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MULTICRITERION ANALYSIS OF GROUNDWATER MANAGEMENT ALTERNATIVES

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ANALYSE MULTICRITERE

APPLIQUEE A LA GESTION DES EAUX SOUTERRAINES

Résumé. On présente une application de quatre méthodes d'aide multicritère à la décision à un problème caractéristique de la gestion des eaux souterraines. Pour générer une représentation discrète de l'ensemble de solutions non dominées, on utilise un modèle de gestion des eaux exprimé sous forme multicritère. Ce modèle est basé à la fois sur la méthode des éléments finis pour modéliser le système physique et sur une combinaison de la méthode des réponses (response matrix method) et la méthode implicite (embedding method) pour formuler un programme mathématique. On analyse alors, en tant qu'outils potentiels d'aide à la décision pour le rangement et enfin le choix du schéma de gestion approprié, quatre méthodes multicritères : Compromise Programming, ELECTRE III, la théorie de l'utilité multi-attribut et la méthode UTA représentant, respectivement, les approches basées sur la distance, sur le surclassement et sur l'utilité multiplicative et additive. La comparaison des résultats obtenus montre que ces méthodes, quoique différentes, aboutissent aux mêmes solutions. Enfin, on étudie l'adéquation de ces méthodes aux différents problèmes de gestion des eaux souterraines.

MULTICRITERION ANALYSIS

OF GROUNDWATER MANAGEMENT ALTERNATIVES

Abstract. An application of four multicriterion decision making (MCDM) techniques to a typical groundwater management problem is presented. The management model, expressed in a multicriterion form and based on the finite element method and a combined embedding/response matrix method, is used to generate a discrete representation of a nondominated solution set. Four MCDM techniques : Compromise Programming, ELECTRE III, Multiattribute Utility Function and UTA method representing, respectively, distance-based, outranking and utility (multiplicative and additive forms) approaches are examined as potential decision aid tools to rank and finally to select the appropriate management scheme. Comparison of the results shows that the quite different MCDM methods lead to a similar subset of recommended solutions. Adequacy of these techniques for groundwater management is discussed from several points of view.

1. INTRODUCTION

The purpose of this paper is to examine how several multicriterion decision making (MCDM) techniques may be used as decision aid tools in groundwater management. Groundwater resources management problems require methods that are well adapted to their complex structure as well as acceptable by a Decision Maker (DM) confronted to these problems. Simulation techniques such as groundwater flow simulation models, which for a long time have been used in management problems, are not proper tools for decision aiding. As shown in El Magnouni and Treichel [1992] the role of simulation techniques in groundwater management aid is limited since the determination of the set of parameters which must be done before any simulation constitutes itself an essential decision making problem. The simulation is practically reduced to serving as a feasibility test for an infinity of actions that can be undertaken and as a way of forecasting groundwater system reactions. More adapted optimization methods, such as embedding and response matrix methods [Gorelick, 1983], are capable of solving management problems but only in a relatively simple, single-objective version. The optimal solution resulting from these methods has a little chance to be accepted for implementation by the DM since (s)he has other criteria which should be taken into account. New approaches in Bogardi et al. [1991], Shafike et al. [1992] and El Magnouni and Treichel [1992] allow us to treat the groundwater management problems in a multiobjective framework.

In this paper we examine the suitability of four MCDM techniques to groundwater management, namely :

- Compromise Programming, a distance-based technique;
- ELECTRE III based on a fuzzy outranking relation;
- Multiattribute Utility Function, assuming the fulfilment of utility theory axioms;
- UTA in which a set of piecewise linear, additive utility functions is assessed.

We use the multiobjective groundwater management model developed in El Magnouni and Treichel [1992] to formulate the problem and generate a discrete representation of a nondominated, Pareto-solution set. This model based on finite elements and a combined embedding/response matrix method is applied here to a typical three-objective groundwater resources management problem in an island concerning the maximization of total pumping rates, minimization of operating costs and minimization of risk. A brief presentation of the model and case study example are given in the next section. In the subsequent section we describe the algorithm used to generate the Pareto set. Then the four MCDM techniques are applied to this set. The results produced by each technique and their evaluation from several points of view are discussed in the last section.

2. GROUNDWATER MANAGEMENT MODELING

The groundwater management model presented in this paper couples a hydrodynamic groundwater flow simulation model with a multi-criteria decision aid one. It was developed in El Magnouni and Treichel [1992] and illustrated by an idealized but typical example of groundwater resources management in an island which is also used here; the DM is represented by an experienced colleague of the B.R.G.M.

Let us consider a circular island with diameter $d = 50 \text{ km}$. Groundwater resources are accumulated in a single water layer and are to be used for water supply of a distribution center (a town) located in the middle of the island. Aquifer transmissivity and recharge rate units are assumed to be uniform and equal to $5 \text{ m}^2/\text{s}$ and $3.2 \cdot 10^{-7} \text{ m/s}$, respectively. The management problem consists in finding the location of the wells and corresponding pumping rates, satisfying the groundwater flow law (the hydrodynamic model), all the constraints on the hydraulic heads and the minimal demand at the water distribution center. Moreover, the solution should reflect the DM's preference system and the decision aid model should possibly yield the best compromise among multiple objectives formulated by the DM, here maximization of total pumping rates, minimization of operating costs and minimization of risk.

We consider a two dimensional, steady-state groundwater flow model which is given by the following equation :

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{R + Q(x,y)}{T} \quad (1)$$

where

- h - hydraulic head , [m];
- x, y - Cartesian coordinates , [m];
- R - recharge , [m/s];
- $Q(x,y)$ - pumping rates , [m/s];
- T - aquifer transmissivity , [m²/s],

subject to Dirichlet boundary condition :

$$h(x, y) = 0 \quad (2)$$

for all (x, y) lying on the border of the island.

Using a 2D finite element approximation with 41 nodes and 70 elements (Figure 1) and Galerkin's method [Wang and Anderson, 1982] we transform the differential groundwater flow equation into a linear system :

$$[G] [h] = [R] + [D] [Q] \quad (3)$$

In equation (3) G is a square coefficient matrix called the global conductance matrix; h is the unknown hydraulic head vector of nodal heads; R represents the boundary condition vector; Q is a decision variable vector and D is the 0-1 matrix representing a variety of water supply well locations.

We allow each node inside the island to be a potential location of a water supply well, excluding the nodes lying on the border (nodes $n^\circ 32$ to 41 in Figure 1.). Apart from boundary condition (2), which now in the discrete representation is transformed to $h_i = 0$ for $i = 32, 33, \dots, 41$, we require a number of supplementary constraints to be satisfied. For example, to avoid salt water intrusion the hydraulic head at the nodes lying close to the border should be greater than 0.01 (since $h_i \geq 0.01$ for $i = 22, 23, \dots, 31$) and the hydraulic head at the other nodes should be non-negative (then $h_i \geq 0$ for $i = 1, 2, \dots, 21$). Moreover, the minimal water distribution center demand, equal to $12.5 \text{ m}^3/\text{s}$, must be provided (then $\sum_i Q_i \geq 12.5$).

