Methods and Tools for Public Policy Evaluation

Alexis Tsoukiàs

LAMSADE - CNRS, Université Paris-Dauphine
tsoukias@lamsade.dauphine.fr

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Outline

1. Introduction
   - Public Decision Processes
   - What is Evaluation?
   - Decision Aiding

2. Basics

3. Cost-Benefit Analysis

4. Multi-attribute Value Functions

5. Further Reading
A legitimization issue

Policy makers feel lacking legitimation in their policy making process.

- Mistrust between public opinion, experts and policy makers.
- Information society and information circulation and availability.
- Social fragmentation.
- Short agendas vs. long term concerns.
What is a public decision process?

- Distributed Decision Power (several stakeholders).
- Different Rationalities.
- Participation “de facto”.
- Public Deliberation.
- Social Outcomes.
- Long Time Horizon
What is specific in Public Policies?

- Different types of Actors:
  - Political actors (short term political agendas).
  - Officials and Experts (medium term knowledge based agendas).
  - Social groups more or less fragmented.

- Different types of stakes.
  - From long term and/or affecting large parts of territory and population, to
  - short term individual “opportunistic” stakes.

- Heterogeneous resources such as: knowledge, trust, money, land, authority, power etc. are committed in the process.
Consequences

What are the consequences?
- Conflicting opinions, priorities, actions.
- Conflicting information and interpretations.
- Different languages and communication patterns.
- Mutually adaptive behaviour along time.

What does it mean?
Accountability, Legitimation, Consensus, Evidence
Consequences

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Advantages

- Common language.
- Improved accountability.
- Basis for participative decision making.
- Exploring less “obvious aspects” (better insight).
- Avoiding intuitive errors.
Formal models in Public Policy Assessment

Drawbacks

- Possible loss of a global insight.
- Possible loss of creative thinking.
- Too much structuring of the decision process.
- Does everybody understands formal models?
- Cost of using formal models.
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What is Evaluation?

- What does it mean evaluation?
- *Measuring value*
- What does it mean measuring?
- What is value?
Values?

- Value of what?
- Value for whom?
- Value for doing what?
- Is there an objective value?
Values?

- Value as a social agreement.
- Economic value and money.
- Value of use and marginal value.
- Personal values.
- Values as ethics and norms.
Did the air quality improved?

<table>
<thead>
<tr>
<th>pollutant</th>
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<th>SO₂</th>
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<th>dust</th>
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The air quality improved, but for the ATMO index did not. Who tells the truth?
Meaningfulness

**Theoretical Soundness**
Information needs to be manipulated in a coherent and consistent way (measurement theory).

**Operational Completeness**
Information needs to be manipulated in order to be useful for who is using it and for those purposes for which has been designed. It should allow to reach a conclusion.
What does it mean?

95% of rural households in Burkina Faso do not have tap water available

- For us this is a serious problem and evidence of poverty, but for the locals is not.
- For the local men this is not a problem, while it is for the local women.
Differences of perspective

- Different standards and thresholds.
- Different cultures.
- Different stakeholders.
- Different concerns.
- Different resources.
Is it good or bad?

The h-index of X is 19. Is (s)he a good researcher?

- Who is a good researcher?
- What good research means?
- Who decides and for what purpose about research quality?
What do we take into account?

- Values and preferences of relevant stakeholders.
- Individual values and social values.
- Judgements (experts, politicians, opinions).
Who is the winner?

10 voters have preferences $aPbPc$,
6 voters have preferences $bPcPa$
and 5 voters have preferences $cPbPa$. 

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Most electoral systems will choose $a$, which is the one the majority does not want. Actually the Condorcet winner is $b$. 
Different ways to construct evidence

- Different ways to establish a majority.
- Different ways to compute an average.
- Different ways to take into account the importance of...
- Positive and Negative reasons/arguments.
Evaluation and Decision Aiding

Not easy ...
Evaluating is less easy intuitive from what it appears to be

... to aid to decide
Evaluating is a Decision Aiding activity
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Evaluating is a Decision Aiding activity
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Deciding ...

- Decision Maker
- Decision Process
- Cognitive Effort
- Responsibility
- Decision Theory
... and Aiding to Decide

- A client and an analyst
- Decision Aiding Process
- Cognitive Artifacts
- Consensus
- Decision Aiding Methodology
What is a Decision Aiding Process?

The interactions between somebody involved in a decision process (the client) and somebody able to support him/her within the decision process.

Consensual construction of shared cognitive artifacts

A Decision Aiding Process makes sense only with respect to a Decision Process in which the client is involved and with respect to which demands advice.
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A Decision Aiding Process is a Decision Process where at least two actors are involved: the client and the analyst, with at least two concerns: the client’s “problem” and the analyst’s job, mobilising at least the following resources: the client’s domain knowledge and the analyst’s methodological knowledge.

A Decision Aiding Process becomes part of the Decision Process for which it has been established. The analyst enters as an actor such a Decision Process.
Its Cognitive Artifacts

- Representation of the problem situation
- Problem Formulation
- Evaluation Model
- Final Recommendation
Representing a Problem Situation

- Who has a problem?
- Why this is a problem?
- Who other is affected by the decision process?
- Who decides?
- Who pays for the consequences and the bill?
- What I am doing here?
A Problem Situation

\[ PS = \langle A, O, RS \rangle \]

\( A \) actors, participants, stakeholders

\( O \) objects, concerns, stakes

\( RS \) resources, commitment
Formulating a Problem

*Constructing a first formal representation of the client’s concerns, applying an abstract and formal language, using a model of rationality.*

- What objects do we consider in formulating “the problem”?
- What do we know or are we looking for such objects?
- What do we want to do with such objects?
A Problem Formulation

\[ \Gamma = \langle A, V, \Pi \rangle \]

- **A** Actions
- **V** Points of view
- **\Pi** Problem statement
Constructing an Evaluation Model

- Fixing alternatives.
- How to describe them?
- Are there any preferences?
- Are we sure about the information?
- How to put all this information together?
Evaluation Model

\[ \mathcal{M} = \langle A, D, E, H, U, R \rangle \]

- **A** alternatives, decision variables, ...
- **D** dimensions, attributes, ...
- **E** scales associated to attributes,
- **H** criteria, preference models, ...
- **U** uncertainty, epistemic states, ...
- **R** procedures, algorithms, protocols ...
Establishing a final Recommendation

- Going back to reality.
- What do we put in the final report?
- Is it valid?
- Is it legitimated?
- It works?
- Are we satisfied?

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Meaningfulness ...

- Do we use the information correctly?
- Is it meaningful for the analyst? *(Measurement Theory)*
- Does it make sense for the decision process?
- Is it meaningful for the client? *(Client Satisfaction)*
... and Legitimation

- Ownership
- Organisational Dimension
- Culture
- Decision Process
Notation

- Sets: $A, B \ldots$ of cardinality $n, m, k \ldots$
- Variables $x, y, z \ldots$
- Numbers $\mathbb{N}, \mathbb{Z}, \mathbb{R}$
- Vector Spaces $\mathbb{N}^n, \mathbb{R}^n$
- Binary Relations $\succeq, \succsim, \sim$ possibly subscribed
- The usual logical notation $\land, \lor, \rightarrow, \neg, \forall, \exists$
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What are the problems?

