STOCHASTIC PROGRAMMING

November 20, 2024

Today (Lecture 4/5)

- Exercise(s)
- Risk measures
- Intro to chance constraints (time permitting)

0) Exercise

$$F(x,\xi) = \begin{cases} -x_1 & \text{if } \xi = \xi_1 \\ 0 & \text{if } \xi \neq \xi_1 \end{cases} \rightarrow \xi \in (\xi_1, \xi_2]$$

$$\mathbb{P}(\xi = \xi_1) = p \qquad \mathbb{P}(\xi = \xi_2) = 1-p$$

$$\xi \in [\xi_1, \xi_2] = \{ \alpha \xi_1 + (1-\alpha)\xi_2 \mid \alpha \in [0,1] \}$$

1) (P)

$$\underset{x \in \mathbb{R}^2}{\text{minimize}} \quad \mathbb{E}_\xi [F(x,\xi)] \quad \text{s.t.} \quad x_1 + x_2 = 1, \quad x \geq 0$$

a) <u>Solution of (P)</u>

$$\mathbb{E}_\xi [F(x,\xi)] = \mathbb{P}(\xi = \xi_1) \times F(x,\xi_1) + \mathbb{P}(\xi = \xi_2) \times F(x,\xi_2)$$
$$= p \times (-x_1) + (1-p) \times 0$$
$$= -p x_1$$

$$\underset{x \in \mathbb{R}^2}{\text{minimize}} \quad -p x_1 \quad \text{s.t.} \quad x_1 + x_2 = 1, \quad x \geq 0$$

$\forall x \in \{ x \mid x_1 + x_2 = 1, x \geq 0 \}$,

$$-p x_1 \geq -p$$

because

$$\left. \begin{array}{l} x_1 + x_2 = 1 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{array} \right| \Rightarrow 0 \leq x_1 \leq 1$$

lower bound on the objective

Setting $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ achieves the lower bound, hence

RP: Value of the recourse problem

$$\left[\min_{x \in \mathbb{R}^2} \left\{ -p x_1 \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\} = -p \right.$$

Risk-neutral solution

$$\left[\operatorname*{argmin}_{x \in \mathbb{R}^2} \left\{ -p x_1 \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \right.$$

b) Compute the EVPi and the VSS for (P)

- EVPi = RP − WS where

$$WS = \mathbb{E}_{\xi} \left[\min_{x \in \mathbb{R}^2} \left\{ F(x, \xi) \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\} \right]$$

$$= \mathbb{P}(\xi = \xi_1) \times \min_{x \in \mathbb{R}^2} \left\{ F(x, \xi_1) \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\}$$

$$+ \mathbb{P}(\xi = \xi_2) \times \min_{x \in \mathbb{R}^2} \left\{ F(x, \xi_2) \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\}$$

$\xi = \xi_1$: $F(x, \xi_1) = -x_1$

$$\min_{x \in \mathbb{R}^2} \left\{ F(x, \xi_1) \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\}$$

$$= \min_{x \in \mathbb{R}^2} \left\{ -x_1 \text{ s.t. } x_1 + x_2 = 1,\ x \geq 0 \right\}$$

$$= -1$$ (follows from the results of question a))

$\xi = \xi_2$: $F(x, \xi_2) = 0$

$$\min_{x\in\mathbb{R}^2} \{F(x,\xi_2) \text{ s.t. } x_1+x_2=1,\ x\geq 0\}$$

$$= \min_{x\in\mathbb{R}^2} \{ \underset{\substack{\uparrow \\ \text{constant objective}}}{0} \text{ s.t. } \underset{\substack{\uparrow \\ \text{nonempty feasible set}}}{x_1+x_2=1,\ x\geq 0} \}$$

$$= 0$$

Hence, $WS = \overbrace{\mathbb{P}(\xi=\xi_1)}^{p} \times (-1) + \overbrace{\mathbb{P}(\xi=\xi_2)}^{1-p} \times 0 = -p + 0 = -p$

$$EVPI = RP - WS = (-p) - (-p) = 0$$

Interpretation: For that problem, deciding x before or after observing ξ does not change the objective function on average