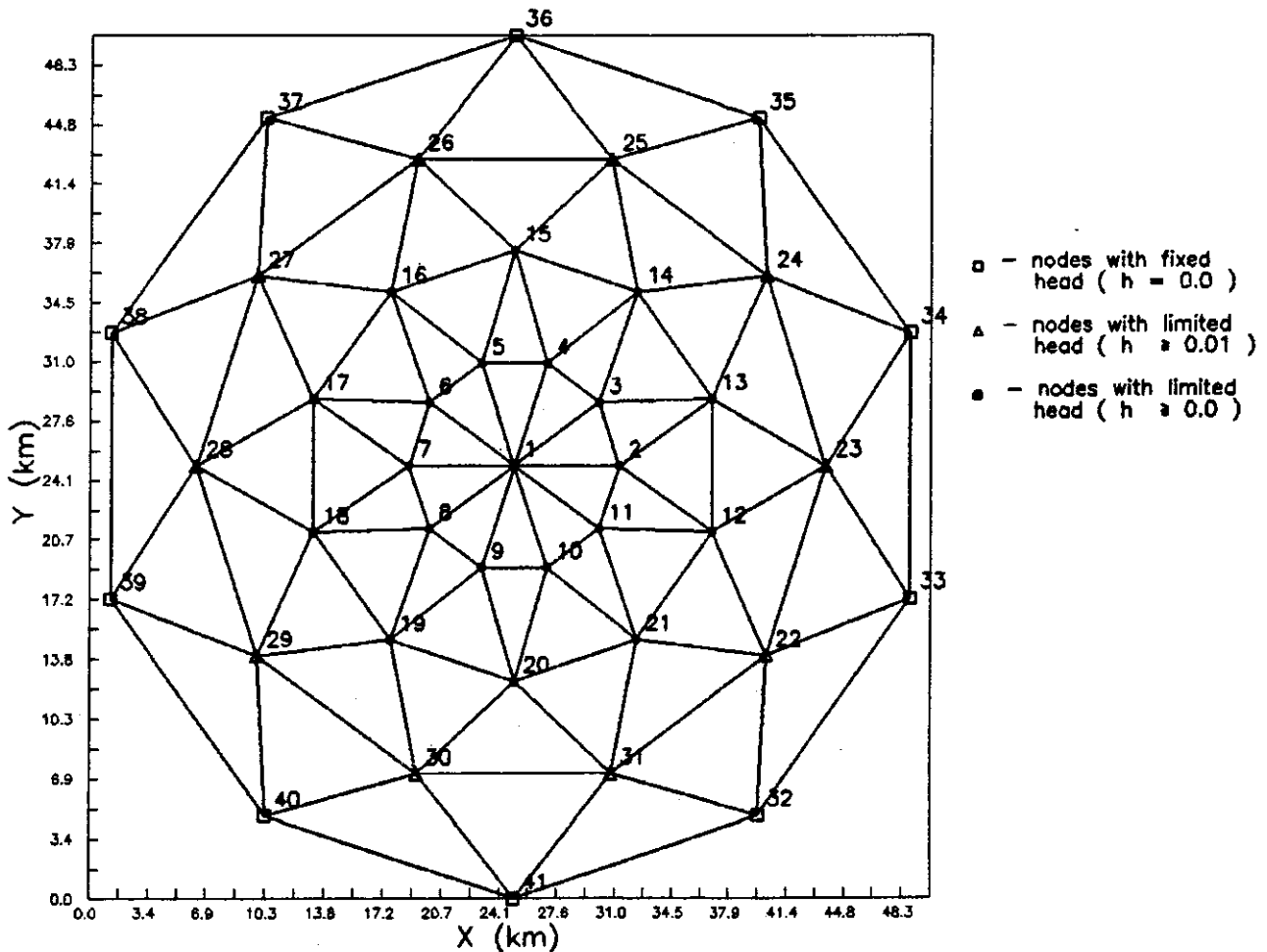


Figure 1. Triangular finite element mesh used in the example.

However, equation (1) representing the groundwater flow behavior and approximated by linear system (3) is overdetermined; it has two unknowns h and Q . In order to overcome these difficulties we have turned the problem into a multiobjective optimization one [El Magnouni and Treichel, 1992].

The following three conflicting objectives have been articulated by the DM :

1. maximization of total pumping rates over the aquifer;
2. minimization of operating costs including water pumping and water transport costs;
3. minimization of risk interpreted as the percentage uncertainty in water quantity at the exploitation stage.

Because the groundwater management problem involves long-term planning where several parameters have an approximative character, we are of the opinion that making a linear approximation of the objective functions does not introduce significant errors in the model. We calculate the unit costs of exploitation, which should be defined at each node, as a linear combination of the water pumping costs depending on well depth and water transport costs depending on the distance from the well to the distribution center. The costs are assumed to be larger towards the boundaries.

It is assumed that in the groundwater system under consideration three risk zones are present and correspond to the knowledge of the system parameters and their uncertainty. In the *NE* part of the island which is quite heterogeneous and where only very few observation are available the risk coefficient is equal to 30% , in the *NW* part to 1% and in the *South* part, 5%. In the present study the risk zones and coefficients were defined in a simplified, arbitrary way to illustrate the possibility to take it into account. For real-case studies a general methodology of construction of this kind of criteria based on geographical information system is developed [Hrkal, 1992] but its explanation goes beyond limits of this paper.

Then the three objective functions considered in our problem are as follows:

$$\max (g_1 = \text{total pumping rate}) = \max \sum_i Q_i \quad (4)$$

$$\min (g_2 = \text{operating costs}) = \min \sum_i c_{ci} Q_i \quad (5)$$

$$\min (g_3 = \text{risk}) = \min \sum_i c_{ri} Q_i \quad (6)$$

The coefficients in the objective function (5) take on values from 0.1 to 2.0 , and in the function (6) the values 0.3 , 0.01 and 0.05 , depending on the location of potential water supply wells. The index i corresponds to all potential locations.

Expressing the constraints mentioned above in an integrated, mathematical form we obtain :

$$[G_1] [Q] \geq [Q_0] \quad (7)$$

$$[G_2] [h] \geq [h_0] \quad (8)$$

$$[G] [h] - [D] [Q] = [R] \quad (9)$$

where G_1 and G_2 are the 0-1 matrices; Q_0 and h_0 correspond to the minimal water distribution center demand and desired limits on hydraulic heads, respectively.

Problem (4) - (9) is a multiobjective linear programming (MOLP) problem where Q and h are variables. However, using the combined embedding and response matrix method [Herrling and Hecke, 1986] we solve equation (9) in a preliminary calculation step applying the classical Cholesky algorithm adapted to the n vectors on the right hand side. We obtain the following relation:

$$[h] = [R^*] + [D^*] [Q] \quad (10)$$

which permits to reformulate the constraint (8) as follows

$$[G_2^*] [Q] \geq [h_0^*] \quad (11)$$

In the resulting problem (4) - (7) and (11), Q constitutes a vector of the only decision variables.

There are two classes of methods of solving MOLP problems : interactive methods and methods with pre-generation of Pareto set. In an interactive method [Vanderpooten, 1989] two phases, articulation of preferences and calculation of a corresponding solution, are repeatedly executed until the best compromise solution is reached. Methods of second class usually consist of two separate steps : generation of Pareto solutions (nondominated solution set) and then application of one or several decision aid techniques to rank the solutions or/and to find a solution(s) yielding the best compromise(s) among criteria. The interactive methods, very convenient from DM's point of view, require an efficient solver to calculate a new proposal for each iteration. However, the real-case groundwater management problems result in MOLP formulation which is too large and numerically hard for microcomputer available optimization codes. Then we have proposed a two-steps method to overcome these difficulties and to give the possibility to use a more performance solver on mainframe if necessary. In the next sections we will present the procedures and techniques chosen to implement these two steps.

3. GENERATION OF A REPRESENTATION OF THE PARETO SET

Frequently, the first step in multiobjective optimization techniques consists in determining a subset of feasible alternatives that are of interest for the DM. Usually, this set is composed of nondominated (efficient) solutions. The nondominated solution set, also known as the Pareto set, is a subset of the feasible region that contains solutions for which any improvement of one objective function can be achieved only at the expense of at least one other objective function. This set, or a good representative subset of it, is presented to the DM whose aim is to choose, possibly using a decision aid technique, a subset of alternatives considered as the best compromises or "satisficing" solutions. This procedure may lead to the selection of a given alternative.

There are several possible methods of generating a representation of the Pareto set of a MOLP problem. Let us only recall the weighting (parametric) method, ϵ -method, multicriterion simplex method [Goicoechea et al., 1982]. In this paper, however, we shall use a method proposed in Choo and Atkins [1980] which is very a convenient version of ϵ -method, from the user's point of view [Jacquet-Lagrèze et al., 1987].

The algorithm starts by generating the first efficient point of the MOLP problem, y^0 , which is as close as possible to the ideal point G^* . The next section provides a definition of the ideal point. As a measure of closeness, the weighted Chebyshev norm or L_∞ -norm in the criterion space is used. It is defined as the maximum weighted deviation from the ideal point G^* , which is to be minimized

$$\min_x \max_i w_i |G^* - g_i(x)| \quad x \in X \quad (12)$$

The weights w_i in this Chebyshev norm are chosen so as to keep the y^0 point in the direction of the line joining the ideal point and the point F^* defined by the worst values of each criterion in the pay-off table; F^* is usually called the nadir point. This locates y^0 in the central part of the feasible region. The weight coefficients are calculated from the following expression :

$$w_i = \frac{1}{|G_i^* - F_i^*|} \quad (13)$$

The min-max problem (12) can be transformed into the following minimization problem :

$$\begin{aligned} & \min z & (14) \\ \text{s.t.} & \\ & z \geq w_i |G_i^* - g_i(x)| & i = 1, 2, \dots, I \\ & x \in X \end{aligned}$$

Solution of this problem, x_0 , is used to calculate the first efficient point $y^0 = (y_1^0, y_2^0, \dots, y_I^0) = (g_1(x_0), g_2(x_0), \dots, g_I(x_0))$. Next, this point is taken as a starting point for generating other efficient solutions. For this purpose we take several extra points regularly distributed on the line segment between y^0 and F^* ; $y^1, y^2, \dots, y^R$.

