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Multicriteria decision-aid : Two applications in education management

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R E S U M E

Dans la première section les conditions et la formalisation du choix trichotomique sont établies sur la base des concepts de relation de surclassement et d'ensembles de références. Ils permettent la modélisation des préférences dans le contexte décisionnel étudié.

Deux applications suivent. La première appliquée à la décision du jury de sélection des candidats à une institution d'enseignement commercial, permet, grâce à un exemple numérique de préciser l'utilisation du modèle.

La seconde décrit une application faite à la répartition de subventions aux écoles d'une académie. Nous montrons que le type d'aide à la décision choisi répond au contexte organisationnel en ce qui concerne la gestion du système d'information et de relations.

SUMMARY

In the first section the conditions for and formalism of trichotomic choice problem - formulation are stated on the basis of the fondamental concepts of outranking relations and reference sets, used to the modelling of global preferences within the given context . Two applications follow. The first one, applied to the decision of the admission committee of a business school, makes the explanation of the procedure more concrete. The second one is the description of an implementation for ressource allocations to schools of an academic region. It is shown that the approach to decision aid we propose fits to the organisational background regarding the management of information systems as well as the human relations and organisational developpment.

INTRODUCTION

The procedures resulting from the operational research are usually concerned to show up, in consideration of a given criteria, an optimal solution out of a set of possible solutions which is given beforhand. The procedure for decision aid dealt with in this paper does not come within this traditional problematic. In fact this one does not seem to be suitable to aid a decision maker who, out of documents or conversations, proceeding from multiple applicants, has to answer to the same type of question (eventualy spread over time). The demand may regard a credit or a diploma grant, the attribution of a promotion by selection, an increase of wages, the launching of a new product, of a research project, the processing of a damage, or a claim file a.s.o...

In that kind of problems each demand, and consequently each decision, is in competition with the others only in the range of a rather fuzzy or elastic constraint which limits (for physical, financial, technical human reasons) the global capacity of acceptance for a given period. In such conditions the optimisation problematic leads to the implementation of quite artificial or difficult solutions (1).

Furthermore the criterias upon which the decision maker bases his judgment to give to each demand a favorable or unfavorable answer may involve quite various factors which are not necessarily quantifiable. The available information may be inexact or vague. The particular influence of each criteria and more generally the way in which the combination of their values influences the judgment of the decision maker (in terms of "better and worse" or "bad and good") may only be partially known according to the decision maker's good will.

In such conditions the decision maker may wish to rely on a procedure which out of the only information available during each processing, helps him to chose in terms of acceptance, refuse, or postponement for further information.

(1) B. ROY. *A conceptual frame work for a normative theory of "decision aid" Management Science Special issue on Multiple Criteria Decision Making 1976*

How can we conceive such a procedure of trichotomic choice ? That is the problem we are studying there (section one). Can such a procedure be implemented in the context of administrative organisation ? That is what we are trying to show by describing two applications in the field of education management (section two and three).

The first application will enable us to make the explanation of the procedure and of the model used more concrete by studying the well known case of the decision in an admission committee in a business school.

The second application regarding a decision procedure for ressource allocations to schools takes place in a large and rather bureaucratic administration. By describing how the procedure has been implemented we shall try to show that such an approach to decision aid particularly fits to the organisational background regarding the management of information system as well as of the human relations and organisational development.

I. THE METHODOLOGY OF MULTICRITERIA TRICHOTOMIC SEGMENTATION

1. Type of aid intended

Given, actions, objects or candidates (we are now going to speak of objects), which are not fundamentally in competition, we want to help an identified decision maker by saying whether such or such object is compatible with his requirement.

We have thus to define and model a referential system which will allow us to establish by comparison (on the basis of n criterias) the intrinsic value of each object. Thanks to this particular modelling effort we don't need to consider the whole objects at once to be able to come to a conclusion on each of them.

The procedure lends itself to a sequential examination of the objects. Nevertheless to insure a sufficient reliability we shall admit that in some cases the intrinsic value of the object can not be difined.

More precisely, on the base of just the initial information introduced in the model, the main point of the trichotomic segmentation is, to :

- accept all the objects declared without the intervention of the decision maker as sufficiently good (category A_1)
- refuse all the objects declared without the intervention of the decision maker as too bad (category A_3)
- demand a further examination by the decision maker for the others (category A_2)

By assigning each object to one of these categories we discriminate beetween

- those for which initial information is sufficient to found a solid assumption about the intrinsic value of the object (category A_1 and A_3)
- those for which initial information does not allow us to come to a conclusion.

For these objects we must have recourse to the decision maker ; a search for new information and a deeper reflexion are necessary.

When the initial information alows us to build a discriminating trichotomy (population of A_1 and A_3 not too small), the implementation of these principles leads to a better organisation of the decision maker's reflections and effort by directing, in a selectiv way, his reflexion, the mobilisation of his experience and of all diffuse information he can have, toward the examination of the most difficult cases.

To implement such a trichotomic segmentation we have to model in three stages (1) :

1- In the first stage we have to represent and consider each object "a," the set A ($a \in A$), and the consequences or main attributs of each "a" evaluated on n scales E_i and represented by the vectors $\gamma(a) = \gamma_1(a) \ldots \gamma_n(a)$

$\gamma_i(a) \in E_i$

2- In a second stage we have to model the decision maker's preference : choice of a consistent family of criterion (2) $g_1 \ldots g_n$ adapted to the discriminating power on each scale E_i, formal definition of the global preference to be able to establish on the basis of the n criterias the decision maker's preference : "a" is best or worst than a', or "a" can't be compared to a'

3- In the last stage we have to build the rules to decide wether an object a must be assigned to A_1, A_2 on A_3. We shall see that this assignment is chosen on the basis of "a" comparison to particular objects of reference.

2. Fuzzy out ranking relation

Let us assume that for the objects we consider the study of the relevant consequences and attributs has been made and formalised so that we can sum them through a consistent family of n criterias (2). So let us assume that to each object a is associated a vector :

$\underline{g}(a) = g_1(a), \ldots g_n(a)$

Let us agree that "a" is better as $g_i(a)$ is greater.

(1) J. MOSCAROLA *Aide à la décision en présence de critères multiples fondée sur une procédure trichotomique. Thèse PARIS IX DAUPHINE 1977*

(2) B. ROY *Outranking and fuzzy outranking a concept making partial order analysis - Decision Making with Multiple objectives IIASA Vienna 1976*

This vector represents the initial information. Each component g_i (a) grasps more or less precisely an aspect of the intrinsic value of "a." To compare two objects on the base of that initial information we use the concept of fuzzy outranking.

It can be introduced in a quite natural way if we accept that the modelling of preferences is not complete and only depends on the part of preferences which we are able to know with sufficient objectivity and confidence (see table 1).

The term outranking refers to those of the preferences thus modelled ; given two potential actions a and a' :

- a' outranks "a" signifies that the scientist has good reasons to admit that in the eyes of the decision-maker a' is at least as good as "a" (consequently a' is indifferent from or preferred to "a") ;

- a' does not outrank "a" signifies that the arguments in favour of proposition "a' is at least as good as a", are judged insufficient, that there exists or not militant arguments in favour of the proposition ""a" is at least as good as a' " (consequently a is incomparable or preferred to a').

Within the framework of such a conceived modelling, the scientist may be more or less exigeant (take more or less risks) to accept the outranking, whence the concept of fuzzy outranking.

A fuzzy outranking relation S can be characterised by the definition of a degree of outranking d associating with each couple (a',a) a number d (a', a) : d being a criterion destined to fix the more or less high credibility of the outranking of "a" by a'. More precisely, the degree of credibility must possess the following properties :

1°) The number d (a', a) only calls into play a' and "a" through their evaluations on the consistent criterion family, otherwise :

$$d(a', a) = d\left[\underline{g}(a'), \underline{g}(a)\right]$$

2°) d(a', a) increases with the reliability of the outranking of a by a', thus in particular :

d (a', a) is a non-decreasing function of g_i (a') $\forall$ i

and non-increasing of g_i (a) $\forall$ i .

