

# CAHIER DU LAMSADE

Laboratoire d'Analyse et Modélisation de Systèmes pour l'Aide à la Décision  
(Université Paris-Dauphine)  
Unité de Recherche Associée au CNRS UMR 7024

## MULTIOBJECTIVE FUZZY LINEAR PROGRAMMING FOR SUSTAINABLE IRRIGATION PLANNING: AN INDIAN CASE STUDY

CAHIER N° 186  
décembre 2001

K. SRINIVASA RAJU <sup>1</sup>  
Lucien DUCKSTEIN <sup>1</sup>

received: July 2000.

---

<sup>1</sup> GRESE, ENGREF, 19, avenue du Maine, 75732 Paris Cedex 15, France (ksraju@bits-pilani.ac.in, Duckstein@Engref.fr). Presently with Department of Civil Engineering, Birla Institute of Technology and Science, Pilani, Rajasthan, India (Email: ksraju@bits-pilani.ac.in).

## CONTENTS

|                                                                                   | <u>Pages</u> |
|-----------------------------------------------------------------------------------|--------------|
| Résumé . . . . .                                                                  | 2            |
| Abstract . . . . .                                                                | 2            |
| 1. Introduction . . . . .                                                         | 3            |
| 2. Mathematical modelling . . . . .                                               | 4            |
| 3. Methodolog of Multiobjective Fuzzy Linear Programming (MOFLP) . . . . .        | 12           |
| 4. Application of multiobjective fuzzy linear programming to the case study . . . | 13           |
| 5. Discussion and conclusions . . . . .                                           | 15           |
| Acknowledgements . . . . .                                                        | 15           |
| References . . . . .                                                              | 16           |

## **Programmation linéaire multicritère floue pour la gestion d'une surface irriguée : Un cas d'espèce indien**

### Résumé

On applique la programmation multicritère floue pour gérer le système de deux réservoirs qui fait partie du projet d'irrigation Jayakwadi dans la province de Maharashtra en Inde. Trois objectifs sont à maximiser : le bénéfice net, la production agricole et la main d'œuvre employée. Ces trois objectifs, qui correspondent à des critères de durabilité, sont modélisés à l'aide de fonctions d'appartenance linéaires. On trouve des valeurs de compromis avec un bon degré de satisfaction de 0.58.

Mots clés : Planification d'un système d'irrigation, programmation mathématique floue, programmation linéaire floue programme Jayakwadi, Inde.

## **Multiobjective fuzzy linear programming for sustainable irrigation planning: an Indian case study**

### Abstract

Multiobjective Fuzzy Linear Programming (MOFLP) irrigation planning model is formulated for the evaluation of management strategy for the case study of the two reservoir systems of Jayakwadi irrigation project, Maharashtra, India. Three objectives, net benefits, agricultural production and labour employment, are considered in the irrigation planning scenario. All three objectives are sustainability related and are to be maximised. These objectives are quantified by linear membership functions in a fuzzy multiobjective framework. It is observed from MOFLP solution that net benefits, agricultural production and labour employment are 2.031 Million Rupees, 2.1186 Million tons, 35.858 Million man-days, respectively, with degree of satisfaction ( $\lambda$ ) 0.58.

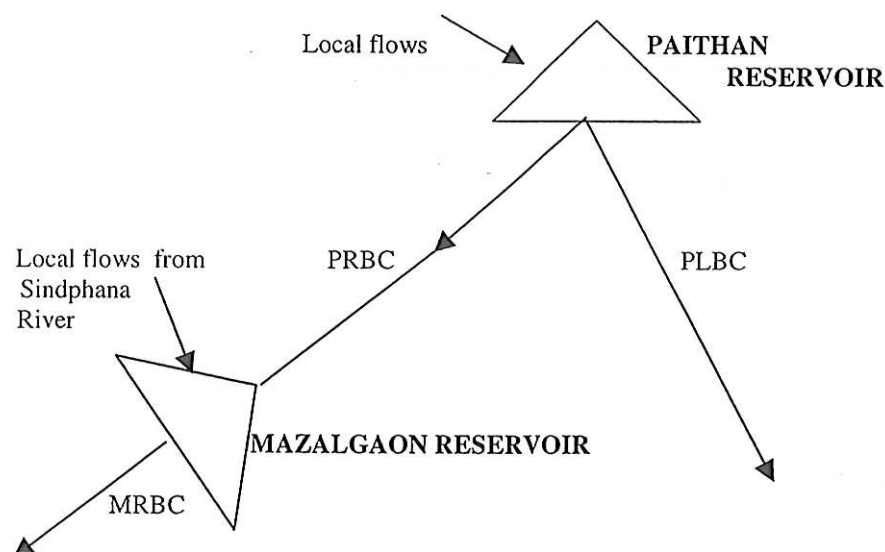
Keywords: Fuzzy mathematical programming; Fuzzy linear programming; Jayakwadi irrigation project; India.