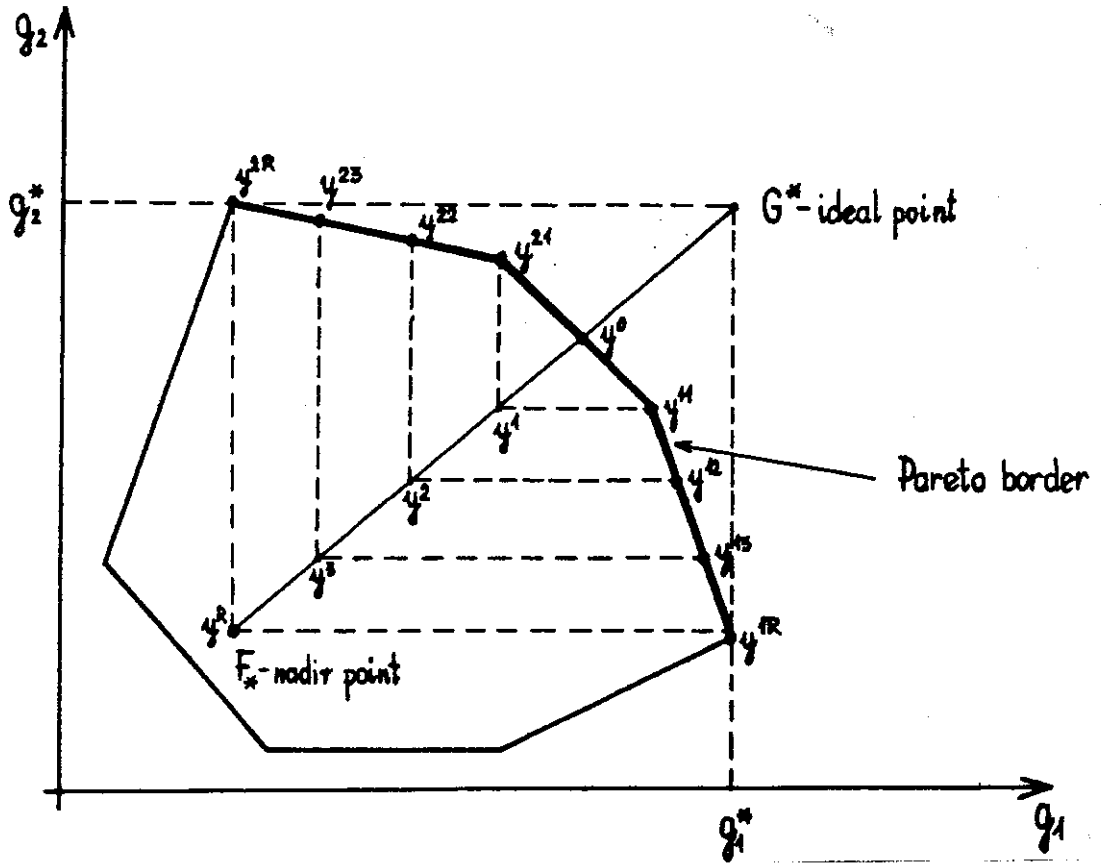


Figure 2. Sketch of the Pareto set generated by means of the algorithm of Choo and Atkins [1980].

Then the following series of one-criterion optimization problems is solved:

for a given $r = 1, 2, \dots, R$, and $k = 1, 2, \dots, K$

$$\begin{aligned} & \max_x g_k(x) \\ \text{s.t.} & \quad g_i(x) \geq y_i^0 - r/R |y_i^0 - F_i^*| \quad i = 1, \dots, I; i \neq k \\ & \quad x \in X \end{aligned} \tag{15}$$

In this way we obtain a representation $\{y^{kr}; k=1, \dots, K, r=1, \dots, R\}$ of the efficient border. The procedure is illustrated in Figure 2 for a bicriterion maximization problem.

Following this procedure we generated a sample of Pareto set points for our three-objective linear programming problem (4)-(7) and (11). Table 1 shows these nondominated alternatives which are then presented to the DM so that he/she can choose the one that best fits his/her desirata. Making this choice is supported by a decision-aid technique. In the next sections we shall examine the adequacy of four MCDM techniques as decision aids for groundwater management problems : Compromise Programming, ELECTRE III, Multiattribute Utility Function and the UTA method.

Table 1. Generated Pareto alternatives, ideal, nadir and anti-ideal solutions.

Alternative N°	Total pumping rate <i>g1</i>	Costs <i>g2</i>	Risk <i>g3</i>
1	32.1072	23.8851	2.5198
2	36.8617	32.1272	3.4380
3	41.3418	40.3693	4.3563
4	44.8920	48.6113	5.2744
5	44.8920	56.8534	6.1926
6	27.2054	15.5633	3.4380
7	22.3036	8.1764	3.2292
8	17.4018	4.5128	2.8017
9	12.5000	2.3882	2.0996
10	27.2054	27.3084	0.4983
11	22.3036	19.4461	0.2336
12	17.4018	16.9711	0.1740
13	12.5000	18.7500	0.1250
Ideal	44.8920	2.3882	0.1250
Nadir	12.5000	56.8534	6.1926
Anti-ideal	12.5000	67.3379	7.5452

4. MCDM TECHNIQUES - METHODOLOGY AND RESULTS

4.1. Compromise Programming (CP)

Compromise programming (CP), first used in a multiple objective linear programming context [Zeleny, 1973, 1974], is a distance-based technique designed to identify so-called compromise solutions which are determined to be the closest, by some distance measure, to

an ideal solution. This technique has also been used in Duckstein and Opricovic [1980] for multiobjective optimization in river basin system development, in Tecle et al. [1988] for multicriterion selection of wastewater management scheme, and in Duckstein et al. [1991] to rank estimation techniques for fitting extreme floods. In those applications of the CP method, the compromise solutions were compared to solutions obtained using other MCDM techniques.

For an explanation of what is called by a compromise solution in this technique, we define what is meant by an ideal solution and specify a measure of closeness to be used. The ideal solution can be defined as the vector $G^* = (G_1^*, G_2^*, \dots, G_I^*)$ where the G_i^* are the best (optimal) values of each objective and are the solutions to the optimization (in our case maximization for $i=1$ and minimization for $i=2, 3$) problems

$$G_i^* = \min_x g_i(x), \quad x \in X, \quad i=1, 2, \dots, I \quad (16)$$

The ideal solution is generally not feasible. The notion of anti-ideal is also used to define the vector $G_* = (G_{1*}, G_{2*}, \dots, G_{I*})$ where G_{i*} are the worst values of each criterion on the feasible domain; note that this point G_* is different in general from the nadir point defined earlier.

One of the most frequently used measures of closeness is a family of weighted L_p metrics, defined as follows :

$$L_p = \left(\sum_{i=1}^I w_i^p \left| \frac{g_i(x) - G_i^*}{G_{i*} - G_i^*} \right|^p \right)^{1/p} \quad (17)$$

where w_i are the criterion weights, $i=1, 2, \dots, I$ and $1 \leq p \leq \infty$. The compromise solution x_{CP} , a feasible solution as close as possible to the ideal in sense the chosen metric (with given p and w_i), then results from the following optimization problem :

$$\min_x L_p(x) \quad \text{for } x \in X \quad (18)$$

For given set of weights $\{w_i\}$ and for all $p : 1 \leq p \leq \infty$ the set of all compromise solutions, called the compromise set, is obtained. Usually, and in the present paper also, only three points of the compromise set are calculated : the extremes for $p=1$ and $p=\infty$, and an intermediate point for $p=2$. It is worth to note that CP method, for $1 \leq p < \infty$, always produces a nondominated solution; for $p=\infty$ the uniqueness of the solution, which is the same as the Chebyshev solution, is not guaranteed.

The results of applying this method to generated set of Pareto alternatives for given set of weights $\{w_1 = 0.22, w_2 = 0.42, w_3 = 0.36\}$ are displayed in Table 2. The weights that we have used in CP have normalized values corresponding to values of importance

coefficients expressed in ELECTRE III (cf. Table 3) and scaling and weighting coefficients assessed in MUF and UTA methods (cf. formula (28) and Fig. 6 , respectively).

Table 2. Lp-distance from ideal solution and ranking of alternatives resulting from Compromise Programming.