TABLE 1 : SITUATIONS TO WHICH MAY LEAD

THE COMPARISON OF TWO POTENTIAL ACTIONS a, a'

	Situations	Definitions
Four fundamental exclusive situations	Indifference	The two actions are indifferent in the sense that there exist clear and positive reasons to choose equivalence example : $g(a) = g(a')$ $\forall_i$ some of the equalities not being rigorous but only a approximate
	strict preference	One of the two actions (which one being known) is strictly preferred to the other example : $g_i(a) = g_i(a')$ $\forall_i \neq k$, $g_k(a)$ a significant difference
	large preference	One of the two actions (which one being known) is not strictly preferred to the other but it is impossible to say if the other is strictly preferred to or indifferent from the first one because neither of the two former situations dominates example : $g_i(a) = g_i(a')$ $\forall_i \neq k$, $g_k(a') = g_k(a)$ neither sufficiently small to justify indifference nor sufficiently large to justify strict preference
	incomparability	The two actions are not comparable in the sense that neither of the three former situations dominates example : $g_i(a) > g_i(a')$ for $i = 1, \ldots, p$, $g_i(a') > g_i(a)$ for $i = p+1, \ldots, n$ the majority of the differences being significant

3°) d (a',a) = 1 implies a certain outranking of "a" by a' whereas d (a', a) = 0 implies either a certain non-outranking of a by a', or the total absence of arguments in favour of such an outranking ; it follows that :

$$0 \leqslant d(a', a) \leqslant 1.$$

Given a fuzzy outranking relation S. It is interesting to introduce the outranking relation (non fuzzy) defined by:

$$a \, S^{\lambda} a' \Leftrightarrow d(a,a') \geqslant \lambda$$

λ can be interpreted as a threshold generaly near 1 and which is in all cases at least egal to 1/2. The outranking through S^{λ} can be stated as soon as the credibility degree has reached this threshold. To take into consideration a decreasing sequence of values λ comes down to introducing a nested family of outranking relations (non fuzzy) more and more richer but more and more risky. The scientist can in this way test the behavior of the model, in function of requirements of variable security and severity.

In the Annexe we describe a way to build such an outranking relation.

3. The references sets

Given two particular objects (real or fictitions) $b \in B$ and $c \in C$ called reference objects and characterized by the vectors $\underline{g}(b)$ and $\underline{g}(c)$.

These vectors indicate on the set of criteria the "combined limits" of what we consider sufficiently good and sufficiently bad.

B and C must lead to the following properties :

- if an object is such that for a $b \in B$, the credibility degree d(a, b) is sufficient to garantee (without too much risk) the preference of "a" compared to b, then "a" is presumed to be "sufficiently" good and merits acceptance on the basis of initial information

- if an object "a" is such that for a $c \in C$ the credibility degree d (c, a) is sufficient to garantee (without too much risk) the preference of c on "a", then "a" is presumed to be "for too" bad and merits rejection on the basis of initial information.

The elements of the set R = B U C may be real objects, if not they are introduced as combinations of limits on the n criterions.

4. Segmentation procedure

λ Is the minimum threshold which indicates the limit above which we have a right to garantee through the model the preference of "a" componed to a' (a S^{λ} a'). This threshold is fixed by the decision maker.

Thanks to natural graphic convention we may represent S^{λ} with an outranking graph (1). The analysis of such a graph (fig. 2) built on the objects of "a" U R is useful for the scientist to locate in the context of the decision maker's preferences, the object which is being studied.

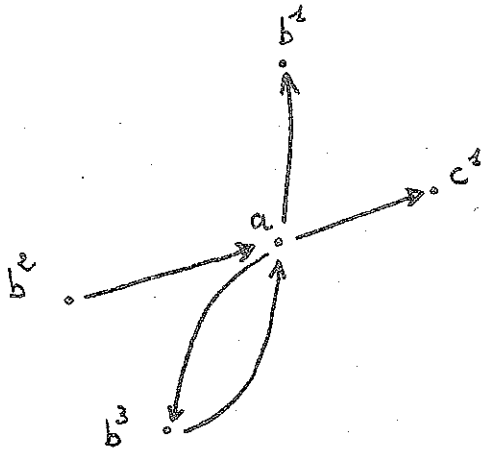


Figure 2

Thus on the example of figure 2

a is at least as good as b^1 and c^1

a is less good as b^2

a can't be compared to c^2

a is equivalent to b^3

To analyse this outranking graph let us define the subsets B^+, B^-, B° & C^+, C^-, C° :

$b \in B^+$ if $d(a, b) \geqslant \lambda$ thus $aS^{\lambda} b$

$b \in B^-$ if $d(b, a) \geqslant \lambda$ thus $bS^{\lambda} a$

$b \in B^\circ$ if $d(a, b) < \lambda$ and $d(b, a) < \lambda$ thus non $aS^{\lambda} b$ and non $bS^{\lambda} a$

(1) B. ROY - *critères multiples et modélisation des préférences (l'apport des relations de surclassement)* [illegible] n° 1-1974

$c \in C^+$ if $d(c, a) \geqslant \lambda$ thus $cS^\lambda a$

$c \in C^-$ if $d(a, c) \geqslant \lambda$ thus $aS^\lambda c$

$c \in C^\circ$ if $d(a, c) < \lambda$ and $d(c, a) < \lambda$ thus non $aS^\lambda c$ and non $cS^\lambda a$

Figure 3 is an illustration of these definitions.

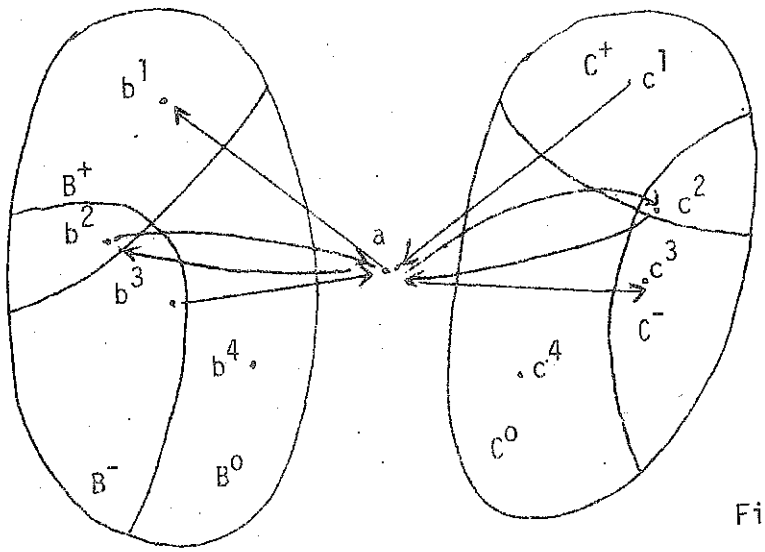


Figure 3

When the relation S^λ is antisymetric these subsets form a partition of B and a partition of C :

$$B^+ \cap B^- = \emptyset \ , \quad C^+ \cap C^- = \emptyset$$

This partition (to which it is always possible to refer (1)) characterizes the situation of a and the scientist uses it to work out the rules which are necessary to assign a to A_1 , A_2 or A_3

Given a partition of B and C, it can be characterized by

$$x^+ = |B^+| \ , \quad x^- = |B^-|$$

$$y^+ = |C^+| \ , \quad y^- = |C^-|$$

(1) J. MOSCAROLA - B. ROY *Procédure automatique d'examen de dossiers fondée sur une segmentation trichotomique en présence de critères multiples* - R. A. I. R.O. Opérations Research Vol 11 n° 2 Mai 1977

To each a $\in$ A we can therefore associate the vector $E_\lambda(a) = (x^+, x^-, y^-, y^+)$. Following the configuration of this vector we shall assign "a" to A_1, A_2 or A_3. The tree in figure 4 shows an example of decision rules.

