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On the Choquet multiple criteria preference aggregation model: Theoretical and practical insights from a real-world application

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Abstract

We consider the use of the Choquet integral for evaluating projects or actions in a real-world application dealing with the re-qualification of an abandoned quarry. Despite the Choquet integral being a very well-known preference model for which there is a rich and well developed theory, its application in a multiple criteria decision aiding perspective requires some specific methodological developments. This led us to work out and implement, in practice, two new procedures: One to build interval scales with the objective of assigning utility values on a common scale to the criteria performances. And a second to construct a ratio scale for assigning numerical values to the capacities of the Choquet integral. This article discusses the strengths and weaknesses of the Choquet integral as appearing in the case study, and proposing as well insights related to the interaction of the experts within a focus group.

Keywords: Multiple criteria analysis, Decision support systems, Choquet integral, Focus group.

1. Introduction

The starting point for this research is the work developed in a previous study (see Bottero et al., 2015) in which we handled the interaction between some pairs of criteria within the context of an outranking method, namely Electre III. Interaction among criteria is a crucial aspect of decision aiding that is attracting more and more attention. That is why we took advantage of the opportunity we had (it should be underlined that this was a very exceptional situation) to work with the same experts as in the previous study, in order to evaluate the same five alternative re-qualification projects or actions for an abandoned quarry, by making use now of the multiple criteria preference aggregation Choquet integral (Choquet, 1953) model. It should be noticed that the purpose of this article is not to make a comparison of the results previously obtained in Bottero et al. (2015) with those we obtained with the current methodology. Our purpose is rather to test the potential of the Choquet integral in a real-world decision problem that led us to propose two new

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procedures, one which allows to assign utility values on a common scale to the the corresponding criteria performances, and the other which allows to assign numerical values to the capacities of the Choquet integral. From this perspective, we highlight the strengths and weaknesses of a decision aiding approach, which is based on the Choquet integral, where we first asked the experts to work separately, and then put them to work together within a focus group context.

The paper is organized as follows. Section 2 presents a short description of the case study in Bottero et al. (2015). Section 3 is devoted to the new procedures for building an interval scale with the objective of placing the performances of the criteria on a common utility scale, and to construct a ratio scale for assigning numerical values to the capacities of the Choquet integral. Section 4 is devoted to the way the two procedures were implemented in practice, *i.e.*, the interaction with the experts and the focus group to assign numerical values to the required data for the application of the Choquet integral. Section 5 presents the design of the experiments, the construction of two specific sets of threshold functions, and the results obtained. Finally, Section 6 presents some conclusions according to the results obtained in the previous section and provides some directions for future research.

2. The case study

This section proposes a brief description of the context of the application. It then introduces the stakeholders and their representatives who participated in the process, illustrates the actions and criteria under analysis, and finally, discusses some aspects referring to the interaction between some pairs of criteria.

2.1. The context

The decision aiding problem under analysis concerns the evaluation of alternative projects for the requalification of an abandoned quarry located in Northern Italy. The quarry has been abandoned since 1975 and covers a total surface of $65\ 000m^2$, with a depth of approximately $25m$ from the ground level. From the environmental point of view, the area under analysis represents both a territorial weakness, due to its abandoned state which has caused uncontrolled vegetation growth and water-filled pits, and an opportunity, due to it being part of the Provincial ecological system of environmentally valuable sites. Five alternative projects (hereafter called *actions*) for the re-qualification of the area have been identified and discussed by the Municipal Authority (for more details see Bottero et al., 2014).

In particular, the five re-qualification actions have been analyzed and compared in order to rank them from the best to the worst. The data concerning the actions as well as the performance criteria and their interaction effects come from two previous investigations carried out by the authors involved in the present research. The first study by Bottero et al. (2014) proposed the use of the Choquet integral to compare the alternative re-use actions for the abandoned quarry taking into account synergies (mutual-strengthening) and redundancies (mutual-weakening) among criteria. The second study, recently carried out by Bottero et al. (2015), developed an extension of the ELECTRE III method to model multiple interaction effects between criteria and tested it using the abandoned quarry re-qualification problem.

2.2. The stakeholders and their representatives

In public policy making the actors and their behaviors represent the core of any possible theoretical model (Boerboom and Ferretti, 2014; Dente, 2014). The *actors* are those individuals or

organizations that make the actions able to influence the decisional outcomes and do so because they pursue goals regarding the problem and its possible solution, or regarding their relations with other actors (Dente, 2014). In particular, any actor having a vested interest in the decision process, either directly affecting or being affected by its resolution, including experts and the public, is named a stakeholder. The first, essential, step of a decision aiding process to support public policy formulation thus consists of identifying the stakeholders and their objectives (*e.g.*, Ferretti, 2016). In the present study, the decision to re-use the abandoned quarry is characterized by the presence of multiple stakeholders with different and frequently conflicting objectives. In what follows, the relevant stakeholders who can have a role in the process under investigation are presented for different administrative levels (for more details see Bottero et al., 2015).

1. *National level*: The Forestry Corps, *i.e.*, the National Police Force in charge of the protection of natural heritage and landscape.
2. *Regional level*: (*i*) the Regional authority, *i.e.*, the organisation responsible for territorial planning and management across the whole Piedmont Region, and (*ii*) the Regional Environmental Authority, *i.e.*, the authority responsible for environmental protection in the area.
3. *Provincial level*: The Provincial authority, *i.e.*, the authority responsible for territorial planning and management in the Province of Novara.
4. *Local level*: (*i*) the municipal technical office, *i.e.*, the authority responsible for the monitoring all construction activities in the municipality, (*ii*) the mayor, *i.e.*, the actor responsible for approving or rejecting any transformation project in the municipality, (*iii*) local practitioners, *i.e.*, professionals such as architects and urban planners working in the area under analysis, (*iv*) inhabitants, *i.e.*, the local population affected by the transformation, and finally, (*v*) private entrepreneurs, *i.e.*, private actors who could invest in the transformation projects.

The decision aiding process in this study was based on a participative approach that has been developed through the focus group technique (*e.g.*, Morgan, 1988; Stewart and Shamdasani, 1990). In order to include the concerns and points of view of the relevant stakeholders, three experts were involved in our *focus group*: An expert in the field of economic evaluation (expert `e1`); an expert in the field of environmental engineering (expert `e2`); an expert in the field of landscape ecology (expert `e3`). The use of a panel of experts expands the knowledge basis and may serve to avoid the possible biases which characterize the situation with a single expert. In our case, particular attention was thus dedicated to the panel composition in order to have it balanced and representative of the key stakeholders involved in the planning and evaluation process. It is worth highlighting that the experts with whom the authors of the paper worked in the present application are the same as in the study by Bottero et al. (2015).

2.3. Actions, criteria, and the performance table

Five alternative actions were considered by the Municipal Administration for the re-qualification of the abandoned quarry. All five actions consider the arrangement of security measures for the banks of the quarry but they differ regarding the specific recovery hypothesis.

In particular, the first action consists of basic reclamation of the quarry. In this project the quarry would be completely filled in with materials coming from building demolition and the topsoil would be left to the natural evolution of the vegetation.

The second action concerns the establishment of a new forest. More specifically, the project considers the complete filling in of the quarry with material coming from building demolition, the laying of topsoil and the establishment of a new oak-hornbeam wood.

The third action refers to the creation of a wetland in the area. Under this hypothesis the quarry would be partially filled in with material from building demolition work and the formation of a lake would be allowed, including the planting of wetland vegetation in the surrounding area.

The fourth action considers converting the quarry into a node of the provincial ecological network. This project consists of partially filling in the quarry with material from building demolition work, the formation of a lake and setting out pathways and recreational areas for visitors to the site.

The fifth action consists of developing a multi-functional structure. In this case, the quarry would be filled in to allow the construction of a recreational structure well integrated with the surrounding landscape and with high energy and environmental performances.

To summarise, the five actions under consideration are:

1. *Basic reclamation* (a_1).
2. *Valuable forest* (a_2).
3. *Wetland* (a_3).
4. *Ecological network* (a_4).
5. *Multi-functional area* (a_5).

To support comparison of the five actions for re-qualification of the quarry, a family of six criteria has been built. The criteria represent all the concerns pertaining to the sustainability of the decision aiding problem and they can be summarized as follows:

1. *Investment costs* (g_1): This criterion considers the building costs for carrying out the re-qualification project; the scale unit is Euros and this criterion is to be minimized.
2. *Profitability* (g_2): This criterion evaluates the income that the investment is likely to generating on the local economic system; the criterion scale is a seven-level qualitative scale where level 1 means "very bad", 2 "bad", 3 "rather bad", 4 "average, 5 "rather good", 6 "good", and 7 "very good"; this criterion is to be maximized.
3. *New services for the population* (g_3): This criterion concerns the effects of the re-qualification projects in terms of generating new services for the local population, including sport facilities, recreational areas, green parks, *etc*; its scale is the aforementioned seven-level qualitative scale and it is to be maximized.
4. *Naturalized surface* (g_4): This criterion considers the impacts of the projects on the landscape quality and the conservation of bio-diversity; its scale is in hectares of naturalized surface and it is to be maximized;
5. *Environmental effects* (g_5): This criterion considers the consequences that the projects have on the environmental system, including minimization of geo-technical and hydro-geological; its scale is the aforementioned seven-level qualitative scale and it has to be maximized.
6. *Consistency with local planning requirements* (g_6): This criterion considers the existence of urban planning constraints that could affect the administrative feasibility of the project; the criterion scale distinguishes between two situations, *i.e.*, feasible or "yes" (corresponding to level 1) and not feasible or "no" (corresponding to level 0); this criterion is to be maximized.

The performances of actions under analysis according to the criteria considered are presented in Table 1.

Table 1: Performance table

	Costs (g₁)	Profitability (g₂)	Services (g₃)	Surface (g₄)	Environment (g₅)	Consistency (g₆)
a_1	30 000	rather bad (3)	very bad (1)	2	average (4)	yes (1)
a_2	45 000	rather bad (3)	rather good (5)	5	rather good (5)	yes (1)
a_3	90 000	very bad (1)	good (6)	3.2	very good (7)	yes (1)
a_4	120 000	very bad (1)	very good (7)	3.5	good (6)	yes (1)
a_5	900 000	very good (7)	very good (7)	1	rather bad (3)	no (0)

2.4. Why do some criteria interact?

Two different forms of interaction between criteria have been considered in the present application: The mutual-strengthening effect and the mutual-weakening effect (Figueira et al., 2009). A mutual-strengthening effect between two criteria is present when the overall weight of these two criteria is greater than the sum of the weight of the two criteria considered separately, while a mutual-weakening effect between two criteria is present when their overall weight is less than the sum of the weight of the two criteria considered separately.

In the context of the present application, starting from the interaction between criteria that were identified in the previous study (Bottero et al., 2015), the research considers the following interactions:

- Mutual-strengthening between criteria g_1 (costs) and g_5 (environment).

- Mutual-weakening between criteria g_4 (surface) and g_5 (environment).

In particular, according to the experts involved in the focus group to discuss the interaction between the evaluation criteria, there is a mutual-strengthening effect between criteria g_1 (costs) and g_5 (environment): Indeed , a project with negative environmental effects very often also has low costs (due for example to the use of cheap technology with pollutant emissions). Therefore, a project with positive environmental effects and low costs is very well appreciated. Consequently, the overall weight of this pair of criteria is greater than the sum of their individual weights.

Moreover, during the focus group the experts identified a mutual-weakening effect between the criteria g_4 (surface) and g_5 (environment): Indeed, a project of high ecological quality is often also of high environmental quality. Consequently, the overall weight of this pair of criteria is less than the sum of their individual weights.

Mention should be made of the fact that the antagonistic effect identified in the focus group for the ELECTRE III application has not been considered in the present research as this effect cannot be modeled using the basic Choquet integral approach. It is, however, possible to model such an effect by applying the bipolar Choquet integral (Grabisch and Labreuche, 2005; Greco and Rindone, 2014). To simplify the interaction protocol with the experts, we used the basic Choquet integral.

3. Choquet integral and procedures for determining capacities and common scales

This section presents fundamental concepts about the Choquet integral. This aggregation function requires the assignment of a weight to each subset of criteria by means of a function called capacity. Moreover, the Choquet integral also requires that the evaluations or utilities of each action on the considered criteria are expressed on the same scale. New procedures for computing capacities and utilities on a common scale are also introduced in this section.

3.1. The Choquet integral

Let A denote a set containing m actions, $a_1, \ldots, a_j, \ldots, a_m$, and G denote a set with n criteria, $g_1, \ldots, g_i, \ldots, g_n$ (for the sake of simplicity we can also identify the criteria by their indices). For an action a and a criterion g_i, $g_i(a)$ is the performance of action a on criterion g_i, and $u_i(g_i(a))$ is the utility of the performance $g_i(a)$; again for the sake of simplicity we will henceforth use $u_i(a)$ instead of $u_i(g_i(a))$.