Alt.	p = 1	Rank	p = 2	Rank	p = ∞	Rank
1	0.34203	8	0.20091	2	0.13901	1
2	0.40758	10	0.25650	10	0.19231	8
3	0.47500	11	0.32101	11	0.24561	11
4	0.54874	12	0.38956	12	0.29890	12
5	0.64658	13	0.45903	13	0.35220	13
6	0.36606	9	0.21800	5	0.16074	4
7	0.34145	7	0.21822	6	0.15342	2
8	0.33031	6	0.22784	7	0.18671	6
9	0.31580	4	0.23995	8	0.22000	9
10	0.29938	3	0.20181	3	0.16115	5
11	0.26899	1	0.18903	1	0.15342	2
12	0.28339	2	0.20918	4	0.18671	6
13	0.32580	5	0.24412	9	0.22000	9

4.2. ELECTRE III

The ELECTRE III technique, developed in Roy [1978], is based on a fuzzy outranking relationship. It deals with a finite set of alternatives evaluated on a consistent family of criteria. The concept of pseudocriterion is used to model DM's preferences. It allows to model an indifference relation, a weak preference relation and a strict preference relation as a result of combining indifference and preference thresholds with a traditional criterion. The method aggregates these partial preferences in a fuzzy outranking relation, and then uses this relation to set up a partial preference preorder of the alternatives. Applications of ELECTRE III include the comparative study on the hypothetical siting of a nuclear power plant in Roy and Bouyssou [1986], the determination of which Paris Metro stations should be renovated in Roy et al. [1986], the programming of rural water supply systems in Treichel [1987] and Roy et al. [1992].

ELECTRE III starts by comparing each pair of alternatives, (a, b) , with the aim of assessing a credibility level of the affirmation that " a outranks b " ($a S b$) which means that a is considered to be at least as good as b . The calculation of the credibility $d(a, b)$ of this affirmation is essentially based on two concepts called concordance and discordance [Roy, 1990] and respects certain qualitative principles, in particular excludes the possibility that a major disadvantage on one criterion might be compensated by a number of minor advantages on other criteria.

The goal of the concordance concept is to use the values of indifference and preference thresholds associated with each criterion in order to characterize a group of criteria considered to be in concordance with the affirmation being studied, and to assess a relative importance of this group of criteria compared with the remainder of the criteria. The formulas for calculating the concordance index (in the case of a maximization problem and constant thresholds) are as follows :

$$c_i(a, b) = \begin{cases} 1 & \text{if } g_i(b) - g_i(a) \leq q_i \\ \frac{p_i - (g_i(b) - g_i(a))}{p_i - q_i} & \text{if } q_i < g_i(b) - g_i(a) \leq p_i \\ 0 & \text{if } g_i(b) - g_i(a) > p_i \end{cases} \quad (19)$$

where q_i and p_i are fixed indifference and preference thresholds, respectively. Next, using the coefficients of importance k_i determined for each criterion, we calculate :

$$C(a, b) = \frac{\sum_{i=1}^I k_i c_i(a, b)}{\sum_{i=1}^I k_i} \quad (20)$$

The index $C(a, b)$ expresses the concordance degree of the affirmation being studied. Let us note that the coefficients of importance k_i are not used in the sense of trade-off or scalarizing (utility function) coefficients.

The other concept, discordance, is applicable to characterize which criteria are not in concordance with the affirmation under consideration, the ones whose opposition is strong enough to reduce the credibility which would result from taking into account just the concordance. A possible reduction should be also calculated. The formula for calculating this discordance index is :

$$D_i(a,b) = \begin{cases} 0 & \text{if } g_i(b) - g_i(a) < p_i \\ \frac{(g_i(b) - g_i(a)) - p_i}{v_i - p_i} & \text{if } p_i \leq g_i(b) - g_i(a) < v_i \\ 1 & \text{if } g_i(b) - g_i(a) \geq v_i \end{cases} \quad (21)$$

where v_i is a "veto threshold" defined as the minimum value of discordance which gives criterion i the power to take all credibility away from the affirmation being studied even when opposed to all the other criteria in concordance with the affirmation. The threshold functions and importance coefficients used in our case study example are presented in Table 3.

Table 3. Threshold functions and weights used in ELECTRE III.

Criterion	Indifference threshold q_i	Preference threshold p_i	Veto threshold v_i	Importance coefficient
1	2	10	35	2
2	$0.05 g_2(x)$	$0.2 g_2(x) + 2$	$0.6 g_2(x) + 6$	4
3	$0.1 g_3(x) + 0.2$	$0.25 g_3(x) + 1.0$	$0.5 g_3(x) + 3$	3.5

Finally, the fuzzy outranking relation index $d(a,b)$ is calculated using the following formula:

$$d(a,b) = \begin{cases} C(a,b) & \text{if } I_0 = \{i: D_i(a,b) > C(a,b)\} = \emptyset \\ C(a,b) \prod_{i \in I_0} \frac{1 - D_i(a,b)}{1 - C(a,b)} & \text{if } I_0 \neq \emptyset \end{cases} \quad (22)$$

The set of values of this index $d(a,b)$ summarizes the results of the comparison of all possible pairs of alternatives. These results are established from the data available using principles such as non-complete compensation that determine the credibility of the outranking [Roy, 1985]. Full compensation means that the sum of marginal decreases in

some objectives is fully compensated by the sum of marginal increases in the other objectives. This may be true for money but not for problems involving risk or damages.

In the next step ELECTRE III constructs two complete preorders on the set of alternatives using a technique called "distillation". The final partial preorder resulting from the method is a set-theoretic intersection of these two preorders. The possibility of not having a priority relation between two alternatives which is allowed in the partial preorder, is related to the fact that in certain cases it is impossible to come to a conclusion in a priority relation within a multicriterion framework. The preorders resulting from the ELECTRE III method are presented in Figure 3.

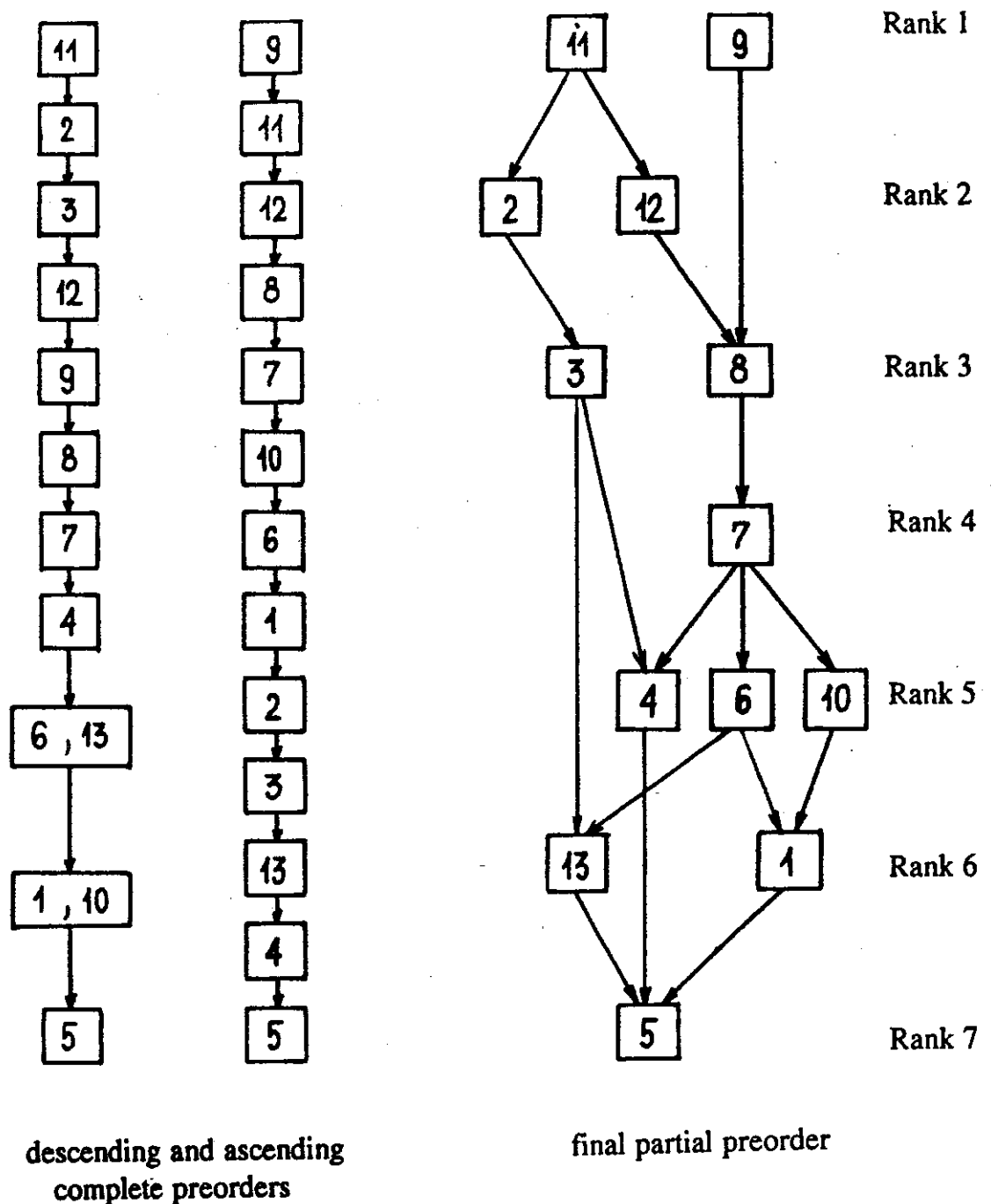


Figure 3. Preorders resulting from ELECTRE III.