- How to learn preferences?
- How to model preferences?
- How to aggregate preferences?
- How to use preferences for recommending?
Binary relations

- \( \succeq \): binary relation on a set \((A)\).
- \( \succeq \subseteq A \times A \) or \( A \times P \cup P \times A \).
- \( \succeq \) is reflexive.

What is that?

If \( x \succeq y \) stands for \( x \) is at least as good as \( y \), then the asymmetric part of \( \succeq \) (\( \succ \): \( x \succeq y \) \& \( \neg(y \succeq x) \)) stands for strict preference. The symmetric part stands for indifference (\( \sim_1 \): \( x \succeq y \) \& \( y \succeq x \)) or incomparability (\( \sim_2 \): \( \neg(x \succeq y) \) \& \( \neg(y \succeq x) \)).
We can further separate the asymmetric (symmetric) part in more relations representing hesitation or intensity of preference.

\[ \succ = \succ_1 \cup \succ_2 \cdots \succ_n \]

We can get rid of the symmetric part since any symmetric relation can be viewed as the union of two asymmetric relations and the identity.

We can also have valued relations such that:

\( \nu(x \succ y) \in [0, 1] \) or other logical valuations ...
Binary relations have specific properties such as:

- **Irreflexive**: \( \forall x \lnot(x \succ x); \)
- **Asymmetric**: \( \forall x, y \ x \succ y \rightarrow \lnot(y \succ x); \)
- **Transitive**: \( \forall x, y, z \ x \succ y \land y \succ z \rightarrow x \succ z; \)
- **Ferrers**: \( \forall x, y, z, w \ x \succ y \land z \succ w \rightarrow x \succ w \lor z \succ y; \)
One dimension

\[ x \succeq y \iff \Phi(u(x), u(y)) \geq 0 \]

where:
\[ \Phi : A \times A \mapsto \mathbb{R} \]. Simple case \( \Phi(x, y) = f(x) - f(y); \ f : A \mapsto \mathbb{R} \)

Many dimensions

\[ x = \langle x_1 \cdots x_n \rangle \quad y = \langle y_1 \cdots y_n \rangle \]

\[ x \succeq y \iff \Phi([u_1(x_1) \cdots u_n(n)], [u_1(y_1) \cdots u_n(y_n)]) \geq 0 \]
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More about \( \Phi \) in Measurement Theory
Numbers

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More about \( \Phi \) in Measurement Theory
Preference Structures

A preference structure is a collection of binary relations $\sim_1, \cdots \sim_m, \succ_1, \cdots \succ_n$ such that:

- they are pair-disjoint;
- $\sim_1 \cup \cdots \sim_m \cup \succ_1 \cup \cdots \succ_n = A \times A$;
- $\sim_i$ are symmetric and $\succ_j$ are asymmetric;
- possibly they are identified by their properties.
 Independently from the nature of the set $A$ (enumerated, combinatorial etc.), consider $x, y \in A$ as whole elements. Then:

**If $\succeq$ is a weak order then:**

- $\succ$ is a strict partial order,
- $\sim_1$ is an equivalence relation and $\sim_2$ is empty.

**If $\succeq$ is an interval order then:**

- $\succ$ is a partial order of dimension two,
- $\sim_1$ is not transitive and $\sim_2$ is empty.
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Preference Structures

If $\succeq$ is a PQI interval order then:

$\succsim_1$ is transitive, $\succsim_2$ is quasi transitive, $\succsim_1$ is asymmetrically transitive and $\succsim_2$ is empty.

If $\succeq$ is a pseudo order then:

$\succsim_1$ is transitive, $\succsim_2$ is quasi transitive, $\succsim_1$ is non transitive and $\succsim_2$ is empty.
Preference Structures

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What characterises such structures?

Characteristic Properties

Weak Orders are complete and transitive relations. Interval Orders are complete and Ferrers relations.

Numerical Representations

w.o.  \iff \exists f : A \rightarrow \mathbb{R} : x \succeq y \iff f(x) \geq f(y)

i.o.  \iff \exists f, g : A \rightarrow \mathbb{R} : f(x) > g(x); x \succeq y \iff f(x) \geq g(y)
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PQI Interval Orders are complete and generalised Ferrers relations.
Pseudo Orders are coherent bi-orders.

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p.o. $\iff \exists f, t, g : A \rightarrow \mathbb{R} : f(x) > t(x) > g(x); x \succ_1 y \iff g(x) > f(y); x \succ_2 y \iff g(x) > t(y)$
More about structures

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The Problem

- Meaningful numerical representations.
- Putting together numbers (measures).
- Putting together binary relations.
- Overall coherence ...
- Relevance for likelihoods ...
The Problem

Suppose we have $n$ preference relations $\succeq_1 \cdots \succeq_n$ on the set $A$. We are looking for an overall preference relation $\succeq$ on $A$ “representing” the different preferences.

\[
\begin{align*}
\succeq_i (x, y) & \quad \text{if} \quad f_i(x), f_i(y) \\
\succeq (x, y) & \quad \text{if} \quad F(x, y)
\end{align*}
\]
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What is measuring?

Constructing a function from a set of “objects” to a set of “measures”.

Objects come from the real world.

Measures come from empirical observations on some attributes of the objects.

The problem is: how to construct the function out from such observations?
1. Real objects \((x, y, \cdots)\).
2. Empirical evidence comparing objects \((x \succeq y, \cdots)\).
3. First numerical representation \((\Phi(x, y) \geq 0)\).
4. Repeat observations in a standard sequence \((x \circ y \succeq z \circ w)\).
5. Enhanced numerical representation \((\Phi(x, y) = \Phi(x) - \Phi(y))\).
Any of the above could be a numerical representation of this empirical evidence. Ordinal Scale: any increasing transformation of the numerical representation is compatible with the EE.
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Ordinal Scale: any increasing transformation of the numerical representation is compatible with the EE.
Consider putting together objects and observing:

\[ \alpha_1 \circ \alpha_5 > \alpha_3 \circ \alpha_4 > \alpha_1 \circ \alpha_2 > \alpha_5 > \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1 \]

Consider now the following numerical representations:

<table>
<thead>
<tr>
<th></th>
<th>( L_1 )</th>
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<td>99</td>
<td>29</td>
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\( L_1, L_2 \) and \( L_3 \) capture the simple order among \( \alpha_1 - \alpha_5 \), but \( L_2 \) fails to capture the order among the combinations of objects.
Further Example

**NB**

For $L_1$ we get that $\alpha_2 \circ \alpha_3 \sim \alpha_1 \circ \alpha_4$

while for $L_3$ we get that $\alpha_2 \circ \alpha_3 > \alpha_1 \circ \alpha_4$.

We need to fix a “standard sequence”.

**Length**

If we fix a “standard” length, a unit of measure, then all objects will be expressed as multiples of that unit.
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**Length**

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Scales

Ratio Scales
All proportional transformations (of the type $\alpha x$) will deliver the same information. We only fix the unit of measure.