The combination of the four basic exclusive situations (Table 1) on the set of the pairs (a, c) and (a, b) and the non necessary transitiorty of S^λ, leads to quite complex configurations, some of which may appear as surprising.

So on figure 5 a appears both as less good and better than two different elements of B, and at the same time less good than one of the references which is a limit of the "far too" bad.

Although particular conditions imposed on the preference modelling allows us to reduce such incoherencies (1) it is nevertheless necessary to build assignment rules (1) which are able to take such incoherencies into account. So in the case of figure 5, if we apply the decision tree figure 4 "a" will be assigned to A_2.

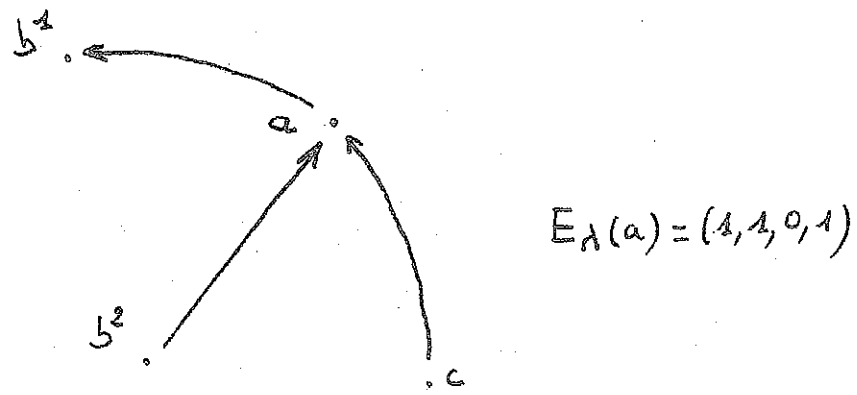
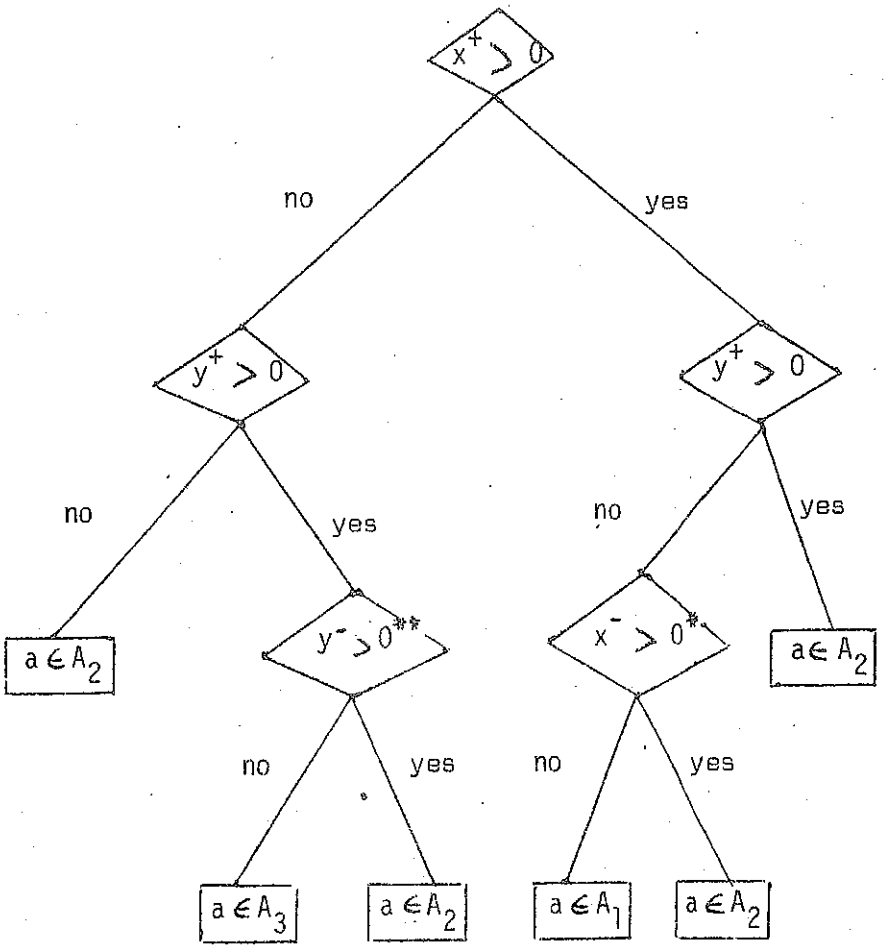


Figure 5

(1) J. MOSCAROLA - B. ROY *Procédure automatique d'examen de dossiers fondée sur une segmentation trichotomique en présence de critères multiples* - RAIRO Opérations Research Vol 11 n° 2 Mai 1977

Figure 4 : Assignment rules built out of the configuration analysis of $E_{\lambda}(a) = (x^+, x^-, y^-, y^+)$

$x^+ > 0 \Rightarrow \exists b \in B : aS^{\lambda} b$
$x^- > 0 \Rightarrow \exists b' \in B : b'S^{\lambda} a$
$y^+ > 0 \Rightarrow \exists c \in C : cS^{\lambda} a$
$y^- > 0 \Rightarrow \exists c' \in C : aS^{\lambda} c'$



* This 0 can be replaced by x^+ ** This 0 can be replaced by y^+

II. THE SELECTION OF CANDIDATES FOR A BUSINESS SCHOOL ADMISSION

1. The problem of selection

The admission to the school is pronounced by anadmission commitee of about 6 members (professors and professionals).
The number of candidates is superior than the admissions offered. The selection rate is about 1/3.

For each candidate the admission commitee has got a dossier including information about his curriculum, his past school results and his psychological profile.

The commitee's task is made quite difficult for the following reasons : the dossiers are quite numerous the quality of information, regarding each candidate is not very good and the decision criterias to take into account are various. As a consequence we can observe that

- there is an implicit tendency to privelege some criterias such as school results, although every body doubts the pertinency of this information
- the behavior of the commitee is not always the same : some criteria only play an incidental part
- sometimes exceptional information not included in the dossier are brought in by a member.

This distorsions within the procedure mainly appear during the examination of the candidates who are just at the limit of admission conditions.

The following solution can be considered

1/ selection from dossier : we try to get a partition of the candidates set A in three groups

A_1 official admission from dossier
A_2 convocation to a selection interview
A_3 officially refusal from dossier

2/ Final selection during an interview of the candidates affected to the group A_2, and eventually ranking on a complementary list.

2. The modelling

To affect a candidate "a" to one of the three groups A_1, A_2 or A_3 we use the model which has been discribed before

2.1. The consistent family of criteria

The admission commitee is looking for candidates who will be able to succeed in their studies as well as in their future professional integration. The candidate should therefore be able to adapt himself both to quantitative and relatively litterary matters, but his own human qualities and motivations are considered as important to his futur professional integration. This initial and quite confused perception can be precised by the definition of dimensions considered as useful to value as well as possible the consequences of the admission. By defining a scale E_i upon each dimension i we can build a state indicator γ_i (a) which is the formal information taken into account. According to the variation of the decision maker's preference along E_i we can build g_i out of γ_i

The table 6 gives an example of that modelling work.

2.2. The references

The admission commitee chooses two references for the "good" candidates b_1 and b_2. The vertices $\underline{g}$ (b_1) and $\underline{g}$ (b_2) mark the combined limits from which the candidate can be considered as sufficiently "good" to be officially admitted on dossier.

These two references are usefull to express in a formal way the fact that the commitee considers that good pupil without any particular personality (b_1), should be admitted as well as a hard working candidate whose personality seems to be particulary suitable, but whose school results are rather mediocre (b_2).