## 1. Introduction

Irrigation planning problems require techniques for planning that are well adapted to their complex structure and are acceptable by a decision maker (DM) as planning objectives may be numerous and acceptable plans may differ from one another depending on the relative importance given to each objective. A crop production program that provides the greatest net benefits is not likely to create the highest employment of labour, nor may it produce the maximum amount of food grains. For developing countries these objectives are also more important than mere maximisation of net benefits alone, so as to attain sustainability of irrigation systems. Each development objective draws upon the same available resources, but not equally. It is not possible to maximise all the objectives simultaneously. Trade-offs are necessary when formulating a plan that achieves an appropriate compromise among various irrigation development objectives [Loucks *et al.* (1981)]. In the present study fuzzy linear programming methodology suggested by Zimmermann [1978,1996] is applied for a multiobjective irrigation planning problem in the Indian context and termed as Multiobjective Fuzzy Linear Programming (MOFLP). The methodology involves fuzzy sets to model the degree of attainment of goals defined on particular objectives.

Application of Fuzzy Programming to reservoir operation have been presented by numerous authors including Chang *et al.* [1997], Pesti *et al.* [1996], Bardossy and Duckstein [1995], Shreshta *et al.* [1996]. Excellent descriptions of Fuzzy Linear Programming (FLP) are reported by Korhonen *et al.* [1989], Slowinski [1997], Mohan and Nguyen [1998]. Multiobjective Fuzzy Linear Programming has advantages over the other existing multiobjective optimisation methods, namely, Constraint and Weighting methods [Cohon (1978)] such as 1) these may become computationally disadvantageous when the number of objectives are greater than three, 2) Weighting method requires specification of weights that are difficult to quantify, and sometimes the solution is sensitive to the weights. On the other hand MOFLP approach requires only one additional constraint for each additional objective function. Also flexibility to convert the fuzzy model into existing optimisation software makes the approach more attractive. But no efforts have been made to study the application of Multiobjective Fuzzy Linear Programming (MOFLP) to irrigation planning scenarios. The present study considers a Multiobjective Fuzzy Linear Programming (MOFLP) framework by incorporating three objectives: net benefits, agricultural production and labour employment for selection of the compromise irrigation plan, i.e., cropping pattern, reservoir storages, surface water allocation policies, evaporation loss, overflow, etc., for the case study of Jayakwadi irrigation project, Lower Godavari region, Maharashtra, India. Jayakwadi irrigation project is a major irrigation project consisting of a two reservoir system, namely, Paithan and Mazalgaon located on the Godavari river. The project is mainly meant for irrigation. The site lies within latitudes  $18^{\circ}46' N$  to  $19^{\circ}30' N$  and longitude  $75^{\circ} 20' E$  to  $77^{\circ}45' E$ . Two canal systems are originating from Paithan reservoir, namely, Paithan left bank canal (PLBC) and Paithan right bank canal (PRBC) having culturable command areas of 1,42,000 ha and 42,000 ha. After some distance downstream (along the length of PRBC), Mazalgaon reservoir exists with the source of supply from Sindphana river, a tributary of Godavari river. There is 93, 885 ha command area under Mazalgaon reservoir and the canal system is termed as Mazalgaon right bank canal (MRBC). Fig.1 presents the schematic diagram of this two reservoir system. Gross and live storage

Figure 1  
Schematic diagram of Jayakwadi irrigation project



PLBC = Paithan Left Bank Canal  
 PRBC = Paithan Right Bank Canal  
 MRBC = Mazalgaon Right Bank Canal

capacities for Paithan reservoir are 2909 Million Cubic Meter (MCM) , 2170 MCM. These are 453.64 MCM, 311.30 MCM for Mazalgaon reservoir, respectively. Overall conveyance efficiency of the project is 49%. Crops in the command area are Sugar-cane, Banana, Chillies, Cotton, Sorghum, Paddy, Wheat, Gram and Groundnut. Project covers five districts (district means cluster of villages), namely, Aurangabad, Ahmednagar, Bhir, Parbhani, Nanded in Maharashtra state, India. The topography for the command area consists mainly of rolling and undulating country, i.e., series of ridges and valleys interspersed with low hill ranges [Deshpande *et al* (1999)]. The paper is divided into four sections. In the next section mathematical modelling is discussed in detail along with the results. Section 3 describes the Multiobjective Fuzzy Linear Programming (MOFLP) methodology. Section 4 demonstrates the application of MOFLP to the above case study followed by discussion and conclusions.

## 2. Mathematical modelling

Linear Programming (LP) based optimisation model is employed in the present analysis. The three sustainable planning objectives considered are maximisation of net benefits, maximisation of agricultural production and maximisation of labour employment. Mathematical modeling of the three objectives and the corresponding constraints are explained below.