Interval Scales
All affine transformations (of the type $\alpha x + \beta$) will deliver the same information. Besides the unit of measure we fix an origin.
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**Interval Scales**

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More complicated

Consider a Multi-attribute space:

\[ X = X_1 \times \cdots \times X_n \]

to each attribute we associate an ordered set of values:

\[ X_j = \langle x_j^1, \ldots, x_j^m \rangle \]

An object \( x \) will thus be a vector:

\[ x = \langle x_1^l, \ldots, x_n^k \rangle \]
Generally speaking ... 

\[ x \succeq y \]

\[ \iff \]

\[ \langle x_1^l \cdots x_n^k \rangle \succeq \langle y_1^i \cdots y_n^j \rangle \]

\[ \iff \]

\[ \Phi(f(x_1^l \cdots x_n^k), f(y_1^i \cdots y_n^j)) \geq 0 \]
### What that means?

<table>
<thead>
<tr>
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<tr>
<td>( a_2 )</td>
<td>25</td>
<td>80</td>
<td>C</td>
<td>700</td>
</tr>
</tbody>
</table>

For what value of \( \delta_2 \) are \( a_1 \) and \( a_2 \) indifferent?
What that means?

<table>
<thead>
<tr>
<th></th>
<th>Commuting Time</th>
<th>Clients Exposure</th>
<th>Services</th>
<th>Size</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>20</td>
<td>70</td>
<td>C</td>
<td>500</td>
<td>1500</td>
</tr>
<tr>
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<td>80</td>
<td>C</td>
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<td>1500</td>
</tr>
<tr>
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<td>C</td>
<td>700</td>
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<td></td>
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### Introduction Basics

Cost-Benefit Analysis

Multi-attribute Value Functions

Further Reading

Preferences

Measurement

Social Choice Theory

Uncertainty

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</tr>
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The trade-offs introduced with \(\delta_1\) and \(\delta_2\) allow to get \(a \sim a_1 \sim a_2\)
Standard Sequences

**Length**: objects having the same length allow to define a unit of length;

**Value**: objects being indifferent can be considered as having the same value and thus allow to define a “unit of value”.

*Remark 1*: indifference is obtained through trade-offs.

*Remark 2*: separability among attributes is the minimum requirement.
What do we get?

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The easy case

IF

1. restricted solvability holds;
2. at least three attributes are essential;
3. \( \succeq \) is a weak order satisfying the Archimedean condition
   \[ \forall x, y \in \mathbb{R}, \exists n \in \mathbb{N} : ny > x. \]

THEN

\[ x \succeq y \iff \sum_{j} u_j(x) \geq \sum_{j} u_j(y) \]
The above ideas apply also in:

- Economics (comparison of bundle of goods);
- Decision under uncertainty (comparing consequences under multiple states of the nature);
- Inter-temporal decision (comparing consequences on several time instances);
- Social Fairness (comparing welfare distributions among individuals).
Outline

1 Introduction

2 Basics
   - Preferences
   - Measurement
   - Social Choice Theory
   - Uncertainty

3 Cost-Benefit Analysis

4 Multi-attribute Value Functions

5 Further Reading
Four candidates and seven examiners with the following preferences.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>1</td>
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<td>B</td>
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<td>4</td>
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<td>2</td>
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</tbody>
</table>
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<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>(B(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
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<td>2</td>
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<td>1</td>
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<td>B</td>
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<td>D</td>
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<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>25</td>
</tr>
</tbody>
</table>

The Borda count gives \(B>A>C>D\)
Borda vs. Condorcet

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<table>
<thead>
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<tr>
<td>B(x)</td>
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<td>14</td>
<td>15</td>
<td></td>
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If D is not there then A>B>C, instead of B>A>C
Borda vs. Condorcet

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The Condorcet principle gives $A > B > C > A$.
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The Condorcet principle gives A > B > C > A !!!!
Arrow’s Theorem

Given $N$ rational voters over a set of more than 3 candidates can we found a social choice procedure resulting in a social complete order of the candidates such that it respects the following axioms?

- Universality: the method should be able to deal with any configuration of ordered lists;
- Unanimity: the method should respect a unanimous preference of the voters;
- Independence: the comparison of two candidates should be based only on their respective standings in the ordered lists of the voters.
There is only one solution: the dictator!!

If we add no-dictatorship among the axioms then there is no solution.
Gibbard-Satterthwaite’s Theorem

When the number of candidates is larger than two, there exists no aggregation method satisfying simultaneously the properties of universal domain, non-manipulability and non-dictatorship.
# Why MCDA is not Social Choice?

<table>
<thead>
<tr>
<th>Social Choice</th>
<th>MCDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Orders</td>
<td>Any type of order</td>
</tr>
<tr>
<td>Equal importance of voters</td>
<td>Variable importance of criteria</td>
</tr>
<tr>
<td>As many voters as necessary</td>
<td>Few coherent criteria</td>
</tr>
<tr>
<td>No prior information</td>
<td>Existing prior information</td>
</tr>
</tbody>
</table>

Social Choice MCDA
Given a set $A$ and a set of $\succeq_i$ binary relations on $A$ (the criteria) we define:

$$x \succeq y \iff C^+(x, y) \succeq C^+(y, x) \text{ and } C^-(x, y) \succeq C^-(y, x)$$

where:
- $C^+(x, y)$: “importance” of the coalition of criteria supporting $x$ wrt to $y$.
- $C^-(x, y)$: “importance” of the coalition of criteria against $x$ wrt to $y$. 

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Methods and Tools for Public Policy Evaluation
Specific case 1

Additive Positive Importance

$$C^+((x, y)) = \sum_{j \in J^+} w^+_j$$

where:

$$w^+_j: \text{"positive importance" of criterion } i$$

$$J^+ = \{ h_j: x \succeq_j y \}$$

Then we can fix a majority threshold $$\delta$$ and have

$$x \succeq^+ y \iff C^+((x, y)) \geq \delta$$

Where "positive importance" comes from?
Specific case 1

Additive Positive Importance

\[ C^+(x, y) = \sum_{j \in J^\pm} w^+_j \]

where:

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Where “positive importance” comes from?

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Specific case 2

Max Negative Importance

\[ C^-(x, y) = \max_{j \in J} -w_{-j} \] where:
\[ w_{-j} \]: "negative importance" of criterion
\[ J = \{ h_j : v_j(x, y) \} \]

Then we can fix a veto threshold \( \gamma \) and have
\[ x \succeq y \iff C^-(x, y) \geq \gamma \]

Where "negative importance" comes from?
Specific case 2

Max Negative Importance

\[ C^-(x, y) = \max_{j \in J^-} w_j^- \]

where:
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- \( J^- = \{ h_j : v_j(x, y) \} \)
Max Negative Importance

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Where “negative importance” comes from?
The United Nations Security Council

Positive Importance

15 members each having the same positive importance

\[ w_j^+ = \frac{1}{15}, \delta = \frac{9}{15}. \]

Negative Importance

10 members with 0 negative importance and 5 (the permanent members) with \( w_i^- = 1, \gamma = 1 \).
Example

The United Nations Security Council

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Outranking Principle

\[ x \succeq y \iff x \succeq^+ y \text{ and } \neg(x \succeq^- y) \]

Thus:

\[ x \succeq y \iff C^+(x, y) \geq \delta \land C^-(x, y) < \gamma \]
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Thus:

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**NB**

The relation \( \succeq \) is not an ordering relation. Specific algorithms are used in order to move from \( \succeq \) to an ordering relation \( \succeq^\ast \).
What is importance?