The Choquet integral (Choquet, 1953) is an aggregation function that permits the aggregation of utilities on the considered criteria taking into account interactions among criteria. It is based on the concept of capacity or fuzzy measure (see Grabisch, 1996). A *capacity* is a set function, $\mu : 2^G \to [0,1]$, on the power set, 2^G (*i.e.*, the set of all subsets of G) satisfying the following properties:

i) $\mu(\emptyset) = 0$ and $\mu(G) = 1$ (boundary conditions).

ii) $\forall\, S \subseteq T \subseteq G\ \mu(S) \leqslant \mu(T)$ (monotonicity condition).

For any $T \subseteq G$, $\mu(T)$ represents the total capacity (weight) of criteria from T. The idea behind the capacity μ is that, for $R, S \subseteq G$, with $R \cap S = \emptyset$,

- if $\mu(R \cup S) > \mu(R) + \mu(S)$, then there is a mutual-strengthening (synergy) between sets R and S.

- if $\mu(R \cup S) < \mu(R) + \mu(S)$, then there is a mutual-weakening (redundancy) between sets R and S.

Since in any case $\mu(\emptyset) = 0$ and $\mu(G) = 1$, the values $\mu(S)$ assigned by the capacity μ to all other $2^{|G|} - 2$ subsets S of G have to be defined. For the sake of simplicity, we call the values $\mu(S)$ as the capacities of set S. Given an action $a \in A$ and a capacity μ on 2^G, the Choquet integral can be defined as follows:

$$C_\mu(a) = \sum_{i=1}^{n} \left(u_{(i)}(a) - u_{(i-1)}(a) \right) \mu\left(G_i\right), \tag{1}$$

where $u_{(1)}, \ldots, u_{(n)}$ are the utilities of criteria from G, reordered in such a way that $u_{(1)}(a) \leqslant \cdots \leqslant u_{(i)}(a) \leqslant \cdots \leqslant u_{(n)}(a)$, and $G_i = \{(i), \ldots, (n)\}$, for $i = 1, \ldots, n$, with $u_{(0)}(a) = 0$.

Given a capacity μ on 2^G, its Möbius representation is a function $m : 2^G \to \mathbb{R}$ such that, for all $S \subseteq G$,

$$\mu(S) = \sum_{T \subseteq S} m(T), \tag{2}$$

we have that,

$$m(S) = \sum_{T \subseteq S} (-1)^{|S-T|} \mu(T), \tag{3}$$

and the above properties $i)$ and $ii)$ can be reformulated as follows,

$i')$ $m(\emptyset) = 0, \ \sum_{T \subseteq G} m(T) = 1.$

$ii')$ $\forall \, i \in G$ and $\forall R \subseteq G \setminus \{j\}, \ m(\{i\}) + \sum_{T \subseteq R} m(T \cup \{i\}) \geqslant 0.$

The Choquet integral can now be expressed in terms of the Möbius representation m of the capacity μ as follows,

$$C_\mu(a) = \sum_{T \subseteq G} m(T) \min_{i \in T} \{u_i(a)\}. \tag{4}$$

In real-world decision problems, it seems reasonable to focus on the interactions of a small set of pairs of criteria. Let O denote the set of interacting pairs of criteria, $\{i, j\}$; thus, for all $S \subseteq G$ we have

$$\mu(S) = \sum_{i \in S} m(\{i\}) + \sum_{\{i,j\} \subseteq S, \{i,j\} \in O} m(\{i,j\}), \tag{5}$$

and,

$$\mu(G) = \sum_{i \in G} m(\{i\}) + \sum_{\{i,j\} \in O} m(\{i,j\}) = 1, \tag{6}$$

so that the Choquet integral can be reformulated as

$$C_\mu(a) = \sum_{i \in G} m(\{i\}) u_i(a) + \sum_{\{i,j\} \in O} m(\{i,j\}) \min\{u_i(a), u_j(a)\}. \tag{7}$$

It should be noted that for $\{i, j\} \in O$, $m(\{i, j\}) > 0$ if there is mutual-strengthening between i and j and $m(\{i, j\}) < 0$ if there is mutual-weakening between these two criteria.

Henceforth, for the sake of simplicity, we shall use m_i instead of $m(\{i\})$ for all $i \in G$ and m_{ij} instead of $m(\{i, j\})$ for all $\{i, j\} \in O$. The same applies to μ_i and μ_{ij}.

Example 1. *Consider only two interacting pairs of criteria, $\{i, j\}$ (characterized by a mutual-strengthening effect) and $\{k, j\}$ (characterized by a mutual-weakening effect). From the sign of these interaction effects and property $ii')$, we have*

$a)$ $m_{ij} \geqslant 0$ $\qquad\qquad$ $b)$ $m_{kj} \leqslant 0$
$c)$ $m_i + m_{ij} \geqslant 0$ $\qquad\quad$ $d)$ $m_j + m_{ij} \geqslant 0$
$e)$ $m_k + m_{kj} \geqslant 0$ $\qquad\quad$ $f)$ $m_j + m_{kj} \geqslant 0$
$g)$ $m_j + m_{ij} + m_{kj} \geqslant 0$

From the Möbius representation (3) we have $m_i = \mu_i$, $m_j = \mu_j$, $m_k = \mu_k$, $m_{ij} = \mu_{ij} - \mu_i - \mu_j$, and $m_{kj} = \mu_{kj} - \mu_k - \mu_j$. By replacing them in the previous system we obtain the following two consistency conditions,

$$\mu_{ij} \geqslant \mu_i + \mu_j, \tag{8}$$

and

$$\max\{\mu_k, \mu_j\} \leqslant \mu_{kj} \leqslant \mu_k + \mu_j. \tag{9}$$

The first condition comes from a) while the second comes from b), e), f), and g) (note that from g), $\mu_{kj} \geqslant \mu_i + \mu_k + \mu_j - \mu_{ij}$, but since from a), $\mu_i + \mu_j - \mu_{ij} \leqslant 0$, a non-negative amount is removed from μ_k; we can thus discard the expression obtained from g)).

A fundamental requirement when applying the Choquet integral is that the criteria utilities must be on the same scale. Indeed, in formulations (1), (4), and (7), the computation of the Choquet integral requires a comparison of the utilities on all the criteria. More precisely, when applying formulation (1) it is necessary to rank order the utilities of criteria in G, from the smallest to the largest, and to compute the differences of utilities on the different criteria, while when applying formulations (4) and (7) it is necessary to o compute the minimum of the utilities for all pairs of interacting criteria. Therefore, a fundamental condition to the application of the Choquet integral is that the utilities on the considered criteria have to be expressed on a common scale.

3.2. Determining the capacities

Figueira and Roy (2002) proposed a modified version of Simos' deck of cards method (see Maystre et al., 1994) for determining the weights of criteria within the context of outranking methods (for a list of applications see Siskos and Tsotsolas, 2015). In the conclusion of their work Figueira and Roy (2002) stated that the method could be adapted to build other ratio scales as well as to build interval scales. The purpose of the current and the next subsection is to present two extensions of the deck of cards method for building ratio and interval scales, respectively. With such a purpose in mind, instead of considering criteria, we can take into account more general objects (for instance, actions, projects, scale levels). In any case, the dialog between the analyst(s) and the decision-maker(s) or expert(s) should take into account the following aspects:

1. The analyst should provide the decision-maker with a first *set of cards*, each one containing the name of each object and some additional information (if necessary).
2. The analyst should also provide the decision-maker with a large enough *set of blank cards*.
3. The analyst must then ask the decision-maker for a *ranking* of the cards in the first set - the decision-maker should rank the objects from the least to the most important (if some are tied, they should occupy the same position in the ranking).
4. The analyst should call the attention of the decision-maker to the fact that two consecutive positions in the ranking may be more or less close. This greater or smaller closeness can be modeled through the *insertion of blank cards* in the intervals of the consecutive positions in the ranking.
5. Finally, the analyst should obtain from the decision-maker information allowing to fix the *ratio z* between the value of the most appreciated and the value of the least appreciated object (*i.e.*, how many times the most appreciated object is more important than the least appreciated one). Two other reference objects can be considered for the definition of this ratio.

It should be remarked that the information obtained in the last point is important in order to build ratio scales. When building interval scales, it is replaced by the definition of at least two reference values, as can be seen in subsection 3.3.

The construction of a ratio scale for capacities requires, in general, the consideration of very specific objects, which we will call them *projects*. Marichal and Roubens (2000) proposed a method for determining the capacities of the Choquet integral from a reference set (of projects). Based on their idea, and assuming that the minimal and the maximal utilities on each criterion are 0 and 1, respectively, we propose to build the reference set of projects as follows:

- n projects (as many as the number of criteria), denoted by p_j, for all $j \in G$, which can be characterized by a vector of the form $(0, \ldots, 0, u_j(p_j) = 1, 0, \ldots, 0)$, where p_j has the highest evaluation on criterion j and the lowest elsewhere.

- $|O|$ projects (as many as the number of interacting criteria pairs) denoted by $p_k = p_{ij}$, $k = n + 1, \ldots, |O|$, which can also be characterized by a vector of the form $(0, \ldots, 0, u_i(p_i) = 1, 0, \ldots, 0, u_j(p_j) = 1, 0, \ldots, 0)$, for all $\{i, j\} \in O$, where p_k has the highest utilities on criteria i and j, and the lowest elsewhere.

The method we propose for assessing the capacity, μ, necessary to compute the Choquet integral can be presented, as a step-by-step procedure, as follows (it also allows to compute the Möbius representation m of the capacity μ to be computed):

1. Consider the following finite set of reference projects: $P = \{p_1, p_2, \ldots, p_k, \ldots, p_t\}$.
2. Consider the ranking of the projects provided by the decision-maker and denoted by $R_1, \ldots,$ $R_h, \ldots, R_v$, the equivalence classes in the ranking (R_1 containing the least preferred projects, R_2 containing the second least preferred projects, and so on, until R_v, containing the most preferred projects). Let us denote by r_h a project representative of projects in the equivalence class R_h, $h = 1, \ldots, v$. Of course, all the projects in class R_h will have the same value of r_h. Let e_h denote the number of blank cards between the equivalence classes R_h and R_{h+1}, $h = 1, \ldots, v - 1$. Note that in the ranking there are as many units between the first and the last position as the total number of blank cards plus the number of intervals in the ranking.
3. Assign a value to project r_1 (and consequently to all the projects in R_1), say $w(r_1) = \ell$ (it is frequent to consider $w(r_1) = 1$).
4. Compute the value of each unit as follows:
$$\alpha = \frac{\ell(z-1)}{s}$$
where
$$s = \sum_{h=1}^{v-1}(e_h + 1),$$
that is, α is obtained by dividing the difference between the values of the most preferred objects ($w(r_v) = \ell z$) and the least preferred objects ($w(r_1) = \ell$), s being the number of units between R_1 and R_v (if two other reference projects are taken into account these formulas must be modified accordingly).
5. Compute the values $w(r_h)$, $h = 2, \ldots, v$, as follows:
$$w(r_h) = \ell + \alpha \left(\sum_{j=1}^{h-1}(e_h + 1) \right).$$

6. Compute the value of each project, $w(p_k) = w(r_h)$ for all $p_k \in R_h$, $h = 1, \ldots, v$.
7. Compute the modified values $\overline{w}(p_k)$ as follows: $\overline{w}(p_k) = w(p_k)$ if $k = i \in G$; $\overline{w}(p_k) = w(p_k) - w(p_i) - w(p_j)$ if $k = ij, \{i, j\} \in O$.
8. Compute the Möbius coefficients, m_k, and the capacities, μ_k, for $k = 1, \ldots, t$:

$$m_k = \frac{\overline{w}(p_k)}{\displaystyle\sum_{j=1}^{t} \overline{w}(p_j)},$$

and

$$\mu_k = \frac{w(p_k)}{\displaystyle\sum_{j=1}^{t} \overline{w}(p_j)},$$

where the coefficients m_k must fulfill conditions i') and ii') and must be consistent with the sign of the interactions (mutual-strengthening and mutual-weakening) provided by the decision-maker. Otherwise, a *non-conformity case* occurs.

In Section 4, we shall show how the method proposed in this section can be applied to elicit the interaction aspects with the experts in order to define the values for the capacities of the Choquet integral.

3.3. Building interval scales

The utility values of the Choquet integral are the levels of a common interval scale, in general, within the range $[0, 1]$. The translation from the original scales of the criteria to a single common interval scale requires the use of a procedure that should account for the intensity of preferences between consecutive intervals of the scale. In this section we present a new procedure for defining an interval scale based on the concepts of the deck of cards method (Figueira and Roy, 2002). The procedure presented here allows scales not necessarily within the range $[0, 1]$ to be constructed.

In order to build an interval scale we need to define at least two reference levels (instead of the definition of z, as in the case of ratio scales), to anchor the computations. If more than two reference levels are defined, we can replicate the procedure for every two consecutive reference levels.

