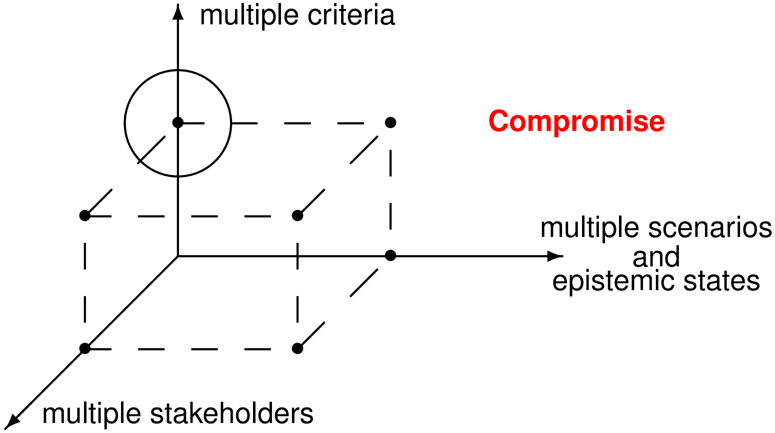

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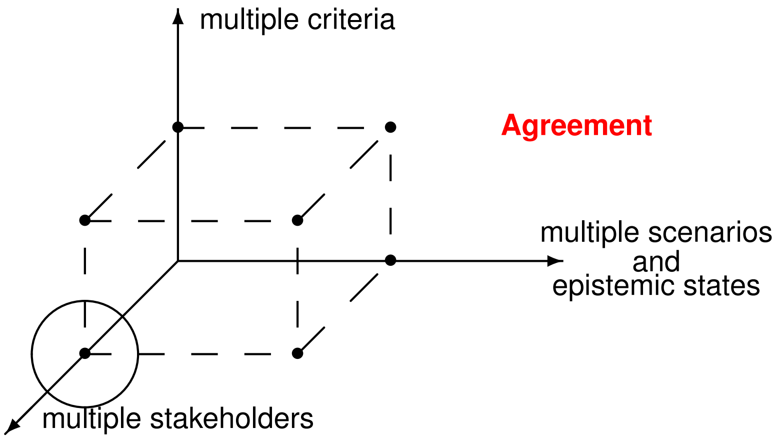

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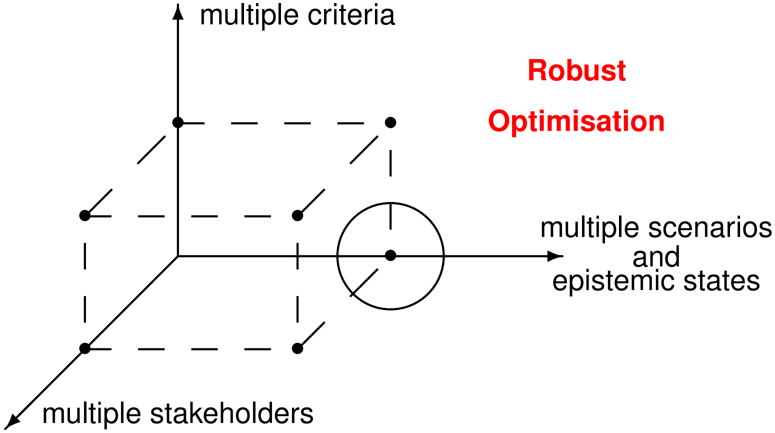
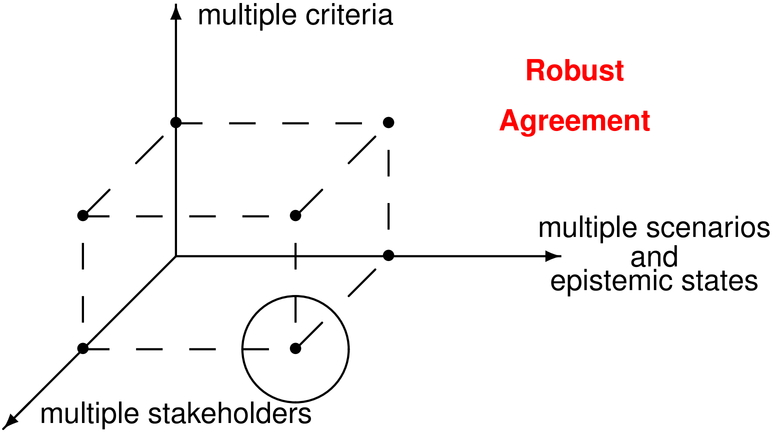