. $VSS = EEV - RP$

$$EEV = \mathbb{E}_\xi \left[F(x(\bar{\xi}), \xi) \right]$$

where $x(\bar{\xi}) \in \operatorname{argmin}_{x\in\mathbb{R}^2} \left\{ F(x,\bar{\xi}) \text{ s.t. } x_1+x_2=1,\ x\geq 0 \right\}$

$$\bar{\xi} = \mathbb{E}_\xi[\xi] = \mathbb{P}(\xi=\xi_1)\times\xi_1 + \mathbb{P}(\xi=\xi_2)\times\xi_2$$
$$= p\,\xi_1 + (1-p)\,\xi_2$$

If $p=1$, $\bar{\xi}=\xi_1$, otherwise $p\xi_1+(1-p)\xi_2 \in (\xi_1, \xi_2]$

Case 1 : $p = 1$

Then $\underset{x \in \mathbb{R}^2}{\text{minimize}} \{ F(x, \bar{\xi}) \text{ s.t. } x_1 + x_2 = 1, x \geq 0 \}$

$\Leftrightarrow \underset{x \in \mathbb{R}^2}{\text{minimize}} \{ F(x, \xi_1) \text{ s.t. } x_1 + x_2 = 1, x \geq 0 \}$

$\Leftrightarrow \underset{x \in \mathbb{R}^2}{\text{minimize}} \{ -x_1 \text{ s.t. } x_1 + x_2 = 1, x \geq 0 \}$

Hence $\underset{x \in \mathbb{R}^2}{\text{argmin}} \{ F(x, \bar{\xi}) \text{ s.t. } x_1 + x_2 = 1, x \geq 0 \} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

$$x(\bar{\xi}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} EEV = \mathbb{E}_{\xi} \left[F(x(\bar{\xi}), \xi) \right] &= \mathbb{P}(\xi = \xi_1) \, F(x(\bar{\xi}), \xi_1) + \mathbb{P}(\xi = \xi_2) \, F(x(\bar{\xi}), \xi_2) \\ &= p \times \left(-[x(\bar{\xi})]_1 \right) + (1 - p) \times 0 \\ &= -p = RP \end{aligned}$$

In that case, $VSS = EEV - RP = 0$: No benefit of SP compared to replacing with ξ with $\bar{\xi}$

(Makes sense: $\mathbb{P}(\xi = \xi_1) = 1$!)

Case 2 : $p \in [0, 1)$

In that case, $\bar{\xi} \in (\xi_1, \xi_2]$ and

$$F(x, \bar{\xi}) = 0$$

Hence $\underset{x \in \mathbb{R}^2}{\text{minimize}} \; F(x, \bar{\xi}) \quad \text{s.t.} \quad x_1 + x_2 = 1, \quad x \geq 0$

$$\Leftrightarrow \quad \underset{x \in \mathbb{R}^2}{\text{minimize}} \quad 0 \quad \text{s.t.} \quad x_1 + x_2 = 1, \; x \geq 0$$

$$\min_{x \in \mathbb{R}^2} \{ F(x, \bar{\xi}) \;\; \text{s.t.} \;\; x_1 + x_2 = 1, x \geq 0 \} = 0$$

$$\underset{x \in \mathbb{R}^2}{\text{argmin}} \{ F(x, \bar{\xi}) \;\; \text{s.t.} \;\; x_1 + x_2 = 1, x \geq 0 \} = \underbrace{\left\{ \begin{bmatrix} t \\ 1-t \end{bmatrix} \middle| \, t \in [0,1] \right\}}_{\text{entire feasible set}}$$

→ Picking different values for $x(\bar{\xi})$ leads to different values for VSS / EEV

$$x(\bar{\xi}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow VSS = 0 \quad (\text{back to the case } p = 1)$$

$$x(\bar{\xi}) \neq \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow VSS \neq 0$$

Worst case : $x(\bar{\xi}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$EEV = \mathbb{E}_\xi \left[F(x(\bar{\xi}), \xi) \right] = \mathbb{P}(\xi = \xi_1) \times F(x(\bar{\xi}), \xi_1) + \mathbb{P}(\xi = \xi_2) F(x(\bar{\xi}), \xi_2)$$

$$= p \times \left(- [x(\bar{\xi})]_1 \right) + (1-p) \times 0$$

$$= 0$$

$EEV = 0 \qquad RP = -1 \qquad \Rightarrow \qquad VSS = 1$: loss in replacing ξ by $\bar{\xi}$