1. Consider the following discrete scale of criterion g: $E_g = \{l_1, l_2, \ldots, l_k, \ldots, l_t\}$, where $l_1 \prec l_2 \prec \cdots \prec l_k \prec \cdots \prec l_{t-1} \prec l_t$ ($\prec$ means "strictly less preferred than").
2. Define two reference levels, say l_p and l_q and assign two utility values to these reference levels. We frequently use

   $u(l_p) = 0$.
   $u(l_q) = 1$.

   Other values can be assigned to l_p and l_q. Observe that very often levels l_p and l_q coincide with l_1 and l_t, respectively.
3. Consider the ranking of the levels with a certain number of blank cards, e_k, in the intervals between every two consecutive levels, l_k and l_{k+1}, $k = 1, \ldots, t - 1$:

$$l_1 \; e_1 \; l_2 \; e_2 \; \cdots \; l_p \; e_p \; l_{p+1} \; e_{p+1} \; \cdots \; l_k \; e_k \; l_{k+1} \; \cdots \; l_{q-1} \; e_{q-1} \; l_q \; \cdots \; l_{t-1} \; e_{t-1} \; l_t.$$

4. Now, consider only the the levels in between l_p and l_q (in between levels l_k and l_{k+1} there are $(e_k + 1)$ units) and compute the unit valuation:

$$\alpha = \frac{u(l_p) - u(l_q)}{h},$$

where

$$h = \sum_{k=p}^{q-1} (e_k + 1),$$

which represents the number of units between levels l_p and l_q.

5. Compute the utility value, $u(l_k)$, for each level, $k = 1, \ldots, t$, as follows:

$$u(l_k) = \begin{cases} u(l_p) - \alpha \left(\sum_{j=k}^{p-1} (e_j + 1) \right), & \text{for} \quad k = 1, \ldots, p - 1, \\ \\ u(l_p) + \alpha \left(\sum_{j=p}^{k-1} (e_j + 1) \right), & \text{for} \quad k = p + 1, \ldots, q - 1, q + 1, \ldots, t. \end{cases}$$

In Section 4 we shall present the details about the interaction elicitation protocol used with the experts to define the common scale, starting from the original quantitative and verbal scales. The original scales were encoded in a common $[0, 1]$ utility scale, but for some scales we defined three reference levels with the utility values of 0, 0.5, and 1. The previous procedure has been applied twice, to encode utility values within the range $[0, 0.5]$, and then within the range $[0.5, 1]$. The formula of Point 5 above becomes simply,

$$u(l_k) = u(l_1) + \alpha \left(\sum_{j=1}^{k-1} (e_j + 1) \right), \quad \text{for} \quad k = 2, \ldots, t,$$

with $u(l_1) = 0$ in the first subinterval and $u(l_1) = 0.5$ in the second.

An attempt to build an interval scale was also proposed by Pictet and Bollinger (2008), but without considering the value of the unit and considering instead the blank cards as positions in the ranking. This makes the method unpracticable when confronted with a large number of blank cards in the intervals between consecutive levels. In addition, the formula to compute the values of each level cannot assign values strictly lower than the value of the lower reference level.

4. Assigning numerical values to the required data for the application of the Choquet integral

For the assignment of a utility value to each one of the considered actions through the application of the Choquet integral it is necessary:

1. To place on a common utility scale the performances of the actions according to each one of the considered criteria. We will describe in subsection 4.1 the procedure we followed when the performances were characterized on a verbal scale. In subsection 4.2 we will deal with the criteria when the performances are characterized on numerical (continuous) scales.

As noted above, this common utility scale must be an interval scale and the utility values thus considered should be commensurable, *i.e.*, these values should be such that for whatever the action a and the criteria g_i and g_j considered, every equality of the type,

$$u_i(a) = u_j(a), \tag{10}$$

is supposed to have the following meaning: *The intensity of satisfaction provided by the action a on criterion g_i is the same as that provided by this action on criterion g_j.*

2. To assign a numerical value to each capacity of the Choquet integral, which is the object of subsection 4.3.

The assignment of these numerical values in a perfectly rigourous manner would require posing a very large number of questions to the experts. The time they have and especially the risk of tiredness restricts the number of questions which it is possible to ask them. This led us to take into account a certain number of empirical hypotheses without being able to verify them.

4.1. Assigning utility values to the verbal scales levels

Concerning criterion g_6 (consistency) we have assigned a utility 1 to the verbal level "yes" and 0 to "no" (see subsection 2.3).

Concerning the seven levels of the verbal scales associated with criteria g_2 (profitability), g_3 (services), and g_5 (environment), we have assigned:

– A utility value 1 to the scale level "very good".

– A utility value 0.5 to the scale level "average".

– A utility value 0 to the scale level "very bad".

When working with the experts we started by explaining that utility value $u_j(a)$ of action a, according to criterion g_j, should be interpreted as the intensity of satisfaction provided by action a with respect to this criterion. Utility value 1 is associated with a performance which provides a *total intensity of satisfaction*, no better performance level would be likely to improve this intensity of satisfaction. Utility value 0 is associated with a performance level which provides a *total intensity of dissatisfaction*: All the performances levels below this level provide the same total intensity of dissatisfaction.

Subsequently, we called the attention of the experts for the meaning of Equation (10). Finally, we put them at work separately to assign a utility value to each one of the four intermediate scale levels. The experts have admitted (with some reservations from expert `e2`) that, for each of these intermediate levels, the utility values should be the same whatever the criterion considered, among the three that share this seven-level verbal scale. Abandoning this hypothesis would lead applying the procedure presented hereafter three times successively (the justification for this can be found in subsection 3.3). Furthermore, abandoning this hypothesis would increase considerably the number of computations to be performed. As will be seen in subsection 5.3.1, this hypothesis is with no consequences.

We gave the expert `ek` (`k=1,2,3`) 7 cards, each carrying the name of each one of the seven levels. These cards were ranked according to the order of the scale levels. The expert `ek` was invited to work on the lower part of the verbal scale, then on its upper part. We also provided the expert with 15 blank cards for each part of the scale. The expert was asked to insert blank cards in each

of the intervals of the lower part of the scale in such a way that the number of cards introduced should represent the higher or lower intensity of satisfaction between two consecutive levels. The same procedure was applied to the upper part of the scale.

The experts reacted in the following way when applying this procedure:

- At first, they were unsure (*i.e.*, they hesitate) as to the differences between two consecutive levels because in their minds all of these differences were equal.

- Then, when trying to insert the blank cards in the different intervals, they acknowledged (except for `e1`) that these differences could not all be equal.

- It should be noted that the three experts appreciated both parts of the scale in a symmetric way. Nevertheless, their utilities are many contrasting (see Table 2: In this table l_1 means "very bad", l_2 means "bad", and so on. For the sake of readability the initials of the verbal levels are also in brackets).

Table 2: Number of blank cards in the intervals between consecutive levels by expert.

Experts	l_1(vb)	e_1	l_2(b)	e_2	l_3(rb)	e_3	l_4(a)	e_4	l_5(rg)	e_5	l_6(g)	e_6	l_1(vg)
e1		0		0		0		0		0		0	
e2		3		2		1		1		2		3	
e3		5		3		7		7		3		5	

This led us to introduce three possible encodings (by applying the method of subsection 3.3), denoted by `b1`, `b2`, and `b3`, for the seven-level verbal scale. Each of them is linked to an expert `e1`, `e2`, and `e3`, respectively (see Table 3).

Table 3: Utility values for the scale levels by encoding (expert).

Experts	Encoding	$u(vb)$	$u(b)$	$u(rb)$	$u(a)$	$u(rg)$	$u(g)$	$u(vg)$
e1	b1	0.0000	0.1667	0.3333	0.5000	0.6667	0.8333	1.0000
e2	b2	0.0000	0.2222	0.3889	0.5000	0.6111	0.7778	1.0000
e3	b3	0.0000	0.1667	0.2778	0.5000	0.7222	0.8333	1.0000

The utility value of each action with respect to criteria g_2, g_3, g_5, and g_6, for each one of the three encodings above is presented in Table 5 in Appendix A.

4.2. Assigning utility values to the numerical (continuous) scales levels

Two criteria are concerned: g_1 (the investment cost) and g_2 (the surface of the naturalized area). For these two criteria, the performances used to characterize the 0 and 1 utility values cannot be defined in an obvious way. For each one of the two criteria we considered two possible encodings.

a) With respect to the *cost*:

- *Option* 1: Utility value 1 is defined by the cost of the less expensive action (30 000), and utility value 0 is defined by the cost of the most expensive action (900 000).

– *Option* 2: Utility value 1 is defined by 70% of the cost of the less expensive action (21 000), and utility value 0 is defined by 130% of the cost of the most expensive action (1 117 000).

b) With respect to the *surface*:

– *Option* 1: Utility value 1 is defined by the action which leads to the naturalization of the largest surface (5 *ha*), and utility value 0 is defined by the action which leads to the naturalization of the lowest surface (1 *ha*).

– *Option* 2: Utility value 1 is defined by the largest surface that could be naturalized (6.5 *ha*), and utility value 0 is defined by the lowest surface that could be naturalized (0 *ha*).

On this basis we defined three encodings for each criterion. The first two encodings, denoted by c1 and c2, for the *cost criterion* (g_1), and by s1 and s2, for the *surface criterion* (g_4), are defined by taking into account the above Options 1 and 2. The utility values assigned to the intermediate performances of criterion g_j ($j = 1, 4$) in a given subinterval ($g_j^\ell < g_j < g_j^u$) are defined by linear interpolation, through the application of the following formula:

$$u_j(g_j) = u_j(g_j^\ell) + \frac{g_j - g_j^\ell}{g_j^u - g_j^\ell} \left(u_j(g_j^u) - u_j(g_j^\ell) \right).$$

(11)

These encodings were presented to the experts, who considered them relevant despite the element of arbitrariness they may contain. Then, we proposed that the experts work collectively to define only two other new encodings (c3 and s3) to avoid multiplying the number of questions and to reduce fatigue for the experts. Only Options 2 were considered to define the performances characterizing extreme utility values of these new encodings (other options could be introduced later to take into account more encodings if it was proved to be necessary).

We started by asking the experts to work together to define encoding s3. The following procedure was used. We first reminded the experts that the surface of 6.5 *ha* had been chosen to characterise a total intensity of satisfaction, and 0 *ha* to characterize a total intensity of dissatisfaction. Then, we asked them to define a surface m that was able to characterize an average level of intensity of satisfaction in the considered scale. We added that an average level indicates a surface m such that moving from 0 *ha* to m *ha* brings the same variation of intensity of satisfaction as the variation that brings the transition from m *ha* to 6.5 *ha*. We asked them if $m = 3.25$ *ha* (the middle of the interval) would be suitable. They judged this value very high, and after some discussion they proposed $m = 3$ *ha*. We then continued the same exercise by taking the interval 0 *ha* − 3 *ha*; and, finally we did the same for the interval 3 *ha* − 6.5 *ha*. The experts quickly agreed to keep the values 1 *ha* and 5 *ha*, respectively. In each of these intervals thus defined, we proceeded by linear interpolation to assign a utility value to each of the considered surfaces.

Concerning the definition of encoding c3 we asked the experts to work in the same way as in the previous case. However, they found this new exercise much more difficult given the large difference of the costs which characterize the utility values 1 and 0, 21 000 and 1 117 000, respectively. Nevertheless, a general agreement was reached to keep the values 150 000, 400 000, and 800 000.

For each action the utility values associated with their performances on the criteria g_1 and g_4 were defined by linear interpolation, for each of the three encodings considered (see Table 6 in Appendix A).

4.3. Assigning numerical values to the capacities of the Choquet integral

We followed the procedure described in subsection 3.2. A mutual-strengthening effect and a mutual-weakening effect being taken into account, eight capacities must be considered. This is the reason that led us to introduce eight types of dummy projects, as follows:

- Projects of type p_j, for $j = 1, \ldots, 6$, which are characterized by a utility value of 1 on the criterion g_j, and a utility value of 0 on all of the other projects.

- Project of type p_7 (also denoted by p_{45}), which is characterized by a utility value of 1 on the criteria g_4 and g_5, and a utility value of 0 on all of the other projects.

- Project of type p_8 (also denoted by p_{15}), which is characterized by a utility value of 1 on the criteria g_1 and g_5, and a utility value of 0 on all of the other projects.

For the characterizations of each type of project it is necessary to take into account the actual performances which are encoded by the 0 and 1 utility values with respect to the considered project. Concerning criteria, g_2, g_3, g_5, and g_6, such performances are those corresponding to the extreme levels of the verbal scales. For each one of the two remaining criteria, g_1 and g_4, two options were envisaged (see subsection 4.2). They correspond to encodings, `c1`, `c2`, and `s1`, `s2`, respectively. This lead us to introduce four variants to each one of the eight types of projects defined above. In what follows these variants are denoted as follows: `c1s1`, `c1s2`, `c2s1`, and `c2s2`. Let us remark that the types of dummy projects previously introduced make use of no role of the performances which led to define the utility values other than the extreme. Consequently, encodings `b1`, `b2`, `b3`, `c3`, and `s3`, have no intervention in the procedure which we followed to assign numerical values to the capacities.