Where $w_j^+$, $w_j^-$ and $\delta$ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.
What is importance?

Where $w_j^+$, $w_j^-$ and $\delta$ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

Example

Given a set of criteria and a set of decisive coalitions ($J^{\pm}$) we can solve:

$$\max \delta$$

subject to

$$\sum_{j \in J^{\pm}} w_j \geq \delta$$

$$\sum_j w_j = 1$$
Lessons Learned

- We can use social choice inspired procedures for more general decision making processes.
- Care should be taken to model the majority (possibly the minority) principle to be used. The key issue here is the concept of “decisive coalition”.
- We need to “learn” about decisive coalitions, since it is unlike that this information is available. Problem of learning procedures.
- The above information is not always intuitive. However, the intuitive idea of importance contains several cognitive biases.
- A social choice inspired procedure will not deliver automatically an ordering. We need further algorithms (graph theory).
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   - Social Choice Theory
   - Uncertainty

3. Cost-Benefit Analysis

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5. Further Reading

Alexis Tsoukiàs
Methods and Tools for Public Policy Evaluation
What is Probability?

A measure of uncertainty, of likelihood ... of subjective belief ...

Consider a set $N$ and a function $p : 2^N \mapsto [0, 1]$ such that:
- $p(\emptyset) = 0$;
- $A \subseteq A \subseteq N$, then $p(A) \leq p(B)$;
- $A \subseteq A \subseteq N$, $A \cap B = \emptyset$, then $p(A \cup b) = p(A) + p(B)$;

Then the function $p$ is a “probability".
What is Probability?

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A probability is an additive measure of capacity
Decision under risk

<table>
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<th>$\theta_2$</th>
<th>$\theta_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$x_{11}$</td>
<td>$x_{1n}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$x_{21}$</td>
<td>$x_{2n}$</td>
</tr>
<tr>
<td>actions</td>
<td>$\cdots$</td>
<td>outcomes</td>
</tr>
<tr>
<td>$a_m$</td>
<td>$x_{m1}$</td>
<td>$x_{mn}$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_n$</td>
</tr>
</tbody>
</table>

$\langle p_1, x_{i1}; p_2, x_{i2}; \cdots; p_n, x_{in} \rangle$

is a lottery associated to action $a_i$. 
Von Neuman and Morgenstern Axioms

A1  There is a weak order \( \succeq \) on the set of outcomes \( X \).

A2  If \( x \succ y \) implies that \( \langle x, P; y, 1 - P \rangle \succ \langle x, Q; y, 1 - Q \rangle \), then \( P > Q \).

A3  \( \langle x, P; y, Q; z, 1 - Q \rangle, 1 - P \rangle \sim \langle x, P; y, Q(1 - P); z, (1 - Q)(1 - P) \rangle \)

A4  If \( x \succ y \succ z \) then \( \exists P \) such that \( \langle y, 1 \rangle \sim \langle x, P; z, 1 - P \rangle \)

If the above axioms are true then

\[ \exists v : X \mapsto \mathbb{R} : a_l \succeq a_k \iff \sum_{j=1}^{n} p_j x_{lj} \geq \sum_{j=1}^{n} p_j x_{kj} \]
Expected Utility Theory is falsifiable under several points of view

- Gains and losses induce a different behaviour of the decision maker when facing a decision under risk.
- Independence is easily falsifiable.
- Rank depending utilities.
- What happens if probabilities are “unknown”?
- Where probabilities come from?
- What is subjective probability?
Probability does not exist!!!

Ramsey and De Finetti

If the option of $\alpha$ for certain is indifferent with that of $\beta$ if $p$ is true and $\gamma$ if $p$ is false, we can define the subject's degree of belief in $p$ as the ratio of the difference between $\alpha$ and $\gamma$ to that between $\beta$ and $\gamma$ (Ramsey, 1930, see also De Finetti, 1936).
Probability does not exist!!!

Ramsey and De Finetti

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Savage will give a normative characterisation of von Neuman’s expected utility, but the axioms remain empirically falsifiable.
How do we build a value function?

Consider the following possible outcomes:
10, 20, 30, 40, 50, 60, 70
Without loss of generality we can consider that:
\( v(10) = 0 \) and \( v(70) = 1 \).
Then if the decision maker validates that he is indifferent between a sure outcome of 40 and a lottery \( \langle 70, 0.8; 10, 0.2 \rangle \) we get \( v(40) = v(70) \times 0.8 + v(10) \times 0.2 \), that is \( v(40) = 0.8 \). With the same protocol we can obtain the following value function.
A risk adverse value function
An Example: New Product Development

The R/D department of your company applies for a grant aiming to develop a new “100% fat free” chocolate. They ask for 100K €. There is a 30% probability that they will succeed. If it is the case you face the problem of what type of production you should undertake. If you opt for a mass production and there is a positive reply from the market you can expect 500K € profit, otherwise the profit will be 50K €. If you make just an experimental production the figures will be respectively 100K € and 40K €. There is 50% probability that such a product will meet a positive reply from the market. What are your decisions? Consider the monetary outcomes as a value function.
Decision Trees

Invest?  
0

YES
NO

-17.5  

YES
NO

-100

175  

success?
what inv.?
mass.
exp.
market?

+ 50%
- 50%
+ 50%
- 50%

-100
400
-50
0
-60

0

14

Alexis Tsoukiàs
Methods and Tools for Public Policy Evaluation
Decision Trees

Boxes are decisions. Circles are lotteries.

The value of the lottery “market reaction to mass production” is 175. The value of the lottery “market reaction to exp. production” is -30. The decision therefore is “mass production”. The value of the lottery “success of the R/D” is -17.5, while the no investment has a value of 0. Therefore the decision is not to give the grant to the R/D dept.
Conditional Probabilities

Suppose a serious invalidating illness affecting 1/10000 of the population. There is an examination with 1/100 possibility of error. You undergo such an examination and the result is positive!! What are your chances to be really ill?
\[ P(I) \]: probability of being ill
\[ P(NI) \]: probability of not being ill
\[ P(+|I) \]: probability of having a positive result if you are ill;
\[ P(I|+) \]: probability of being ill if the result is positive.

\[
P(I|+) = \frac{P(I)P(+|I)}{P(I)P(+|I) + P(NI)P(+|NI)} = 0.01
\]
Bayes’s theorem

Given a set of events $X = \{x_1, \cdots, x_n\}$ and the knowledge $A$ then:

$$P(x_k | A) = \frac{P(x_k)P(A|x_k)}{\sum_{i=1}^{n} P(x_i)P(A|x_i)}$$
Outline

1. Introduction
2. Basics
3. Cost-Benefit Analysis
   - Net Present Value
   - Cash Flow Example
   - Net Present Social Value
   - An Example
4. Multi-attribute Value Functions
5. Further Reading
Let’s start with some intuitive hypotheses.

- An investment should take place only if the expected benefits outperform the expected costs.
- The money used for the investment is either borrowed (from the money market) or if it is used from the investor’s treasure it should be at least as profitable as if it was borrowed.
Thus, if $K$ is an investment, then:

$$B(K) - C(K) \geq 0$$

where $B(K)$ ($C(K)$) represent the overall benefits (costs) of the investment (of course we may usually expect to have some profit which implies having a difference more than simply non-negative, but for our presentation this is irrelevant).

