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COOPERATION IN CONFLICT ANALYSIS ¹

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LA COOPERATION DANS L'ANALYSE DES CONFLITS

RESUME

On présente des définitions et des procédures pour modéliser et analyser la coopération qui peut avoir lieu dans un conflit. En particulier, on propose de meilleures définitions de la coopération pour des situations dans lesquelles deux décideurs peuvent coopérer dans un conflit entre deux décideurs ou plus. En utilisant une application simple à une procédure de négociateurs dans le travail, on montre un moyen d'appliquer facilement ces définitions. On propose de nouvelles directions de recherche pour le futur qui comprennent l'intégration de la coopération dans un SIAD existant pour effectuer à terme des études d'analyse des conflits.

COOPERATION IN CONFLICT ANALYSIS

ABSTRACT

Definitions and procedures are presented for modelling and analyzing cooperation that can take place in a dispute. In particular, improved definitions on cooperation are given for the situation where two decision makers can cooperate with one another in a conflict consisting of two or more decision makers. Using a simple application in labor-management negotiations, the way in which the definitions can be easily applied is explained. Future research directions are suggested, including the incorporation of cooperation into an existing decision support system for carrying out conflict analysis studies.

1. INTRODUCTION

Not all conflicts are strictly competitive. In bargaining and negotiation, decision makers must cooperate in order to come up with a contract that is both beneficial and acceptable to all parties concerned. For example, in negotiations for arms reductions by the superpowers, both sides must cooperate to be able to reach a resolution which is better for both parties. Likewise, in disputes ranging from toxic chemical pollution to labor-management negotiations, cooperation is needed to resolve the disputes optimally. These situations are dealt with theoretically by defining cooperative options, which are options that can only be taken by joint action involving two or more parties.

One type of cooperation which commonly arises is the situation where two decision makers have one or more cooperative options shared between themselves. For example, consider the policy of Great Britain with respect to the balance of power in Europe during the past four centuries. Britain would usually cooperate with the second most powerful country in continental Europe in order to counteract the influence of the strongest nation. This shrewd policy eventually led to the military defeats of Spain, France and Germany at different times in the past when each of these nations pursued expansionist policies in Europe. Another example of cooperation between two decision makers is when labor and management jointly select cooperative options to allow a legal contract to be signed by both sides.

Because of the importance of the above type of cooperation among two decision makers taking part in a conflict, the objective of this paper is to present theoretical definitions and related procedures that will allow this situation to be modelled and analyzed in practice. More specifically, in a game or conflict having two or more decision makers, the upcoming theory permits one group of two decision makers to cooperate with one another. The results constitute an extension of the conflict analysis methodology [4, 5, 10] for studying real world conflict in which cooperation takes place. Previously, some research had been carried out for incorporating the ability to handle cooperation into conflict analysis [1, 2, 3, 5, 8]. However, in this paper more precise definitions are presented and the theory is significantly improved.

The systematic study of an actual dispute using conflict analysis consists of the two main stages of modelling and analysis [1-11]. At the modelling

step, one must determine the decision makers involved in the conflict, each decision maker's options or available courses of action, and each decision maker's preferences among the possible outcomes or scenarios. At the stability analysis stage, one calculates the stability of each outcome from each player's point of view. In general, an outcome is stable for a given decision maker if it is not advantageous for the decision maker to move unilaterally to a more preferred outcome by changing his selection of options. A particularly robust solution concept for describing human behavior and hence calculating stability, is the sequential stability solution concept [4, 5, 11]. When a given outcome is stable for every decision maker, it constitutes an equilibrium or possible compromise resolution to the game.

In the next section, definitions are given for modelling cooperation and noncooperation in a dispute having a total of n -decision makers, where $n \geq 2$, and two decision makers can cooperate with one another. Subsequently, the sequential stability solution concept is expanded for use when cooperation is taking place. Before the conclusions, a simple example on labor - management negotiations is presented for explaining how the foregoing ideas are applied in practice.

2. DEFINITIONS

In order to be able to study cooperation among two decision makers in an n-decision maker conflict, various ideas must be clearly defined. Following the presentation of all required definitions needed to develop a game model, the procedure for stability analysis is explained. The authors would like to emphasize that although some versions of the definitions given here appeared in earlier research [1, 2, 3, 5, 8], the current paper contains theoretical improvements, fills in gaps in the definitions, eliminates some inconsistencies that are present in the other related research, and presents the research in a systematic manner within a single document. Furthermore, because the proofs for Theorems 1 and 2 in this paper follow the same line of reasoning for the corresponding theorems when cooperation is not present [5, 11], the details of the proofs are not given.

2.1 Model Definitions

Cooperation in bargaining and negotiation requires that two types of outcomes be defined. These outcomes are called *cooperative outcomes* (CO's) and *non-cooperative outcomes* (NCO's), and are defined below for the case of an n-decision maker game in which one set of two decision makers, called i and j, can cooperate. After establishing the preferences for each decision maker, a stability analysis can be carried out for the resulting game.

Definition 1. - Decision Makers. The participants or players in a conflict are called the decision makers. The set of n decision makers is given by

$$N = \{1, 2, \dots, i, \dots, n\} \quad (1)$$

Definition 2. - Options. A given decision maker i possesses a set of available options defined by

$$O_i = \{o_{1i}, o_{2i}, \dots, o_{m_i i}\} \text{ for all } i \in N \quad (2)$$

Definition 3. - Strategies. Any subset of options that can be taken from the set O_i by a particular decision maker i is called a strategy. A specific strategy for decision maker i is denoted by s_{ri} . The strategy $s_{ri} \in P(O_i)$, where $P(O_i)$ is the power set (set of all subsets) of O_i . The set of all possible strategies for decision maker i is denoted by S_i , where $s_{ri} \in S_i$ for all $i \in N$.

For a particular strategy, s_{ri} , decision maker i can select or reject an option o_{ki} . Keep in mind that in the practical implementation of a strategy, the options which are not selected by a decision maker also form part of the decision maker's course of action contained in the particular strategy.

Let $f(o_{ki})$ be a function mapping an option o_{ki} to the symbols Y or N . Option o_{ki} is said to be selected by decision maker i if it appears in his strategy s_{ri} and it is not taken if it is not part of the strategy s_{ri} . Hence:

$$f(o_{ki}) = \begin{cases} Y, & \text{if } o_{ki} \in s_{ri} \\ N, & \text{if } o_{ki} \notin s_{ri} \end{cases} \quad (3)$$

where Y means "yes" the option is taken and N indicates "no", it is not selected.