NB: with $x(\xi) = \begin{bmatrix} t \\ 1-t \end{bmatrix}$ with $0 \leq t < 1$,

$$EEV = -pt > -p = RP \qquad VSS = (1-t) > 0$$

c) Compare the solution of (P) with that of

$$\underset{x \in \mathbb{R}^2}{\text{minimize}} \ \mathbb{V}_\xi \left[F(x,\xi) \right] \quad \text{s.t.} \quad \begin{array}{l} x_1 + x_2 = 1 \\ x \geq 0 \end{array}$$

where $\mathbb{V}_\xi \left[F(x,\xi) \right]$ denotes the variance

$$\mathbb{V}_\xi \left[F(x,\xi) \right] = \mathbb{E}_\xi \left[F(x,\xi)^2 \right] - \mathbb{E}_\xi \left[F(x,\xi) \right]^2$$

From a), we know that $\mathbb{E}_\xi \left[F(x,\xi) \right] = -px_1$

$$\mathbb{E}_\xi \left[F(x,\xi)^2 \right] = \mathbb{P}(\xi = \xi_1) \times F(x,\xi_1)^2 + \mathbb{P}(\xi = \xi_2) \times F(x,\xi_2)^2$$
$$= p\, x_1^2 + (1-p) \times 0 = p x_1^2$$

Hence $\mathbb{V}_\xi \left[F(x,\xi) \right] = px_1^2 - (-px_1)^2 = p(1-p)x_1^2$

and the problem is

$$\underset{x \in \mathbb{R}^2}{\text{minimize}} \ x_1^2 \quad \text{s.t.} \quad x_1 + x_2 = 1, \ x \geq 0$$

(NB: Quadratic program)

Since $x_1^2 \geq 0 \ \forall x \in \mathbb{R}^2$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is feasible, it follows that

$$\underset{x \in \mathbb{R}^2}{\operatorname{argmin}} \left\{ x_1^2 \text{ s.t. } x_1 + x_2 = 1, x \geq 0 \right\} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$\min_{x \in \mathbb{R}^2} \left\{ x_1^2 \text{ s.t. } x_1 + x_2 = 1, x \geq 0 \right\} = 0$$

$\mathbb{E}_\xi [F(x, \xi)]$	$\mathbb{V}_\xi [F(x, \xi)]$
$x^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, optimal value is $-p$	$x^* = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ optimal value is 0
Risk neutral formulation • Actual value $F(x^*, \xi)$ can be either -1 $(\xi = \xi_1)$ or 0 $(\xi = \xi_2)$ $\Rightarrow$ Variability in the outcome	Risk averse formulation • Regardless of the value of ξ, $F(x^*, \xi) = 0$ $\Rightarrow$ No variability in the outcome • Very conservative solution

1 Risk measures and dispersion

Reminder from lecture 3: Markowitz portfolio

$\xi \in \mathbb{R}^n$ uncertain returns on n assets

$x \in \mathbb{R}^n$ portfolio of investments $\quad (x_i \geq 0 \ \forall i, \ e^T x = 1)$, $e = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Goal: Get the best return of investment possible without knowing the value of ξ

Markowitz problem

$\gamma > 0$

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \ \frac{\gamma}{2} x^T V x - \mu^T x \quad \text{s.t.} \quad e^T x = 1, \quad x \geq 0$$

$$\mu = \mathbb{E}_\xi [\xi] \in \mathbb{R}^n, \quad V = \underbrace{\mathbb{E}_\xi \left[(\xi - \mu)(\xi - \mu)^T \right]}_{\text{With } n = 1, \ V = \mathbb{V}_\xi [\xi]} \in \mathbb{R}^{n \times n}$$

γ expresses a trade-off between the expected return on investment $\mu^T x$ and the variability of the portfolio $\frac{1}{2} x^T V x$

Large γ: More weight on variability, prefer to invest in assets that have small variability (less chances of huge gains, less chances of huge losses)

Small γ: More weight on expected returns, more likely to gain on average but ignore the extreme cases in which you may suffer a huge loss

NB: Since 2008, obligation for portfolio trading companies to assess risks in their investment

$\longrightarrow$ Variability is the most simple metric

$\longrightarrow$ Value-at-Risk (see below) has become more popular

Generalizations:

- For a general objective $F(x, \xi)$ with ξ uncertain, the variability / variance is defined as $\mathbb{V}_\xi [F(x, \xi)]$

$\Longrightarrow$ For linear $F(\cdot, \cdot)$, typically gives rise to a

- convex quadratic program (like Markowitz's portfolio)

$\Rightarrow$ Efficiently solvable up to 1000s of variables in the deterministic setting (Ex: CVX in Julia)

$\Rightarrow$ With a scenario approach, can apply progressive hedging to solve convex quadratic multistage programs

$\rightarrow$ Using $\mathbb{V}_\xi[F(x,\xi)]$ as the objective instead of $\mathbb{E}_\xi[F(x,\xi)]$ is one example of a dispersion risk measure.