4.3. Multiattribute Utility Function (MUF)

The Multiattribute Utility Function (MUF) method, developed in Keeney and Raiffa [1976], represents a global utility function modeling and is derived from multiattribute utility theory (MAUT). This theory is based on a set of axioms which should be satisfied in order to represent the DM's preference system by a utility function. The axioms, such as pre-existence of a complete system of preferences, transitivity of DM's preferences, preference independence and utility independence are equivalent to assumptions about rationality and consistency of the DM. It is also assumed that probabilistic lottery comparisons can always be used to assess the different parameters involved in the definition of the global utility function ; that is, the single attribute utility functions corresponding to each point of view and the scaling coefficients that allow their aggregation into a global model can be assessed. In the present paper we do not discuss whether or not the utility theory axioms are reasonable; for that discussion see for example Roy and Bouyssou [1986]. We simply assume that the axiomatic basis that underlies multiattribute utility theory is approximately satisfied and thus assume the existence of a utility function representing the DM's preference system.

Application of MUF method can be found in Keeney and Wood [1977] to a problem in river basin planning and in Keeney and Raiffa [1976] to a number of decision problems in miscellaneous branches of activity. On the other hand, Sakawa [1982] has proposed another interactive MCDM technique incorporating certain desirable features of the MUF method.

The MUF assessment process consists of three following steps [Keeney and Wood, 1977] :

step 1 : Investigation of the qualitative preferences structure and choice of a functional form of a global utility function;

step 2 : Construction of single-attribute, marginal utility functions;

step 3 : Assessment of scaling coefficients and global multiattribute utility function.

In the first step the qualitative structure of DM's preferences is examined by checking the preference independence and utility independence of each objective (attribute) or pairs of attributes. To check whether or not the pair of attributes, for example $\{g_1, g_2\}$ (where $g_i = g_i(x)$), is preferentially independent of the other attributes, in our case $\{g_3\}$, we examine if preferences among $\{g_1, g_2\}$ or trade-offs between g_1 and g_2 held constant, do not depend on the level of the other attribute, g_3 . In a similar manner we verify that each pair of attributes, $\{g_1, g_3\}$ and $\{g_2, g_3\}$, is preferentially independent of the other, g_2 , and g_1 , respectively.

To examine if each of g_1, g_2, g_3 is utility independent of the other attributes we check if preferences among lotteries over each attribute g_i for fixed values of the other attributes, do not depend on the constant levels of those attributes. For example, the certainty equivalent gE_2 for the 50-50 lottery yielding either $g_2=2.4$ with probability 0.5 , or

$g_2=67.3$ with probability 0.5, equal to 20.0, does not change significantly when only the levels of g_1 and g_2 are varied. Next, the certainty equivalent for the 50-50 lottery yielding 2.4 or 20.0, respectively, found out to be 8.0, also does not depend on the levels of the other attributes. Consequently, we assume that the attributes g_1 , g_2 , g_3 are preferentially and utility independent.

Hence, only additive or multiplicative multiattribute utility functions are to be considered. To decide which one of these two forms is appropriate for our problem, we investigate whether there is a preference or an indifference between the three pairs of lotteries A_i and B_i shown in Figure 4.

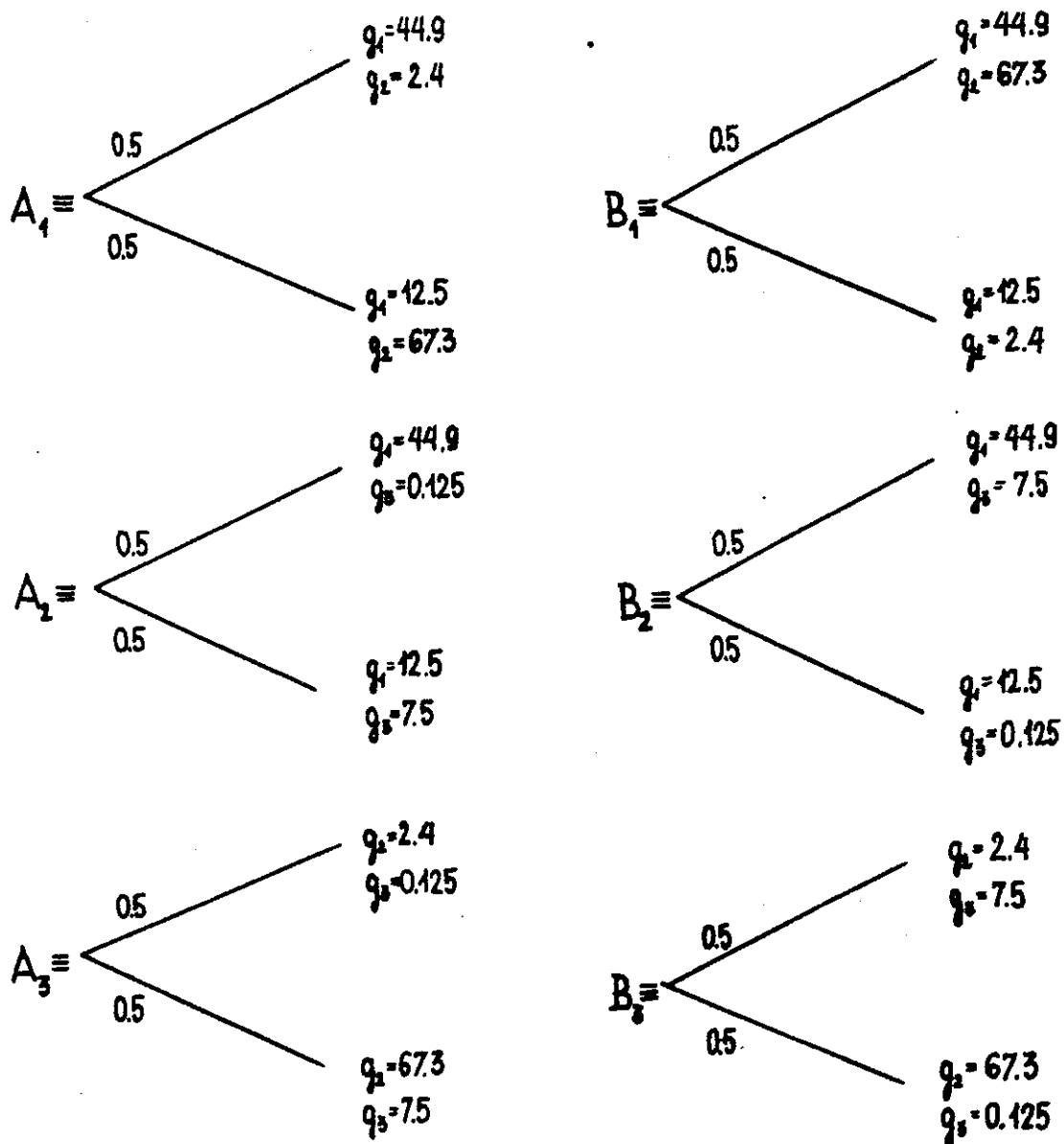


Figure 4. Lotteries for determining the functional form of multiattribute utility function.