Each variant was presented to the experts using a set of 8 cards (a set *per* variant). On each card we wrote down the 6 performances which characterized the variant of the considered project (the corresponding utilities were not on the card). The experts were presented with the set of cards associated with the variant `c1s1`. We explained that they first should rank the cards by a non-decreasing order of the satisfaction level associated with to the considered projects. Then, they should insert blank cards to differentiate the amplitude of the differences that separate these satisfaction levels. Finally, we asked the experts to assign a numerical value (denoted by z) to the ratio between the satisfaction levels of the projects ranked in the best position and those ranked in the worst position. In a first phase, the experts worked separately, successively for each one of the four variants, then they worked collectively (denoted by expert `e4`).

The experts accepted the work with the cards with no difficulties. They quickly understood they should express their higher or lower great satisfaction with respect to each of the the dummy projects as they were characterized on the cards with the purpose of re-qualifying the abandoned quarry. To the question posed by the analyst: "What do you think about the approach followed?", the experts declared that the use of the cards was very well adapted when they worked separately as well as in the multi-actor work they were submitted to in the focus group: "The possibility of handling the cards led to successive discussions related with their relative positions as well as with respect to the number of blank cards to be inserted into the intervals", "the discussion were of a great help for reaching a *consensus*". The experts specified that in the cases were the disagreements persisted regarding the ranking of some pairs of cards, they decided to consider such pairs in the same position. Then, the analyst asked them to compare the ranking finally obtained (expert `e4`) with the rankings selected when working separately (see Table 4).

Table 4: Rankings of projects and blank cards by expert.

		Ranks and Blank Cards															$z^{\min}$	$z^{\max}$
		R_1	e_1	R_2	e_2	R_3	e_2	R_4	e_4	R_5	e_5	R_6	e_6	R_7	e_7	R_8	$z^{\min}$	$z^{\max}$
Expert e1	c1s1	p_6	0	p_1	0	p_5	0	p_2,p_3	0	p_4	0	p_{45}	2	p_{15}			80	80
	c1s2	p_6	0	p_1	0	p_5	0	p_2,p_3	0	p_4	0	p_{45}	2	p_{15}			80	80
	c2s1	p_6	0	p_1	0	p_5	0	p_2,p_3	0	p_4	0	p_{45}	4	p_{15}			100	100
	c2s2	p_6	0	p_1	0	p_5	0	p_2,p_3	0	p_4	0	p_{45}	4	p_{15}			100	100
Expert e2	c1s1	p_6	1	p_3,p_4,p_5	1	p_{45}	4	p_1	2	p_2	1	p_{15}					70	70
	c1s2	p_6	1	p_3,p_4,p_5	1	p_{45}	4	p_1	2	p_2	1	p_{15}					70	70
	c2s1	p_6	1	p_3,p_4,p_5	1	p_{45}	5	p_1	3	p_2	1	p_{15}					70	70
	c2s2	p_6	1	p_3,p_4,p_5	1	p_{45}	4	p_1	4	p_2	1	p_{15}					70	70
Expert e3	c1s1	p_6	3	p_2	2	p_1	2	p_4	4	p_3	2	p_5	3	p_{45}	4	p_{15}	20	25
	c1s2	p_6	3	p_2	2	p_1	3	p_4	3	p_3	2	p_5	4	p_{45}	2	p_{15}	15	20
	c2s1	p_6	3	p_2	3	p_1	2	p_4	3	p_3	2	p_5	3	p_{45}	5	p_{15}	20	25
	c2s2	p_6	3	p_2	3	p_1	3	p_4	2	p_3	2	p_5	5	p_{45}	4	p_{15}	20	30
Expert e4	c1s1	p_6	3	p_3	1	p_4,p_5	3	p_1	4	p_2,p_{45}	4	p_{15}					90	90
	c1s2	p_6	1	p_3	1	p_4,p_5	3	p_{45}	1	p_2	5	p_1	3	p_{15}			90	90
	c2s1	p_6	5	p_1	2	p_3	2	p_4,p_5	2	p_2	0	p_{45}	5	p_{15}			90	90
	c2s2	p_6	5	p_1	1	p_3	3	p_4,p_5	3	p_2,p_{45}	5	p_{15}					100	100

After underlining that projects p_{45} and p_{15} always occupied the best positions in the ranking and project p_6 occupied the worst position, the analyst highlighted the most important divergences and asked the experts to explain them. More precisely, expert e1 (expert in economic evaluation) indicated she favored project p_4 because she judged it particularly important to improve the landscape ecology of a large naturalized surface. In the focus group she realized that the question was a bit different since expert e2 (environmental engineer) explained that the covered surface for the landscape for the naturalized area was not particularly large (at most $6.5\,ha$). Expert e1 also drew the attention about the fact that project p_1 was very cheap, but since it did not produce any effect, ranking it in a good position was not justified. This explanation led e2 to accept the ranking finally obtained. Other very interesting discussions took place, particularly regarding project p_2, but for reasons of space they are not described in this article.

The experts were unanimous in underlining that the excessively unrealistic features of some projects made it difficult to define a ranking. Other rankings could, according to them, be justified. These unrealistic features were due to large difference in cost which separated the cheapest projects from the most expensive. They were also due to the poverty of information that characterized the projects. Finally, regarding the z value, the experts appreciated the possibility offered of expressing the values through an interval, except for expert e1 who had no hesitations in providing a single value for z.

On these grounds and by applying the procedure in subsection 3.2 several sets of numerical values were assigned to the capacities (see Table 7 in Appendix A).

5. Computational experiments and results

This section starts by fixing up the design of the experiments, then it introduces two specific sets of threshold functions, and finally, it presents the results obtained.

5.1. The design of the experiments

Each computational experiment, which leads to the assignment of a utility value to each one of the five actions, $a_1, \ldots, a_5$, with the Choquet integral, requires the precise definition of the conditions in which such an experiment has been performed. Therefore, a particular choice of these conditions (or options) constitutes what we call a *configuration*. A configuration is defined by the following elements:

— The way in which the utilities have been assigned to the *numerical scale levels* of criteria g_1 (*i.e.*, the levels of costs) and g_4 (*i.e.*, the levels of surfaces). For each criterion, three ways of encoding the scale levels have been considered: `c1, c2, c3` and `s1, s2, s3` (see Section 4).

— The way in which the utilities have been assigned to the *verbal scale levels* of criteria g_2, g_3, and g_5. Three ways of encoding the scale levels have been considered: `b1, b2, b3` (see Section 4).

— The *expert*, who, in the considered conditions, has defined the way in which s\he ranks the projects, $p_1, \ldots, p_8$ (see Section 4), where $p_7 = p_{45}$ and $p_8 = p_{15}$. Four experts have been taken into account: `e1, e2, e3, e4`.

— The chosen *z ratio value* (from now on "`z`" will be used instead) for performing the computations. Two values for `z` have been considered: The minimum (`min`) and the maximum (`max`) values provided by the considered expert.

— The *number of blank cards* placed in each one of the seven intervals in the ranking of the projects, $p_1, \ldots, p_8$. Firstly, we started by taking into account the number of blank cards (with its respective position in the ranking) as it has been proposed by the considered expert (from now on this number will be identified as `onc` - original number of cards). Secondly, for the reasons presented in subsections 5.2 and 5.3, below, we took into account a modification of the blank cards (from now on this number will be identified as `ace` - adding one card everywhere, *i.e.*, in every interval).

Thus, every configuration is characterized by a code of the type: `c(i)s(j)b(k)e(h)z(min,max)` `onc\ace`, with $i = 1, 2, 3$, $j = 1, 2, 3$, $k = 1, 2, 3, 4$, and $h = 1, 2, 3, 4$ (note that `b4=b1`).

The purpose of the performed experiments is to examine to which extent the results are affected or influenced by the following three aspects: (1) the way of encoding the different numerical and verbal scales levels during the individual and collective work of the experts; (2) the ill-determined value of `z`; and, (3) the number (and place) of the blank cards inserted into the intervals of the ranking of projects, $p_1, \ldots, p_8$.

At first, we have been interested in the results obtained with the 192 configurations characterized as follows: `c(i)s(j)b(k)e(h)z(min,max)onc` and `ace`, for $i, j = 1, 2$, $k = 1, 2, 3, 4$ and $h = 1, 2, 3, 4$.

Starting from the conclusions that we reached in examining the results obtained with these 192 configurations, in a second stage, we considered the encodings `c3` and `s3` along with the new `ace`

encoding. This process was done, not by merging the new encodings with the 192 configurations, as analyzed in the first stage, but only with some (well defined) of them. The purpose of this second phase was to confirm or to inquire the results obtained in the first stage of the study based on the examination of the 192 configurations.

Before presenting the sets of results obtained (see subsection 5.3), it is necessary to precise the way we proceed to judge if the utility values assigned to the actions, $a_1, \ldots, a_5$, in two different configurations should be seen as *significantly different*. For such a purpose, it is *necessary* to check to what extend the utility value of a given action could be influenced in a non significant way by some difficulties and hesitations experienced by the expert during the decision aiding process (see Section 4). It is important to recall that this process led her\him to build the *required data* necessary to assign a utility value to each one of the five actions by using the Choquet integral. These data are related with both the ranking of the projects, $p_1, \ldots, p_8$, and the value assigned to `z`. The purpose of the next subsection is to show how we have addressed this task by building two specific sets of threshold functions.

5.2. Two sets of threshold function

The analyses of the impact of both the ill-determination of `z` and the number and position of blank cards led us to build two sets of threshold function. These analyses are presented in the next three subsections.

5.2.1. Analyzing the impact of the ill-determination of `z`

In order to analyze the influence of the ill-determination of `z`, it was necessary to compare the values of the utilities assigned to each one of the five actions in the pairs of configurations characterized in the same way, but for the `z` value. To accomplish this task we started by taking the pairs of the obtained results as follows: `c(i)s(j)b(k)e(h)onc` with $i, j = 1, 2$, $k = 1, 2, 3, 4$, and $h = 1, 2, 3, 4$. Each pair is formed by the results obtained with `zmin` and `zmax`. There are 48 pairs of possible configurations. We focused our analysis on the ones which associate:

- With each one of the experts `e(h)` ($h = 1, 2, 3, 4$), the encoding `b(k)` ($k = 1, 2, 3, 4$) s\he has defined to assign a utility to each level of the verbal scale (see Section 4).

- With the expert `e4`, each one of the three encodings `b1=b4, b2, b3`.

- With the experts `e2` and `e3`, the uniform encoding `b1`, which has been considered relevant by them.

There are thus 32 pairs of configurations. It is also necessary to draw the attention of the reader to the fact that some pairs (more precisely 8 of them) did not allow to assign a utility to the five actions. This impossibility was related to the cases where the way in which the expert ranked the projects, $p_1, \ldots, p_8$, by inserting blank cards in the intervals proved to be *non conform* to the hypotheses established regarding the interaction between criteria (see Section 2). This non conformity occurs because one of the consistency conditions as defined in expressions (8) or (9) is not fulfilled by the values assigned to the μ coefficients of the Choquet integral. These pairs, which led to a particular analysis (see subsection 5.3.5), are not considered in the current section.

According to the definition of the discriminating threshold functions (see Roy et al., 2014) the set of functions to be built aims at understanding the part of arbitrariness, due to the ill-determination of `z`, which affects the utility values assigned to the actions by the Choquet integral.

This arbitrariness is highlighted by the differences which separate, for each action, these utilities when they are assigned under identical conditions, but with different **z** values. These values, which are 24 $(32 - 8)$ for each action a_i, are denoted by $u^{zmin}(a_i)$ and $u^{zmax}(a_i)$. In order to build direct threshold functions, we have to look at the way the 120 (5×24) differences of utility in absolute value, $|u^{zmin}(a_i) - u^{zmax}(a_i)| = \Delta^z(a_i)$, vary with respect to the lowest of the two values, *i.e.*, $\min\{u^{zmin}(a_i), u^{zmax}(a_i)\} = \min^z\{a_i\}$ (see Figure 1 in Appendix C).

Surprisingly, we observed that the hypothesis of direct thresholds, *i.e.*, thresholds which are independent from the utility values in the abscissa axis, does not seem to be acceptable. The regression line (see Figure 1) confirms this observation. This line highlights the fact (which is clearly visible in the graph) that, in most cases, when the utility values increase, the absolute values of the differences tend to decrease. This led us to conclude that there was a need to build affine threshold functions. With the aim of constructing such functions, we observed the dispersion of the differences. This distribution is presented in Table 8 in Appendix B (where $Nb.$ is the number of cases, and the repeated values are in between parenthesis). The examination of this table allows to state that:

- 102 of the smallest values of $\Delta^z(a_i)$ are at most equal to 0.0029.