Definition 4. - Outcomes. An outcome is formed when each decision maker selects a strategy. Let s_{ri} be a specific strategy for decision maker i , where $s_{ri} \in S_i$. A particular outcome is given by $q = \{s_{a1}, s_{b2}, \dots, s_{ri}, \dots, s_{in}\}$ where $s_{a1} \in S_1$, $s_{b2} \in S_2$, $\dots$, $s_{ri} \in S_i$, $\dots$, $s_{in} \in S_n$. The set of all mathematically possible outcomes, Q , is defined as the Cartesian product of all the decision makers' strategy sets:

$$Q = \{S_1 \times S_2 \times \dots \times S_i \times \dots \times S_n\} \quad (4)$$

In practice, there are some outcomes which are infeasible due to reasons given below for the case of cooperation as well as elsewhere for non-cooperation. For example, a given decision maker may have some options under his control that are mutually exclusive and, hence, he can only choose at most one of them at any one point in time. Accordingly, any outcome that contains a choice of more than one of these options is also infeasible. When modelling and analyzing a game, one wishes only to deal with feasible outcomes. Let the set of feasible outcomes be denoted by Q^* where

$$Q^* \subseteq Q \quad (5)$$

Definition 5. - Cooperative Options. Let decision makers i and j in an n -decision maker game be the two decision makers who can cooperate. Also, let $\bar{o}_{ki}$ and $\bar{o}_{hj}$ be two specific cooperative options controlled by decision makers i and j , respectively, where the overbar (-) indicates these options are cooperative. Options $\bar{o}_{ki}$ and $\bar{o}_{hj}$ are said to be cooperative if

$$f(\bar{o}_{ki}) = f(\bar{o}_{hj}) \quad (6)$$

for all $q \in Q^*$.

Comments: According to the definition in Equation 6, for options $\bar{o}_{ki}$ and $\bar{o}_{hj}$ to be cooperative, decision makers i and j must either select option $\bar{o}_{ki}$ and $\bar{o}_{hj}$, respectively (i.e. $f(\bar{o}_{ki}) = f(\bar{o}_{hj}) = Y$), or the decision makers must not take either option (i.e. $f(\bar{o}_{ki}) = f(\bar{o}_{hj}) = N$). The situation where $f(\bar{o}_{ki}) \neq f(\bar{o}_{hj})$ is not allowed.

In practice, these two cooperative options are often defined exactly the same. For example, in a labor-management negotiation conflict, these options may be written as "compromise" for both i and j . In some situations, however, options $\bar{o}_{ki}$ and $\bar{o}_{hj}$ may be defined differently. For example, $\bar{o}_{ki}$ may be the option where decision maker i "sells a car for \$15,000" whereas $\bar{o}_{hj}$ is the option where decision maker j "buys the car for \$15,000". Because of this, the condition where $\bar{o}_{ki} \equiv \bar{o}_{hj}$ (i.e. $\bar{o}_{ki}$ and $\bar{o}_{hj}$ are defined the same) is not attached to the definition in Equation 6.

Definition 6. - Noncooperative Options. If one option does not depend cooperatively on another as defined in Equation 6, the option is called noncooperative. The total set of options, O_i , available to decision maker i consists of either both the cooperative options (Definition 5) and the noncooperative ones (Definition 6) or else just one of these two types.

Definition 7. - Cooperative Strategies. Let $\bar{o}_{ki}$ be a cooperative option for decision maker i according to the definition in Equation 6. Strategy $\bar{s}_i$ is a cooperative strategy for decision maker i when there exists a cooperative option $\bar{o}_{ki}$ such that

$$f(\bar{o}_{ki}) = Y \quad (7)$$

Likewise, one can define a cooperative strategy for decision maker j who cooperates with decision maker i . Let $\bar{o}_{hj}$ be the cooperative strategy for decision maker j according to Equation 6. Strategy $\bar{s}_{pj}$ is a cooperative strategy for decision maker j when there exists a cooperative strategy $\bar{o}_{hj}$ such that

$$f(\bar{o}_{hj}) = Y \quad (8)$$

One can combine Equations 7 and 8 to obtain an equivalent definition for cooperative strategies. For cooperative strategies $\bar{s}_{pi}$ and $\bar{s}_{pj}$ to be formed for decision makers i and j , respectively, there exists a pair of cooperative options $(\bar{o}_{ki}, \bar{o}_{hj})$ such that

$$f(\bar{o}_{ki}) = f(\bar{o}_{hj}) = Y \quad (9)$$

Comments: In the above definitions, $\bar{o}_{ki}$ represents a cooperative option for decision maker i which is contained in decision maker i 's strategy $\bar{s}_{pi}$. Likewise, $\bar{o}_{hj}$ is a cooperative option for decision maker j which is part of j 's strategy $\bar{s}_{pj}$. According to Equations 7 to 9, for a cooperative outcome to be formed, both cooperative options must be taken by the decision makers (i.e. $f(\bar{o}_{ki}) = f(\bar{o}_{hj}) = 1$).

For a specific application, one may wish to place further restrictions on the pairs of cooperative options. For example, it may only be possible for one cooperative option pair to be selected at any one point in time. In labor-management negotiations, for instance, only one contract can be accepted from a range of proposed contracts. These situations can be identified using standard procedures for specifying which feasible combinations of options are possible for forming strategies.

Definition 8. - Noncooperative Strategies. Strategies which do not satisfy Definition 7 are referred to as noncooperative strategies. The total set of strategies, S_i , available to decision maker i consists of either both the cooperative strategies (Definition 7) and the noncooperative ones (Definition 8) or else just one of these two kinds of strategies.

Definition 9. - Cooperative Outcomes (CO's). Let decision makers i and j in an n -decision maker game be the two decision makers who can be cooperative because they share at least one pair of cooperative options. An outcome $\bar{q}$ is defined to be cooperative when these two decision makers decide to select at least one pair of cooperative options according to Equation 9. An overbar is used to indicate that decision makers i and j have selected cooperative strategies. Because the cooperation is restricted to the two decision makers i and j , the remaining decision makers, denoted by $N-i-j$, must select noncooperative strategies. Hence,

$$\bar{q} = \{\bar{s}_i, \bar{s}_j, s_{k,N-i-j}\} \quad (10)$$

where $s_{k,N-i-j}$ is the joint noncooperative strategy selected across the remaining decision makers.

Comments: An example of a cooperative outcome in labor-management negotiations is the situation where labor and management agree upon a final contract settlement while other interested parties such as government bodies no longer have to intervene in the negotiations in order to help labor and management reach an agreement.

Definition 10. - Noncooperative Outcomes (NCO's). Outcomes which do not satisfy definition 9 are referred to as noncooperative outcomes. The total set of feasible outcomes Q^* in Equation 5 can consist of both cooperative and noncooperative outcomes, only cooperative outcomes, or only noncooperative outcomes.

Definition 11. - Individual Preference Function for NCO's. For each decision maker i , the preference function, M_i , for decision maker i maps elements from the set Q^* to elements in the set $P(Q^*)$, the power set of Q^* . Closely related to the preference function is a number of useful sets. For the case of NCO's let

$$M_i^+(q) = \{\text{set of all NCO's preferred by } i \text{ to outcome } q \in Q^*\} \quad (11)$$

and

$$M_i^-(q) = \{\text{set of all NCO's not preferred by } i \text{ to outcome } q\} \quad (12)$$

Outcome $q \in M_i^-(q)$ if q is a NCO.