TABLE 6 : SUMMARY OF THE MODELLING WORK FROM THE CLOUD OF ATTRIBUTES TO THE CRITERIAS

loud of attributes	Dimension i	Commentary	E_i	Criteria
School level former school cursus Level in general subjects	1. A level subjects	The commitee ranks the A Level subjects according to 5 levels of increasing value	1 to 5	$g_1(a) = \gamma_1(a)$
	2. Marks of the past school year in math	These matters enable us to value the quality of the candidate's knowledge in general subjects	0 to 20	$g_2(a) = \gamma_2(a)$
	3. Marks of the past school year in French		0 to 20	$g_3(a) = \gamma_3(a)$
	4. Marks of the past school year in English		0 to 20	$g_4(a) = \gamma_4(a)$
Personality school integration	5. Improvement abilities of the candidate during the past school years	Based on the variation of the past marks in the 3 former matters. 9 cases of increasing improvement are defined	1 to 9	$g_5(a) = \gamma_5(a)$
	6. Appraisal of the head of the school attended last year	This global appraisal included in the candidate's dossier. It is evaluated on a 5 levels scale (bad... excellent)	1 to 5	$g_6(a) = \gamma_6(a)$
Personality	7. Further experiences	Experienced candidates in professionnal or social activities are looked for-3 level evaluation	1 to 3	$g_7(a) = \gamma_7(a)$
	8. Motivation	EValuation based on results of psychometric tests	1 to 4	$g_8(a) = \gamma_8(a)$
	9. Professional interest	Evaluation based on results of psychometric tests	1 to 3	$g_9(a) = \gamma_9(a)$

This choice enable us to give a chance to candidates whose personnality is considered as very positive for a good professional integration.

One reference only is defined for the "bad" candidate. Not to penalize candidates on the base of psychological tests which may be contested g_8 (c) and g_9 (c) are fixed at the lowest grade of the scales E_8 and E_9.

The Table 7 gives an example for possibles values to give to $\underline{g}$ (b_1), g (b_2) and g (c).

2.3. Fuzzy, outranking relation

This relation can be built using nested outranking relations as shown in the Annexe at the end of this paper.

The Table 7 indicates

- an example of weights for each criteria : p_i
- an example of admittable discordance intervals : D_i

2.4. Segmentation procedure

On the base the outranking graph which S^λ defined on the set (a, b_1, b_2, c) we shall try to affect "a" to one of the three categories

A_1 : to be admitted

A_3 : to be rejected

A_2 : to be invited to a selection interview

Let us assume that we may apply the decision tree of the figure 4 and that, to come to a partition B^+, B^- of B and C^+, C^- of C we decide

- to affect c to C^- when both d (a, c) $\geqslant \lambda$ and d (c, a) $\geqslant \lambda$
- to affect b to B^+ when both d (a, b) $\geqslant \lambda$ and d (b, a) $\geqslant \lambda$

which means (1) that the admission commitee prefers to be mistaken in admitting a candidate rather than in rejecting him.

(1) J. MOSCAROLA *Aide à la décision en présence de critères multiples fondée sur une trichotomie - Thèse Paris IX Dauphine 1977*

TABLE 7

Criteria	A.LEV	Math	French	English	Improvment	Appraisal	Experience	Motivation	Professio int.
Scale E_i	1 to 5	1 to 20	1 to 20	1 to 20	1 to 9	1 to 5	1 to 3	1 to 4	1 to 3
Reference : $g(b_1)$	4	12	12	14	5	3	1	2	1
$g(b_2)$	2	9	9	10	8	4	3	4	3
$g(c)$	1	8	8	10	3	2	1	1	1
Weight p_i	1	2	2	2	1	1	1	1	1
Discordance interval D_i	3	5	6	5	3	2	2	3	2

TABLE 8

Concordance Table

	b_1	b_2	c
b_1		0.58	1
b_2	0.41		1
c	0.16	0.16	

Discordance Table

	b_1	b_2	c
b_1		1	0
b_2	0.8		0
c	0.8	1.6	

3. Numerical application

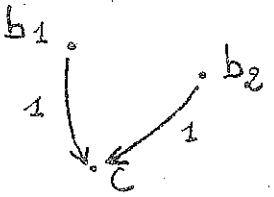
Applying the rules defined in annexe and refering ourselves to concordance and discordance tables (Table 8) we get

$d(b_1, b_2) = 0.58$ $d(b_2, b_1) = 0$ $d(c, b_1) = 0$

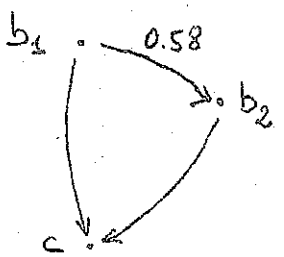
$d(b_1, c) = 1$ $d(b_2, c) = 1$ $d(c, b_2) = 0$

Then the outranking graph associated to S^{λ} is the following

$\lambda_1 = 0.9$, $\lambda_2 = 0.7$, $\lambda_3 = 0.6$



$\lambda_4 = 0.55$



We can see that the good pupil (b_1) is prefered with a quite a low degree of credibility to the candidate with a high individual value (b_2) . So if we are exacting in establishing the preference between b_1 and b_2 these two types of candidate appear as non comparable which correctly expresses the commitee's opinion.

3.1. Illustration of the evaluation of candidates.

The three following examples (Tables 9, 10, 11) illustrate how the method works.

TABLE 9

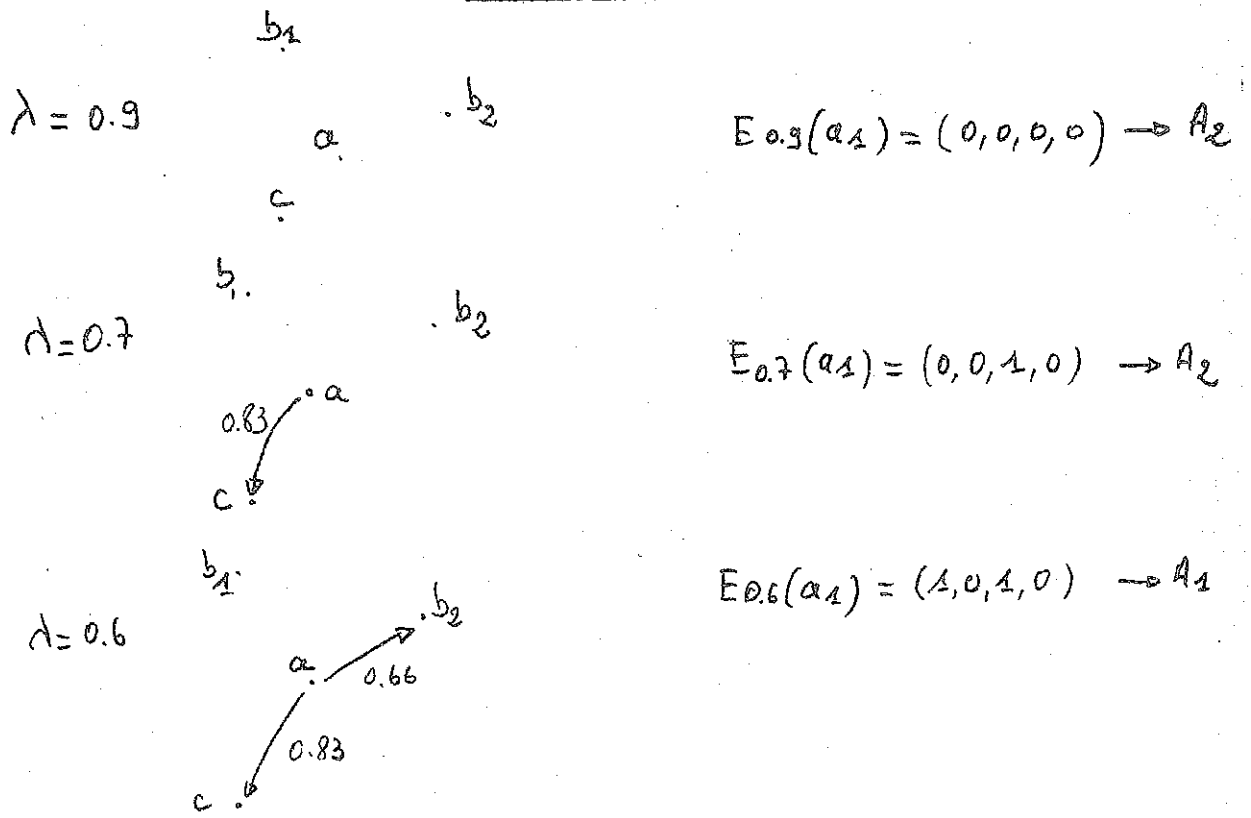
Table of criterias value

	A.LEV	Math	French	English	IMP.	APP.	EXP.	MOT.	INP
a_1	5	15	12	8	7	4	3	2	3
b_1	14	12	12	14	5	3	1	2	1
b_2	2	9	9	10	8	4	3	4	3
c	1	8	8	10	3	2	1	1	1