Fixing a time horizon $T$ (divided in $i$ time periods) within which we may verify the profitability of the investment we get:

$$\sum_{i=1}^{T} B_i(K) - C_i(K) \geq 0$$
If you borrow 1 € today under an interest rate of $r$ for a period $i$ then at the end of that period you have to return $1 + r$ €.

If you know that at the end of the period you can return $X$ € then at the beginning of that period you can borrow not more than $\frac{X}{1+r}$ €.

If the periods are $n$ then you can borrow at most $\frac{X}{(1+r)^n}$ €.
On this basis the real value of the investment has to be discounted to the interest rate to which the money is borrowed (at net of the inflation rate if any). If such a discount rate is named $r$ we get:

$$\sum_{i=1}^{T} \frac{B_i(K) - C_i(K)}{(1 + r)^n} \geq 0$$

We call this formula the NET PRESENT VALUE (NPV) of the investment and we expect it to be positive in order to make the investment interesting over the time horizon considered.
Outline

1. Introduction
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Consider an investment consisting in buying some new machinery (estimated cost 100000 €). The time horizon is fixed to three years with annual operating costs 10000 €. At a fix discount rate of 5%, what should be the annual income (consider it fix every year) in order to make the investment interesting?
Let's write down the cash flow over the three years

\[
\frac{X - 110000}{1.05} + \frac{X - 10000}{1.05^2} + \frac{X - 10000}{1.05^3}
\]

where \( X \) is the unknown annual income.
Putting the cash flow non negative and resolving for $X$ we get: $X \geq 45000$ approximately.
This means that we have to generate approximately a constant annual income of 45000 € in order to be the investment interesting.
Consider now the same investment and the same operating costs as with the previous example. However, you know now that the first year you can expect an income of 5000 €, the second an income of 45000 € and the third an income of 75000 €. At what discount rate this investment will be interesting (always in a three years horizon)?
Let’s write down the cash flow over the three years

\[
-\frac{95000}{1 + r} + \frac{35000}{(1 + r)^2} + \frac{65000}{(1 + r)^3}
\]

where \( r \) is the unknown discount rate
Putting the cash flow non negative and resolving for $X = 1 + r$ we get:

$X \geq 1.03$ approximately.

This means that for a discount rate of approximately 3% this investment becomes interesting.

The reader will note that in order to solve the cash flow equation it is necessary to solve a non linear equation (in this case quadratic). In order to do so he should remind to consider only the positive solutions.
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Alexis Tsoukiàs
Methods and Tools for Public Policy Evaluation
Consider a project part of a public policy. First hypothesis: there are multiple and qualitatively different costs and benefits instead of single ones.

\[
B_i(K) = \sum_j b_{ij}(K)
\]
\[
C_i(K) = \sum_j c_{ij}(K)
\]

where \(b_{ij}(K)\) is the \(j\)the benefit of project \(K\) at time \(i\) and \(c_{ij}(K)\) is the \(j\)the cost of project \(K\) at time \(i\)
Multiple Costs and Benefits

Consider a project part of a public policy. Second hypothesis: *each cost and each benefit should be commensurable, possibly in monetary terms.*

There are two ways to obtain that:

- either there is a market (direct or proxy) where these costs and benefits can be priced;
- or there exist suitable trade-offs between each cost and benefit with a reference (cost or benefit) expressed in monetary terms.
## A small example: a highway project

<table>
<thead>
<tr>
<th>COSTS</th>
<th>BENEFITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>Accessibility</td>
</tr>
<tr>
<td>Maintenance</td>
<td>Time reduction</td>
</tr>
<tr>
<td>Landscape</td>
<td>Area development</td>
</tr>
<tr>
<td>Pollution</td>
<td>Less accidents</td>
</tr>
<tr>
<td>Yard disturbances</td>
<td>Workforce employment</td>
</tr>
</tbody>
</table>

### Methods and Tools for Public Policy Evaluation

Alexis Tsoukiàs
A small example: a highway project

Time reduction

In order to calculate the monetary equivalent of time reduction we can consider the value of time resulting from the job market.

Landscape

In order to calculate the monetary cost of Landscape we can consider the extra construction cost required to avoid each specific landscape deterioration the highway may create (trade-off).
At this point we can calculate the Net Present Social Value of project $K$.

\[
NPSV(K) = \sum_{i=1}^{T} \frac{\overline{b}_i(K) - \overline{c}_i(K)}{(1 + r)^i} = \frac{\sum_k h_k b_{ik}(K) - \sum_j p_j c_{ij}(K)}{(1 + r)^i}
\]

where $h_k$ and $p_j$ represent the trade-offs among the different costs and benefits.
Further implicit hypotheses we did

1. The society is seen as a collection of consumers of goods affected by the project realisation.
2. Any cost and benefit have a price (there is a direct or proxy market where this is fixed).
3. Cost and benefits can compensate one the other.
4. Further generations will still value the projet as we do today (in case the project time horizon spans over several generations).
5. There is no uncertainty as far as the outcomes of the project are concerned.
Procedure Summary

1. Identify a set of potential costs of the project.
2. Identify a set of potential benefits of the project.
3. Establish appropriate prices for each cost and for each benefit.
4. Establish appropriate trade-offs among the different costs and benefits.
5. Fix an appropriate time horizon within which the project should be evaluated as well as the time periods discretising the time horizon.
6. Choose an appropriate discount rate homogenising the future costs and benefits to the present prices.
Results

1. If $NPSV(K) > 0$ then project $K$ is socially profitable. If several projects compete then their $NPSV$ could be used to rank them.

2. The ratio $\frac{B(K)}{C(K)}$ (where $B(K)$ is the overall discounted benefits and $C(K)$ the overall discounted costs represents the project effectiveness. If it superior to 1 then the project is socially effective. If several projects compete this ratio could rank them.

3. Rankings according to $NPSV$ and according to effectiveness may be different.

4. Solving for $NPSV(K) = 0$ with $T$ unknown establishes the payback period.

5. Solving for $NPSV(K) = 0$ with $r$ unknown establishes the internal return rate (at what discount rate the project is profitable).
If $NPSV(K) > 0$ then project $K$ is socially profitable. If several projects compete then their $NPSV$ could be used to rank them.

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The following example is borrowed from the EU Manual of Cost-Benefit Analysis, see reference in “Further Readings”.