In terms of NCO's, the overall preference of decision maker i is written as

$$M_i = \{M_i^+(q) \mid \forall q \in Q^*\} \quad (13)$$

Definition 12. - Unilateral Movements for NCO's. Let s_i stand for a strategy for decision maker i and s_{N-i} a strategy for the remaining decision makers except for decision maker i . A NCO outcome q is formed by the strategy pair (s_i, s_{N-i}) . When a strategy is underlined, this means that the strategy remains fixed. Let the set of NCO's that are accessible to decision maker i from outcome q by decision maker i changing his or her strategies and the other decision makers, $N-i$, remaining at a fixed strategy, $\underline{s}_{N-i}$, be given by the set

$$m_i(q) = \{ (s_i, \underline{s}_{N-i}) : s_i \in S_i, \underline{s}_{N-i} \in S_{N-i} \} \quad (14)$$

The set $m_i(q)$ is called the set of *unilateral movements* by i from outcome q .

An important subset of the unilateral movements is the set of *unilateral improvements* (UI's) defined as

$$m_i^+(q) = m_i(q) \cap M_i^+(q) \quad (15)$$

The set of unilateral movements not preferred by decision maker i to outcome q is given as

$$m_i^-(q) = m_i(q) \cap M_i^-(q) \quad (16)$$

This set is referred to as the set of *unilateral disimprovements*.

Finally, the overall set of unilateral movements, $m_i(q)$, consists of both the UI's, $m_i^+(q)$, and the unilateral disimprovements $m_i^-(q)$. Hence,

$$m_i(q) = \{m_i^+(q) \cup m_i^-(q)\} \quad (17)$$

Comments: The outcomes contained in the sets defined by $m_i(q)$, $m_i^+(q)$ and $m_i^-(q)$ are called unilateral movements, unilateral improvements and unilateral disimprovements, respectively. Likewise, the strategies of decision maker i which form outcomes in these three sets are referred to by the same name.

For example, when decision maker i has a UI from outcome q to outcome p , both outcome p and decision maker i 's strategy in p are called UI's.

Definition 13. - *Individual Preference Function for CO's.* Let

$$\bar{M}_i^+(q) = \{\text{set of CO's preferred by } i \text{ to outcome } q\} \quad (18)$$

and

$$\bar{M}_i^-(q) = \{\text{set of CO's not preferred by } i \text{ to outcome } q\}. \quad (19)$$

Outcome $q \in \bar{M}_i^-(q)$ if q is a CO. In terms of CO's, the overall preferences of decision maker i are

$$\bar{M}_i = \{\bar{M}_i^+(q) \vee q \in Q^*\} \quad (20)$$

The overall preferences for decision maker i are based on preferences for both NCO's and CO's, and are defined as

$$M_i^*(q) = \{M_i^+(q), \bar{M}_i^-(q) \vee q \in Q^*\} \quad (21)$$

Definition 14. - *Joint Preference Functions for CO's.* Because decision makers i and j can cooperate, one must consider their joint preferences. Therefore,

define

$$\bar{M}_{i,j}^+(q) = \bar{M}_i^+(q) \cap \bar{M}_j^+(q) \quad (22)$$

as the set of CO's that both decision makers i and j prefer to outcome q .

Also, define

$$\bar{M}_{i,j}^-(q) = \bar{M}_i^-(q) \cup \bar{M}_j^-(q) \quad (23)$$

as the set of CO's that either decision maker i or j does not prefer to outcome q . Outcome q is contained in this set if it is a CO.

Definition 15. - Joint Movements for CO's. Let $\bar{s}_i$ and $\bar{s}_j$ stand for cooperative strategies for decision makers i and j , respectively. Also, let $\underline{s}_{N-i-j}$ represent a fixed noncooperative strategy of the remaining decision makers. Keep in mind that only two decision makers, say i and j , are allowed to cooperate with one another. For this situation, a CO, say $\bar{p}$, is written as

$$\bar{p} = (\bar{s}_i, \bar{s}_j, \underline{s}_{N-i-j}), \quad (24)$$

where $\bar{p} \in Q^*$.

The set of CO's that are accessible jointly to decision makers i and j from outcome q is given by

$$\bar{m}_{ij}(q) = \{(\bar{s}_i, \bar{s}_j, \underline{s}_{N-i-j}): \text{all possible cooperative strategies for } i \text{ and } j \text{ are considered}\} \quad (25)$$

For the above and other related definitions, decision makers i and j can be moving away from either a NCO q or a CO outcome $\bar{q}$ to a CO $\bar{p}$. The outcomes contained in $\bar{m}_{ij}(q)$ are called *cooperative movements*.

The set of *cooperative improvements* (CI's) for i and j is defined as

$$\bar{m}_{ij}^+(q) = \bar{m}_{ij}(q) \cap \bar{M}_{ij}^+(q) \quad (26)$$

The set of *cooperative disimprovements* for either i or j is given as

$$\bar{m}_{ij}^-(q) = \bar{m}_{ij}(q) \cap \bar{M}_{ij}^-(q) \quad (27)$$

Finally,

$$\bar{m}_{ij}(q) = \bar{m}_{ij}^+(q) \cup \bar{m}_{ij}^-(q) \quad (28)$$

Comments: The sets of outcomes defined by $\bar{m}_{ij}(q)$, $\bar{m}_{ij}^+(q)$ and $\bar{m}_{ij}^-(q)$, are called cooperative movements, cooperative improvements and cooperative disimprovements, respectively. Likewise, the strategies of decision makers i and j which form outcomes in these three sets are referred to by the same name.

A CI always involves a change in selection for at least one cooperative option by decision makers i and j . However, a CI could also include changes in the choice of one or more noncooperative options by i , j or both decision makers, as long as the resulting outcome is jointly more preferred (see Equations 20 and 24) by both i and j .

Definition 16. - Non-cooperative Improvements. Decision maker i is said to possess a non-cooperative improvement (NCI) from a CO $\bar{q}$ to a more preferred NCO p or CO $\bar{p}$, if he can unilaterally change his strategy to end up at either a more preferred NCO or CO. This definition assumes that decision maker i only changes the selection of his noncooperative and/or cooperative options based on his own preferences (see Definitions 11 and 13) and not his joint preferences with j (see Definition 14). In addition, the strategies of decision makers $N-i$ remain fixed and only decision maker i changes his strategy selection. However, decision maker i 's strategy change can involve noncooperative option changes, cooperative option changes or both. For example, decision maker i may select all of the same cooperative options but only alter his choice of a noncooperative option in order to move using a NCI from CO $\bar{q}$ to cooperative outcome $\bar{p}$. Another possibility is that he may drop only his previous cooperative option choices in order to improve via a NCI from a CO $\bar{q}$ to a NCO p . Keep in mind that for this situation, decision maker j automatically drops any corresponding cooperative options that i no longer takes. The set of NCI's is given by

$$m_i^*(\bar{q}) = \{(m_i(\bar{q}) \cap M_i^*(\bar{q})) \cup (\bar{m}_i(\bar{q}) \cap \bar{M}_i^*(\bar{q}))\} \quad (29)$$

Comments: The outcomes contained within the first pair of brackets on the right hand side are the NCO's to which i can improve from CO $\bar{q}$. The outcomes in the second pair of brackets are the CO's to which i can improve from the CO $\bar{q}$. The term NCI is used to refer to any outcome contained in $m_i^*(\bar{q})$ as well as the new strategy of decision maker i used to reach that outcome. Figure 1 displays NCI's as well as the CI's (Definition 15) and the UI's (Definition 12) given earlier. Once again, the three types of movements refer either to outcomes or the appropriate strategy changes used to achieve those outcomes.