	concordance	discordance *
a_1, b_1	0.83	1.2 (English)
a_1, b_2	0.66	
a_1, c	0.83	

	concordance	discordance *
b_1, a_1	0.25	
b_2, a_1	0.58	1.2 (math)
c, a_1	0.16	

outranking graphs



Notice : This candidate is too bad in english to be considered as a good pupil (discordance effect). With a quite low credibility degree he could be considered as better than the reference b_2 which might allow to admit him.

* In table 9, 10, 11 the discordance indicate is notice only when it is superior to 1

TABLE 10

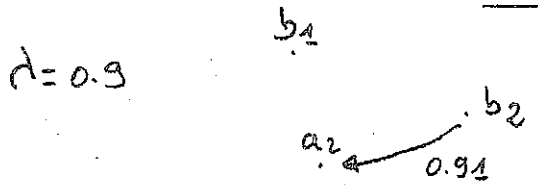
Table of criterias value

	A.LEV	Math	French	English	IMP.	APP.	EXP.	MOT.	INP
a_2	4	6	7	9	2	1	3	1	2
b_1	4	12	12	14	5	3	1	2	1
b_2	2	9	9	10	8	4	3	4	3
c	1	8	8	10	3	2	1	1	1

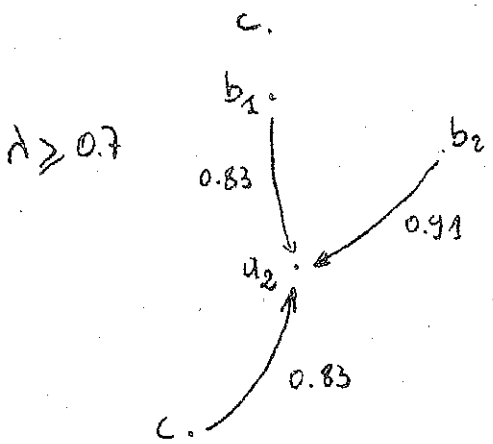
	concordance	discordance
a_2, b_1	0.25	
a_2, b_2	0.16	
a_2, c	0.33	

	concordance	discordance
b_1, a_2	0.83	1
b_2, a_2	0.91	
c, a_2	0.75	1

outranking graphs



$E_{0.9}(a_2) = (0,1,0,0) \rightarrow A_2$



$E_{0.7}(a_2) = (0,2,0,1) \rightarrow A_3$

Notice : This candidate can be rejected. The outranking of a_2 by c is obtained with a high credibility degree.

TABLE 11

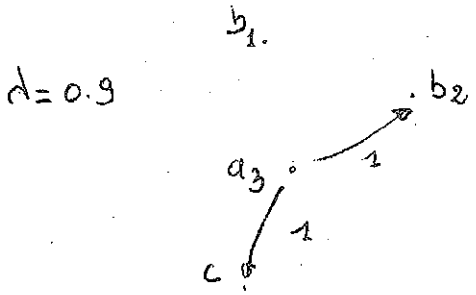
Table of criterias value

	A.LEV	MAT	FREN	ENG	IMP.	APP.	EXP.	MOT.	INP
a_3	3	10	10	11	8	4	3	4	3
b_1	2	12	12	14	5	3	1	2	1
b_2	4	9	9	10	8	4	3	4	3
c	1	8	8	10	3	2	1	1	1

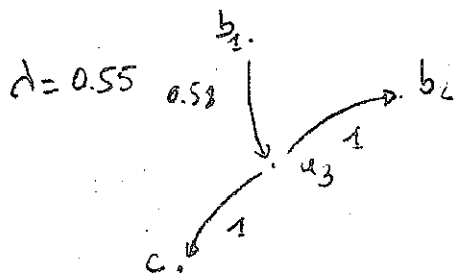
	concordance	discordance
ab_1	0.41	
ab_2	1	
ac	1	

	concordance	discordance
b_1 a	0.58	
b_2 a	0.50	
c a	0	

outranking graphs



$E_{0.9} = (1, 0, 1, 0) \rightarrow A_1$



$E_{0.55} = (1, 1, 1, 0) \rightarrow A_2$

Notice : As for candidate a_1 this examplereveals the advantage of the use of two non comparable references. Here although he obtains middle school results, the candidate has a personnality which justify his admission. With a lower credibility degree this candidate appears as less good than the good pupils, which explains that, if we apply the decision tree of figure 4, (based on a majority rule) he will be affected to A_2 unless with a higher credibility degree he would be admitted. This may suggest to apply an other decision tree based apon the most significant elements of B on C (1)

(1) J. MOSCAROLA - *Aide à la décision en présence de critères multiples fondée sur une trichotomie - Thèse Paris IX Dauphine 1977*

4. The use of the procedure by the commitee

The choice of criterias as well as other elements of the model (the vectors of references $\underline{g}(b_1)$, $\underline{g}(b_2)$, $\underline{g}(c)$ the weight p_i, the threshold ...) explicitly expresses the policy of the commitee (during a first meeting), which is certainly one of the most important contributions of the procedure. With respect to this formal elicitation let us point out that the use of the model may bring out some underlying dissensions among the members of the commitee regarding the admission policy. Nevertheless thanks to the modelling effort the consensus to which the commitee debate must necessarily leads become clearer and more transmissible.

In this way the model is also a report of the decisions taken by the commitee.

During the second meeting the commitee members have a look at the results of the procedure and decide wether the resulting partition A_1, A_2, A_3 can in all cases be applied.

Let us notice that at this stage the commitee remains entirely free of taking the final decision.

Thus if during the first meeting the members of commitee have not come to any agreement regarding the admission policy, the scientist may indicate what are the consequences of each policy in view.

In this way the final arbitration no more appeals to principles but rather to the consequences of these principles applied to the case of some candidates.

It is also possible to rank (using Electre Method (1) for example) the candidates assigned to A_2 which enables us in case of desistances of candidates assigned to A_1, the admission of the first candidates of A_2.

(1) P. BERTHIER - B. ROY *La méthode Electre II Une application au média planning VII Conférence internationale de recherche opérationnelle Dublin 1972*

III. RESSOURCE ALLOCATION TO THE SCHOOLS OF AN ACADEMIC REGION

1. The distribution procedure

1.1. The financing of the schools of an academic region : the initial situation

The french administration of the national educational system is centralized and mainly financed by public subsidies.

It is centralised: at the top the central administration controls about 20 academic regions, each of which administers about 300 colleges. The colleges receive their resources (teachers and money) from the academic regional authority, which itself receives its resources from the central authority.

We are interested here in the allocation of funds from the academic authority to the school. These subsidies are necessary to the schools to balance its budget (1) (figure 2) and results from a division of the total subsidy allocated by the central authority to the academic region. This division is made by the financial department of the regional authority and should take into account the situation and needs of each school.

An initial study on the financial situation of the schools and on the division procedure showed that :

- the most important criterion upon chich the financial director officialy bases his decision is the number of pupils attending the school ; however, that criterion does not reflect very well the operating costs (the direct charges - increasing with the number of pupils - do not represent the fourth of the subvention)

- the financial director adjusts the theoritical amount of money computed from the number of pupils. Therefore he bases his judgement on diffuse information which is not systematically taken into account and so he tends to favour some schools, more than others often due to subjective factors such as personal relations (manager of the school, financial service)

(1) *the payment of teachers is not part of that budget*

- there are very important disparities of financial exigencies between schools. These disparities can most frequently be explained by the history of the school and the principle of status quo.