The project consists in constructing a new motorway by-passing a densely populated area in order to decrease traffic congestion and air pollution, besides improving accessibility and safety. Two options are considered, a free motorway and a tolled one. It is not expected to observe major increases in traffic, since the area is already heavily developed. It is rather expected to observe traffic diversion, moving from the present local network to the new motorway.
Highway project: hypotheses

- The length of the new motorway is 72km.
- The technical life is 70 years and thus the assessment time horizon has been fixed at 30 years (approximately 40%). The discretised time line has been established in years.
- The social discount rate has been fixed at 5.5%.
- Traffic forecast has been established using conventional traffic and transportation models.
## Highway project: COSTS and BENEFITS

<table>
<thead>
<tr>
<th>COSTS</th>
<th>BENEFITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investments</td>
<td>Consumer’s Surplus</td>
</tr>
<tr>
<td>Work</td>
<td>Time reduction</td>
</tr>
<tr>
<td>Land</td>
<td>Vehicle Operating Costs Reduction</td>
</tr>
<tr>
<td>Junctions</td>
<td>Gross Producer and Road User Surplus</td>
</tr>
<tr>
<td>General</td>
<td>Tolls (in case)</td>
</tr>
<tr>
<td>Operating</td>
<td>Vehicle Operating Costs</td>
</tr>
<tr>
<td>Maintenance</td>
<td>State Revenues</td>
</tr>
<tr>
<td>Other</td>
<td>Environmental Benefits</td>
</tr>
<tr>
<td></td>
<td>Accident Reduction</td>
</tr>
</tbody>
</table>
### Free Highway, 0-15 years

<table>
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<tr>
<th>Benefits</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>12</th>
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<tbody>
<tr>
<td>Consumer’s surplus</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>53.7</td>
<td>56.1</td>
<td>58.4</td>
<td>60.8</td>
<td>63.2</td>
<td>65.6</td>
<td>68.0</td>
<td>70.3</td>
<td>72.7</td>
<td>75.1</td>
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<tr>
<td>Time Benefits</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>69.9</td>
<td>62.5</td>
<td>65.0</td>
<td>67.6</td>
<td>70.1</td>
<td>72.6</td>
<td>75.2</td>
<td>77.7</td>
<td>80.3</td>
<td>82.8</td>
<td>85.3</td>
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<tr>
<td>Vehicle Operating Costs (perceived)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.8</td>
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<tr>
<td>Gross Producer and Road User Surplus</td>
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<td>0.0</td>
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<td>-10.3</td>
<td>-10.6</td>
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<td>-12.1</td>
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<td>-12.6</td>
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<tr>
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<tr>
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<td>0.0</td>
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<td>-10.3</td>
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<td>-10.8</td>
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<tr>
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<td>0.0</td>
<td>0.0</td>
<td>10.3</td>
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<td>12.6</td>
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<td>Net Environmental Benefits</td>
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<td>Accident reduction</td>
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<table>
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<tr>
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<tr>
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### Introduction

**Basics**

**Cost-Benefit Analysis**

**Multi-attribute Value Functions**

**Further Reading**

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## Net Present Value

### Cash Flow Example

### Net Present Social Value

### An Example

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### Free Highway, 16-30 years

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## Tolled Highway, 16-30 years

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### Results

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It is clear that only the Free Motorway is socially profitable.
Introduction
Basics
Cost-Benefit Analysis
Multi-attribute Value Functions
Further Reading

Outline

1 Introduction

2 Basics

3 Cost-Benefit Analysis

4 Multi-attribute Value Functions
   - What is a value function?
   - How Better?
   - Comparing apples to peaches
   - Example

5 Further Reading

Alexis Tsoukiàs
Methods and Tools for Public Policy Evaluation
Consider a regional plan which is expected to affect the economy, the landscape, the environment and quality of life of citizens.

In order to choose among competing projects we need to compare the consequences that these may have against several different dimensions and identify the “best” ones.

This is not straightforward as it may appear
First Questions

What does it mean better?

- Better for whom?
- How do we measure better on landscape esthetics?
- How do we compare better on landscape esthetics with better on costs?
Cost-Benefit Analysis claims that there is a “better” for the society as a whole and this is established computing the consumers’ surplus for each project. However, this implies that consumers’ all have the same preferences and that for all possible consequences there exist markets (direct or proxy) allowing the consumers’ to express such preferences.

It is reasonable to argue both such hypotheses. Consumers/Citizens have conflicting preferences (opinions) on many issues and is unlike that any observable externality has an associable market revealing the preferences.
The Value Functions Hypotheses

Collective Rationality

It makes no sense to try to fix society’s preferences (Arrow’s theorem). Instead we can look to model a specific decision maker/stakeholder preferences since she could have consistent values.

Subjective Values

If this is true then we can try to “measure” the consequences of any project or policy against such values: this is a subjective value function.
The Value Functions Hypotheses

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Outline

1. Introduction
2. Basics
3. Cost-Benefit Analysis
4. Multi-attribute Value Functions
   - What is a value function?
   - How Better?
   - Comparing apples to peaches
   - Example
5. Further Reading
How do we measure better?

Let’s go more formal.

- Let \( x, y, z \ldots \) be competing projects within set \( A \);
- Let \( d_j(x) \) representing the consequences of project \( x \) on dimension \( d_j \);
- Let \( d_j(A) \) representing the set of all consequences for all projects in \( A \).

The first step consists in verifying that:

\[
\forall j \in D \ \exists \succeq_j \subseteq d_j(A)^2
\]

such that \( \succeq_j \) is a weak order (consequences should be completely and transitively ordered).
How do we measure better?

If the previous hypothesis is verified then

$$\forall j \in D \ \exists h_j : A \mapsto \mathbb{R} : d_j(x) \succeq d_j(y) \iff h_j(x) \geq h_j(y)$$

In other terms for each dimension we can establish a real valued function respecting the decision maker’s preferences.

This function is ONLY an ordinal measure of the preferences.
Suppose you have 4 projects $x, y, z, w$ of urban rehabilitation and an assessment dimension named “esthetics”. You have:
- $d_e(x) = \text{statue}$;
- $d_e(y) = \text{fountain}$;
- $d_e(z) = \text{garden}$;
- $d_e(w) = \text{kid’s area}$;
Preferences expressed could be for instance:
$d_e(x) \succ d_e(y) \succ d_e(z) \sim d_e(w)$
A possible numerical representation could thus be:
$h_e(x) = 3$, $h_e(y) = 2$, $h_e(z) = h_e(w) = 1$
Example-2

Suppose you have 4 projects $x, y, z, w$ of urban rehabilitation and an assessment dimension named “land use”. You have:
- $d_i(x) = 100$sqm;
- $d_i(y) = 50$sqm;
- $d_i(z) = 1000$sqm;
- $d_i(w) = 500$sqm;
Preferences expressed could be for instance (suppose the decision maker dislikes land use):
$d_e(y) \succ d_e(x) \succ d_e(w) \sim d_e(z)$
A possible numerical representation could thus be:
$h_e(y) = 4, h_e(x) = 3, h_e(w) = 2, h_e(z) = 1$, but also:
$h_e(y) = 50, h_e(x) = 100, h_e(w) = 500, h_e(z) = 1000$
Is this sufficient?

For the time being we have the following table:

\[
\begin{array}{cccc}
 & d_1-h_1 & d_2-h_2 & \ldots & d_n-h_n \\
x & & & & \\
y & & & & \\
z & & & & \\
w & & & & \\
\vdots & & & & \\
\end{array}
\]

The consequences of each action and the numerical representation of the decision maker’s preferences (ordinal).
Is this sufficient?

NO!

We need something more rich. We need to know, when we compare $x$ to $y$ (and we prefer $x$) if this preference is “stronger” to the one expressed when comparing (on the same dimension) $z$ to $w$.

We need to compare differences of preferences.
An example

For instance, if the above function represents the value of “land use” it is clear that the difference between 50sqm and 100sqm is far more important from the one between 500sqm and 1000sqm.
First Summary

Let’s summarise our process until now.

- We get the alternatives.
- We identify their consequences for all relevant dimensions.
- These consequences are ordered for each dimension using the decision maker’s preferences.
- We compute the value function measuring the differences of preferences (for each dimension).
First Summary

 Alternatives

 A

 x

 y

 z

 .