Robust

Optimisation

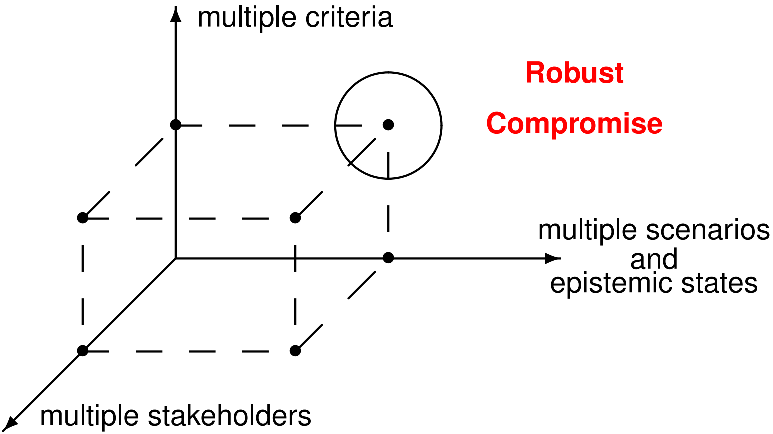
Partitioning? How?



Robust

Agreement

Partitioning? How?



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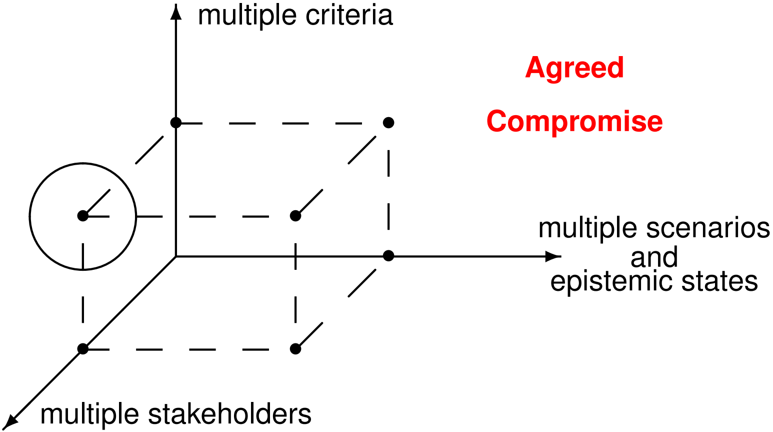
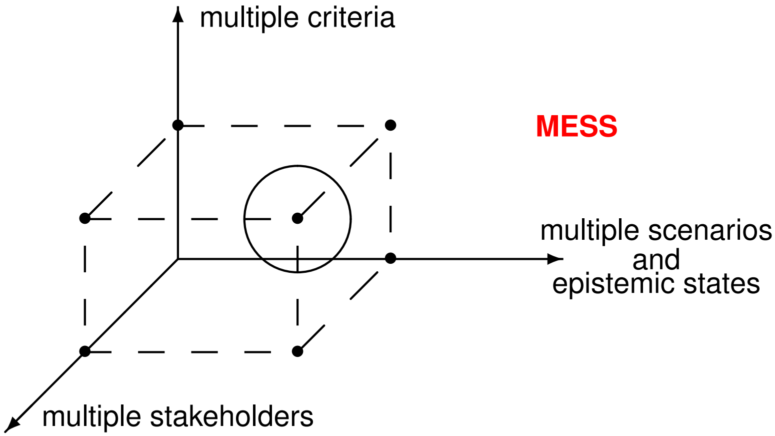


Agreed

Compromise

Partitioning? How?



MESS

Is that all?

- Behind a criterion other criteria may be considered in a hierarchy of criteria (objectives);
- Behind a stakeholder other actors may have to be considered, that precise stakeholder being a speaker for a community;
- Behind a state of the nature other uncertainties may have to be considered;
- Any combination of the above may in reality occur as complex as possible.

Partitioning? For what?