- The interval $[0.003, 0.0065]$ does not contain any value for $\Delta^z(a_i)$, and the same occurs for the interval $[0.0085, 0.0109]$.

- 7 of the 120 values of $\Delta^z(a_i)$ belong to the interval $[0.0066, 0.0084]$.

- $\Delta^z(a_i)$ is greater than or equal to 0.0011 for 9 of the 120 values.

Let us consider two actions, a_i and a_j, for which the Choquet integral assigns two values (in the same configuration) such that $u(a_i) < u(a_j)$. When these utilities correspond to those observed in one of the 101 first cases mentioned above, we assumed that the ill-determined knowledge of the **z** value that was considered to define these utility values, lead to a difference $\Delta^z(a_i)$ that is not significant of a preference. Moreover, we considered that only in the last 9 cases it would be possible to have a strict preference of a_j compared with a_i. On these bases, we put: $q^z(u(a_i)) = 0.006$ for $u(a_i) = 0.250$, and $q^z(u(a_i)) = 0.004$ for $u(a_i) = 0.500$. This led to the following indifference threshold function:

$$q^z(u(a_i)) = -0.008u(a_i) + 0.008.$$

It is easy to check that with this function there is indeed indifference for the first 101 values of Table 8 and preference for the remaining ones.

Certainly, this function is not the only one that allows to reach this objective. However, given the use that is made of this threshold function (see subsection 5.2.3), the reader will understand with no difficulties, that if this function was replaced by other threshold functions leading to the same cases of indifference and preference, it could not significantly affect the subsequent conclusions resulting from the choice of the threshold function we considered.

To define a preference threshold function able to take into account the above referred objective, we consider: $p^z(u(a_i)) = 0.011$ for $u(a_i) = 0.375$, and $p^z(u(a_i)) = 0.009$ for $u(a_i) = 0.500$. This led to the following indifference threshold function:

$$p^z(u(a_i)) = -0.016u(a_i) + 0.017.$$

This function also allows to achieve the objective. Here again we can notice that this function is not the only one, but what has been said for the choice of the indifference threshold function also applies for the selection of a preference threshold function.

5.2.2. Analyzing the impact of the ill-determination of the number and position of the blank cards
In order to analyze this impact, we need to compare the values of the utilities assigned to each one of the five actions in the pairs of configurations, which were characterized in the same way, but for the number and location of blank cards in the ranking of projects, $p_1, \ldots, p_8$. It is, however, necessary that the two dispositions of these blank cards be enough close to reflect the difficulty of the expert when s\he makes the decision about the number of blank cards to be inserted into each one of the seven intervals. This led us to introduce a first modification, denoted by `ace`, from the number and initial disposition `onc` of the blank cards as they were inserted by the expert in the seven intervals of the projects ranking. The configuration `ace` is deduced from `onc` by adding a blank card in each one of the seven intervals.

In order to make possible the comparison, we started by taking the pairs of the obtained results as follows: `c(i)s(j)b(k)e(h)zmin` with $i, j = 1, 2$, $k = 1, 2, 3$, and $h = 1, 2, 3, 4$.

Each pair is formed by the results obtained with the configurations `onc` and `ace`. Only the 24 of the 48 pairs taken into account in subsection 5.2.1 were again considered here (for the same reasons). Therefore, for each action a_i, we obtained 24 pairs denoted by $u^{onc}(a_i)$ and $u^{ace}(a_i)$. Here again we can observe that, the greater the lowest of these two values, $\min^{bc}\{a_i\}$, the more the absolute value, $|u^{onc}(a_i) - u^{ace}(a_i)| = \Delta^{bc}(a_i)$, tends to diminish (see Figure 2 in Appendix C). Thus, it is necessary, as before, to build affine threshold functions. For such a purpose, we looked at the repartition of the new $\Delta^{bc}(a_i)$ values (see Table 9 in Appendix B). The examination of this table allows to conclude that:

- 104 of the smallest values of $\Delta^{bc}(a_i)$ are at most equal to 0.0148.

- The interval $[0.0149, 0.0171]$ does not contain any value for $\Delta^{bc}(a_i)$, and the same occurs for the interval $[0.0249, 0.0316]$.

- 12 of the 120 values of $\Delta^{bc}(a_i)$ belong to the interval $[0.0172, 0.0248]$.

- $\Delta^{bc}(a_i)$ is greater than or equal to 0.0317 for 4 of the 120 values.

When the Choquet integral assigns (in the same configuration) to two actions the utilities that correspond to those utilities observed in the first above 104 cases, we considered that the ill-determination of the number and position of the blank cards, which have been taken into account to assign these values, lead to a difference $\Delta^{bc}(a_i)$ that is not significant of a preference, and only for the last 4 cases it would be possible to have a strict preference of one of the actions with respect to the other. On these bases, we consider: $q^b(u(a_i)) = 0.017$ for $u(a_i) = 0.250$, and $q^b(u(a_i)) = 0.014$ for $u(a_i) = 0.400$. This led to the following indifference threshold function:

$$q^b(u(a_i)) = -0.020u(a_i) + 0.022.$$

Analogously, we consider: $q^b(u(a_i)) = 0.028$ for $u(a_i) = 0.250$, and $q^b(u(a_i)) = 0.020$ for $u(a_i) = 0.400$. This led to the following preference threshold function:

$$p^z(u(a_i)) = -0.032u(a_i) + 0.036.$$

The reader can also check that with this set of threshold function the objective is achieved.

5.2.3. On the way the two threshold functions have been used

In this article the Choquet integral was used to assign utility values to each one of the five actions, $a_1, \ldots, a_5$, with the purpose of making possible their comparison. The implementation of the Choquet integral was developed according to two subsequent steps. Firstly, it was necessary to assign utilities to the different criteria scale levels, on which the performances of criteria are characterized. In this first step several options are available (see Section 3). In a second step, it was necessary the intervention of an expert (see Section 4) to assign numbers to the μ parameters of the Choquet integral. Our purpose is to highlight the influence that the choice of the options, which is considered in the first step, and the choice of the expert, which is considered in the second step, may have on the way the obtained utility values by the Choquet integral lead to the comparison of the actions among them. We can perform this task by simply comparing the way the different analyzed configurations rank the actions in a complete order by a decreasing order of their utilities. This way leads to highlight that the non significant differences come only from the two ill-determination factors that have been analyzed in the previous two subsections. It is consequently necessary to take into account the two sets of threshold functions to highlight only the significant differences. With such a purpose in mind, to each one of the complete orders considered, we considered a pseudo order preference structure (Roy and Vincke, 1984), which assigns, for every pair of actions, the most relevant of the following three binary relations: *indifference* (I), *weak preference* (Q), and *strict preference* (P).

For a given configuration, the two sets of threshold function do not necessarily provide the same pseudo order. Always having in mind the necessary relation of avoiding non significant differences, we only chose to consider the indifference or the preference when there is unanimity with the two sets of functions in favor of one or the other action. Figure 3 (see Appendix C) shows that it is impossible that one of two functions leads to an indifference when the other leads to a strict preference. This figure also shows that:

- There is indifference with the two sets of threshold functions if and only if there is indifference for $q^z(u(a_i))$.

- There is strict preference with the two sets of threshold functions if and only if there is preference for $p^b(u(a_i))$.

These elements led us to associate with each configuration a unique pseudo order.

5.3. Presentation of the obtained results

After a brief introduction, this section presents the analyses of the influence or impact on the results of the following elements: The experts, the choosing encodings for the cost and surface criteria, and the number and position of the blank cards.

5.3.1. Preliminaries

As we have explained at the beginning of subsection 5.1, the application of the Choquet integral for assigning a utility value to each one of the five actions, $a_1, \ldots, a_5$, needs the selection of one option, for each one of the considered characteristics. This leads to the definition of what we called a configuration. This is done with the purpose of establishing the conditions under which the computations are performed. It is also important to recall that the purpose of this paper is to highlight the less or more significant impact related with the choice of the way the actions under analysis can be ranked.

Consider two configurations, which are only different in a single option selected with respect to one of the characteristics. We say that the choice between one or the other of these two options is *with no impact in this configuration* if the resulting pseudo orders in one and the other case are identical. We state that such a choice is *with no significant impact* when it is with no impact in a set of configurations judged representative. This set may contain only a small number of configurations for which the pseudo orders are not rigorously identical, but they are only different for the replacement of a weak preference either by an indifference or by a strict preference.

The obtained results allow to assert that the choice between the options `zmin` and `zmax` is with no significant impact. For such a reason the results presented in the next subsections are all related with the option `zmin`.

Let us also recall that three encodings were considered to obtain the utility values for the verbal scale levels. More precisely, these encodings are the uniform encoding `b1` of expert `e1`, the encoding `b2` of expert `e2`, and the encoding `b3` of expert `e3` (see Section 4). The obtained results show that in many configurations the choice of the encoding is with no impact. However, in some of them the obtained pseudo orders may differ not only for the replacement of a weak preference either by an indifference or by a strict preference, but also for the replacement of an indifference by a strict preference and even by a reversal in the order in which the two consecutive actions are ranked. Thus, it is not anymore possible to consider that the way of assigning utility values to the verbal scale levels is with no significant impact. On the one hand, the impact of this choice remains low. For this reason we will not systematically analyze it in the following subsections, while we will only mention certain occurrences. On the other hand, the obtained results show that the resulting pseudo orders have big changes according to the considered expert (see subsection 5.3.2) and also according to the way of assigning utility values to the criteria g_1 (cost) and g_4 (surface) (see subsection 5.3.3). The subsection 5.3.4 is devoted to the examination of the impact on the pseudo orders of the number and position of the blank cards in the ranking of the projects, $p_1, \ldots, p_8$. As we have mentioned in subsection 5.1, for some configurations it is not possible to assign utility values to the actions since the way the expert ranked the projects, $p_1, \ldots, p_8$, and inserting blank cards into the intervals, has proved to be inconsistent (or non conform) with the interaction between criteria. Subsection 5.3.5 is thus devoted to the examination of the non conformity cases.

5.3.2. Analyzing the influence of the experts

In order to highlight this influence or impact of the experts, we took into account the following set of configurations: `c(i)s(j)b(h)e(h)z(min)onc` and `ace`, for $i, j = 1, 2$ and the considered expert, `h`, with her\his encoding of blank cards, `h` (remember that `b4=b1`). This led to the construction of 8 configurations *per* expert. For each one of these configurations, Table 10 in Appendix B shows the obtained results successively for each one of the 4 experts. For the sake of simplicity, in this table, we considered the transitivity of P for non successive pairs; for example, the situation in which aQb, aPc, and bPc is represented by $aQbPc$.

On the one hand, let us observe that in each one of the 8 configurations, when there is conformity, action a_2 is always placed in rank 1. On the other hand, there is no configuration for which the 4 experts rank in the same way the other 4 actions. Moreover, in each configuration there is at least one expert that ranks in position 2 an action, which is ranked in the last position by another expert. For example, this happens in the first 6 configurations, between experts `e1` and `e2`, with reference to action a_1. The same happens for the configurations `c2s2`, between experts `e4` and `e3`, with reference to action a_5. In rank 2, we find 6 times a_1, 4 times a_5, 7 times a_4, and 4 times a_3, when in the last position of the rank we find 12 times a_1, 8 times a_3, and 6 times a_5.

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In total, with these 32 configurations we obtained 6 non conformity cases and 13 different pseudo orders: 3 for each one of the experts, `e1`, `e3`, `e4`, and 5 for the expert `e2` (for a total 14 of pseudo orders). Let us remark that there is only one common pseudo order for two experts: This is the pseudo order in configuration `c1s2ace` for `e4` and in configuration `c2s2onc` for `e1`.

The computations performed, for each expert, with the reference set, took into account, for each one of them, the way the verbal scale has been encoded. We also performed other computations with the purpose of examining the impact the choice of the encoding could have on the obtained pseudo orders. For this purpose, we started to perform the computations by replacing the encoding `b1` by the encodings `b2` and `b3`, for the experts `b2` and `b3`, respectively. This choice was motivated by the fact that the encoding `b1` considered by the expert `e1` and adopted by the expert `e4` (representing the group of the three experts) may be seen as a *reference encoding*, since it consists of assigning equal variations of utility between two consecutive scale levels, whatever their position in the scale. The obtained results highlight the following aspects:

a) For the expert `e2`, as well as for the expert `e3`, most of the modifications (which are in a large number) consist of replacing a weak preference either by an indifference or by a strict preference. These modifications represent the situations that we considered (see subsection 5.3.1) with impacts that deserve to be judged as non significative.

b) For the expert `e2`, moving from `b2` to `b1` only highlights a significant modification: With the `c1s1ace` configuration (see Table 10) action a_1, which was in position 2, moves to rank 3, and action a_4, which was in position 3, moves to position 2 (with a strict preference).

c) For the expert `e3`, moving from `b3` to `b1` does not highlight any significant modification.