Definition 17. - Types of Games. A game G consists of the strategy sets and preference functions for all of the decision makers. When there are no cooperative options (see Definition 5), and, hence, all of the options are noncooperative (see Definition 6), the game is called a noncooperative game. If there is at least one cooperative option, the game is referred to as a cooperative game.

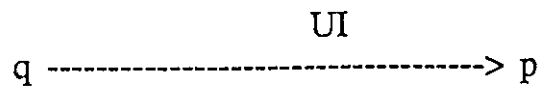


Fig. 1a. For a unilateral improvement (UI), decision maker i changes his strategy to move from NCO outcome q to a more preferred NCO p .

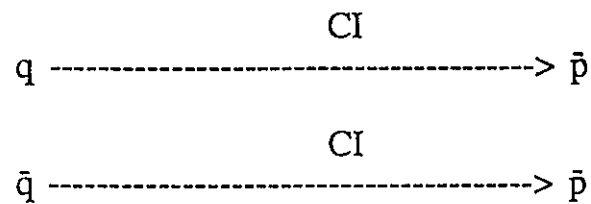


Fig. 1b. To form a cooperative improvement (CI), two decision makers i and j change the selection of at least one of their cooperative options and perhaps also one or more noncooperative options to move from either a NCO q or a CO $\bar{q}$ to a more preferred CO $\bar{p}$.

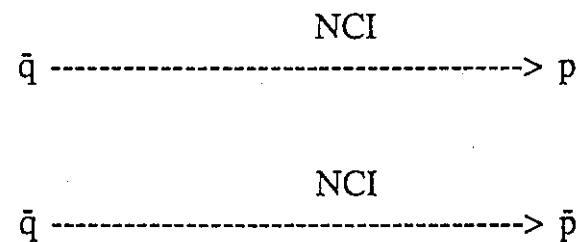


Fig. 1c. For a non-cooperative improvement (NCI), decision maker i changes the selection of his noncooperative and/or cooperative options to move from a CO $\bar{q}$ to a more preferred NCO p or CO $\bar{p}$.

Figure 1: Important types of movements

2.2 Stability Analysis

The definitions of the previous section are required for defining the theoretical idea of a game. When these theoretical constructs are applied to a real world problem, they are collectively referred to as a conflict model. The main components in a conflict model are the decision makers, their options and their preferences among the resulting outcomes.

After developing a conflict model, the next step is to carry out a stability analysis in order to determine the equilibria. This is done by calculating stability for each outcome for every decision maker. Generally speaking, an outcome is stable for a given decision maker if it does not benefit that decision maker to move from that outcome to another either by changing his strategy unilaterally or doing so in cooperation with another decision maker. An outcome that is stable for all decision makers is an equilibrium and constitutes a possible resolution of the conflict.

Depending upon the situation, human beings involved in a conflict can react in a number of different ways. Accordingly, the stability of an outcome for a decision maker can be defined appropriately in order to predict behavior in different situations. Below, specific stability definitions are given for use with cooperative and noncooperative games. Collectively, these stability definitions are referred to as a solution concept.

Definition 18. - *Cooperatively Rational Outcomes.* This definition is subdivided into two parts according to whether or not the outcome examined for stability is a NCO or a CO.

Case 1 - *Cooperatively Rational NCO:* For a decision maker $i \in N$, a NCO $q \in Q^*$ is cooperatively rational for i iff there exist no UI's and CI's for i from q .

Comments: Notice in Figure 1 that the only types of movements away from a NCO are the UI's and CI's. In order for the NCO q to be cooperatively rational for decision maker i , the decision maker can have no UI's or CI's available for use. When a game is noncooperative (see Definition 17), this definition of cooperatively rational outcome is identical to that of a rational outcome. Recall that in a noncooperative game, outcome q is rational for i iff decision maker i has no UI's available from q .

Keep in mind that a NCI (see Figure 1c) does not enter the definition for Case 1 because a NCI refers to movement away from a CO.

Case 2 - *Cooperatively Rational CO*: For a decision maker $i \in N$, a CO $\bar{q} \in Q^*$ is cooperatively rational for i iff there exist no CI's and NCI's for i from $\bar{q}$.

Comments: From Figure 1, one can see that the only kinds of movements away from a CO are the CI's and NCI's. In order for CO $\bar{q}$ to be cooperatively rational for decision maker i , this decision maker can have no CI's or NCI's available for use. A UI does not enter the definition for a cooperatively rational CO in Case 2 because a UI refers to unilateral movement away from a NCO (see Figure 1a).

Overall Comments: For an outcome to be cooperatively rational for i , there can be no available UI's, CI's or NCI's that would allow decision maker i to improve his position on his own or else in cooperation with another decision maker. For convenience of interpretation, cooperatively rational outcomes are categorized according to the two cases given above.

The next type of stability to be defined is called sequential sanctioning. However, before defining what constitutes a sequentially sanctioned outcome, a number of preliminary definitions must first be given.

Definition 19. - Credible Response. Let $\tilde{q}$ stand for either a NCO or a CO. Recall that N and Q^* represent the set of decision makers (Definition 1) and the set of outcomes (Definition 4), respectively, in a game. For $\tilde{q} \in Q^*$ and $H \subseteq N$, $H \neq \emptyset$, a credible response of H to $\tilde{q}$ is a member of $C_H(\tilde{q}) \subseteq Q^*$, defined inductively by

(i) $\tilde{q} \in C_H(\tilde{q})$,

(ii) if $\tilde{p} \in C_H(\tilde{q})$, $i \in H$ and strategy $\tilde{s}_i$ is a UI for i from $\tilde{p}$ or $\tilde{s}_i$ is a CI for i (and in this case also for a second decision maker j) or $\tilde{s}_i$ is a NCI, then the resulting outcome after $\tilde{s}_i$ is selected belongs to $C_H(\tilde{q})$.