Objective 1: The net benefits ( $BE$ ) under different crops from command areas of PLBC, PRBC, MRBC are to be maximised. These are obtained by subtracting the cost of surface water from gross benefits of crops (excluding costs of fertilizers, labour employment etc). Mathematically it can be expressed as

$$BE = \sum_{i=1}^{10} BL_i AL_i + \sum_{i=1}^{10} BR_i AR_i + \sum_{i=1}^{10} BM_i AM_i - C_w \sum_{t=1}^{12} (RLR_t + RM_t) \quad (1)$$

where  $i$  = Crop index [1 = Sugar-cane(P), 2=Banana(P), 3=Chillies(TS), 4=Cotton (TS), 5=Sorghum(S), 6=Paddy(S), 7=Sorghum (W), 8=Wheat (W), 9=Gram (W), 10=Groundnut(HW)], S = Summer, W = Winter, TS =Two season, HW =Hot weather, P = Perennial,  $t$  = Time index (1=January, ....., 12=December).  $BE$  = Net benefits from the whole planning region (Indian Rupees);  $BL_i, BR_i, BM_i$  = Gross benefits from the crops (excluding costs of fertilisers, labour employment etc) from the command areas of PLBC, PRBC, MRBC, respectively (Indian Rupees);  $AL_i, AR_i, AM_i$  = Area of crop  $i$  grown in the command areas of PLBC, PRBC, MRBC (ha);  $C_w$  = Cost of surface water (Rs/MCM);  $RLR_t$  = Total water releases from Paithan reservoir to command areas of PLBC and PRBC (MCM);  $RM_t$  = Water releases from Mazalgaon reservoir to command area of MRBC (MCM).

Objective 2: Agricultural production (AP) of all the crops for the whole planning region is to be maximised.

$$AP = \sum_{i=1}^{10} Y_i (AL_i + AR_i + AM_i) \quad (2)$$

where  $AP$  = Agricultural production (Tons);  $Y_i$  = Yield of the crops (Tons / ha).

Objective 3: Labour employment for each crop  $i$  for the whole year for the whole planning region is to be maximised. Mathematically this can be expressed as

$$LA = \sum_{t=1}^{12} \sum_{i=1}^{10} L_{it} (AL_i + AR_i + AM_i) \quad (3)$$

where  $LA$  = Labour requirement for whole planning horizon (Man-Days);  $L_{it}$  = Labour requirement for crop  $i$  in month  $t$  (Man-Days).

The above three objectives are subject to the following constraints :

### Paithan reservoir scheme

#### 1. Continuity equation

Reservoir operation includes water transfer, storage, inflow and spillage activities. Water transfer activities consider transport of water from the reservoir to the producing areas through canals to meet the water needs. A monthly continuity equation for the reservoir storage (MCM) can be expressed as

$$SLR_{t+1} = SLR_t + I_t - ELR_t - RLR_t - OLR_t; t = 1, 2, \dots, 12 \quad (4)$$

where  $SLR_{t+1}$  = End of month reservoir storage in the Paithan reservoir (MCM);  $I_t$  = Monthly net inflows into the Paithan reservoir (MCM);  $ELR_t$  = Monthly net evaporation volume (MCM);  $OLR_t$  = Overflows from Paithan reservoir (MCM).

The above constraint assumes that the monthly inflows into the reservoir are known with certainty. When stochasticity is incorporated into the inflow terms, the above equation changes to

$$\Pr(SLR_{t+1} - SLR_t + ELR_t + RLR_t + OLR_t \geq I_t) \geq \alpha; t = 1, 2, \dots, 12 \quad (5)$$

or else

$$SLR_{t+1} - SLR_t + ELR_t + RLR_t + OLR_t \geq I_t^\alpha; t = 1, 2, \dots, 12 \quad (6)$$

where  $I_t^\alpha$  is inverse of the cumulative distribution of inflows at reliability level  $\alpha$  (in which stochastic considerations are included).

## 2. Crop area restrictions

The total cropped area allocated for different crops in PLBC command area in a particular season should be less than, or equal to, the Culturable Command Area (CCA).

$$\sum_i AL_i \leq CCA \quad ; i=1,2,3,4,5,6 \quad \text{Summer season} \quad (7)$$

$$\sum_i AL_i \leq CCA \quad ; i=1,2,3,4,7,8,9 \quad \text{Winter season} \quad (8)$$

$$\sum_i AL_i \leq CCA \quad ; i=1,2,7,8,9,10 \quad \text{Hot weather season} \quad (9)$$

Crops 1,2 are perennial and thus included into all the seasons; Crops 3,4 are of two-season crops and occupy the land both in the Summer and Winter seasons; Crop 10 is a hot weather crop. Other Winter crops 7,8,9 are also included into equation (9) because crops 7,8,9 occupy the land in January, whereas crops 7,8 occupy the land also in February, whereas crop 10 starts in January and ends by May. So crop 10 shares the same CCA with other Winter, Perennial crops for some portion of time. Similar analysis is employed for command areas of PRBC and MRBC, respectively.