We then examined, for the expert `e4`, the impact it could have the fact of replacing either `b2` or `b3` by `b1`. We also observed that several little significant modifications occur. The replacement of `b2` by `b1` in the `c1s2onc` configuration (see Table 10) leads to substitute an indifference between a_5 and a_3 by a strict preference in favor of a_5 with respect to a_3.

Let us observe that the encodings `b1`, `b2`, and `b3` associate the 0 and 1 utilities with the same (extreme and median) scale levels. The differences are only related with the other (intermediate) scale levels. In order to highlight the advantage of the little significant impacts, it is undoubtedly necessary to perform some computations with other encodings than the ones with 0 and 1 utility values for the extreme scale levels. We thought it may not necessary to undertake such a type of calculations for these scales because they appeared to us to be too arbitrary.

5.3.3. Analyzing the influence of choosing encodings for the cost (g_1) and the surface (g_4) criteria scales

For this analysis we took into account the reference set of configurations, after completing it with the following two configurations: `c3s3b(h)e(h)zminonc` and `ace`, $h = 1, 2, 3, 4$.

Table 11 in Appendix B shows successively, for each expert, the obtained pseudo orders along with the configurations which led to obtain such pseudo orders. This table highlights that in many configurations the way of assigning utility values to the numerical scale levels, that were used to characterize the performances of the actions on criteria g_1 and g_4, significatively affects the obtained pseudo orders, for each one of the four experts. Let us start by considering the encodings `c1` and `c2`. They differ from the costs considered for the characterization of the 0 and 1 utilities, respectively.

In these two scales, the utility value assigned to a performance g_j in a given interval $(g_j^\ell < g_j < g_j^u)$ is defined in the same way by linear interpolation (see Equation (11)).

We observed that, all other things being equal, when the expert works with c1 or c2, the obtained results may differ. It is, for example, the case (see Table 11):

  − For the experts, e1, e2, and e4 with the configurations c1s2onc and c2s2onc.

  − For the expert e3; in this case, it was possible to define a pseudo order, with configurations c2s2onc and ace, while this did not happen either with c1s2onc or with c1s2ace.

Let us consider now the encodings s1 and s2 for which the differences are analogous to the ones that differentiate c1 from c2. Also in this case we observed that the results may be significantly different whether the expert works with s1 or s2 (all other things being equal). It is, for example, the case:

  − For the experts, e1, e2, and e2, with the configurations c2s1onc and c2s2onc.

  − For the expert e4; in this case, it was not possible to define a pseudo order with configurations c1s1onc and ace, while this did not happen with the configurations c1s2onc and ace.

The performances that were selected as references for defining the 0 and 1 utility values are the same as in c2 and c3, on one side, and as in s2 and s3, on the other side. Moreover, while in c2 and in s2 the intermediate utility values were defined by linear interpolation (as it was recalled below), in c3s3 the intermediate utility values were obtained from the interaction with the group of experts, e4. It is the only change that originated the differences (where they exist) between the pseudo orders found with the configurations c2s2(onc,ace), on one side, and the pseudo orders found with the configurations c3s3(onc,ace), on the other side. The examination of Table 11 highlights only two significant modifications:

  − With expert e1, the movement from c2s2onc to c3s3onc leads to a change from $a_4 P(a_3 I a_5)$ into $a_4 I a_5 Q a_5$.

  − With expert e3, the movement from c2s2onc to c3s3onc leads to a change from $a_4 P a_3$ into $a_3 Q a_4$.

In conclusion, with these 10 configurations we now observe 5 different pseudo orders for e1, 6 for e2, 3 for e3, and 3 for e4 (in total 17). The consideration of the two additional configurations, c3s3onc and ace, highlights 3 new pseudo orders (16 among the 17 are different).

5.3.4. Analyzing the influence of the number and position of the blank cards
The difference of the utility values assigned to the actions when (all other things being equal) we added (in a uniform way) a blank card into the intervals of the ranking of projects, $p_1, \ldots, p_8$, was used to define the second of the two sets of threshold functions.

It is important to start by examining the influence such a uniform change (moving from onc to ace) may have on the obtained pseudo orders. The examination of Table 11 shows that:

  − The pseudo orders obtained for two configurations that are only different for the option (onc,ace) are frequently different, but in the majority of the cases there is only a replacement of a weak preference either by an indifference or by a strict preference.

- There are only two significant differences, one with expert `e1` and the other with the expert `e3` (see in Table 11 the configurations, `c2s2b1e1z80onc` and `c2s2b3e3z20onc`, respectively).

Taking into account the way the above mentioned set of threshold functions has been defined, it appeared to us interesting to examine the impact the ill-determination of the number of blank cards could have on one and only one of the intervals of the ranking of projects, $p_1, \ldots, p_8$. Would such a hesitation of the expert leave the pseudo order unchanged? If there were cases where it was otherwise, would the changes remain non significant? It appeared to us important to examine these two questions for the expert `e4` (group of the experts). Six configurations are concerned: `c1s2`, `c2s2`, `c3s3` combined with `onc` and `ace` (the configurations `c1s2` and `c2s2` provide non conform results, see subsection 5.3.5).

Let us consider two projects, p_x and p_y, which are in the same position or in consecutive positions in the ranking. In the case where they are consecutive (in that order), adding a supplementary blank card, requires the examination of two new options (denoted in an obvious way) `oncpx1py` and `acepx1py`. In the case where p_x and p_y are in the same position, it is necessary to make a separation of both, *i.e.*, to create an interval between these two projects (which does not necessarily requires the introduction of a blank card between p_x and p_y). There are two ways of making this separation: Ranking them in the order $p_x p_y$ or in the reverse order $p_y p_x$. Consequently, there are $2 \times 2 = 4$ new options, denoted `oncpx0py`, `acepx0py`, `oncpx1py`, and `acepy1px`.

We have actually seen that the addition of a single blank card often left the pseudo order either unchanged or with non significant modifications. The change is, however, significant in the following configurations:

- `c1s2oncp61p1`, the pseudo order becomes $a_2 P(a_4 I a_3) Q a_5 P a_1$ instead of $a_2 P a_4 P(a_5 I a_3) P a_1$.

- `c1s2oncp31p4`, the pseudo order becomes $a_2 P a_4 Q a_5 Q a_3 P a_1$ instead of $a_2 P a_4 P(a_5 I a_3) P a_1$.

- `c1s2oncp40p5`, the pseudo order becomes $a_2 P a_4 P a_5 P a_3 P a_1$ instead of $a_2 P a_4 P(a_5 I a_3) P a_1$.

- `c3s3oncp71p2`, the pseudo order becomes $a_2 P a_4 P a_3 P a_1 P a_5$ instead of $a_2 P a_5 P a_4 P a_3 P a_1$.

- `c3s3acep71p2`, the same modification as in the previous configuration.

Taking into account these 5 significant results, we decided it would be useless to do the same kind of analysis for the experts, `e1`, `e2`, and `e3`. We observed that the experts frequently hesitate on the number of blank cards to be inserted not only into a single interval of the ranking of projects, $p_1, \ldots, p_8$, but also into several intervals. Thus, there is a strong indication that in order to take into account the effect of such conjoint hesitations, we can only increase the number of cases in which the obtained pseudo orders are significantly different.

It should be noted that the following pseudo orders are new (*i.e.*, they are not in Table 11): $a_2 P(a_4 I a_3) Q a_5 P a_1$, $a_2 P a_4 Q a_5 Q a_3 P a_1$, and $a_2 P a_4 P a_5 P a_3 P a_1$.

5.3.5. Analyzing the cases of non conformity

The non conformity (*i.e.*, the violation of the mutual-strengthening and\or mutual-weakening conditions) has been observed in the following cases (see Table 11):

- With the expert `e3`, the mutual-strengthening condition is violated for the configurations `c1s2onc` and `c1s2ace`.

– With the expert $e4$, the mutual-weakening condition is violated for the configurations `c1s1onc` and `c1s1ace`, and the mutual-strengthening condition is violated for the configurations `c2s1onc` and `c2s1ace`.

We have started by finding the cases for which it was possible to restore the conformity either by separating two projects in the same position or by adding one more blank card into one of the intervals of the ranking of projects, $p_1, \ldots, p_8$. The only possible cases are the following:

– With the expert $e3$, the only configuration that permits the restoration of the mutual-strengthening condition is `c1s2oncp71p8`; we obtain thus the pseudo order $a_2 P a_3 P a_4 P a_1 P a_5$.

– With the expert $e4$, the only configuration that permits the restoration of the mutual-strengthening condition is `c2s1oncp11p8`; we obtain thus the pseudo order $a_2 P a_4 P a_1 Q a_3 P a_5$.

In the remaining cases of non conform configurations, we investigated the minimum number of modifications to be performed in order to restore the conformity. The results are as follows:

– With the expert $e3$, only a single configuration is involved, *i.e.*, `c1s2ace`. In this case we needed to add two blank cards between p_7 and p_8 to restore the conformity; the obtained pseudo order was the same as in the above configuration `c1s2onc`.

– With the expert $e4$, three configurations were involved:

  i) For the configurations `c1s1onc` and `ace` the mutual-weakening condition was violated (we can recall that the projects were ranked as follows: $p_6 3 p_3 1 (p_4, p_5) 3 p_1 4 (p_2, p_7) 4 p_8$). We started by examining if it was possible to avoid the violation of the mutual-weakening condition by placing p_7 before p_2 and by adding a high enough number of blank cards between them: Unfortunately, this was impossible. The final solution found was the following: To place p_1 and p_7 is the same position and insert a blank card between these two projects and p_2 (when adding a single blank card the mutual-weakening condition is no longer violated, but the mutual-strengthening condition becomes violated). In this case the pseudo order found is the following: $a_2 P a_4 P (a_5 I a_1) P a_3$.

  ii) For the configuration `c2s1ace` the mutual-strengthening condition was violated. In order to restore the conformity it is now necessary to add two blank cards in between p_1 and p_8, and not only a single blank card as it was in the case of the configuration `c2s1onc`. The ranking remains the same as in the previous case.

It should be noted that the following pseudo orders are new (*i.e.*, they are not in Table 11): $a_2 P a_4 P a_1 Q a_3 P a_5$ and $a_2 P a_4 P (a_5 I a_1) P a_3$.

6. Conclusions

The results presented in Section 5 lead to the following conclusions:

a) The application of the Choquet integral to justify a ranking of the five actions, $a_1, \ldots, a_5$, based on the work performed by the experts, leads to the identification of a_2 as being the significatively better ranked action, and this happened for all the experts and the data considered for the implementation of the Choquet integral. However, with respect to the other four actions only very weak or partial conclusions (which differ from the experts and the configurations considered) can be drawn (see Table 11). It is not surprising that experts `e1`, `e2`, and `e3` (who have been working separately) obtain different results. They do not necessarily assess the role of each criterion and the intensity of the interactions in the same way. This is why we decided to make them work together (expert `e4`). The experience exchanges that took place (exchanges that the experts found to be constructive - see Section 4), have resulted in a *consensus*. With the resulting data of this *consensus*, the Choquet integral does provide a ranking of the actions for 6 of the 10 configurations studied. Three rankings were obtained: In the second position, behind a_2, we have either a_4 or a_5; action a_1 always occupies the last position; and, a_3 always occupies the penultimate position. However, the position of a_3 appears to be very sensitive to the number of blank cards (see subsection 5.3.4). Indeed, it is sufficient to add a single blank card in certain intervals to rank a_3 either in the third position or even in the second position tied with a_4. Finally, in the 4 non conformity configurations, it may be enough to add one blank card in one and only one interval of the ranking of projects, $p_1 \ldots, p_8$, to restore the conformity. This can lead to having a_5 in the last position instead of a_1, which moves to the third position (see subsection 5.3.5).

b) This application of the Choquet integral with four experts highlights the following two points.

   1. The way the initial performances have been transformed to be encoded on a common utility scale:
  - It has a non significant influence on the results when the initial performances are modeled in a verbal way on a seven-level scale (from "very bad" to "very good") since we established that the levels "very bad", "average", and "very good" should naturally be encoded on the utility scale with the values 0, 0.5, and 1, respectively (see subsection 5.3.1).
  - It has a significant influence on the results when the initial performances are modeled on a numerical scales which refer to costs and surfaces (see subsection 5.3.3). With these scales the choice of the extreme values, which were used as reference levels to encode the 0 and 1 utility values, are more arbitrary. It is thus the choice of such performances that has the most significant influence on the results. The way the intermediate performances are encoded may also have an impact, but in a less systematic way.

   2. The experts worked together to assign numerical values to the capacities of the Choquet integral, this has shown that:
  - The choice of a value for the ratio `z` within the indetermination interval defined by the experts has a non significant impact on the results (see subsection 5.3.1).
  - The way the projects, $p_1, \ldots, p_8$, were ranked as well as the number of blank cards inserted in the intervals has, in most of the configurations studied, a significant impact on the results (see subsections 5.3.4 and 5.3.5).

c) The experts' reactions during the working phase that was performed to build the required data for the implementation of the Choquet integral, led us to the following findings:

1. The deck of cards method that was presented to assign utility values to the verbal scale levels was quickly understood by each of the experts, who subsequently have used it with no difficulties (see subsection 4.1). As for the numerical scales the approach that was followed to define the encodings `c3` and `s3` (see subsection 4.2) did not lead to any difficulty.