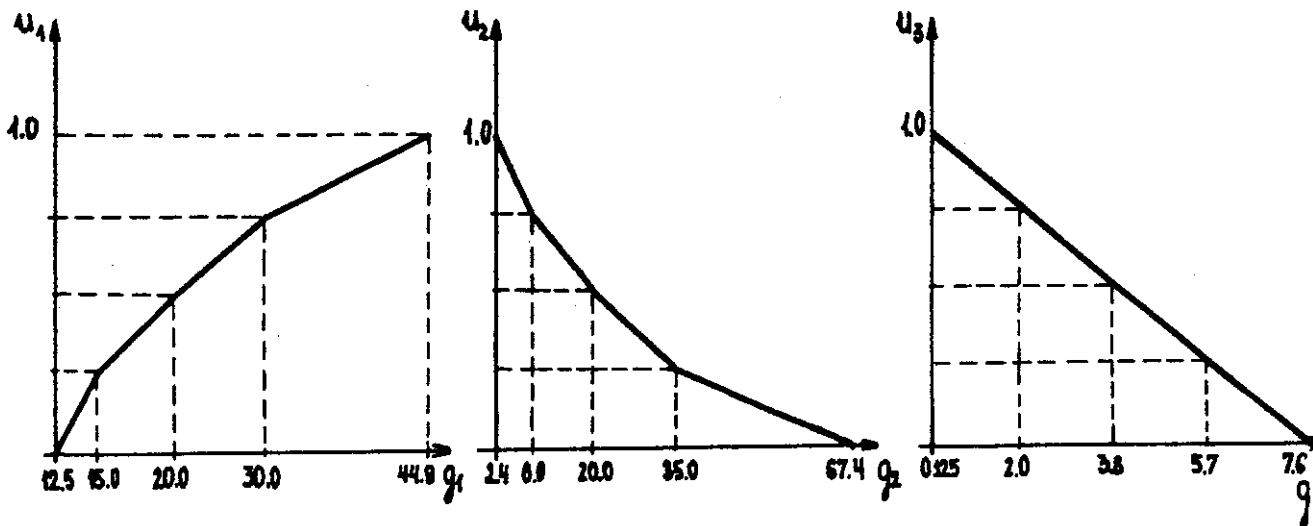
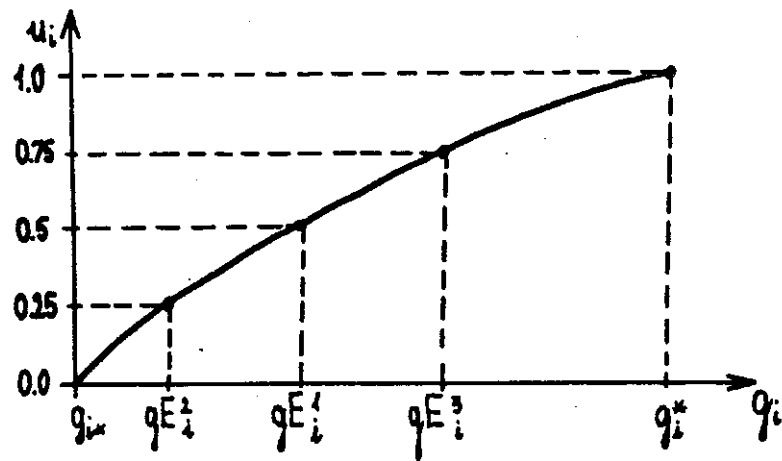
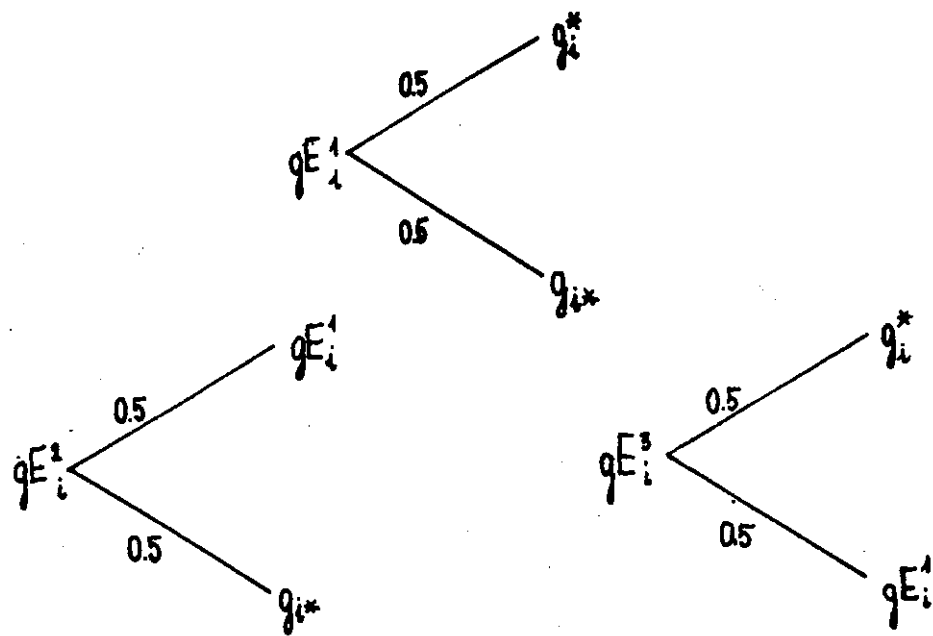


Figure 5. Selected lotteries for finding the shape of marginal utility functions.

It is noted that lotteries A_i yield either the best or worst of both attributes, whereas lotteries B_i yield the best of one and the worst of the other. We find that lotteries of the type B_i are always preferred. This, together with preferential and utility independence assumptions, leads to the multiplicative form of our MUF :

$$1 + ku(g_1(x), g_2(x), g_3(x)) = \prod_{i=1}^3 (1 + kk_i u_i(g_i(x))) \quad (23)$$

where utility function u and marginal utility functions u_i are scaled from 0 to 1; the scaling coefficients $k_i \in (0, 1)$ and k satisfies the equation :

$$1 + k = \prod_{i=1}^3 (1 + kk_i) \quad (24)$$

In the second step the single-attribute, marginal utility functions are assessed. For this purpose the following procedure is applied to each attribute-criterion function.

First we assess the deterministic value (the certainty equivalent) gE_i^1 which provides indifference to 50-50 lottery in which one can win the best value of $g_i = g_i^*$ with probability 0.5, or the worst value of $g_i = g_i^*$ also with probability 0.5. Next, we search the values gE_i^2 and gE_i^3 , the certainty equivalents for 50-50 lotteries yielding gE_i^1 and g_i^* , and gE_i^1 and g_i^* , respectively. We repeat this several times for finding the shape of i -th marginal utility function. The procedure is illustrated in Figure 5.

Then we use a piecewise linear approximation for calculating any intermediate value of the utility function. A more careful procedure for assessing a utility function may be found in Krzysztofowicz and Duckstein [1979].

The third and last step concerns in determination of scaling coefficients k_i and k in equation (23). First, an order of importance of the criteria is searched. The operating costs criterion, g_2 , shows to be the most important and the total pumping rates, g_1 , the least important. Then, the trade-offs between two criteria at a time are evaluated. For this, the DM is successively asked to determine levels gE_2 and gE_3 of criteria g_2 and g_3 such that the DM is indifferent between two situations :

$$(g_1^*, g_2^*, g_3^*) \text{ and } (g_1^*, gE_2, g_3^*)$$

and

$$(g_1^*, g_2^*, g_3^*) \text{ and } (g_1^*, g_2^*, gE_3) .$$

Then the values $gE_2 = 5.5$ and $gE_3 = 3.0$ are found. Since $u(g_1^*, g_2^*, g_3^*) = u(g_1^*, gE_2, g_3^*)$ then from equation (23) we obtain

$$u(g_1^*, g_2^*, g_3^*) = \{(1 + kk_1 u_1(g_1^*)) (1 + kk_2 u_2(g_2^*)) (1 + kk_3 u_3(g_3^*)) - 1\} / k =$$

$$= \{(1 + kk_1 0) (1 + kk_2 0) (1 + kk_3 1) - 1\} / k = k_3$$

and

$$u(g_1^*, gE_2, g_3^*) = \{(1 + kk_1 u_1(g_1^*)) (1 + kk_2 u_2(gE_2)) (1 + kk_3 u_3(g_3^*)) - 1\} / k =$$

$$= \{(1+kk_10)(1+kk_2u_2(5.5))(1+kk_30)-1\}/k = k_2u_2(5.5)$$

Then

$$k_3 = k_2 u_2(5.5)$$

and using the assessed marginal utility function u_2 (see Figure 5) we finally obtain the equation :

$$k_3 = 0.86137 k_2 \quad (25)$$

In a similar way, from the second indifference relation between (g_1^*, g_2^*, g_3^*) and $(g_1^*, g_2^*, g_{E3}=3.0)$ we obtain

$$k_1 = k_3 u_3(3.0)$$

and finally

$$k_1 = 0.61254 k_3 \quad (26)$$

Equations (25) and (26) establish relations between the values of scaling coefficients k_i . To know the specific values of these coefficients we ask the DM to decide on a probability p for the lottery yielding either (g_1^*, g_2^*, g_3^*) with probability p or (g_1^*, g_2^*, g_3^*) with probability $(1-p)$ which is indifferent to the situation (g_1^*, g_2^*, g_3^*) . We find $p = 0.7$ for this lottery. Then

$$u(g_1^*, g_2^*, g_3^*) = 0.7 u(g_1^*, g_2^*, g_3^*) + 0.3 u(g_1^*, g_2^*, g_3^*)$$

and next

$$k_2 = 0.7 (k_1 + k_2 + k k_1 k_2) \quad (27)$$

Equations (25) - (27) and equation (24) constitute a system of four equations with four unknowns : k_1 , k_2 , k_3 and k . Solving this system of equations we obtain the following values of the scaling coefficients :

$$\begin{aligned} k_1 &= 0.25626 ; & k_2 &= 0.48569 ; & k_3 &= 0.41836 ; \\ k &= -0.38653 \end{aligned} \quad (28)$$