Comments: A credible response by an individual decision maker to an outcome is simply the outcome resulting from a UI by that decision maker or

a CI by that decision maker plus another decision maker or a NCI. Hence, when a decision maker makes a UI or NCI or is involved in a CI, a credible response by a second decision maker to the outcome resulting from the UI or CI or NCI is the subsequent outcome after a UI or CI or NCI by the second decision maker. The relationships of UI's CI's and NCI's with respect to NCO's and CO's are depicted in Figure 1. Credible responses constitute possible sequences of improvements formed either individually or cooperatively among decision makers. For example, in an n-decision maker game, decision maker i may have a UI from q to outcome p from which decision makers j and k have a CI from p to outcome m. From this outcome, another decision maker may have a NCI to some other outcome etc.

Theorem 1. If $\tilde{q} \in Q^*$, $H' \subseteq H \subseteq N$ and $H' \neq \emptyset$, then $C_{H'}(\tilde{q}) \subseteq C_H(\tilde{q})$.

Comments: Recall from Definition 19 that H is some subset of decision makers from the total set of decision makers N that take part in the credible response given in the previous definition. When this subset of decision makers H remains the same or some decision makers leave H to form H' , then the credible responses available to the decision makers in H' either decrease or remain the same. In other words, when more decision makers are added to H' to form H, the credible responses available to the decision makers either increase or stay the same.

Theorem 2. Let $\tilde{q} \in Q^*$ and $H \subseteq N$, $H \neq \emptyset$. If outcome $\tilde{q}' \in C_H(\tilde{q})$, then $C_H(\tilde{q}') \subseteq C_H(\tilde{q})$.

Definition 20 - Sanctions. As in Definition 19, let $\tilde{q} \in Q^*$ represent either a NCO or a CO. Let $\tilde{s}_i \in \tilde{S}_i$ stand for a UI, CI or NCI for decision maker i from outcome $\tilde{q}$, where the types of movements that are allowed are shown in Figure 1. Let the payoff of outcome $\tilde{q}$ to decision maker i be denoted as $v_i(\tilde{q})$. Hence, outcome $\tilde{p}$ is less preferred than outcome $\tilde{q}$ iff $v_i(\tilde{p}) \leq v_i(\tilde{q})$.

If $\tilde{s}_i$ is a UI, NCI, or part of a CI, one must examine strategy changes by the remaining decision makers denoted by N-i. Suppose that $\tilde{s}_{N-i}$ represents a strategy selection by the other decision makers. Then, strategy $\tilde{s}_{N-i} \in \tilde{S}_{N-i}$ is a sanction against $\tilde{s}_i$ iff

$$v_i(\tilde{s}_i, \tilde{s}_{N-i}) \leq v_i(\tilde{q})$$

Comments: Recall that two decision makers are allowed to cooperate in a n -decision maker game. Let these decision makers be denoted by i and j . After decision maker i takes advantage of a UI, NCI or CI, only the remaining decision makers $N-i$ take part in forming the sanction $\tilde{s}_{N-i}$. However, decision maker j is not able to take advantage of any CI available to him because this would require action by decision maker i . Since decision maker i would not sanction himself, CI's by j for sanctioning are not allowed. If, on the other hand, decision maker i were not one of the two cooperating decision makers, then the two cooperating participants would be members of $N-i$ and they could use their CI's for sanctioning purposes.

Definition 21 - Credible Sanctions. Let $\tilde{q} \in Q^*$ represent either a NCO or a CO. Also, as in Definition 20 let $\tilde{s}_i \in \tilde{S}_i$ stand for a UI, CI or NCI for decision maker i from outcome $\tilde{q}$ where the types of movements that can take place are shown in Figure 1. Then, $\tilde{s}_{N-i}$ is a credible sanction against $\tilde{s}_i$ iff $\tilde{s}_{N-i}$ is a sanction against $\tilde{s}_i$ (Definition 20) and outcome $(\tilde{s}_i, \tilde{s}_{N-i})$ is a credible response of $N-i$ to $(\tilde{s}_i, \tilde{s}_{N-i})$ (Definition 19).

Definition 22 - Sequential Stability. Outcome $\tilde{q} \in Q^*$ is sequentially stable for decision maker i iff for every UI, CI and NCI, given by $\tilde{s}_i$, for i from q , there exists a credible sanction against $\tilde{s}_i$.

Definition 23 - Unstable Outcome. An outcome $\tilde{q}$ is unstable for decision maker i if it is not cooperatively rational (Definition 18) or sequentially stable (Definition 22) for i .

Comments: Figure 1 displays the three types of improvements a decision maker can take either on his own or cooperatively from outcome $\tilde{q}$. Recall that the superscript " $\sim$ " above q indicates that the outcome can be either a CO or NCO. If a credible sanction (Definition 21) cannot be levied by the other decision makers against at least one improvement by i from $\tilde{q}$, then $\tilde{q}$ is unstable. Decision maker i will certainly take advantage of his unsanctioned improvement.

Definition 24 - Equilibrium. Outcome $\tilde{q}$ is an equilibrium if it is either cooperatively rational (Definition 18) or sequentially sanctioned (Definition 22) for each of the decision makers.

3. LABOR-MANAGEMENT CONFLICT

In the previous section many new definitions, algorithms and procedures are presented. The purpose of this section is to use a hypothetical labor-management conflict to clarify the practical meaning of these concepts. For convenience, each example is directly linked to the definitions of the last section by referring to the definition numbers.

3.1 Conflict Model (*Definitions 1 to 10*)

In order to explain some of the ideas presented in Definitions 1 to 10, consider the labor - management conflict in Table 1. This conflict consists of two decision makers called the union and management (Definition 1). The union and management are negotiating a new contract which will be valid for the next two years. In order for an agreement to be reached, both sides must sign either contract A or B. Because a given contract must be agreed to by both parties, it forms a cooperative option. Hence, in Table 1 contract A and contract B are cooperative options according to Definition 5. For convenience of interpretation, these two options have been given the same option numbers in Table 1.

Table 1 *Labor - Management Conflict*

<u>Union</u>	Cooperative Outcomes		Noncooperative Outcomes			
	<u>$\bar{a}$</u>	<u>$\bar{b}$</u>	<u>c</u>	<u>d</u>	<u>e</u>	<u>f</u>
1. Contract A	Y	N	N	N	N	N
2. Contract B	N	Y	N	N	N	N
3. Strike	N	N	Y	N	Y	N
<u>Management</u>						
1. Contract A	Y	N	N	N	N	N
2. Contract B	N	Y	N	N	N	N
4. Lock Out	N	N	N	Y	Y	N

The remaining two options in Table 1 are noncooperative (Definition 6). The union controls the noncooperative option of striking while management can lock out labor if a contract is not signed.

In Table 1, the two outcomes on the left are examples of cooperative outcomes (Definition 9) whereas the four outcomes on the right are noncooperative outcomes (Definition 10). Recall that a *Y* indicates that the option opposite the *Y* is selected by the decision maker controlling it whereas a *N* means that an option is not taken. In order to be able to refer conveniently to outcomes within the text and in later examples, each outcome is assigned a letter name in Table 1. The bars above outcomes $\bar{a}$ and $\bar{b}$ indicate that they are cooperative outcomes.