## 3. Crop water diversions

Monthly crop water diversions  $CWR_{it}$  are obtained from the project reports. In absence of any crop activity,  $CWR_{it}$  is taken as zero. Total water releases from Paithan reservoir must satisfy the irrigation demands of PLBC, PRBC.

$$RLR_t - \sum_{i=1}^{10} CWR_{it} AL_i - \sum_{i=1}^{10} CWR_{it} AR_i = 0; t = 1, 2, \dots, 12 \quad (10)$$

where  $CWR_{it}$  = Crop water diversions for crop  $i$  in month  $t$  (meters).

#### 4. Canal capacity restrictions

Overflows from Paithan reservoir and irrigation demands of PRBC cannot exceed the right bank canal capacity. Also irrigation demands of PLBC cannot exceed the left bank canal capacity.

$$\sum_{i=1}^{10} CWR_{ir} AR_i + OLR_t \leq CCR; t = 1, 2, \dots, 12 \quad (11)$$

$$\sum_{i=1}^{10} CWR_{il} AL_i \leq CCL; t = 1, 2, \dots, 12 \quad (12)$$

where  $CCL, CCR$  = Canal capacities of PLBC, PRBC (MCM)

#### 5. Live storage restrictions

Reservoir storage volume  $SLR_t$  in any month  $t$  must be less than, or equal to, live storage of Paithan reservoir.

$$SLR_t \leq LSP; t = 1, 2, \dots, 12 \quad (13)$$

where  $LSP$  = Live storage of Paithan reservoir (MCM)

#### Mazalgaon reservoir scheme

##### 6. Continuity equation

$$SM_{t+1} = SM_t + IM_t - EM_t - RM_t - OM_t + OLR_t; t = 1, 2, \dots, 12 \quad (14)$$

where  $SM_{t+1}$  = End of month reservoir storage in the Mazalgaon reservoir (MCM);  $IM_t$  = Monthly net inflows into the Paithan reservoir (MCM);  $EM_t$  = Monthly net evaporation volume (MCM);  $OM_t$  = Overflows from Mazalgaon reservoir (MCM).

The above constraint assumes that the monthly inflows into the reservoir are known with certainty. When stochasticity is incorporated into the inflow terms, the above equation changes to

$$SM_{t+1} - SM_t + EM_t + RM_t + OM_t - OLR_t \geq IM_t^\alpha; t = 1, 2, \dots, 12 \quad (15)$$

where  $IM_t^\alpha$  = Inverse of the cumulative distribution of inflows at reliability level  $\alpha$ .

##### 7. Reservoir releases from Mazalgaon must satisfy the MRBC irrigation demands.

$$RM_t - \sum_{i=1}^{10} CWR_{ir} AM_i = 0; t = 1, 2, \dots, 12 \quad (16)$$

##### 8. Irrigation demands of MRBC must be less than canal capacity CCM

$$RM_t \leq CCM; t = 1, 2, \dots, 12 \quad (17)$$

where  $CCM$  = Canal capacity of MRBC (MCM).



## 9. Live storage restrictions

Reservoir storage volume  $SM_t$  in any month  $t$  must be less than, or equal to, live storage of Mazalgaon reservoir.

$$SM_t \leq LSM ; t = 1, 2, \dots, 12 \quad (18)$$

where LSM= Live storage of Mazalgaon reservoir (MCM).

The other constraints incorporated into the model are, for example, crop diversification considerations and labour availability. All the input parameters including inflows are obtained from the Jayakwadi project report [1985] and will be available from authors on request. Some additional information is obtained from agricultural department and Marketing society, etc.

The three cases related to maximisation of net benefits, maximisation of agricultural production and maximisation of labour employment are denoted as BM, PM and LM respectively. Initially three objective functions, i.e., net benefits, agricultural production and labour employment are maximised separately as single objective Linear Programming (LP) problems to determine the maximum and minimum values that can be achieved for each objective. Individual optimal cropping plans for regions PLBC, PRBC, MRBC for three cases BM, PM and LM are presented in Tables 1, 2 and 3, respectively. Salient parameters including net benefits, agricultural production and labour employment are presented in Table 4. The notations 'u' and 'L' represent the upper and lower bounds for each objective. Summarised points as observed from Tables 1 to 4 are given below:

1. In region PLBC, irrigated areas are 174600 ha, 168750 ha, 176130 ha for BM, PM and LM with irrigation intensities of 123%, 119%, 124% ,respectively, as evident from Table 1.
2. It is observed from region PLBC and for BM case that crop acreages are consistent for Cotton (TS), Sorghum(S), Wheat (W), Groundnut (HW) whereas Chillies (TS), Sorghum (W), Gram (W) occupied the land 1.53, 1.09, 1.41 times as compared to the PM case and Banana (P), Gram (W) occupied the land 1.53, 1.41 times as compared to LM case. Similarly in PM case Sugarcane (TS) is occupying 1.5 times as compared to BM case. Even though Sugarcane (TS) is a cash crop, high yielding capacity causes the model (PM case) to maximise it.
3. In PRBC case irrigated areas are 53180 ha, 48400 ha, 51670 ha for BM, PM and LM cases with irrigation intensities of 127%, 115%, 123%, respectively, as evident from Table 2.