Practically we partition A in n classes. These can be:

	Pre-defined wrt some external norm	Defined only through pairwise comparison
Ordered	Rating	Ranking
Not Ordered	Assigning	Clustering

Two special cases:
- there are only two classes (thus complementary);
- the size (cardinality) of the classes is also predefined.

What is a ranking problem?

Primitive

The primitive is a binary relation on A: $\succeq \subseteq A \times A$ to be read "at least as good as".

Result

The result is a partitioning of A in $[A_1], \cdots [A_n]$ such that:
$[A_j] \geq [A_i] \Leftrightarrow j \geq i$ and
$\forall x \in [A_j], y \in [A_i] : x \succeq' y$

Discussion 1

What is a choice problem?

We partition A in two classes $[A_1] \geq [A_2]$. Thus $[A_1] = \sup_A(\succeq')$.

What is an optimisation problem?

A choice problem for which:
- $\succeq = \succeq'$
- $x \succeq y \Leftrightarrow f(x) \geq f(y)$.
- Thus $[A_1] = \max_A f(x)$

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Discussion 2

Why is $\succeq'$ different from $\succeq$?

Generally speaking $\succeq$ is not an ordering relation since preferences can be partial and or inconsistent. If we have to proceed with some operational procedure we need to transform $\succeq$ to an ordering relation $\succeq'$.

Discussion 2

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How do we learn $\succeq$?

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How do we learn $\succeq$?
What properties should $\succeq'$ fulfill?

What is a clustering problem?

Primitive

The primitive is a set of binary relations on A: $\approx_l \subseteq A \times A$ to be read "similar to".

Result

The result is a partitioning of A in $[A_1], \cdots [A_n]$ such that:
$\exists \approx_l : \forall x, y \in [A_j] \;\; x \approx y$ and
$\forall x \in [A_j], \; y \in [A_i] : \neg(x \approx y)$

Discussion 1

Indiscernibility.

In case $\approx_l$ are equivalence relations then the partitioning of A results in constructing the indiscernibility relation on A. However, this is not generally the case and $[A_j] = \sup_A(\approx_l)$.

In other terms we try to maximise similarity within classes (clusters) and minimise similarity among classes (clusters).

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Discussion 2

Distances.

If $\approx_l$ are nested similarity relations with nice properties then we can establish a metric:
- $s(x, y)$: how similar is x to y?
- $d(x, y)$: how distant is x from y?
Then $[A_y] = \{x | \max_A F(s(x, y))\}$,
F being a measure of the overall similarity of the elements of $[A_y]$ with respect to y.

What properties should F and the metrics fulfill?

What is a rating problem?

Primitive

The primitive is a binary relation on A: $\succeq \subseteq A \times P \cup P \times A$
to be read “at least as good as”.
P being the set of external “norms” characterising the ordered classes $C_1 \triangleright \cdots \triangleright C_n$

Result

The result is to assign each element of A in a C_j such that:
$x \in C_j \;\Leftrightarrow\; x \succeq' p_j, p_{j+1}, \cdots p_n$ and $p_1 \cdots p_{j-1} \succeq' x$

Discussion 1

Constraint Satisfaction

If $\forall x, y \in A \cup P \;\; x \succeq y \;\Leftrightarrow\; f(x) \geq f(y)$.
Then $x \in C_j \;\Leftrightarrow\; f(p_{j-1}) \geq f(x) \geq f(p_j)$.
This is a Constraint Satisfaction Problem.

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What is an assigning problem?

Primitive

The primitive is a set of binary relations on A: $\approx_l \subseteq A \times P \cup P \times A$ to be read "similar to".
P being the set of external "norms" characterising the classes $C_1 \cdots C_n$

Result

The result is to assign each element of A in a C_j such that:
$x \in C_j \Leftrightarrow \exists \approx_l : x \approx_l p_j$

Discussion 1

Constraint Satisfaction

If $\forall x, y \in A \cup P \ \ x \approx_I y \ \Leftrightarrow \ f(x) = f(y)$.
This is once again a Constraint Satisfaction Problem.