As an example of a cooperative outcome, consider the outcome $\bar{a}$ on the far left in Table 1 which is written horizontally in text as (Y N N Y N N). This is a cooperative outcome (Definition 9) because the union and management are cooperating by agreeing to sign contract A. Hence, the union is taking the cooperative strategy of (Y N N) and management is following its cooperative strategy of (Y N N) (Definition 7). These two cooperative strategies form the cooperative outcome (Y N N Y N N).

Outcome e, which is the second column from the right in Table 1, is an example of a noncooperative outcome (Definition 10). Notice that in this outcome no cooperative option is selected because the decision makers are not signing contract A or B. However, the union is taking the noncooperative option of striking (Definition 6) in order to form its noncooperative strategy (N N Y) (Definition 8). Likewise, management is taking its noncooperative option of locking out the workers within the noncooperative strategy (N N Y). The noncooperative strategies of the two decision makers create the noncooperative outcome (N N Y N N Y).

The options of strike and lock out are noncooperative. Because of this, each option can be selected either at the same time or separately by the decision makers. Various combinations for selecting options 3 and 4 are shown in the four noncooperative outcomes on the right in Table 1. For the two cooperative outcomes on the left as well as outcome f, notice that the noncooperative options of strike and lock out are not taken.

As a final note, one should keep in mind that one can identify and then remove standard types of infeasible outcomes from the labor-management

conflict. For instance, options 1 and 2 are mutually exclusive since neither decision maker can take both options together by signing two contracts. In addition the union will not strike and management will not have a lock out if a contract is agreed on. Only the feasible outcomes for the labor-management conflict are shown in Table 1.

3.2 Types of Improvements (*Definitions 12, 15 and 16*)

The important types of movements to more preferred outcome that can take place in a conflict are summarized in Figure 1. These movements consist of UI's (Equation 15 in Definition 12), CI's (Equation 26 in Definition 15) and NCI's (Equation 29 in Definition 16). Some hypothetical examples are given to explain what these kinds of movements look like in practice.

Tables 2 and 3 display the preferences of the union and management respectively, for the labor - management conflict of Table 1. In each of these two tables, the outcomes are ranked from most preferred on the left to least preferred on the right. In terms of the letter notation, the preference ordering of outcomes for the union is ($\bar{a}$ c f $\bar{b}$ e d) whereas the management preference ranking is (f $\bar{b}$ d $\bar{a}$ e c).

Table 2 Preference Ranking of Outcomes for the Union

Outcome Letters	$\bar{a}$	c	f	$\bar{b}$	e	d
1. Contract A	Y	N	N	N	N	N
2. Contract B	N	N	N	Y	N	N
3. Strike	N	Y	N	N	Y	N

Management

1. Contract A	Y	N	N	N	N	N
2. Contract B	N	N	N	Y	N	N
4. Lock Out	N	N	N	N	Y	Y

Movements
to More
Preferred
Outcomes

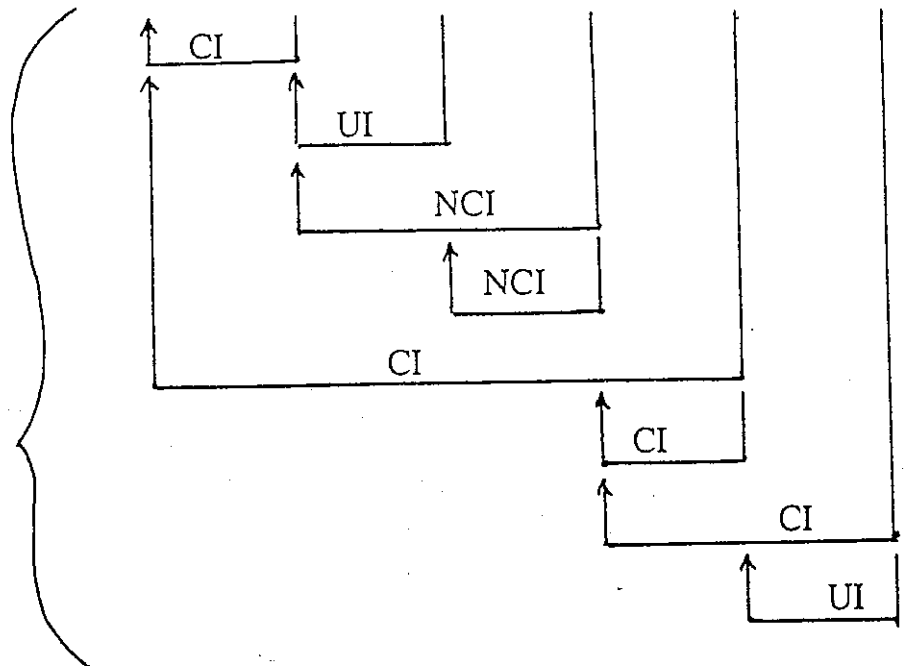


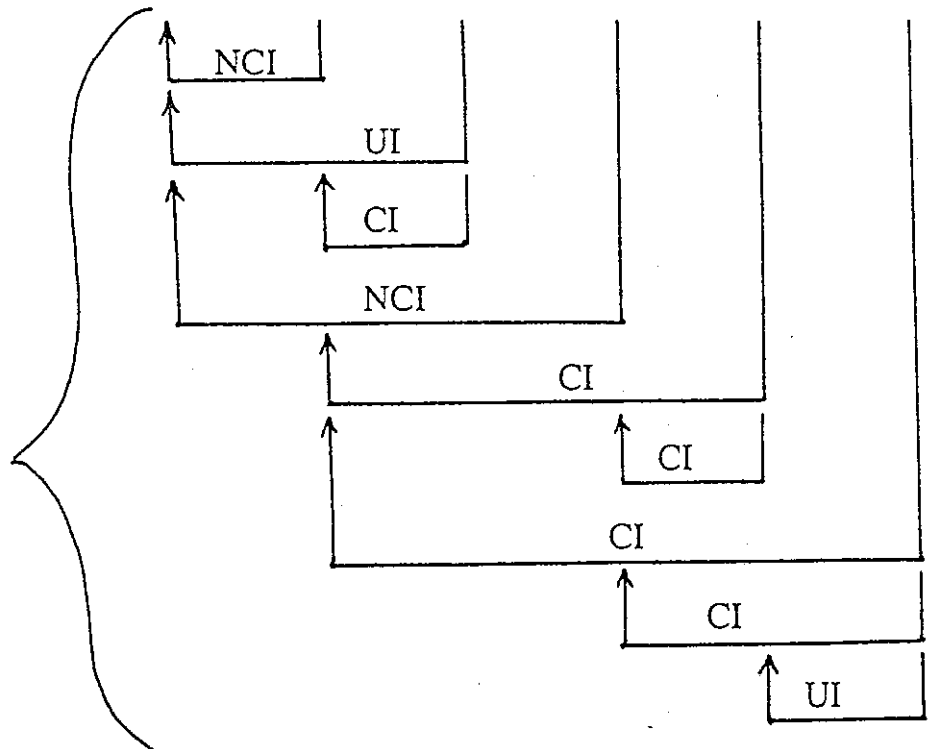
Table 3 Preference Ranking of Outcomes for the Management

Outcome Letters	f	б	d	ā	e	c
1. Contract A	N	N	N	Y	N	N
2. Contract B	N	Y	N	N	N	N
3. Strike	N	N	N	N	Y	Y

Management

1. Contract A	N	N	N	Y	N	N
2. Contract B	N	Y	N	N	N	N
4. Lock Out	N	N	Y	N	Y	N

Movements
to More
Preferred
Outcomes



From the union viewpoint in Table 2, one can see that the union most prefers to have an agreement on contract A. Next the union would like to strike if contract A cannot be reached. Because contract B is not nearly as attractive as contract A to the union, the union even prefers the status quo outcome f, in which no options are selected by either decision maker, over contract B. Finally, if management invokes a lock out, the union prefers to strike (outcome e) over not striking (outcome d).