2. The proposed approach regarding the assignment of numerical values to the capacities highlighted some difficulties (see subsection 4.3). This leads us to call the attention of the reader to the following aspects:
   - Concerning the assignment of numerical values to the ratio `z`, the experts found this exercise quite difficult and they appreciated the possibility of defining only one range for the value of this ratio.
   - When the experts worked separately, they frequently hesitated about adopting a ranking of the eight projects for each of the four families proposed to them. In their opinion, what often made the comparison less significant was, on the one hand, the very unrealistic nature of the projects and, on the other hand, the large differences of their performances on incommensurable scales. Moreover, after adopting a ranking, the number of blank cards to be inserted in each of the intervals could again be a source of hesitation.
   - In the group work (expert `e4`), the experts appreciated the method. They also appreciated the work with the blank cards and considered it a good basis for conducting a decision aiding process with multiple actors. They argued that this way of working allows everyone to clearly explain the way in which they wish to rank the projects and to insert more or less blank cards in the intervals. Moreover, the ability to handle the cards during the discussion was considered very appropriate for reaching a *consensus*.

d) We highlight that we worked with the same experts and the same data used for the study on the ELECTRE III method with interaction between criteria (Bottero et al., 2015). In the previous work, the results obtained were validated by the experts, namely: Project a_2 was the first in the ranking before a_3, which occupied the second position. These two projects being clearly separated from the other three. It should be mentioned that this special position of a_3 did not come from the fact that in ELECTRE III the antagonism effect had been taken into account (note that this effect cannot be considered with the Choquet integral). Although the purpose of this article is not to compare the results obtained with two different methods, we may wish to understand why with the Choquet integral the same project does not always appear in second place (as a legitimate second), behind a_2. It seems to us (but it should be supported by other experiments), that we can present the following argument. The arbitrariness which is inevitably present in the construction of the required data to justify a ranking is less well controlled in the Choquet integral than in the ELECTRE III method with interaction between criteria. In the example studied, if we consider the case of a_3 (see Table 11. The results were highlighted in subsections 5.3.4 and 5.3.5) we observe that the position of a_3 varies in particular due to the following two factors:

– The choice of the extreme performances for criteria g_1 and g_4 which should serve to characterize the 0 and 1 values of the common utility scale.

– The choice of a ranking of the reference (not realistic) projects generates large differences in performances on scales which are difficult to commensurate. Note that such a ranking is used to take into account the interaction effects (mutual-strengthening and mutual-weakening).

The use of the Choquet integral to justify a ranking requires the examination of the impact of the arbitrariness which is present in the two preceding choices. Further work will seek to:

– Encode in terms of utility the performances of the numerical scales in a different way by taking into account the extreme performances of these scales, for example, in verbal terms.

– To define the projects to be ranked in order to obtain the numerical values of the capacities in a more realistic way by avoiding the presence of the extreme performances. Furthermore, we can envisage taking into account more reference projects than those related with the criteria and to the pairs of interacting criteria considered in the Choquet integral approach we have proposed, in order to mitigate the difficulties underlying the choice of the number of blank cards.

From a methodological point of view, in our opinion, the application we proposed shows that:

– The common utility scale and the numerical values for the capacities have to be built using user-friendly and easily understandable procedures, which permit decision-makers to clearly comprehend the problem and actively participate in the co-construction of the evaluation model. In fact, the conclusions stated previously show that the rankings are strongly dependent on these two points.

– The procedures, based on the deck of cards method, proposed in this paper were confirmed as a valid instrument to discuss with the experts and to collect reliable information for building the evaluation model.

– It is important to explore the consequences of the experts' hesitations in the construction of the common utility scale and the capacities of the Choquet integral, because this can provide useful insights into the stability of the rankings.

– From the preceding point, we must conclude that the way the utility values obtained led to ranking the actions in a complete pre-order may not be significant. The way of modeling hesitation and ill-determination through the use of indifference and preference thresholds led to ranking the actions in terms of pseudo orders, which make the results of the Choquet integral more robust. To the best of our knowledge, this is the first time this has been done in a multiple criteria application of the Choquet integral.

A. Appendix

Table 5: Utility values for each action according to criteria g_2, g_3, g_5, and g_6.

Actions (a_i)	utility $u_2(a_i)$			utility $u_3(a_i)$			utility $u_5(a_i)$			utility $u_6(a_i)$		
	b1	b2	b3	b1	b2	b3	b1	b2	b3	b1	b2	b3
a_1	0.3333	0.3889	0.2778	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
a_2	0.3333	0.3889	0.2778	0.6667	0.6111	0.7222	0.6667	0.6111	0.7222	1.0000	1.0000	1.0000
a_3	0.0000	0.0000	0.0000	0.8333	0.7778	0.8333	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
a_4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8333	0.7778	0.8333	1.0000	1.0000	1.0000
a_5	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.3333	0.3889	0.2778	0.0000	0.0000	0.0000

Table 6: Utility values for each action according to criteria g_1 and g_4

Actions (a_i)	utility $u_1(a_i)$			utility $u_4(a_i)$		
	c1	c2	c3	s1	s2	s3
a_1	1.0000	0.9922	0.9825	0.2500	0.3077	0.3750
a_2	0.9828	0.9791	0.9535	1.0000	0.7692	0.7500
a_3	0.9310	0.9399	0.8665	0.5500	0.4923	0.5250
a_4	0.8966	0.9138	0.8080	0.6250	0.5385	0.5625
a_5	0.0000	0.2350	0.1825	0.0000	0.1538	0.2500

Table 7: Capacities of the Choquet integral by expert and *per* option.

		z	Capacities of the Choquet Integral							
			μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_{45}	μ_{15}
Expert e1	c1s1	80	0.0633	0.1782	0.1782	0.2356	0.1207	0.0058	0.2931	0.4655
	c1s2	80	0.0633	0.1782	0.1782	0.2356	0.1207	0.0058	0.2931	0.4655
	c2s1	100	0.0567	0.1598	0.1598	0.2113	0.1083	0.0052	0.2629	0.5206
	c2s2	100	0.0567	0.1598	0.1598	0.2113	0.1083	0.0052	0.2629	0.5206
Expert e2	c1s1	70	0.2987	0.3960	0.0715	0.0715	0.0715	0.0066	0.1364	0.4610
	c1s2	70	0.2987	0.3960	0.0715	0.0715	0.0715	0.0066	0.1364	0.4610
	c2s1	70	0.2929	0.4075	0.0639	0.0639	0.0639	0.0066	0.1212	0.4647
	c2s2	70	0.2643	0.4075	0.0639	0.0639	0.0639	0.0066	0.1212	0.4647
Expert e3	c1s1	20	0.1512	0.0974	0.2949	0.2051	0.3488	0.0255	0.4206	0.5104
	c1s1	25	0.1491	0.0940	0.2959	0.2041	0.3509	0.0206	0.4243	0.5161
	c1s2	15	0.1542	0.1020	0.2935	0.2239	0.3458	0.0323	0.4328	0.4851
	c1s2	20	0.1509	0.0968	0.2951	0.2230	0.3491	0.0247	0.4393	0.4934
	c2s1	20	0.1665	0.0962	0.2895	0.2192	0.3423	0.0259	0.4126	0.5180
	c2s1	25	0.1647	0.0928	0.2904	0.2186	0.3443	0.0210	0.4162	0.5240
	c2s2	20	0.1585	0.0919	0.2750	0.2250	0.3249	0.0254	0.4247	0.5079
	c2s2	30	0.1552	0.0862	0.2759	0.2241	0.3276	0.0172	0.4310	0.5172
Expert e4	c1s1	90	0.2091	0.3114	0.0864	0.1273	0.1273	0.0046	0.3114	0.4136
	c1s2	90	0.1225	0.2990	0.1814	0.2402	0.2402	0.0049	0.3186	0.4363
	c2s1	90	0.4397	0.2771	0.0603	0.1145	0.1145	0.0061	0.2229	0.5481
	c2s2	100	0.1223	0.3188	0.1616	0.2402	0.2402	0.0044	0.3188	0.4367

B. Appendix

Table 8: Distribution of the utility values $\min^z\{a_i\}$

$\Delta^z(a_i)$	$Nb.$	$\min^z\{a_i\}$	$\Delta^z(a_i)$	$Nb.$	$\min^z\{a_i\}$	$\Delta^z(a_i)$	$Nb.$	$\min^z\{a_i\}$
0.000017	1	0.624699	0.000596	1	0.219691	0.002475	1	0.218640
0.000077	1	0.490508	0.000597	1	0.663873	0.002509	1	0.179804
0.000080	1	0.450089	0.000608	1	0.654536	0.002517	1	0.180248
0.000088	2	0.316572	0.000611	2	0.739096, 0.476550	0.002589	1	0.199511
0.000099	1	0.444453	0.000612	1	0.654743	0.002602	1	0.211009
0.000104	1	0.443163	0.000635	1	0.535318	0.002713	1	0.255884
0.000109	1	0.445917	0.000638	1	0.454520	0.002720	1	0.191514
0.000109	2	0.418322(2)	0.000663	1	0.477044	0.002721	1	0.295259
0.000110	1	0.441044	0.000670	1	0.531768	0.002730	1	0.295765
0.000113	3	0.404508, 0.420436(2)	0.000677	1	0.505307	0.002863	1	0.237685
0.000114	1	0.391164	0.000677	2	0.505307(2)	0.006217	1	0.540023
0.000115	1	0.377820	0.000683	1	0.323549	0.006266	1	0.523888
0.000136	1	0.433587	0.000690	1	0.305839	0.006536	1	0.647971
0.000143	1	0.400855	0.000692	1	0.454378	0.006600	2	0.518273(2)
0.000147	1	0.434745	0.000700	1	0.316864	0.006608	1	0.375475
0.000148	2	0.402968, 0.505424	0.000703	1	0.505984	0.007937	1	0.278661
0.000242	1	0.470206	0.000706	1	0.300246	0.008125	1	0.283817
0.000243	2	0.456862, 0.633636	0.000711	2	0.781706, 0.283628	0.008189	1	0.246878
0.000253	1	0.526572	0.000760	1	0.446246	0.008374	1	0.252102
0.000256	1	0.423853	0.000789	1	0.423580	0.010966	2	0.503325(2)
0.000271	1	0.525573	0.000828	1	0.377396	0.013321	1	0.264110
0.000274	1	0.631695	0.000836	2	0.387464(2)	0.013587	1	0.443518
0.000286	1	0.624699	0.000860	1	0.363202	0.013640	1	0.268899
0.000289	1	0.419881	0.000870	2	0.372178(2)	0.013669	1	0.420170
0.000304	1	0.629755	0.001133	1	0.525514	0.014552	1	0.435537
0.000312	1	0.624942	0.001169	1	0.717182	0.017012	1	0.288827
0.000317	1	0.441128	0.001331	2	0.515140(2)	0.026766	1	0.454520
0.000323	1	0.429867	0.001454	1	0.377993			
0.000352	1	0.437578	0.001456	1	0.378144			
0.000377	1	0.739970	0.001483	1	0.359058			
0.000418	1	0.244524	0.001489	1	0.382083			
0.000444	1	0.638146	0.001518	1	0.362573			
0.000469	1	0.631739	0.001524	1	0.757368			
0.000483	1	0.641638	0.001573	1	0.334668			
0.000500	1	0.621695	0.001608	1	0.338183			
0.000519	1	0.629998	0.001935	1	0.715150			
0.000543	1	0.217881	0.002166	1	0.185228			
0.000557	1	0.652823	0.002169	1	0.198819			
0.000573	1	0.707185	0.002291	1	0.326927			
0.000582	1	0.654507	0.002295	1	0.340517			