The financial department was realy conscious of the indue importance given to the number of pupils and of the fact that basing the adjustment of the figures calculated on diffuse informations was a very partial, too subjective, and biaised method. It tried to take more systematically into consideration some elements such as the building structure, and the financial abilities of the school. However, the available information was too extensive and varied to be systematically taken into account and so to balance the very preponderant weight given to pupils number.

1.2. Objectives

The aim of the study is to build a model for decision aid. This model must be used to solve management difficulties resulting from the increasing number of schools (more than 300) and the complexity of information. The solution is based on an enrichment of the formal information system, a diagnosis aid and a selective orientation of the search for further information (regarding the school which can't be evaluated through the formal information system and the diagnosis aid).

More precisely the model must allow us:

- a) to carry out a multidimensionnal analysis of the school and to multiply the information elements systematically taken into consideration for the division and so to lower the importance given to the number of pupils and the appeal to badly controlled diffuse information

- b) to start from the preferences, the objectives and the policy of the decision maker, as he puts it into words ; to combine this with an automatic data processing system used to compute an agregate evaluation of the school. This diagnosis has to reveal the disparities between schools and to call into question the established situations.

- c) to implement a procedure which does not fix the division policy ; and so to forbid any rigid standards. Therefore the procedure must let the decision maker entirely free of his actions, the reciprocal arrangement of which is the setting up of dialogue between the schools and the financial service : possibility for each school to defend its own case on the base of particular information exceptionaly taken into account by the financial service.

1.3. Work method

As there have been already at disposal in the financial service a lot of information which have not really been used, a study constraint has been fixed not to create new basic information from the schools, but to try to make the existing information useful. This constraint led to some difficulties in difining some particular criterias.

The decision maker of the financial service as well as the school managers have been tightly associated within a working group in the definition of the model.

To make the explicitation of objectives, preferences and criterias clearer we systematically try to interact with the decision maker and the school managers on the basis of the study of well known and reckoned cases.

1.4. The solution

To make the understanding of the objectives and of the use of the model clearer we have to describe the whole procedure. It includes two operators and an information system.

1.4.1. The operators (fig. 13)

a) a division function computes for each school a theoritical amount of subsidies. It is computed from :

- the last year subsidies and the budgetary structure
- the variations in the building infrastructure and activities during the past school year
- the external modifications applied to all the schools (total amount of the regional subsidies, increase of prices...)

This computation keeps up the former situations, and disparities between schools.

b) a method of systematic diagnosis gives for each school a global evaluation of its financial situation, and urgency of its needs, based on the examination of the last exercice and of its own contraints. This information supplies the financial director with an aggregate indication about the policy to follow to correct disparities between schools :

so for each school we can automaticaly establish one of the following propositions

- the financial situation and urgency of needs justify an increase of the theoritical amount of subsidies (the school is "poor")

Table 12

The budgetary structure of a school

Charges	Products
Heating and Energy 45 %	Subsidies from state and local public organizations
Utilities maintenance 20 %	
Administration 20 %	Own Ressources from firms
Teaching	Families contribution (1)

(1) Coming from the boardingfees

Table 13

The operators of the procedure

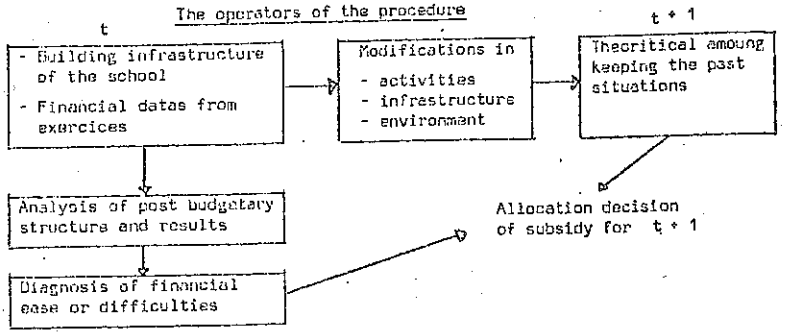
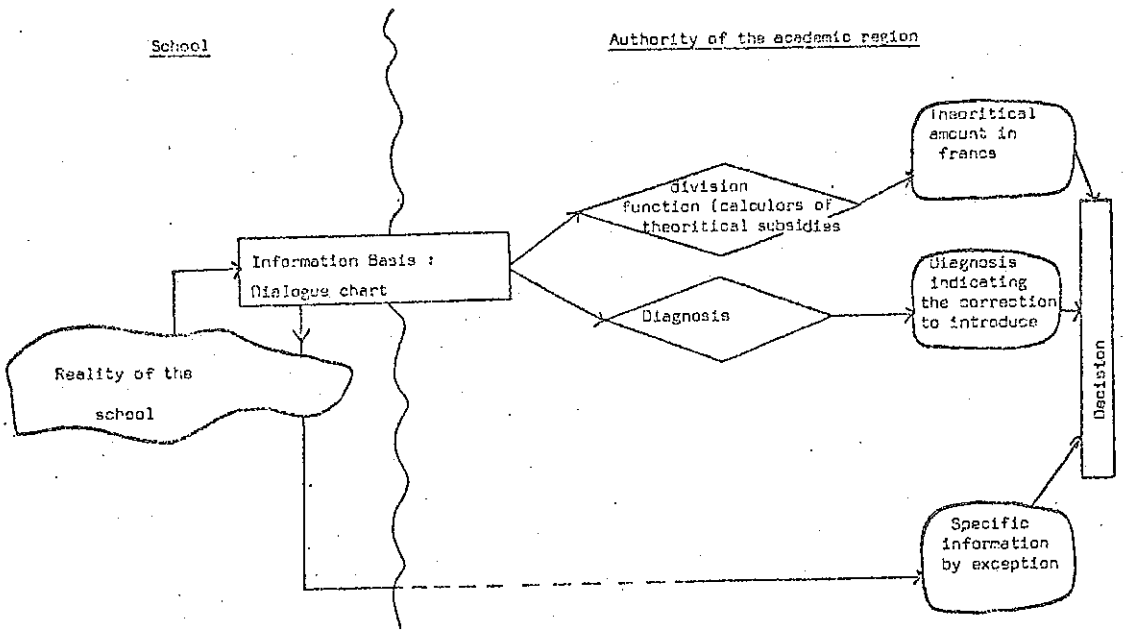


Table 14 : INFORMATION SYSTEM



- the financial situation and urgency of needs justify a decrease of the theoritical amount of subsidies (the school is "rich")

- the available information (introduced in the model) do not enable us to conclude wether the theoritical amount should be corrected.

To the proposition regarding each school is associated a number wich indicates the credibility degree of the presumption of ease or difficulty of the school.

1.4.2. The information system (figure 14)

The theoritical amount of subsidies and the diagnosis are elements of the decision maker's "dashboard". The table 15 shows which information is edited. In the second column one can read the diagnosis. The sign - (+) indicates that the theoritical amount (column 4) should be lowered (highered). The figures indicate the credibility of the proposition(1)

The decision power of the financial director remains entire. He may add to this data his own knowledge of the schools, or a futher information when necessary.

The information basis. The datas used in the operators are reported by the financial service to the school's managers in a document called the "dialogue chart". To each indicator regarding each school is associated the average level calculated from all the schools of the region (or an objective given by the financial service).

The choice of the elements of the information basis, parameters, the division function and the diagnosis method is made in a conference where the school's managers may advise and give their opinions to the financial director.

2. Modelling of the diagnosis method

We apply the methodology described in I. Given a consistent family of n criterias we are able to associate the vector $\underline{g}(a) = g_1(a) \ldots g_n(a)$ to each school. This vector results from the indicators of the information basis and dialogue chart.