As stated in Krzysztofowicz and Duckstein [1979] a negative k reflects a strictly multivariate risk averse attitude of DM. This attitude is in good agreement with the point of view represented by our study and is confirmed by shape of the marginal functions (concave for disutilities and convex for utilities).

We can now calculate the utility of the thirteen Pareto alternatives generated in our case. The marginal and global utility values of the assessed multiattribute utility function are presented in Table 4.

Table 4. Marginal utility values, global MUF values and ranking of generated Pareto alternatives

Alt.	u_1	u_2	u_3	Global utility U	Rank
1	0.78536	0.43525	0.67725	0.63615	7
2	0.86513	0.29788	0.55351	0.55386	10
3	0.94030	0.20849	0.42976	0.48946	11
4	1.00000	0.14477	0.30602	0.43179	12
5	1.00000	0.08105	0.18228	0.35935	13
6	0.68013	0.59243	0.55351	0.63459	8
7	0.55759	0.74633	0.58166	0.68304	6
8	0.37009	0.90535	0.63927	0.73229	2
9	0.00000	1.00000	0.73389	0.73508	1
10	0.68013	0.37819	0.94970	0.68984	5
11	0.55759	0.51154	0.98537	0.72968	3
12	0.37009	0.56310	0.99339	0.71634	4
13	0.00000	0.52604	1.00000	0.63254	9

4.4. UTA technique

UTA (UTilité Additive) technique, developed by Jacquet-Lagrèze and Siskos [1982], belongs to the utility/value function category of MCDM approaches. In general utility function in the sense of von Neumann and Morgenstern [Keeney and Raiffa, 1976] includes a notion of risk whereas a value function is deterministic. In this sense the UTA method is a value function assessment technique and we will use the term "value" in relation to this method. The UTA method consists of assessing a set of piecewise linear, additive value functions using information given by a subjective preference ranking (weak order) on a set of real or imaginary actions/alternatives and the multi-criteria evaluation of these actions. Although the UTA method assumes the fulfillment of utility theory axioms it uses a completely different procedure to build marginal and global value functions. This leads to the assessment of a set of utility functions that are as consistent as possible with the DM's preferences expressed as a weak order of pre-selected actions. A software implementation of this method, PREFCALC, has been developed by Jacquet-Lagrèze [1983]. Real decision making applications can be found in Siskos [1982] for evaluating a system of furniture retail

outlets and in Treichel [1991] for a multi-criteria analysis of rural water supply systems. An extension of the UTA method for MOLP problem is found in Jacquet-Lagrèze et al. [1987].

The UTA technique proceeds in the following two steps:

step 1 : Assessment of an "optimal" value function $U^*(g)$;

step 2 : Assessment of a set of value functions by means of post-optimality analysis.

In the first step an additive piecewise linear global value function is assessed :

$$U(q) = \sum_{i=1}^I u_i[g_i(q)] \quad (29)$$

where $u_i : g_i(q) \rightarrow u_i[g_i(q)] \in [0, 1]$, is a monotone, piecewise-linear function (marginal value function),

and

$$u_i[g_i^*] = 0 \quad \text{for } i=1, \dots, I \quad (30)$$

$$\sum_{i=1}^I u_i[g_i^*(q)] = 1 \quad (31)$$

where g_i^* , g_i^* are extreme values of the criterion i which correspond to the least and most preferred values of g_i , respectively.

The assessed value function should be as consistent as possible with a subjective ranking of several alternatives pre-selected as a reference set. Five alternatives selected as this reference set in our case and their subjective ranking are presented in Table 5.

Table 5. Alternatives selected for the reference subset in UTA and their subjective ranking.

Alt.	$g1$	$g2$	$g3$	Ranking
9	12.50	2.39	2.10	1
8	17.40	4.51	2.80	2
7	22.30	8.18	3.30	3
1	32.11	23.89	2.52	4
4	44.89	48.61	5.27	5

Next, in the second step, a post-optimality analysis allows to delimit ambiguity zones in the marginal value functions, for example, an indifference relation $a I b$ may correspond to $|U(a) - U(b)| < \delta$ rather than $U(a) = U(b)$. For small values of δ there may be ambiguity. The mean functions are proposed but each of the functions lying in the ambiguity zones can be considered as a value function which corresponds well to the reference set ranking. The marginal value functions assessed by the UTA method are shown in Figure 6.

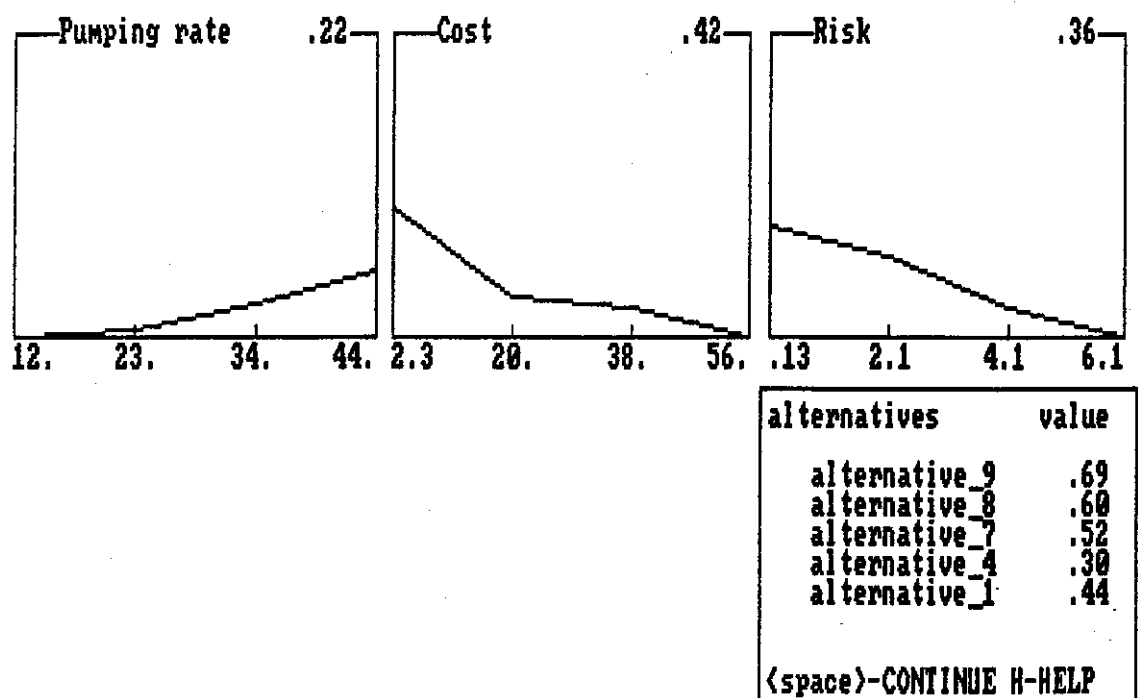


Figure 6. Marginal value functions estimated by UTA and values of reference alternatives.

Next, using equation (29) and the assessed marginal value functions, the global value function coefficients are calculated for each of alternatives under consideration and the final ranking corresponding to these values is reached as presented in Table 6.