Other examples

Quadratic objective
- $\gamma \mathbb{V}_\xi[F(x,\xi)] + \mathbb{E}_\xi[F(x,\xi)]$

not quadratic but minimizing $\sqrt{\mathbb{V}_\xi}$ $\Leftrightarrow$ minimizing $\mathbb{V}_\xi$
- $\sqrt{\mathbb{V}_\xi[F(x,\xi)]}$: standard deviation

- $\gamma \sqrt{\mathbb{V}_\xi[F(x,\xi)]} + \mathbb{E}_\xi[F(x,\xi)]$

$\rightarrow$ Nonlinear objective, cannot be solved in general by quadratic programming solvers (but can still be convex $\Rightarrow$ CVX might help again)

Dispersion measures

(+) Risk-averse formulations (vs $\mathbb{E}_\xi[F(x,\xi)]$: risk neutral)

Take into account the fact that the actual value $F(x,\xi)$ might vary a lot from $\mathbb{E}_\xi[F(x,\xi)]$

(−) Often leads to conservative solutions because of low-probability extreme values

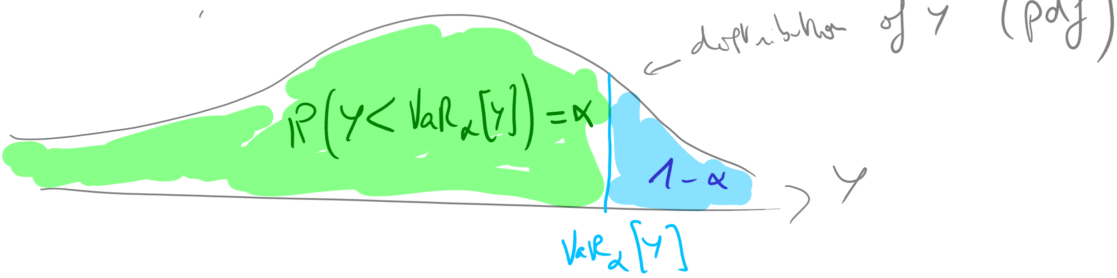
→ Risk measures aim at producing risk-averse formulations that are not too conservative

(2) Value-at-Risk (VaR)

↳ Proposed by J. P. Morgan, still used in their trading models to assess risk

Def: Let Y be a random variable on $\mathbb{R}$ and $\alpha \in (0,1)$. The Value-at-Risk (of level α) $VaR_\alpha[Y]$ is defined as $\delta \in \mathbb{R}$ such that $\mathbb{P}(Y \geq \delta) = 1-\alpha$.

(Equivalently, $\mathbb{P}(Y \geq VaR_\alpha[Y]) = 1-\alpha$)



→ Guarantee that $Y < VaR_\alpha[Y]$ with probability α

(Typical values: $\alpha = 0.95$, $\alpha = 0.99$)

Special case: $Y \in \{y_1, \ldots, y_K\}$, $y_1 \leq \ldots \leq y_K$, $\mathbb{P}(Y = y_k) = p_k$ $\forall k = 1..K$

$VaR_\alpha[Y] = y_{k_\alpha}$, where k_α is the smallest integer such that $\sum_{k=k_\alpha}^{K} p_k \leq 1-\alpha$

General formula

$$\mathrm{VaR}_\alpha[Y] = \underset{\gamma \in \mathbb{R}}{\mathrm{argmin}} \left\{ \gamma + \frac{1}{1-\alpha} \mathbb{E}_Y\left[\max(Y-\gamma, 0)\right] \right\}$$

- $\mathrm{VaR}_\alpha[Y]$ is the solution of a two-stage stochastic program, that can be written as a linear two-stage stochastic program

$$\mathrm{VaR}_\alpha[Y] = \underset{\gamma \in \mathbb{R}}{\mathrm{argmin}} \left\{ \gamma + \mathbb{E}_Y\left[Q(\beta, Y)\right] \right\}$$

where $Q(\beta, Y) = \min_{\beta \in \mathbb{R}} \frac{\beta}{1-\alpha}$ s.t. $\beta \geq Y - \gamma$, $\beta \geq 0$

- Can be approximated using scenarios.