4. In MRBC case irrigated areas are 114400ha, 105800 ha, 114290 ha for BM, PM, LM cases with irrigation intensity of 122%, 113% and 122%, respectively, as evident from Table 3.
5. Surface water releases from Paithan reservoir are 1628.06, 1647.44, 1616.36 MCM for BM, PM and LM cases, respectively. Reverse trend is observed for Mazalgaon reservoir. In this case surface water utilisation in LM case is 805.01 MCM which is more than BM and PM cases. It is observed that in all the three cases reservoir is empty in June for both Paithan and Mazalgaon reservoirs. But at no time the Paithan reservoir reached its maximum live storage level which may be due to higher demands. On the other hand Mazalgaon reservoir reached its maximum live storage in the month of October for BM and LM cases. (Tables are not presented due to space limitation).

Table 1  
Crop plans from the planning model (acreages are '00ha) for Paithan left bank canal

| Crops                | Season  | Solution for         |                      |                               |                                                  |
|----------------------|---------|----------------------|----------------------|-------------------------------|--------------------------------------------------|
|                      |         | Maximis.<br>Benefits | Maximis.<br>Product. | Maximis.<br>Labour<br>Employ. | Multi<br>Objective<br>Fuzzy<br>Linear<br>Program |
| 1.Sugarcane          | P       | 42.60                | 64.00                | 60.20                         | 57.07                                            |
| 2. Banana            | P       | 32.60                | 32.60                | 21.30                         | 30.24                                            |
| 3. Chillies          | TS      | 65.19                | 42.60                | 65.19                         | 65.19                                            |
| 4. Cotton            | TS      | 400.0                | 400.0                | 400.0                         | 400.0                                            |
| 5. Sorghum           | S       | 240.0                | 240.0                | 240.0                         | 240.0                                            |
| 6. Paddy             | S       | 142.0                | 142.0                | 180.0                         | 180.0                                            |
| 7. Sorghum           | W       | 326.0                | 297.7                | 326.0                         | 221.5                                            |
| 8. Wheat             | W       | 355.0                | 355.0                | 355.0                         | 355.0                                            |
| 9. Gram              | W       | 100.0                | 71.00                | 71.00                         | 71.00                                            |
| 10.Groundnut         | HW      | 42.60                | 42.60                | 42.60                         | 42.60                                            |
| Irrigated area       | ('00ha) | 1746.0               | 1687.5               | 1761.3                        | 1662.6                                           |
| Irrigation Intensity | (%)     | 123                  | 119                  | 124                           | 117                                              |

Table 2

Crop plans from the planning model (acreages are '00ha) for Paithan right bank canal

| Crops                | Season  | Solution for         |                      |                               |                                                  |
|----------------------|---------|----------------------|----------------------|-------------------------------|--------------------------------------------------|
|                      |         | Maximis.<br>Benefits | Maximis.<br>Product. | Maximis.<br>Labour<br>Employ. | Multi<br>Objective<br>Fuzzy<br>Linear<br>Program |
| 1.Sugarcane          | P       | 22.43                | 25.20                | 12.60                         | 25.20                                            |
| 2. Banana            | P       | 12.60                | 12.60                | 6.300                         | 12.60                                            |
| 3. Chillies          | TS      | 25.20                | 12.60                | 25.20                         | 25.20                                            |
| 4. Cotton            | TS      | 120.0                | 120.0                | 120.0                         | 120.0                                            |
| 5. Sorghum           | S       | 70.00                | 70.00                | 70.00                         | 70.00                                            |
| 6. Paddy             | S       | 42.00                | 42.00                | 64.00                         | 64.00                                            |
| 7. Sorghum           | W       | 80.00                | 63.00                | 80.00                         | 80.00                                            |
| 8. Wheat             | W       | 105.0                | 105.0                | 105.0                         | 105.0                                            |
| 9. Gram              | W       | 42.00                | 21.00                | 21.00                         | 21.00                                            |
| 10.Groundnut         | HW      | 12.60                | 12.60                | 12.60                         | 12.60                                            |
| Irrigated area       | ('00ha) | 531.8                | 484.00               | 516.70                        | 535.59                                           |
| Irrigation Intensity | (%)     | 127                  | 115                  | 123                           | 128                                              |