Table 9: Distribution of the utility values $\min^{bc}\{a_i\}$

$\Delta^{bc}(a_i)$	$Nb.$	$\min^{bc}\{a_i\}$	$\Delta^{bc}(a_i)$	$Nb.$	$\min^{bc}\{a_i\}$
0.0000	1	0.237685	0.0066	2	0.647956, 0.447756
0.0001	4	0.278661, 0.739970, 0.283817, 0.739096	0.0067	1	0.3754
0.0002	1	0.717182	0.0068	2	0.518273(2)
0.0003	3	0.444453, 0.445991, 0.715150	0.0070	1	0.41706
0.0004	3	0.423580, 0.505545(2)	0.0073	2	0.469786, 0.434129
0.0005	2	0.433587, 0.255884	0.0074	1	0.41277
0.0006	2	0.441044, 0.443163	0.0077	1	0.430190
0.0007	1	0.191514	0.0079	1	0.420170
0.0009	1	0.450089	0.0084	1	0.450207
0.0010	1	0.211009	0.0088	1	0.445752
0.0011	2	0.651679, 0.445917	0.0093	1	0.316572, 0.467253
0.0013	1	0.199511	0.0100	1	0.525954
0.0014	1	0.434745	0.0102	1	0.185228
0.0015	1	0.638146	0.0104	1	0.522015
0.0016	2	0.218640, 0.632009	0.0111	1	0.198819
0.0017	4	0.378144, 0.377993, 0.264110, 0.268899	0.0114	1	0.334668
0.0018	3	0.641638, 0.504873, 0.457105	0.0116	1	0.445456
0.0019	5	0.652823, 0.652595, 0.663873, 0.524762, 0.470448	0.0119	1	0.338183
0.0021	1	0.382083	0.0130	1	0.270598
0.0022	1	0.707185	0.0131	1	0.217881
0.0025	1	0.375413	0.0132	1	0.287046
0.0026	1	0.781706	0.0134	1	0.303494
0.0028	1	0.388449	0.0146	1	0.435503
0.0029	4	0.523888, 0.628838, 0.621844, 0.622084	0.0148	1	0.244524
0.0030	6	0.654507, 0.631739, 0.757368, 0.540023, 0.513441(2)	0.0172	1	0.454520
0.0031	2	0.401485, 0.621604	0.0185	1	0.179804
0.0039	4	0.246878, 0.252102, 0.416507(2)	0.0186	1	0.180248
0.0041	3	0.625667, 0.400855, 0.398896	0.0187	1	0.358703
0.0044	1	0.629998	0.0189	1	0.326927
0.0046	4	0.418322(2), 0.367536(2)	0.0191	2	0.368327(2)
0.0047	1	0.358490	0.0197	1	0.340517
0.0049	1	0.359058	0.0201	1	0.485521
0.0052	1	0.621695	0.0210	1	0.469577
0.0053	1	0.362573	0.0234	1	0.282435
0.0054	1	0.318138	0.0248	1	0.219691
0.0056	1	0.300287	0.0317	2	0.295259, 0.295765
0.0062	2	0.503325(2)	0.0352	1	0.491664
0.0064	1	0.423769	0.0353	1	0.490585

Table 10: Configuration by expert for `bh` and `zmin`

Expert	Configuration	Pseudo order	Configuration	Pseudo order
e1	c1s1onc	$a_2 P a_4 P a_3 P a_5 P a_1$	c2s1onc	$a_2 P a_4 P a_3 P a_5 P a_1$
	c1s1ace	$a_2 P a_4 P a_3 P a_5 P a_1$	c2s1ace	$a_2 P a_4 P a_3 P a_5 P a_1$
	c1s2onc	$a_2 P a_4 P a_3 P a_5 P a_1$	c2s2onc	$a_2 P a_4 P a_5 Q a_3 P a_1$
	c1s2ace	$a_2 P a_4 P a_3 P a_5 P a_1$	c2s2ace	$a_2 P a_4 P(a_3 I a_5) P a_1$
e2	c1s1onc	$a_2 P a_1 P a_4 Q a_5 P a_3$	c2s1onc	$a_2 P a_1 P a_5 Q a_4 P a_3$
	c1s1ace	$a_2 P a_1 P a_4 P a_5 P a_3$	c2s1ace	$a_2 P a_1 P(a_4 I a_5) P a_3$
	c1s2onc	$a_2 P a_1 P(a_4 I a_5) P a_3$	c2s2onc	$a_2 P a_5 P a_1 Q a_4 P a_3$
	c1s2ace	$a_2 P a_1 P(a_4 I a_5) P a_3$	c2s2ace	$a_2 P a_5 P a_1 Q a_4 P a_3$
e3	c1s1onc	$a_2 P a_3 Q a_4 P a_1 P a_5$	c2s1onc	$a_2 P a_3 P a_4 P a_1 P a_5$
	c1s1ace	$a_2 P a_3 P a_4 P a_1 P a_5$	c2s1ace	$a_2 P a_3 P a_4 P a_1 P a_5$
	c1s2onc	mutual-strengthening	c2s2onc	$a_2 P(a_4 I a_3) P a_1 P a_5$
	c1s2ace	mutual-strengthening	c2s2ace	$a_2 P a_3 Q a_4 P a_1 P a_5$
e4	c1s1onc	mutual-weakening	c2s1onc	mutual-strengthening
	c1s1ace	mutual-weakening	c2s1ace	mutual-strengthening
	c1s2onc	$a_2 P a_4 P(a_5 I a_3) P a_1$	c2s2onc	$a_2 P a_5 P a_4 P a_3 P a_1$
	c1s2ace	$a_2 P a_4 P a_5 Q a_3 P a_1$	c2s2ace	$a_2 P a_5 P a_4 P a_3 P a_1$

Table 11: Pseudo order by expert for `bh` and `zmin`

| Expert	Pseudo order	Configurations (complete description)
e1	$a_2Pa_4Pa_3Pa_5Pa_1$	`c1s1b1e1z80onc, c1s2b1e1z80onc, c2s1b1e1z80onc, c1s1b1e1z80ace,`
		`c1s2b1e1z80ace, c2s1b1e1z80ace`
	$a_2Pa_4Pa_5Qa_3Pa_1$	`c2s2b1e1z80onc`
	$a_2Pa_4P(a_3Ia_5)Pa_1$	`c2s2b1e1z80ace`
	$a_2P(a_4Ia_5)Qa_3Pa_1$	`c3s3b1e1z80onc`
	$a_2Pa_4Pa_3Qa_5Pa_1$	`c3s3b1e1z80ace`
e2	$a_2Pa_1Pa_4Qa_5Pa_3$	`c1s1b2e2z70onc`
	$a_2Pa_1P(a_4Ia_5)Pa_3$	`c1s2b2e2z70onc, c1s2b2e2z70ace, c2s1b2e2z70ace`
	$a_2Pa_1Pa_5Qa_4Pa_3$	`c2s1b2e2z70onc`
	$a_2Pa_5Pa_1Qa_4Pa_3$	`c2s2b2e2z70onc, c2s2b2e2z70ace`
	$a_2Pa_1Pa_4Pa_5Pa_3$	`c1s1b2e2z70ace`
	$a_2Pa_5Pa_1Pa_4Pa_3$	`c3s3b2e2z70onc, c3s3b2e2z70ace`
e3	$a_2Pa_3Qa_4Pa_1Pa_5$	`c1s1b3e3z20onc, c2s2b3e3z20ace, c3s3b3e3z20onc, c3s3b3e3z20ace`
	$a_2Pa_3Pa_4Pa_1Pa_5$	`c2s1b3e3z20onc, c1s1b3e3z20ace, c2s1b3e3z20ace`
	$a_2Pa_4Qa_3Pa_1Pa_5$	`c2s2b3e3z20onc`
		(mutual-strengthening violated for the two configurations with `c1s2`)
e4	$a_2Pa_4P(a_5Ia_3)Pa_1$	`c1s2b1e4z90onc`
	$a_2Pa_5Pa_4Pa_3Pa_1$	`c2s2b1e4z90onc, c2s2b1e4z90ace, c3s3b1e4z90onc, c3s3b1e4z90ace`
	$a_2Pa_4Pa_5Qa_3Pa_1$	`c1s2b1ef4z90ace`
		(mutual-weakening violated for the two configurations with `c1s1`; and,
		(mutual-strengthening violated for the two configurations with `c2s1`)

C. Appendix
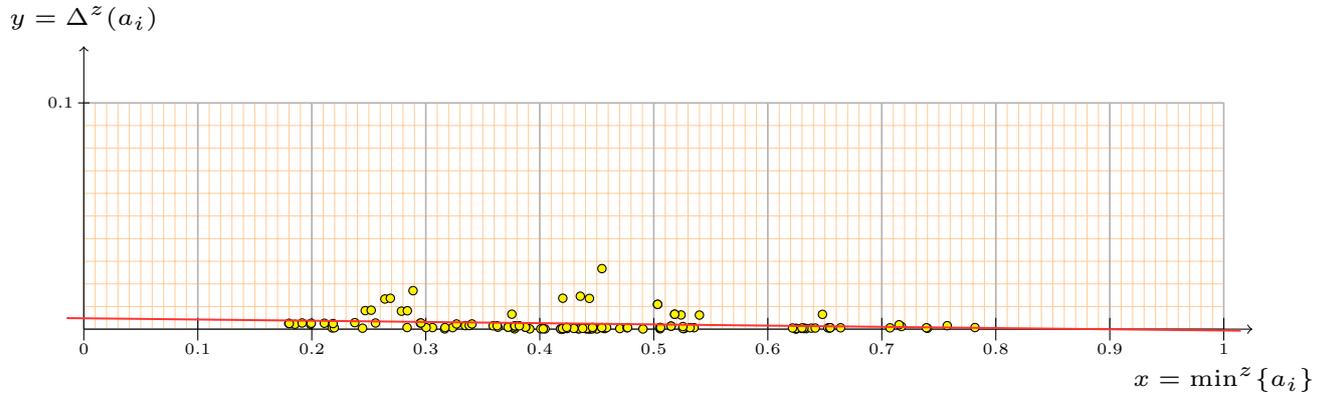
Figure 1: Regression line for the points $(\min^z\{a_i\}, \Delta^z(a_i))$: $y = -0.0054x + 0.0048$
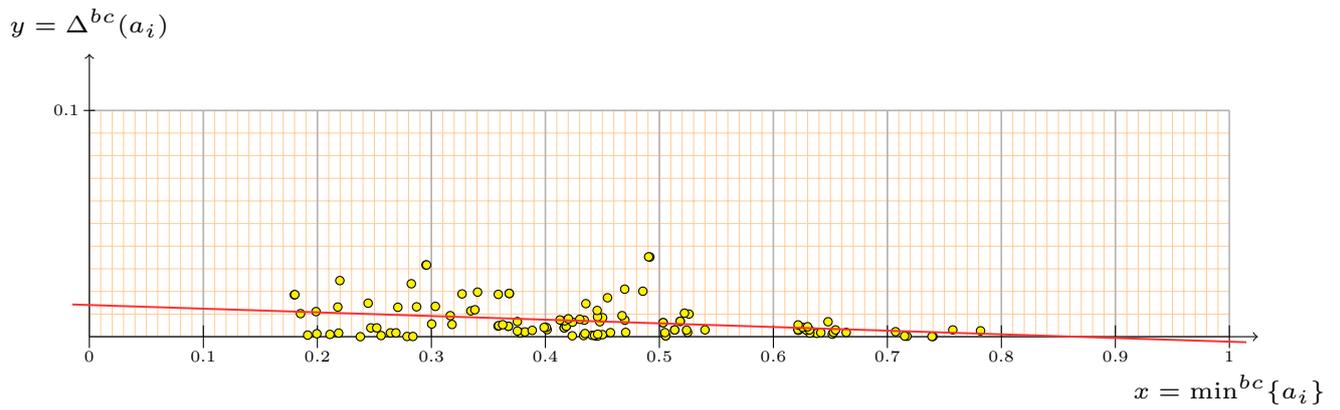

Figure 2: Regression line for the points $(\min^{bc}\{a_i\}, \Delta(a_i))$: $y = -0.0162x + 0.014$
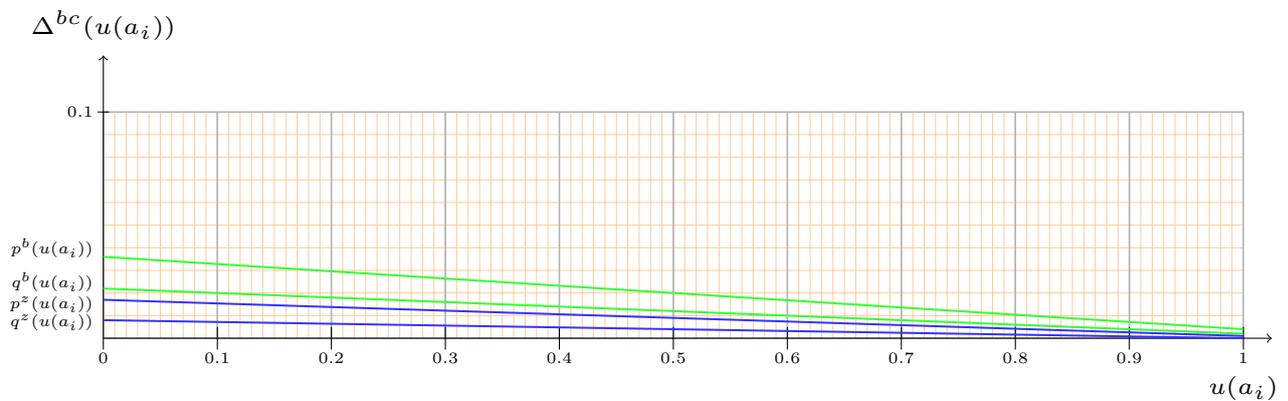

Figure 3: Indifference and preference thresholds lines

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