↳ One can then consider problems of the form

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad \mathrm{VaR}_\alpha\left[F(x, \xi)\right] \quad \text{s.t.} \quad x \in X$$

⇒ Risk averse formulation, tends to be less conservative than dispersion measures

⇒ (Informal) drawback of VaR: Does not characterize how large Y can be beyond the value $\mathrm{VaR}_\alpha[Y]$

⇒ Mathematically, this is related to VaR not being a coherent risk measure

Coherent risk measure :

ρ is coherent if

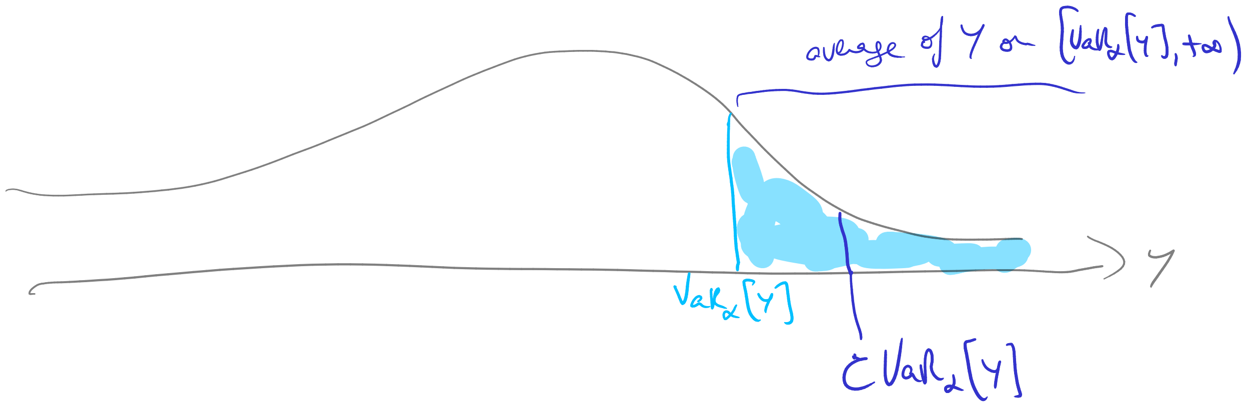
- $Y \geq 0 \implies \rho(Y) \geq 0$ (True for VaR)
- $\rho(cY) = c\,\rho(Y) \quad \forall c > 0$ (True for VaR)
- $\rho(Y + c) = \rho(Y) + c \quad \forall c \in \mathbb{R}$ (True for VaR)
- $\rho(Y + Z) \leq \rho(Y) + \rho(Z)$ NOT true for VaR

$\longrightarrow$ Several coherent risk measures based on VaR have been proposed.

$\Rightarrow$ Most classical example : CVaR Conditional Value-at-Risk (aka Average Value-at-Risk)

Def : Let Y be a random variable in $\mathbb{R}$ and $\alpha \in (0,1)$

$$\mathrm{CVaR}_\alpha[Y] := \mathbb{E}_Y\left[\, Y \mid Y \geq \mathrm{VaR}_\alpha[Y] \,\right]$$



$\rightarrow$ By definition, $\mathrm{CVaR}_\alpha[Y] \geq \mathrm{VaR}_\alpha[Y]$.

$\rightarrow$ CVaR_α is a coherent risk measure

Special case $\quad Y \in \{y_1, \dots, y_K\} \quad P(Y=y_k) = p_k$

$y_1 \leq \dots \leq y_K$

$VaR_\alpha[Y] = y_{k_\alpha}$

$$CVaR_\alpha(Y) = \frac{1}{1-\alpha} \sum_{k=k_\alpha}^{K} p_k y_k \qquad \left(\sum_{k=k_\alpha}^{K} p_k \neq \frac{1}{1-\alpha} \right)$$

Generic formula

$$CVaR_\alpha[Y] = \min_{\gamma \in \mathbb{R}} \left\{ \gamma + \frac{1}{1-\alpha} \mathbb{E}_Y \left[\max(Y-\gamma, 0) \right] \right\}$$

$\rightarrow$ Optimal value of $\underset{\gamma}{\text{minimize}} \left\{ \gamma + \frac{1}{1-\alpha} \mathbb{E}_Y \left[\max(Y-\gamma, 0) \right] \right\}$

(linear) two-stage stochastic program

$\rightarrow$ $VaR_\alpha[Y]$ is the solution to that problem!