6. It is observed from Table 4 that net benefit in BM case is 1.01 times more than in PM case, and 1.1 times more than in LM case. Agricultural production in PM case is 1.14 times more than in BM case, and 1.12 times more than in LM case. Labour utilisation in LM case is 1.04 times compared to BM case, and 1.08 times as compared to PM case. The above conflict in the values of objective functions is attributed to the limitation of resources, i.e., inflows, land availability, canal capacity [Loucks et al. (1981)].
7. However, all three individual objectives for all three planning regions PLBC, PRBC, MRBC have satisfied the minimum crop acreage's (as kept in the model), while optimising the crop acreage's related to corresponding objectives.
8. It is observed from the above analysis that the three planning objectives are conflicting in all aspects as evident from Tables 1 to 4. This demonstrates the necessity to develop trade-off relationships and to select the compromise cropping plan and corresponding water allocation policies in the multiobjective irrigation planning scenario to meet chosen levels of satisfaction as demanded in the decision making process. Multiobjective Fuzzy Linear Programming (MOFLP) planning model is found to be suitable to consider the above situation. Details of the method are given below followed with application to the case study of Jayakwadi irrigation project.

Table 3

Crop plans from the planning model (acreages are '00ha) for Mazalgaon right bank canal

| Crops                | Season  | Solution for         |                      |                               |                                                  |
|----------------------|---------|----------------------|----------------------|-------------------------------|--------------------------------------------------|
|                      |         | Maximis.<br>Benefits | Maximis.<br>Product. | Maximis.<br>Labour<br>Employ. | Multi<br>Objective<br>Fuzzy<br>Linear<br>Program |
| 1. Sugarcane         | P       | 28.20                | 36.40                | 34.21                         | 34.36                                            |
| 2. Banana            | P       | 18.20                | 18.20                | 14.10                         | 14.10                                            |
| 3. Chillies          | TS      | 36.40                | 28.20                | 36.40                         | 36.40                                            |
| 4. Cotton            | TS      | 270.0                | 270.0                | 270.0                         | 270.0                                            |
| 5. Sorghum           | S       | 160.0                | 160.0                | 160.0                         | 160.0                                            |
| 6. Paddy             | S       | 94.00                | 94.00                | 118.00                        | 118.0                                            |
| 7. Sorghum           | W       | 200.0                | 141.0                | 200.0                         | 141.0                                            |
| 8. Wheat             | W       | 235.0                | 235.0                | 235.0                         | 235.0                                            |
| 9. Gram              | W       | 74.00                | 47.00                | 47.00                         | 47.00                                            |
| 10. Groundnut        | HW      | 28.20                | 28.20                | 28.20                         | 28.20                                            |
| Irrigated area       | ('00ha) | 1144.00              | 1058.00              | 1142.9                        | 1084.06                                          |
| Irrigation Intensity | (%)     | 122                  | 113                  | 122                           | 115                                              |

Table 4

Salient parameters for the planning problem

| Parameters                         | Solution for         |                      |                              |                                                  |
|------------------------------------|----------------------|----------------------|------------------------------|--------------------------------------------------|
|                                    | Maximis.<br>Benefits | Maximis.<br>Product. | Maximis.<br>Labour<br>Employ | Multi<br>Objective<br>Fuzzy<br>Linear<br>Program |
| Net Benefits (Million Rupees)      | 2118.7 <sup>u</sup>  | 2094.0               | 1914.0 <sup>L</sup>          | 2031.0                                           |
| Agricultural Produc.(Million Tons) | 1.9570 <sup>L</sup>  | 2.2393 <sup>u</sup>  | 1.9870                       | 2.1186                                           |
| Labour Employ. (Million Man-Days)  | 35.546               | 34.077 <sup>L</sup>  | 37.122 <sup>u</sup>          | 35.858                                           |
|                                    |                      |                      |                              | $\lambda = 0.58$                                 |

### 3. Methodology of Multiobjective Fuzzy Linear Programming (MOFLP)

Multiobjective Fuzzy Linear Programming (MOFLP) problem represents fuzzy input data by fuzzy membership functions. The fuzzy objective function can be maximised or minimised. In Fuzzy Linear Programming the fuzziness of available resources are characterised by the membership function over the tolerance range. In the present study objective functions are considered as fuzzy sets and inflows are considered as probabilistic variables. In conventional LP, the problem is defined as follows (Zimmermann [1996]) :

$$\text{Maximise } Z = C X \quad (19)$$

$$\text{subject to } A X \leq B \quad (20)$$

$$X \geq 0 \quad (21)$$

where  $A=(m \times n)$  matrix of known constants;  $B=(m \times 1)$  vector of constants;  $C=(n \times 1)$  vector of known constants;  $X=(n \times 1)$  vector of decision variables;  $Z=$  Objective function.