Following the method described in I we affect each school to one of the categories A_1, A_2, A_3. More precisely the school a will be assigned to :

(1) For futher explanation see J. MOSCAROLA Thèse Paris IX Dauphine

TABLE 15

ETABLISSEMENTS			ANALYSE GESTIONS		SUBVENTIONS		ANALYSE DES SUBVENTIONS 72 ET 73				OBS.
NUMERO	IDENTIFICATION		DIAGN.	NOTE	SUB.74	SUB.75	SE/EL 73	DSE/EL	SE/M2 73	DSE/M2	DEMANDE
0070001N	LYC.POLY BOISSY D'ANGLAS	ANNONAY	13 -13		376930	619691	163	11	1949	15	545.000
0070002P	CET LES PLATANES	ANNONAY	31 -33		74930	79783	206	8	2867	33	[illegible]
0070003R	LYC.POLY MARCEL GIMOND	AUBENAS	11 -62	-2	250390	286071	90	10	915	22	330.000
0070004S	LYC.TECH ASTIER	AUBENAS	66 -15	2	215030	237071	176	1	985	57	[illegible]
0070004U	CES	BOURG SAINT ANDE	30 -23		90870	302698	176	1	1787	133	[illegible]
0070009X	CET	CHOMERAC	31 -7	1	43500	95661	356	16	726	213	[illegible]
0070011Z	CES GR LES VALENCE	GUILHERAND	26 -29		126460	149161	133	11	1532	133	[illegible]
0070012A	CES	JOYEUSE	3 -05	-1	69759	57031	155	155	1866	1546	[illegible]
0070016E	CET	LARGENTIERE	51 -21	3	45090	28261	323	92	1335	66	[illegible]
0070021K	LYC.POLY VINCENT D'INDY	PRIVAS	22 -27		316333	397673	167	[illegible]	1951	39	[illegible]
0070026R	CET	LE TEIL	27 -33		92033	32180	216	18	2266	357	[illegible]
0070029U	LYC.CL.M GABRIEL FAURE	TOURNON	62 -6	1	136375	177672	263	15	353	60	[illegible]
0070031W	CET	TOURNON	62 -0	1	[illegible]	139518	225	6	621	31	[illegible]
0070037C	CES LEONCE VIELJEUX	LES VANS	15 -6	-2	37375	65989	172	2	1775	18	[illegible]
0070042H	CES LES 3 VALLEES	LE VOULTE	41 -10	2	95169	969695	138	1	1936	33	[illegible]
0070047H	CES M CHAMONTIN	LE TEIL	27 -33		117330	125302	161	2	1761	103	[illegible]
0070048J	CES	TOURNON	27 -18		175190	247929	160	8	1983	267	[illegible]
0070049K	CES	LE CHEYLARD	35 -15		74375	83459	158		886		[illegible]
0260001K	CES	BOURG DE PEAGE	43 -25	3	125257	137938	160	11	1748	260	[illegible]
0260002L	CES	BUIS LES BARONNIES	12 -32		67162	57074	232	29	1319	162	[illegible]
0260004N	CES	LA CHAPELLE EN	39 -39		67357	82861	261	96	887	121	[illegible]

Example - The "lycée polyvalent Marcel Gimond" has got the mark - 2 (column 2) which indicates that the theoritical amount of subsidies (286071 Fr Col. 4) should be lowered. This proposition is established with an average degree of credibility. When in column 2 we don't get any mark that means that the school was assigned to A_2 and that further information is necessary.

A_1 if the initial information introduced in the model is sufficient to fund a presumption of financial ease, which indicates that the theoritical subsidies should be lowered.

A_3 if the initial information introduced in the model is sufficient to fund a presumption of difficulty which indicates that the theoritical subsidies should be highered.

A_2 if the initial information introduced in the model does not allow to cc to any conclusion than a further information is necessary.

Through out the 300 schools the diagnosis leads in this way the attention of the decision makers toward a further examination of the less clear cases.

2.1. The criterias

To build the consistent family of criterias the working group started with the definition of 5 main aspects regarding the evaluation of a school. They refered to the different objectives of the financial department (to assure the schools with sufficient financial autonomy and quality service) and the garantees riquired by the schools (incompressible charges, building infrastructure, financial autonomy). These main aspects were descr bed more precisely by defining n dimensions E_i on which an evaluation sca can be used to compute the state indicators $\gamma_i(a)$.
To build the criterion $g_i(a)$ from the state indicator $\gamma_i(a)$ the decisior makers had to say how their preferences vary along the scale E_i.

For example (table 16) the higher the reserve ratio is the best is the ease of the school ($g_4(a) = \gamma_4(a)$) or the higher the subjective evaluation of building degradation is, the worst is the ease of the school ($g_8(a) = -\gamma_8(a)$).

Let us point out that for some dimensions no criteria can be defined : the differences observed along the scale are not significant(1) or the decision makers (here member of working group) didn't come to any agreement concerning how the preference vary.

2.2. The references

The sets of reference B and C (part I) were reduced in that app cation to one element. Each of them marks the combined limits for each criteria from which we can say
- for the reference c (the "poor" school) that the situation of the school regarding the criteria i justify an increase of the theoritical subsidies
- for the reference b (the "rich" school) that the situation of the school regarding the criteria i justify a decrease of the theoritical subsidies.

(1) *In some cases the concept of pseudo criteria or pre criteria (ROY-JACQUET LAGREZE) may b*

Table 16 : DIMENSIONS AND CRITERIAS

Main aspects of the problem	Dimensions	Scales	
compressible charges	- importance of heating and energy charges - evolution of it	Heating and energy charges - rate : total amount of charges - percent of evolution	$g_1(a) = -\gamma_1(a)$ $g_2(a) = -\gamma_2(a)$
nancial autonomy	- transfer of income from boarding - anticipation capacity - evolution of anticipation capacity - own ressources	- rate of shifting - reserve ratio - percent of evolution of reserve ratio - own ressources ratio	$g_3(a) = -\gamma_3(a)$ $g_4(a) = \gamma_4(a)$ $g_5(a) = \gamma_5(a)$ neglected (disagreement up the variation sensus)
ality of Management	- cost of heating - evolution - quality of budget estimates	- cost per m2 - percent of evolution of it - budget estimates margin rate	$g_6(a) = \gamma_6(a)$ $g_7(a) = -\gamma_7(a)$ neglected (non discriminat
ality of service	- building degradation - quality of living - quality of utilies maintenance - quality of administrative service - quality of teaching service	- subjective evaluation - density : pupils/m2 - cost per m2 - cost per pupil (*) - cost per pupil (*)	$g_8(a) = -\gamma_8(a)$ non monotonic criteria $g_9(a) = \gamma_9(a)$ $g_{10}(a) = \gamma_{10}(a)$ $g_{11}(a) = \gamma_{11}(a)$
ilding infrastructure	- sport installation - lawns, parks and grounds commodities - heat insulation	- ranking following the importance - ranking - subjective evaluation	$g_{12}(a) = -\gamma_{12}(a)$ $g_{13}(a) = -\gamma_{13}(a)$ $g_{14}(a) = -\gamma_{14}(a)$

(*) lacking of other information we asses that the quality of service is proportional to the cost.

In this application the choice of these thresholds must be based upon the examination of the different situations of the school, for the following reasons.

- the concept of intrinsic value, or here of "poverty" or "wealth" must take into account the situation of the schools as a whole and the extend of their disparities

- the whole procedure is based upon the restoration of balance of the theoritical subsidies within the constraints of the regional subsidy.

To make the diagnosis operational the partition A_1 (school to higher) A_3 (school to lower) A_2 (indetermination) have to be balanced.

This objective can be introduced as a constraint in the model. Therefore we can change the values of references so long the rialibility of the final affectation allows it. To make it a shorter trial and error procedure, the working group was given a graphic presentation of the distribution of school on each dimension.