③ Risk measures in stochastic programming

1st approach (similar to Markowitz's portfolio)

Optimize some risk measure of the objective instead of the expected value

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad \mathbb{E}_\xi \left[F(x, \xi) \right] \quad \text{s.t. } x \in X$$

$$\Downarrow$$

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad \rho(F(x, \xi)) \quad \text{s.t. } x \in X$$

ρ risk measure

→ Amenable to scenario decomposition, multi-stage extensions

→ Key: Computing ρ (at least for discrete variables)

→ When ρ is a coherent risk measure, it can typically be expressed as the optimal value of another optimization. In general, we have an expression of the form

$$\rho(F(x,\xi)) = \max_{D \in \mathcal{D}} \mathbb{E}_{\xi \sim D}[F(x,\xi)]$$

↳ $\mathbb{E}$ for ξ following distribution D

Set of probability distributions

$\rho(F(x,\xi))$ is the optimal value of a distributionally robust optimization problem

↳ In between $\mathbb{E}_\xi[F(x,\xi)]$ (1 distribution)

and $\max_{\xi \in \boxed{4}} F(x,\xi)$ $\left(\mathcal{D} = \{\text{Dirac distributions on } \boxed{4}\}\right)$

worst-case perspective (robust optimization)

Set of all possible values for ξ

Dirac distribution

$\mathbb{P}(\xi = \xi_1) = 1$

$\mathbb{P}(\xi \neq \xi_1) = 0$

2nd approach (actually the one used by JP Morgan and others)

Use risk measures in constraints

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \ \mathbb{E}_\xi[F(x,\xi)] \quad \text{s.t.} \ x \in X$$

$\Downarrow$

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad \mathbb{E}_\xi \left[F(x, \xi) \right] \quad \text{s.t.} \quad x \in X$$

$$\rho(x, \xi) \leq \sigma \qquad \sigma \in \mathbb{R}$$

→ Formulation guarantees a maximum level of risk (where the risk is represented by the risk measure ρ)

→ Can lead to more challenging optimization problems

Example: Portfolio allocation

$\xi \in \mathbb{R}^n$ random vector of returns

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad -\mathbb{E}_\xi \left[\xi^T x \right] \quad \text{s.t.} \quad e^T x = 1$$

$$x \geq 0$$

$$x^T V x \leq \sigma^2 \qquad \sigma^2 > 0$$

$$V = \mathbb{E}_\xi \left[(\xi - \mathbb{E}[\xi])(\xi - \mathbb{E}[\xi])^T \right]$$

→ Mathematically equivalent to Markowitz's portfolio allocation with γ chosen as a function of σ^2 (and the distribution of ξ)

⇒ Hard to do the reverse equivalence (for a given γ, what is the value of σ^2 such that the problems are equivalent)

⇒ Value in trying to solve the constrained problem as a standalone problem

→ The problem is a conic program (because of the quadratic constraint). Can be solved with standard solvers but up to 100s of variables rather than 1000s

Example: VaR as a constraint

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad \mathbb{E}_\xi\left[F(x,\xi)\right] \quad \text{s.t.} \quad x \in X$$
$$\mathrm{VaR}_\alpha\left[F(x,\xi)\right] \leq \gamma$$

$$\mathrm{VaR}_\alpha\left[F(x,\xi)\right] \leq \gamma \iff \mathbb{P}\left(F(x,\xi) \geq \gamma\right) \leq 1-\alpha$$

Probabilistic / Chance constraint

(Definition of VaR: $\mathbb{P}\left(F(x,\xi) \geq \mathrm{VaR}_\alpha(F(x,\xi))\right) = 1-\alpha$)

References:

Applied viewpoint
- Cornuéjols, Peña, Tütüncü, Optimization methods in finance

Theoretical
- Shapiro, Ruczynski, Dentcheva, Lectures on stochastic programming

Exercise:

Consider $f(x,\xi) = \mathbb{E}_\xi\left[\left|(\xi - \mathbb{E}[\xi])^T x\right|\right]$

where $\xi \in \mathbb{R}^n$ (as in portfolio allocation)

- Show that $\underset{x \in \mathbb{R}^n}{\text{minimize}} \; f(x,\xi)$ s.t. $e^T x = 1$, $x \geq 0$

  can be reformulated as a linear two-stage stochastic program

- Write it as a linear program when $\xi \in \{\xi_1, \dots, \xi_K\}$ with $\mathbb{P}(\xi = \xi_k) = p_k$.