In the Fuzzy Linear Programming the problem can be restated as

Find  $X$  such that

$$C X \leq Z \quad (22)$$

$$A X \leq B \quad (23)$$

$$\text{and } X \geq 0 \quad (24)$$

Here  $\leq$  denotes fuzzified version of  $\leq$  and has the linguistic interpretation

“essentially smaller than or equal”. The membership function of the fuzzy set decision model  $\mu_D(X)$  is defined as

$$\mu_D(X) = \min_i \{ \mu_i(X) \}; i = 1, 2, \dots, I \quad (25)$$

$\mu_i(X)$  can be interpreted as the degree to which  $X$  fulfils the fuzzy inequality  $C X \leq Z$ .

and  $I$  is the number of objective functions. In the planning scenario, decision maker is not interested in a fuzzy set but in crisp optimum solution, maximising equation (25) becomes

$$\text{Max}_{x \geq 0} \mu_D(X) = \text{Max}_{x \geq 0} \min_i \{ \mu_i(X) \} \quad (26)$$

Membership function  $\mu_i(X)$  is represented as

$$\begin{aligned} \mu_i(X) &= 0 \text{ for } Z \leq Z_L \\ \mu_i(X) &= \left[ \frac{Z - Z_L}{Z_U - Z_L} \right]^\beta \text{ for } Z_L \leq Z \leq Z_U \\ \mu_i(X) &= 1 \text{ for } Z \geq Z_U \end{aligned} \quad (27)$$

where  $Z_U$  = Aspiration level of objective;  $Z_L$  = Lowest acceptable level of objective;  $\beta$  = Exponent indicates the desired shape of membership function. Assignment of value 1 gives rise to linear membership function [Sasikumar and Mujumdar (1998)].  $\mu_i(X)$  reflects the degree of achievement. Value of  $\mu_i(X)$  will be 1 for perfect achievement and 0 for no-achievement of a given strategy and some intermediate values, otherwise. The model can be transformed as follows:

$$\text{Max}_{x \geq 0} \min_i \mu_i(X) \quad (28)$$

subject to

$$A X \leq B \quad (29)$$

$$X \geq 0 \quad (30)$$

The MOFLP model can be formulated as equivalent LP model as below

$$\text{Max } \lambda \quad (31)$$

subject to

$$[\mu_i(X)] \geq \lambda \quad \text{for each objective function } i=1,2,\dots,I \quad (32)$$

$$A X \leq B \quad (33)$$

$$0 \leq \lambda \leq 1 \quad (34)$$

$$X \geq 0 \quad (35)$$

where  $\lambda$  is degree of satisfaction.

#### 4. Application of multiobjective fuzzy linear programming to the case study

Multiobjective fuzzy linear programming (MOFLP) model is applied to the case study of Jayakwadi irrigation project. The formulation of MOFLP model for the irrigation planning problem is as follows :

$$\text{Max } \lambda \quad (36)$$

subject to

$$[\mu_{BE}(X)] \geq \lambda \quad \text{or} \quad \left[ \frac{BE - BE_L}{BE_U - BE_L} \right]^\beta \geq \lambda \quad (37)$$

$$[\mu_{AP}(X)] \geq \lambda \quad \text{or} \quad \left[ \frac{AP - AP_L}{AP_U - AP_L} \right]^\beta \geq \lambda \quad (38)$$

$$[\mu_{LA}(X)] \geq \lambda \quad \text{or} \quad \left[ \frac{LA - LA_L}{LA_U - LA_L} \right]^\beta \geq \lambda \quad (39)$$

$$0 \leq \lambda \leq 1 \quad (40)$$

and all the existing constraints and bounds in the irrigation planning model.  $\mu_{BE}, \mu_{AP}, \mu_{LA}$  are membership functions for net benefits, agricultural production and labour employment, respectively.  $BE_U, AP_U, LA_U, BE_L, AP_L, LA_L$  are the upper and lower bounds for objectives net benefits, agricultural production, labour employment which are obtained from Table 4. Substituting  $BE_U, AP_U, LA_U, BE_L, AP_L, LA_L$  values in above equations and fixing  $\beta=1$  for linear membership function yields

$$-\lambda + 4.885 \times 10^{-9} \text{ BE} \geq 9.35 \quad (41)$$

$$-\lambda + 3.542 \times 10^{-6} \text{ AP} \geq 6.93 \quad (42)$$

$$-\lambda + 3.28 \times 10^{-7} \text{ LA} \geq 11.19 \quad (43)$$

Irrigation planning model consists of 136 constraints including 3 additional constraints for three objective functions (equations 41,42,43). Results of MOFLP are presented in Tables 1 to 4. It is observed from MOFLP solution that:

1. Irrigation intensities for regions PLBC, PRBC, MRBC are 117%, 128%, 115%.
2. For region PLBC, Cotton (TS), Sorghum (S), Wheat(W), Groundnut (HW) have the same acreages as compared to BM, PM, LM cases. Similar trend is observed for regions PRBC and MRBC.
3. For PLBC, it is observed from Table 1 that Cotton (TS), Sorghum(S), Paddy(S), Chillies (TS) reached their maximum targets. Wheat (W), Gram(W) and Groundnut (HW) are at their minimum level, whereas Sugarcane(P) and Banana(P) are between the minimum and maximum targets as defined in the planning model.
4. For PRBC the first 7 crops in Table 2 reached maximum targets where as last three crops are at their minimum level. Similar trend is observed for region MRBC.