2.3. The modelling of global preferences and the outranking relation.

The fuzzy outranking relation used to compare a school "a" to the references b and c is described in the annex.

Let us notice that :

- for the first implementation and to make the understanding of the method easier the scientist did not introduce the discordance concept which was naturally introduced later at the decision maker's request.

- the choice of the weights p_i was made after a discussion based upon several tests where different sets of weights were applied to well known schools.

The choice of the weights as well as those of other elements of preferences modelling (criteria, outranking threshold) is the occasion of an interactive research process which characterise the scientist-decision maker relations.(1)

(1) J. MOSCAROLA - B. ROY *Procédure automatique d'examen de dossiers fondée sur une segmentation trichotomique en présence de critères multiples - RAIRO Operations Research Vol 11 n° 2 Mai 1977*

2.4. The affectation procedure

Several decision trees may be built to decide, on the base of the fuzzy outranking graph on (a, b, c), which proposition should be affected to "a" (1). Through the variations of the credibility degree we can get several diagnosis of decreasing confidence, each of them is characterised by the absolute value of the number in column 2 (table 15)

3. The effects of the model upon the administative environment

The procedure and the diagnosis model we discribed has been used for now 3 years to allocate subsidies to more than 300 schools evaluated on the basis of about 15 criteria.There was some evolutions : some criterias were left, new others were taken into account, the outranking relation, the references were modified, in a word the procedure is adaptative and may develop an apprenticeship process which may reveal a better control of the decision process.

In addition to the information system enrichment due to the formal multidimensionnal analysis, and the use of the diagnosis method as an automaticaly procedure for quick reading of dossier the main effects upon managment methods are :

- a better organisation of the financial department effort to look for a selective information
- a better organisation of the human relations thanks to a better rationalisation of choice and decision, coming from the preferences modelling.

However there is a natural tendancy of the financial department as well as of the school managers to substitute the model to the decision maker by trying to reduce the number of schools for which there is no diagnosis (A_2). This behavior is typical of a bureaucratic behavior, where the refuse of any personnal engagment, the habits of rigid scales and the attachment to the psychological security, they garanty to the decision makers, are real difficulties which must not be underestimated Nevertheless even,if such afuzzy procedure don't really fit to the traditionnal administrative management,we think that they may be useful to promote an organisational developpement toward more adaptative and participative managment methods.

(1) *the decision tree used in this application was more simple than the tree of fig. 4, for further information see J. MOSCAROLA Thèse Paris IX Dauphine*

C O N C L U S I O N

This decision aid approach may be situated in the background of large administrative organisation which can be characterized by :

- the great number of files which is often a "goulot d'étranglement" for the decisional procedures
- the complexity of the problems and the multidimensionality of their consequences
- the weight of too often purely qualitative externalities
- the increasing demand of having decision open and above board

The implementation of the model and the resulting procedure, may undoubtly in quiet numerous situations, contribute to solve some necessities of administrative work without loosing the control of the management.

This may be mainly achieved thanks to

- the increase of cognitive abilities of the decision maker involved by the implementation of the preferences modelling with multiple criteria
- the better management of information system based upon the selective ability introduced by the trichotomy

Finally these improvments involve a better investigation of the solving problems due to any modelling effort.

OUTRANKING RELATION (1)

Given $g_1, g_i, \ldots g_n$ a consistent family of criterias

$P_i \quad i \in F = 1 \ldots n$ the weights assigned to each criteria g_i

$$\sum_{i \in F} P_i = 1$$

D_i the maximal admitted discordant margin for each criteria g_i, $i \in F$

For each pair (a, a') we can build the following partition of F :

$$F = F^+ \cup F^- \cup F^=$$
$$F^+ = \{ i / g_i(a) > g_i(a') \}$$
$$F^- = \{ i / g_i(a) < g_i(a') \}$$
$$F^= = \{ i / g_i(a) \simeq g_i(a') \}$$

Concordance coefficient

We call concordance coefficient the number calculated as follows

$$Con(a, a') = \sum_{i \in F^+} P_i + \sum_{i \in F^=} P_i$$

Calculating for each pair (a, a') of A the value of the coefficients Con (a,a') we build a concordance table

Discordance coefficient

Given d_i the discordant margin observed on the criteria i :

$$g_i(a') - g_i(a) = d_i$$

We call discordance coefficient the number calculated as follows

$$Dis(a, a') = \max_{i \in F^-} \frac{d_i}{D_i}$$

Calculating for each pair (a, a') of A the value of the coefficients Dis (a,a') we build a discordance table.

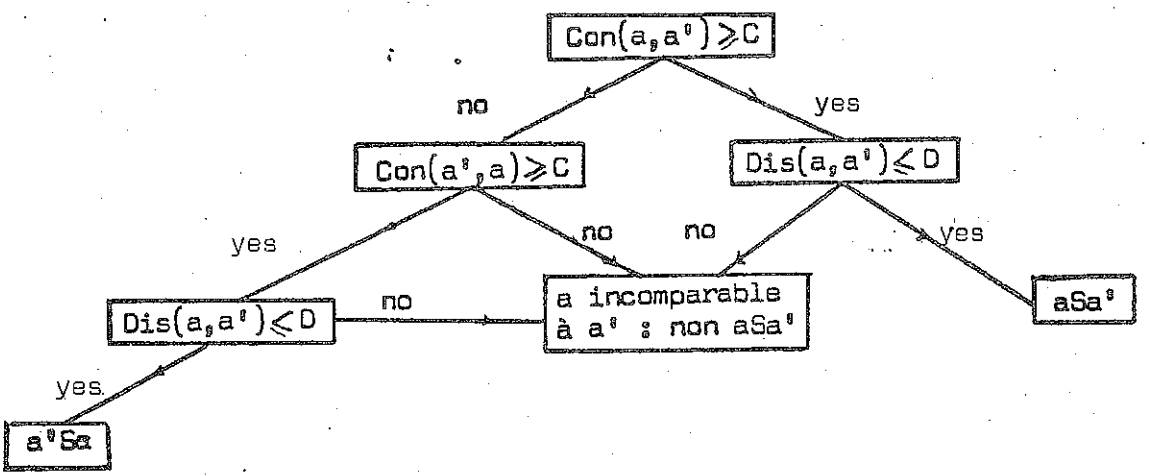
(1) *One can find similar definitions in B. ROY - E. JACQUET LAGREZE and G. HIRSCH - see Bibiography.*

Making up an outranking relation

Given the thresholds of concordance and discordance C and D For example C = 0.6 means that we are at least exacting for the summ of the concording criterias ($i \in F^+$ UF$^=$) to be superior to 0.6 to make up the outranking of a' by a ("a" is prefered to a')

D = 1 means that, if at least one observed margin d_i ($i \in F^-$) is superior to the maximal margin D_i admitted for the criterias i ($i \in F$), then the outranking of a' by a cannot be made up (we cannot say that "a" is prefered to a')

To each pair (a, a') of A, and using the tables of concordance and discordance, we may make up or not the outranking of a' by a out of the following tests.



Fuzzy outranking relation and nested outranking relations

We can make the thresholds C and D vary. We obtain then more or less "severe" outranking relations. They are called nested relations if :

$$S_i \subset S_j \iff a \; S_i \; a' \Rightarrow a \; S_j \; a'$$

The relation S_j is less severe than the relation S_i

In the application regarding the selection of candidates we use 4 nested outranking relations

$S_1 \subset S_2 \subset S_3 \subset S_4$ obtained by making vary the concordance threshold C for a fixed threshold of discordance (D = 1) :

$C_1 = 0.9$ for S_1

$C_2 = 0.7$ for S_2

$C_3 = 0.6$ for S_3

$C_4 = 0.5$ for S_4

Then we are enable to characterize the fuzzy outranking thus obtained taking for the degree of credibility d (a, a') the value of the coefficient of concordance Con (a, a') :

When Dis (a, a') ≤ 1

d (a, a') = Con (a, a')

When Dis (a, a') > 1

d (a, a') = 0

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