Table 6. Marginal and global values and ranking of the generated Pareto alternatives by UTA

Alt.	$u1$	$u2$	$u3$	Global value U	Rank
1	0.08808	0.12311	0.22778	0.43897	8
2	0.13350	0.10370	0.15015	0.38735	10
3	0.18176	0.08011	0.08014	0.34201	11
4	0.22000	0.04005	0.04008	0.30013	12
5	0.22000	0.00002	0.00002	0.22004	13
6	0.04957	0.21123	0.15015	0.41095	9
7	0.01713	0.33027	0.16781	0.51521	6
8	0.00856	0.38931	0.20395	0.60182	2
9	0.00000	0.42355	0.26164	0.68519	1
10	0.04957	0.11505	0.34141	0.50603	7
11	0.01713	0.14866	0.35459	0.52038	4
12	0.00856	0.18855	0.35756	0.55467	3
13	0.00000	0.15988	0.36000	0.51988	5

5. DISCUSSION AND CONCLUSIONS

Groundwater management problems, considering their complexity and the necessity to take into account various aspects/axes of evaluation, require a multiobjective approach and the use of an appropriate MCDM technique. The choice of the technique that should be used constitutes itself a multiobjective problem. Gershon and Duckstein [1984] and next, Tecle and Duckstein [1992] have developed special procedures by which the most suitable MCDM technique can be selected for an arbitrary multiobjective decision problem. However, our purpose in this section is to discuss the "pros" and "cons" of each of the techniques presented in the previous sections in relation to groundwater management. To compare the techniques, we will take into account the following points of view:

- characteristics of each technique including assumptions and theory-axioms on which it is based, limits of its use, kind of data which are necessary for applying the technique (number of coefficients, weights, thresholds, ...), degree of freedom and sensitivity of the technique considering uncertainties and imprecision in the data;

- DM's expectations and degree of participation including the possibility to specify what the DM wants, type of input data required from the DM and their degree of accuracy, form of the final information which (s)he can obtain and explore;
- analyst's understanding and knowledge in the field of decision making, time required to gather the information and calculate results;
- output of each technique, in particular, stability of the ranking in regard to small changes in the parameters, comparison of the alternatives recommended as preferred ones.

First, let us note that our problem consists in evaluating several discrete groundwater management alternatives. These alternatives which are generated automatically from the continuous groundwater flow model satisfy Pareto-optimality conditions with regard to DM's criteria. The criteria have a quantitative, numerical character, as required by MOLP problem formulation; however because groundwater problems extend over a long time period, we should but cannot discuss the results with respect to the uncertainty and imprecision in the data. This is why we are interested in ordering the alternatives for future considerations and in presenting to the DM a few "good alternatives", namely the top ranked ones. A sensitivity analysis is thus required to ascertain the stability of the ranking. The selection of a "best" compromise solution makes sense only if the ranking remains stable.

The four techniques applied to groundwater management in the previous sections belong to different classes of MCDM methods. Compromise Programming is a distance-based technique, which assumes full comparability. The alternatives are evaluated according to their distance to the ideal solution. The L_p - metric used to measure distance implies full compensation for $p=1$ and no compensation for $p=\infty$. The assumption that the distance to the ideal solution represents the DM's preference system is explored to reach a complete preorder of the alternatives, from the best to the worst one. Only the weighting coefficient and the choice of a distance function (with or without compensation) are required and constitute the degrees of freedom of this technique.

The ELECTRE III method is an outranking-based technique. It allows situations where not all alternatives are comparable: there can be alternatives which cannot be compared to each other because of large differences from certain viewpoints. Note that the pairwise comparisons are not performed directly by the DM; using his preference information (inter- and intra-criteria) the method makes these comparisons internally. So, the ability to perform pairwise comparisons is not to be required from the DM. Furthermore, partial compensation is allowed; it depends on concordance and discordance indices calculated from pairwise comparisons. Only a partial preorder resulting from fuzzy outranking relations investigations is obtained. The indifference, preference and veto thresholds, as well as the weights give the DM many degrees of freedom to express his/her preferences. This is also a way to take into account data uncertainty and imprecision, and DM's

hesitations in expressing preferences, which would be a more difficult task in the other techniques considered here.

The MUF method requires that the strict set of axioms of multiattribute utility theory (MAUT) be satisfied and that the DM be a "rational" thinker who can synthesize various attributes into a single dimensional MAUT function. The question has often been posed whether or not such a function exists in the DM's head. For our purpose, we will just say that, within engineering approximation, we may be able to use this concept to model a groundwater problem with conflicting objectives accounting also for DM's risk attitude. A special technique (mixed lotteries) is provided for verifying this attitude. MUF is unique in providing the feature where probabilistic lotteries are used to assess all the coefficients.

The UTA technique, based on the non-probabilistic axioms of MAUT theory, uses an ordinal, subjective ranking of some alternatives and regression analysis to assess an additive value function; it is well adapted to problems where certain alternatives, which can be chosen into a reference set, are relatively well known. The possibility of selecting these alternatives and ordering them constitutes the degrees of freedom of this approach. The sensitivity analysis included in the logic of the technique allows to explore the ambiguous zones of the marginal value functions.

Regarding data accuracy required by each technique, ELECTRE III is the least demanding. To model the DM's preferences it uses a pseudocriterion notion, which tolerates a certain uncertainty and imprecision in the DM's judgments and gives the DM considerable freedom to do what (s)he thinks to be fit. On the contrary, the MUF method which assesses a global utility function by a sequence of probabilistic lotteries, forces the DM to be precise in stating either a certainty equivalent value or a probability in the given lottery. This precision is necessary to specify the shape and coefficients of the assessed global utility function.

From the DM's expectation point of view, each technique under consideration introduces an order on the set of alternatives: CP, MUF and UTA yield a complete preorder, while ELECTRE III results in a partial preorder [Roy, 1985, 1990]. In MUF and UTA the difference between alternatives can be evaluated numerically as the difference in global utility values. The difference in the distances from the ideal point in CP and the final preorder in ELECTRE III have an ordinal character only; this is not always a disadvantage, because the exactness of results (a small difference in the global utility function would be significant) does not correspond to the inaccuracy of available data.

From point of view of an analyst, who is responsible for the interface between a technique and the DM, Compromise Programming is the least time consuming and the easiest technique since a simple model is used to represent the DM's preference system. The other techniques demand from the analyst a good knowledge of the field of decision making for the purpose of persuading the DM of the rightness of sophisticated utility theory

axioms (MUF and UTA) as well as to develop familiarity with relatively new concepts used in ELECTRE III, such as indifference, preference, veto thresholds and the outranking relation itself. Computer programs are available for each of these techniques to facilitate those various tasks.

In Table 7 we present results obtained by each technique considered. The results obtained by ELECTRE III, MUF and UTA show that there are three alternatives: N° 9, 11, 12, selected as "good alternatives" in each of the methods. Alternative N° 11 has been chosen also by Compromise Programming. It is worth to note that Compromise Programming results become distorted because of the algorithm generating the Pareto set (section 3) which yields the alternative N° 1 as being the closest to the ideal point in the sense of the Tchebyshev norm then (L_p -metric with $p = \infty$). Alternative N° 9 which has the best rating in the second criterion (cost) but the worst one from total pumping rate point of view (first criterion) is selected in utility-based methods as the most preferred alternative; whereas ELECTRE III shows it as the best one yet noncomparable to alternative N° 11. In turn, alternative N° 11 has average ratings on all criteria. ELECTRE III thus cannot compare alternatives 9 and 11. At the bottom of each ranking, one finds alternative N° 5 which is dominated by the other alternatives in a weak efficiency sense, as provided by Pareto-set generation algorithm.

Table 7. Comparison of the ranking resulting from various techniques.

Class	Rank	Compromise Programming			ELECTRE III	MUF	UTA
		p = 1	p = 2	p = ∞			
good	1	11	11	1	11, 9	9	9
	2	12	1	11, 7		8	8
	3	10	10		2, 12	11	12
	4	9	12	6		12	11
medium	5	13	6	10	3, 8	10	13
	6	8	7	8, 12		7	7
	7	7	8		7	1	10
	8	1	9	2	4, 6, 10	6	1
	9	6	13	9, 13		13	6
	10	2	2			2	2
poor	11	3	3	3	1, 13	3	3
	12	4	4	4		4	4
	13	5	5	5	5	5	5

Concluding, let us reemphasize that the four MCDM techniques that have been applied in this paper are based on different concepts, yet they lead to a similar subset of recommended solutions (except Compromise Programming which yields slightly different results). The use of one of these techniques in groundwater management problems is conditioned by the nature of the decision situation. In the presence of risk and a kind of rationality in the DM's behavior, the MUF method is well adapted. When some of the alternatives are relatively well known, UTA can be used. In situations of uncertainty and imprecision in the data set and DM's preferences ELECTRE III is the most suitable. In any situation, the purpose of applying an MCDM technique is to aid the DM, not to take his/her place.

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