5. Net benefits, agricultural production and labour employment are 2031 Million Rupees, 2.1186 Million tons, 35.858 Million man-days, respectively, with degree of satisfaction ( $\lambda$ ) 0.58.
6. Net benefits, agricultural production and labour employment per ha on average for the combined culturable command areas (CCA) of PLBC, PRBC, MRBC (i.e., 277 885 ha) are 7308 Rupees, 7.624 tonnes and 129 man-days, respectively.

## 5. Discussion and conclusions

The present paper discussed the application of Multobjective Fuzzy Linear Programming (MOFLP) in the irrigation planning context for the case study of the two reservoir systems of Jayakwadi irrigation project, Maharashtra, India. The methodology provides a suitable mechanism by incorporating sustainability-related aspects into the planning problem. MOFLP methodology is advantageous as compared to single objective planning problems as it can incorporate any number of objectives with any number of reservoirs with ease and without much computational burden. The following conclusions may be drawn from the study.

1. Net benefits, agricultural production and labour employment are 2031 Million Rupees, 2.1186 Million tons, 35.858 Million man-days, respectively, with degree of satisfaction ( $\lambda$ ) 0.58.
2. Net benefits, agricultural production and labour employment per ha on average for the combined culturable command areas (CCA) of PLBC, PRBC, MRBC (i.e., 277 885 ha) are 7308 Rupees, 7.624 tonnes and 129 man-days, respectively.
3. Fuzzy Linear Programming (FLP) is a simple and suitable tool for multiobjective problems as the model can be extended to any number of objectives by incorporating only one additional constraint into the constraint set for each additional objective function.

## Acknowledgements

The first author is grateful to all the authorities of the project for providing necessary information. Special acknowledgements to Prof Arun Kumar and Prof D.V.Morankar for providing additional information.



## References

- Bardossy A., Duckstein L. (1995), *Fuzzy rule based modelling with applications to geophysical, biological and engineering systems*, CRC Press, Boca Raton, Fla.
- Chang N B., Wen C G., Chen Y L. (1997), <<A fuzzy multiobjective programming approach for optimal management of the reservoir watershed>>, *European Journal of Operational Research* **99**, 289-302.
- Cohon J L. (1978), *Multiobjective programming and planning*, Academic Press, New York.
- Deshpande A P., Bhabad A C., Desai D D., Dungarwal N K. (1999) , <<Optimum operating policy for multireservoir system>>, B.E. Thesis, COEKPG, India.
- Jayakwadi Revised Project Report (1985), Government of Maharashtra, India.
- Korhonen P., Wallenius J., Duckstein L.(1989), <<Multiple objective linear programming over a fuzzy feasible set>>, in: G.W.Evans (ed.), *Applications of Fuzzy Set Methodologies in Industrial Engineering*, Elsevier, Amsterdam, 225-235.
- Loucks D P., Stedinger J R., Haith D A. (1981), *Water resources systems planning and analysis*, Prentice Hall, Englewood Cliffs, New Jersey.
- Mohan C., Nguyen H T . (1998), <<Reference direction interactive method for solving multiobjective fuzzy programming problem>>, *European Journal of Operational Research* **107** , 599-613.
- Pesti G., Shreshta B P., Duckstein L., Bogardi I. (1996), <<A fuzzy rule based approach to drought assessment >>, *Water Resources Research* **32**,6, 1741-1747.
- Sasikumar K., Mujumdar P P. (1998), <<Fuzzy optimization model for water quality management of a river system>>, *Journal of Water Resources Planning and Management, ASCE* **124**, 79-88.
- Shreshta B P., Duckstein L., Stakhiv E Z. (1996), << Fuzzy rule based modelling of reservoir operation>> , *Journal of Water Resources Planning and Management, ASCE* **122**, 262-269.
- Slowinski R. (1997), <<Fuzzy multi-objective linear programming>>, *Belgian Journal of Operations Research* **37** , 83-98.
- Zimmermann H J. (1978), << Fuzzy programming and linear programming with several objective functions>>, *Fuzzy Sets and Systems*, **1**, 45-55.
- Zimmermann H J. (1996), *Fuzzy Set Theory and its Applications*, Allied Publishers, New Delhi.