 .

 .

 alternatives
First Summary

What is a value function?
How Better?
Comparing apples to peaches
Example

Alternatives $\{x, y, z, \ldots\}$
Consequences $\{d_1(x), d_1(y), \ldots\}$

Methods and Tools for Public Policy Evaluation
Alexis Tsoukiàs
What is a value function?

Comparing apples to peaches

Example

First Summary

alternatives consequences ordinal

metha

Methods and Tools for Public Policy Evaluation
First Summary

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<table>
<thead>
<tr>
<th>alternatives</th>
<th>consequences</th>
<th>ordinal measures</th>
<th>values</th>
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<tr>
<td>A</td>
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<td>h_j(A)</td>
<td>u_j(A)</td>
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<td>x</td>
<td>d_j(x)</td>
<td>h_j(x)</td>
<td>u_j(x)</td>
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<td>y</td>
<td>d_j(y)</td>
<td>h_j(y)</td>
<td>u_j(y)</td>
</tr>
<tr>
<td>z</td>
<td></td>
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Outline

1. Introduction
2. Basics
3. Cost-Benefit Analysis
4. Multi-attribute Value Functions
   - What is a value function?
   - How Better?
   - Comparing apples to peaches
   - Example
5. Further Reading
Is all that sufficient?

**NO!**

1. The problem is that we need to be able to compare the differences of preferences on one dimension to the differences of preferences on another one (let’s say differences of preferences on land use with differences of preferences on esthetics).

2. At the same time we need to take into account the intuitive idea that for a given decision maker certain dimensions are more “important” than other ones.
Principal Hypotheses

1. The different dimensions are separable.
2. Preferences on each dimension are independent.
3. Preferences on each dimension are measurable in terms of differences.
4. Good values on one dimension can compensate bad values on another dimension.
Principal Hypotheses

Under the previous hypotheses we can construct a global value function \( U(x) \) as follows:

\[
U(x) = \sum_j u_j(x)
\]

and in case we use normalised (in the interval \([0,1]\)) marginal value functions \( \bar{u}_j \) then:

\[
U(x) = \sum_j w_j \bar{u}_j(x)
\]
Principal Hypotheses

where: $w_j$ should represent the importance of the marginal functions;
If $h_j(x)$ represent the ordinal values of dimension $j$ then $u_j(d_j(x)) = 0$ where $d_j(x)$ is the worst value of $h_j$ and in case we use normalised value functions then $u_j(d_j(\bar{x})) = 1$ where $d_j(\bar{x})$ is the best value of $h_j$. 
Standard Protocol

1. Fix arbitrary one dimension as the reference for which the value function will be linear (there is no loss of generality doing so).

2. Fix a number of units diving entirely the reference value function, thus fixing the unit of value $U_1$.

3. Une indifference questions (see later) in order to find equivalent values for the other dimensions.

4. The segments between the equivalent values will shape the other value functions.

5. The ratio of units used to describe each value function with respect to the units for the reference one establishes the trade-offs among the dimensions.
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Indifference Questions

Given \( d_r \) as the reference dimension, \( h_r \) being the ordinal preferences we want to establish a value function for dimension \( d_k \). Consider a fictitious object \( x \) for which we have \( \langle h_r(x), h_k(x) \rangle \). The key question is:

\[
\langle h_r(x), h_k(x) \rangle \sim \langle h_r(\bar{x}), ? \rangle
\]

What should be the measure on dimension \( k \) of an object \( \bar{x} \) whose measure on the reference dimension \( r \) is such that the \( u_r(\bar{x}) = u_r(x) + U_1 \) if \( x \) and \( \bar{x} \) should be indifferent for the decision maker?
Once you get the answer $h_k(\bar{x})$ from the decision maker you go ahead:

$$\langle h_r(x), h_k(\bar{x}) \rangle \sim \langle h_r(\bar{x}), ? \rangle \rightarrow h_k(\bar{x})$$

$$\langle h_r(x), h_k(\bar{x}) \rangle \sim \langle h_r(\bar{x}), ? \rangle \rightarrow h_k(\bar{x})$$

Until the whole set of measures of dimension $k$ has been used.
TIP1 Start considering a point $x$ at the middle of both scales $h_r$ and $h_k$.

TIP2 Then start deteriorating on the reference dimension by one unit of value at time (thus the dimension under construction has to improve) until the upper scale of $h_k$ is exhausted.

TIP2 Then start improving on the reference dimension by one unit of value at time (thus the dimension under construction has to deteriorate) until the lower scale of $h_k$ is exhausted.
What do we get?

We have $U(x) = u_r(x) + u_k(x)$ by definition.
We also have $U(\bar{x}) = u_r(\bar{x}) + u_k(\bar{x})$ after questioning.
And since $x$ and $\bar{x}$ are considered indifferent $U(x) = U(\bar{x})$.
Then we get $u_r(x) + u_k(x) = u_r(x) + U_1 + u_k(\bar{x})$ by construction.
We obtain $u_k(\bar{x}) = u_k(x) - U_1$.

Going ahead recursively we found the point $x$ at the bottom of
the scale for which by definition $u_k(x) = 0$ (by definition). Using
linear segments between all the points discovered we shape
the value function $u_k$. 
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Example

You have to choose among competitive projects assessed against 3 attributes: cost, esthetics and mass. As far as the cost is concerned the scale goes from 5M € to 10M €. Esthetics are assessed on a subjective scale going from 0 to 8. Mass is measured in kg and the scale goes from 1kg to 5kg. In this precise moment you have under evaluation the following four ones:

<table>
<thead>
<tr>
<th>project</th>
<th>c</th>
<th>e</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6,5M €</td>
<td>3</td>
<td>3kg</td>
</tr>
<tr>
<td>B</td>
<td>7,5M €</td>
<td>4</td>
<td>4,5kg</td>
</tr>
<tr>
<td>C</td>
<td>8M €</td>
<td>6</td>
<td>2kg</td>
</tr>
<tr>
<td>D</td>
<td>9M €</td>
<td>7</td>
<td>1,5kg</td>
</tr>
</tbody>
</table>

Which is the “best choice”?
Preferences

First we need to establish appropriate preferences. Suppose in your case the following ones:

- you prefer the less expensive to the more expensive (cost);
- you prefer “pretty” to “less pretty” (esthetics);
- you prefer “heavy” to “less heavy” (mass).
Without loss of generality we establish the cost as reference criterion with a linear value function such that $u_c(5\text{M} \, \€) = 1$ and $u_c(10\text{M} \, \€) = 0$. We fix the value unit $U_1 = 0,5\text{M} \, \€$. 

\[
\begin{array}{c}
\text{Cost Value Function} \\
\end{array}
\]
Esthetics Value Function

In order to construct the value function of Esthetics we proceed with the following dialog:

\[ \langle 7.5M\,€, 4 \rangle \sim \langle 8M\,€, \,? \rangle \]

Consider a project which costs 7.5\,€ and is assessed on esthetics with 4, and a project which costs 8M\,€ (one unit of value less in this case), how much should the second project be improved in esthetics in order to be indifferent to the first one? Suppose we get an answer of 5: \[ \langle 7.5M\,€, 4 \rangle \sim \langle 8M\,€, 5 \rangle \]
We repeat now the question using the new value:

\[ \langle 7.5M\,€, 5 \rangle \sim \langle 8M\,€, \,? \rangle \]