As can be seen in Table 3, management most prefers the maintenance of the status quo and when it prefers to sign an agreement it would like to see contract B come into effect. Management would even prefer to have a lock out (outcome d) more than agreeing upon contract A (outcome $\bar{a}$). However, if the union strikes, management would prefer a lock out (outcome e) over taking no action at all (outcome c).

The types of unilateral improvements that can take place (see Figure 1) are indicated at the bottom of Tables 2 and 3 for the union and management, respectively. As an example of UI (Equation 15 in Definition 12), consider the UI that the union has in Table 2 from outcome f to outcome c. This UI is indicated by an arrow going from outcome f to outcome c. Notice that for the UI the option selections of management remain the same but the union selects its noncooperative option of striking. Another instance of a UI for the union is the UI from outcome d to e in which the management continues with a lock out and the union puts its members on strike.

When looking at a UI from management's viewpoint, the union's option selections remain fixed and management changes the choice of its noncooperative options in order to move from a less preferred noncooperative outcome to a more preferred one. For example, at the bottom of Table 3 notice that management has a UI from outcome d to f. In both of these outcomes, the union chooses none of its options but management ceases a lock out in order to take advantage of its UI from d to f. Management also has a UI from outcome c to e.

In a CI (Equation 26 in Definition 15), both decision makers select at least one cooperative option, and perhaps also some noncooperative ones, in order to move from either a noncooperative outcome or cooperative one to a more preferred cooperative outcome. Because this cooperative outcome is more preferred by both decision makers taking part in the CI, the CI is always indicated for both of them. Consider, for example, the CI that both the union and management have from outcome c to $\bar{a}$. As can be seen in Tables 2 and 3 for the union and management, respectively, this CI is, of course, available

to both decision makers. Notice that the decision makers cooperate by both selecting their cooperative option of accepting contract A and that the union drops its noncooperative option of striking.

Another CI is available for allowing both decision makers to improve their position by moving from outcome e to $\bar{b}$. In this CI, both decisions makers select the cooperative option of contract B. Additionally, the union stops striking and management ceases its lock out. The remaining two examples of CI's are from outcomes e to $\bar{a}$ and d to $\bar{b}$.

From Figure 1, the third type of movement is a NCI (see Equation 29 in Definition 16). To form a NCI, one of the decision makers changes his choice of a noncooperative option and/or cooperative option in order to improve his own position. Notice from Figure 1c that the decision maker taking advantage of an NCI moves from a cooperative outcome to either another more preferred cooperative outcome from his viewpoint or a more preferred noncooperative outcome. Keep in mind that when a decision maker drops a cooperative option, the same thing is done for the other cooperating decision maker. This is because both decision makers must agree to accept any cooperative option but either one on his own can unilaterally drop a cooperative option.

All of the NCI's in Tables 2 and 3 for the union and management constitute NCI's from cooperative to noncooperative outcomes. Consider the NCI for the union in Table 2 from cooperative outcome $\bar{b}$ to noncooperative outcome c . For this NCI, the union decides not to accept contract B and, therefore, it also becomes void for management. In addition, the union chooses its noncooperative option of striking. In its other NCI, the union moves from outcome $\bar{b}$ to the status quo f by backing out of contract B. However, for this NCI, the union does not select its noncooperative option of striking.

As shown in Table 3, management has two NCI's that it can take on its own. One NCI is from outcome $\bar{b}$ to f in which management drops contract B which also forces the union to do likewise. In the other NCI from outcome $\bar{a}$ to f , management backs out of contract A in order to improve its position.

3.3 Stability Analysis (*Definitions 17 to 24*)

If an outcome is stable for a decision maker, it is not advantageous for the decision maker to move to a more preferred outcome either on his own or

else cooperatively with another decision maker. When carrying out a stability analysis, one examines every outcome from every decision maker's point of view. One should also keep in mind that when analyzing outcome q for stability from decision maker i 's viewpoint, all possible improvements that i may have either on his own or else cooperatively with another decision makers must be blocked by one or more other decisions in order for outcome q to be stable for i . In other words, all UI's CI's and/or NCI's from q for i must be sanctioned by others in order for q to be stable for i . Finally, one should remember that after decision maker i considers making an improvement, he does not take part in any further moves or counter moves that may take place by others who can block his improvement. Consequently, if decision maker i , for example, can cooperate with decision maker j , then j cannot block any improvements by i using CI's because this would require i to take part. However, decision makers i and j could use their joint CI's to sanction improvements by another decision maker k when an outcome is being analyzed for stability from k 's viewpoint.

Tables 2 and 3 display the preferences and possible improvements for the union and management, respectively, in the labor-management conflict. For convenience of explanation for the stability analysis of this conflict, all of the information in these tables is transferred to Table 4. Notice in this table that the preferences for the union and management are presented by listing the lettered outcomes from most preferred on the left to least preferred on the right below each decision maker. Also, for a given outcome and decision maker the possible improvements from the outcome are listed below the outcome from most preferred at the top to least preferred at the bottom. Consider, for instance, outcome d from management's viewpoint. Notice from Table 3 that management has a UI from d to f and a less preferred CI from d to $\bar{b}$. Therefore, below outcome d from management's viewpoint outcome f is listed above outcome $\bar{b}$. Additionally, the specific type of improvement is given in brackets beside each improvement. Hence, f and $\bar{b}$ are designated as a UI and CI, respectively, for management below outcome d in Table 4.

Cooperatively rational outcomes, presented in Definition 18, are subdivided into two categories. A cooperatively rational NCO for decision maker i is a NCO outcome for which there are no UI's or CI's for i . A CO is a cooperatively rational CO for decision maker i if i has no CI's or NCI's available to allow him to move away from the CO to a more preferred outcome. Notice that for both types of cooperatively rational outcomes the decision maker cannot improve on his own or jointly to a more preferred outcome.