Basic Claim

- **Any unsupervised decision problem is an optimisation problem.**
- **Any supervised decision problem is a constraint satisfaction problem.**

Since any constraint satisfaction problem can be seen as an optimisation problem, we can definitely focus only to the later ones

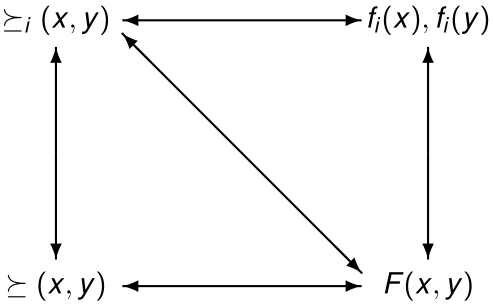
Why $x \succeq y$?

When in reality we just know that:

$$\begin{array}{rcl}
w \succeq_1 z \succeq_1 & x \succeq_1 y & \succeq_1 t \\
w \succeq_2 & y \succeq_2 x & \succeq_2 t \succeq_2 z \\
w \succeq_3 t \succeq_3 & x \succeq_3 y & \succeq_3 z \\
z \succeq_4 & y \succeq_4 x & \succeq_4 t \succeq_4 w \\
 & \vdots &
\end{array}$$

The Problem

Suppose we have n ordering relations $\succeq_1 \cdots \succeq_n$ on the set A.
We are looking for an overall ordering relation $\succeq$ on A "representing" the different orders.



Two fundamental questions

1. **How do we consider differences of preferences along a single criterion/dimension?**
2. **How do we consider differences of preferences among several different criteria/dimensions?**

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1. **How do we consider differences of preferences along a single criterion/dimension?**
2. **How do we consider differences of preferences among several different criteria/dimensions?**

Is everything equally important?

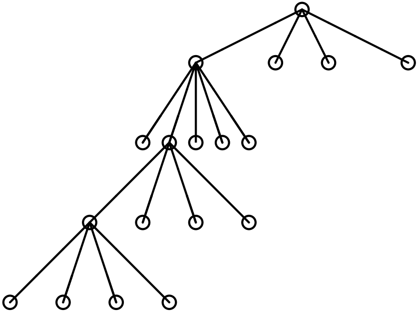
NO! Of course, but ...

It depends on how we put together different differences of preferences:

- Additively with trade offs among independent criteria.
- Non linear functions on less than interval measures.
- Coalition Games.

Hierarchy

A decision problem can be represented as a sequence of preference aggregations along an hierarchy of actors, criteria and states of the nature, combined arbitrarily.



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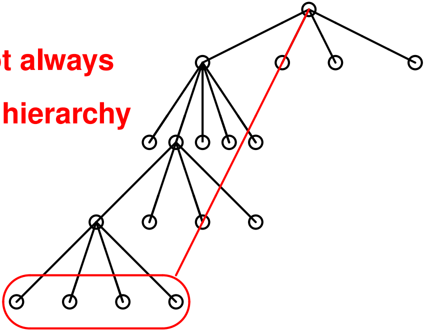
We cannot always exploit the hierarchy



Horse Races Betting

	rain	not rain
horse		
jockey		

$$v^r(t) = 0.5v^r(h) + 0.5v^r(j)$$

$$v^n(t) = 0.5v^n(h) + 0.5v^n(j)$$

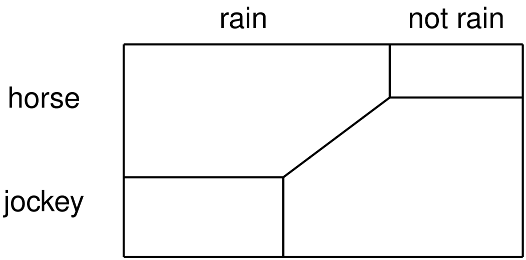
Horse Races Betting

	rain	not rain
team		

$$v(b) = 0.5v^r(t) + 0.5v^n(t)$$

$$v(b) = 0.25v^r(h) + 0.25v^r(j) + 0.25v^n(h) + 0.25v^n(j)$$

Horse Races Betting



What should we do now?

Critical Issues

- The set of alternatives
- Problem statement
- Differences of preferences
- Hierarchy/Separability/Indipendence
- Positive and Negative Reasons

Challenges

- Preference Learning
- Unusual Cases
- Revision and Update
- Construction of Evidence