We now get an answer of 6.
We can summarise the dialog as follows:

\[
\begin{align*}
\langle 7.5 \text{ M} \, \text{€}, 4 \rangle & \sim \langle 8 \text{ M} \, \text{€}, 5 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 5 \rangle & \sim \langle 8 \text{ M} \, \text{€}, 6 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 6 \rangle & \sim \langle 8 \text{ M} \, \text{€}, 7 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 7 \rangle & \sim \langle 8 \text{ M} \, \text{€}, 7.5 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 7.5 \rangle & \sim \langle 8 \text{ M} \, \text{€}, 8 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 4 \rangle & \sim \langle 7 \text{ M} \, \text{€}, 3 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 3 \rangle & \sim \langle 7 \text{ M} \, \text{€}, 1.5 \rangle \\
\langle 7.5 \text{ M} \, \text{€}, 1.5 \rangle & \sim \langle 7 \text{ M} \, \text{€}, 0 \rangle
\end{align*}
\]
The previous dialog will result in the following value function.
Mass Value Function

In order to construct the value function of Mass we proceed with the following dialog:

\[ \langle 7.5\text{ M}\text{ e}, 3.1 \rangle \sim \langle 8\text{ M}\text{ e}, ? \rangle \]

Consider a project which costs 7.5\text{ e} and weighs 3.1\text{ kg} and a project which costs 8\text{ M}\text{ e} (one unit of value less in this case), how much should the second project be improved in mass in order to be indifferent to the first one? Suppose we get an answer of 3.5\text{ kg}: \[ \langle 7.5\text{ M}\text{ e}, 3.1 \rangle \sim \langle 8\text{ M}\text{ e}, 3.5 \rangle \]

We repeat now the question using the new value:

\[ \langle 7.5\text{ M}\text{ e}, 5 \rangle \sim \langle 8\text{ M}\text{ e}, ? \rangle \]

We now get an answer of 3.9.
Mass Indifferences

We can summarise the dialog as follows:

\[
\langle 7.5 \text{M} \text{€}, 3.1 \rangle \sim \langle 8 \text{M} \text{€}, 3.5 \rangle \\
\langle 7.5 \text{M} \text{€}, 3.5 \rangle \sim \langle 8 \text{M} \text{€}, 3.9 \rangle \\
\langle 7.5 \text{M} \text{€}, 3.9 \rangle \sim \langle 8 \text{M} \text{€}, 5 \rangle \\
\langle 7.5 \text{M} \text{€}, 3.1 \rangle \sim \langle 7 \text{M} \text{€}, 2.7 \rangle \\
\langle 7.5 \text{M} \text{€}, 2.7 \rangle \sim \langle 7 \text{M} \text{€}, 2.3 \rangle \\
\langle 7.5 \text{M} \text{€}, 2.3 \rangle \sim \langle 7 \text{M} \text{€}, 1.9 \rangle \\
\langle 7.5 \text{M} \text{€}, 1.9 \rangle \sim \langle 7 \text{M} \text{€}, 1.75 \rangle \\
\langle 7.5 \text{M} \text{€}, 1.75 \rangle \sim \langle 7 \text{M} \text{€}, 1.6 \rangle \\
\langle 7.5 \text{M} \text{€}, 1.6 \rangle \sim \langle 7 \text{M} \text{€}, 1.45 \rangle \\
\langle 7.5 \text{M} \text{€}, 1.45 \rangle \sim \langle 7 \text{M} \text{€}, 1.3 \rangle \\
\langle 7.5 \text{M} \text{€}, 1.3 \rangle \sim \langle 7 \text{M} \text{€}, 1.15 \rangle \\
\langle 7.5 \text{M} \text{€}, 1.15 \rangle \sim \langle 7 \text{M} \text{€}, 1 \rangle 
\]
Mass Value Function

The previous dialog will result in the following value function.
Having obtained the three value functions we can now calculate the values of the four projects for each of them.

\[
\begin{align*}
    u_c(A) &= 0.7 & u_e(A) &= 0.2 & u_m(A) &= 0.875 \\
    u_c(B) &= 0.5 & u_e(B) &= 0.3 & u_m(B) &= 1.160 \\
    u_c(C) &= 0.4 & u_e(C) &= 0.5 & u_m(C) &= 0.625 \\
    u_c(D) &= 0.2 & u_e(D) &= 0.6 & u_m(D) &= 0.330 \\
\end{align*}
\]
Final Results

Finally we get

\[
U_c(A) = 0.7 + 0.2 + 0.875 = 1.775 \\
U_c(B) = 0.5 + 0.3 + 1.160 = 1.960 \\
U_c(C) = 0.4 + 0.5 + 0.625 = 1.525 \\
U_c(D) = 0.2 + 0.6 + 0.330 = 1.130
\]

The project which maximises the decision maker’s value is \( B \).
Where did the weight disappear?

NOWHERE

Suppose we were using normalised value functions which have to be “weighted”. We recall that in such a case we have:

\[ U(x) = \sum_{j} w_j \bar{u}_j(x) \]

Consider the first indifference sentence about esthetics. We had: \( \langle 7.5\text{M}\,\varepsilon, 4 \rangle \sim \langle 8\text{M}\,\varepsilon, 5 \rangle \). We get:

\[ w_c \bar{u}_c(7.5\text{M}\,\varepsilon) + w_e \bar{u}_e(4) = w_c \bar{u}_c(8\text{M}\,\varepsilon) + w_e \bar{u}_e(5) \]

where:
- \( w_c \) and \( w_e \) represent the “weights” of cost and esthetics respectively;
- and \( \bar{u}_c \) and \( \bar{u}_e \) are the normalised value functions.
By construction $u_c(x) = \bar{u}_c(x)$. We get:

$$w_c(\bar{u}_c(7.5M \in) - \bar{u}_c(8M \in)) = w_e(\bar{u}_e(5) - \bar{u}_e(4)).$$

Thus:

$$\frac{w_e}{w_c} = \frac{\bar{u}_c(7.5M \in) - \bar{u}_c(8M \in)}{\bar{u}_e(5) - \bar{u}_e(4)}$$

However, $\bar{u}_c(7.5M \in) - \bar{u}_c(8M \in) = 1/10$ of the cost value function (by construction) and $\bar{u}_e(5) - \bar{u}_e(4) = 1/8$ of the esthetics value function as it results from the dialog. Using the same procedure for mass we get:

- $w_e/w_c = 0.8$ meaning that esthetics represents 80% of the cost value (this is the esthetics trade-off);
- $w_m/w_c = 1.2$ meaning that mass represents 120% of the cost value (this is the mass trade-off);
Conclusion and tips

Tip 1 Not surprisingly the “weight” of each criterion is represented by the maximum value it attains.

Tip 2 It is better not to use any “weights” when constructing value functions, since it can generate confusion to the decision maker. We can explain the relative importance of each criterion using the trade-offs.

So called “weights” are the trade-offs among the value functions and as such are established as soon as the value functions are constructed. They do not exist independently and is not correct to ask the decision maker to express them.
Books


Books

Documents