In Table 4, cooperatively rational outcomes are recognized as outcomes that have no types of improvements written below them. To indicate that an outcome is cooperatively rational, an r is written above the outcome. For the union, $\bar{a}$ is the only type of cooperatively rational outcome. Because $\bar{a}$ is a CO, it falls under the category of cooperatively rational CO (Case 2 in Definition 18). From management's viewpoint, outcome f is the only cooperatively rational outcome. Since f is a NCO, it constitutes a cooperatively rational NCO (Case 1 in Definition 18).

Outcomes that are not cooperatively rational in Table 4 have one or more improvements written below them. To ascertain if a given CO or NCO having improvements is stable for decision maker i , one must check that all of the improvements can be sanctioned by the other decision makers. If decision maker i can eventually end up in a less preferred position if he takes advantage of any of his possible improvements, the outcome is sequentially stable since he is better off to stay at the outcome. Sequential stability is defined in Definition 22 which in turn is based upon Definitions 19, 20 and 21. A sequentially stable outcome is indicated in Table 4 by writing an s above it.

In Table 4, outcomes c and f are two NCO's that are sequentially stable for the union. From outcome c the union has a CI to CO $\bar{a}$. Notice that management possesses an NCI it could take from $\bar{a}$ to f . Because f is less preferred than c for the union, it constitutes a credible sanction (Definition 21). Since outcome $\bar{a}$ is the only improvement for the union from c and it can be blocked by management, outcome c is a sequentially stable outcome (Definition 22) for the union and is marked using the letter s .

Now consider outcome f for the union which has a UI from f to c . Notice that management has CI's from c to $\bar{b}$ and c to $\bar{a}$. However, because a CI also involves the union, these CI's cannot be used for sanctioning purposes (see comments under Definition 20). Nonetheless, management has a UI from

c to e. Because e is less preferred than f by the union, the only improvement from f is credibly sanctioned and, hence, f is sequentially stable.

From management's point of view, outcomes $\bar{b}$, d and $\bar{a}$ are sequentially sanctioned. From outcome $\bar{b}$, management has a NCI to outcome f. However, the union has a UI from f to c and outcome c is less preferred to $\bar{b}$ by the management. Since its only improvement is blocked from outcome $\bar{b}$, outcome $\bar{b}$ is sequentially stable for management.

From outcome d, management has two possible improvements. In order for d to be sequentially stable, both of these improvements must be credibly sanctioned by the union. The UI from d to f is credibly sanctioned by the union unilaterally moving from f to c. When management moves cooperatively with the union from d to $\bar{b}$, the union can take advantage of its NCI by moving unilaterally from $\bar{b}$ to c. Since c is less preferred than d by management, the CI $\bar{b}$ is also credibly sanctioned. Hence, outcome d is sequentially stable for management.

The final outcome which is sequentially stable for management is CO $\bar{a}$. Management has a NCI from $\bar{a}$ to f but the union can credibly sanction this by moving from f to c, which is less preferred than $\bar{a}$ by management.

When an outcome is unstable for a decision maker, at least one of the improvements from the outcome cannot be credibly sanctioned by the other decision makers (Definition 23). An unstable outcome is marked using the letter u. In Table 4, outcomes $\bar{b}$, e and d are unstable for the union while outcome e and c are unstable for management.

As an illustration of how to detect an unstable outcome, consider outcome $\bar{b}$ from the union's perspective. The union has NCI from $\bar{b}$ to c. For sanctioning purposes, management cannot move from c to $\bar{b}$ or $\bar{a}$, since this would require cooperation with the union. However, management has a UI from c to e. Because e is less preferred than $\bar{b}$ by the union, the NCI from $\bar{b}$ to c is credibly sanctioned by management. In order for an outcome to be sequentially stable, all possible improvements must be credibly sanctioned. From $\bar{b}$, the union has NCI to f. Because management has no improvement from f, the NCI for the union from $\bar{b}$ to f cannot be credibly sanctioned. Therefore, overall outcome $\bar{b}$ is unstable for the union and this is indicated by writing a u above outcome $\bar{b}$ in the union's preference ranking.

Outcome e is unstable for the union because neither of its CI's can be credibly sanctioned by management. If the union moves from e to $\bar{a}$, management has a NCI from $\bar{a}$ to f . Because outcome f is more preferred than e by the union, it is not a credible sanction. Since at least one improvement cannot be credibly sanctioned, outcome e is unstable for the union and it is not necessary to check whether or not $\bar{b}$ is credibly sanctioned. However, for explanation purposes the CI from e to $\bar{b}$ is now checked. When the union cooperatively improves from e to $\bar{b}$, management can break away from CO $\bar{b}$ by taking advantage of a NCI from $\bar{b}$ to f . Once again a CI cannot be credibly sanctioned.

Because outcome d is the least preferred outcome for the union, no improvement from d cannot be credibly sanctioned. Therefore, outcome d is unstable for the union. Likewise outcome c is unstable for management.

When examining outcome e for stability from management's point of view, only one of the two CI's can be credibly sanctioned. After management moves from e to $\bar{b}$, the union can go from $\bar{b}$ to c . Since c is less preferred than e for management, the CI from e to $\bar{b}$ is credibly sanctioned. However, when management moves from e to $\bar{a}$ in cooperation with the union, the union will not break away from the cooperation because it has no NCIs from $\bar{a}$. Hence, overall outcome e is unstable for management.

In order for an outcome to be an equilibrium (Definition 24) it must be either cooperatively rational (Definition 18) or sequentially sanctioned (Definition 22) for each decision maker. In other words, an outcome cannot be an equilibrium if it is unstable for at least one decision maker.

The two equilibria in the labor-management conflict are outcomes $\bar{a}$ and f , which are indicated by writing E's above them where they appear in the union's preferences. Outcome $\bar{a}$ is cooperatively rational for the union and sequentially stable for management. Outcome f is sequentially stable and cooperatively rational for the union and management, respectively.

If an equilibrium is reached during the evolution of a conflict, the conflict will remain there until preferences or some other parameters change. This means that if the labor-management conflict is at the status quo outcome f , a cooperative equilibrium, such as outcome $\bar{a}$, cannot be reached until preference or other changes cause the status quo to become unstable for at least one decision maker. The conclusion then is that no contract is signed. The union will not go on strike, however, because it does not want management to retaliate by locking out its members.

4. CONCLUSIONS

Using the precise definitions given in this paper, one can properly model and analyze an n -decision maker conflict in which two of the decision makers can cooperate. As shown by the example, these definitions are not difficult to apply to a practical problem. Nonetheless, to permit widespread utilization by both practitioners and researchers, these definitions should be incorporated into a flexible decision support system. As pointed out by various authors [6, 9], an available decision support system called "DecisionMaker : the Conflict Analysis Program" [6], could be readily expanded to handle cooperation.

The research presented in this paper is restricted to cooperation between two of the n decision makers in a conflict model. In future research this should be extended to include situations where several distinct pairs of decision makers cooperate, where several overlapping pairs of decision makers cooperate, and where sets of decision makers of two or more in size cooperate. Each of these possibilities results in different and challenging kinds of research problems.

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