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AN AGGREGATE PRODUCTION PLANNING MODEL BASED ON MULTIPLE CRITERIA MIXED INTEGER LINEAR PROGRAMMING

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Un modèle multicritère de programmation linéaire entière mixte pour un problème de planification de la production agrégée

Résumé. Ce papier présente un modèle pour la planification de la production agrégée global d'une entreprise portugaise qui produit des matériaux pour la construction. Un modèle multicritère de programmation linéaire entière mixte a été développé. Les critères considérés sont les suivants : (1) maximisation du profit; (2) minimisation des commandes tardives; et, (3) minimisation du changement du nombre d'ouvriers. Ce modèle permet également l'inclusion de certains aspects relatifs à l'inflexibilité partielle de la main-d'œuvre, aux restrictions légales sur la période minimale de travail après embauche, à la dimension de la main-d'œuvre (ouvriers embauchés et mis au chômage), ouvriers en formation et la limitation de la capacité de stockage. Le but de cette étude consiste à définir le nombre d'ouvriers pour chaque catégorie, le nombre d'heures extraordinaires, le niveau de stock pour chaque type de produit et le niveau de produits à sous-traiter en vue de la satisfaction de la demande prévue pour une période de planification de 12 mois. Une approche interactive pour la génération et la sélection de la "meilleure solution" a été proposée.

Mots-clés : Planification Agrégée de la Production, Modèle Multicritère, Approche Interactive, Systèmes pour l'Aide à la Décision.

An Aggregate Production Planning Model Based on Multiple Criteria Mixed Integer Linear Programming

Abstract. This paper presents an aggregate production planning (APP) model applied to a Portuguese firm that produces construction materials. A Multiple Criteria Mixed Integer Linear Programming (MCMILP) model is developed with the following performance criteria: (1) maximize profit; (2) minimize late orders; and (3) minimize work force level changes. It allows the inclusion of certain operational issues such as partial inflexibility of work force, legal restrictions on workload, work force size (workers to be hired and downsized), workers in training, and production and inventory capacity. The purpose is to determine the number of workers for each worker type, the number of overtime hours, the inventory level for each product category, and the level of subcontracting in order to meet the forecasted demand for a planning period of 12 months. Additionally, a Decision Support System (DSS) based on the MCMILP model is proposed. It will help practitioners find the "best" solution for an APP problem without having to familiarize themselves with the mathematical complexities associated with the model. An example to illustrate the use of the DSS is also included.

Keywords: Production Management, Aggregate Planning, Multiple Criteria Model, Interactive Approach, Decision Support Systems.

1. Introduction

The aggregate production planning (APP) problem is about determining the optimum production, work force, and inventory levels for each period of the planning horizon for a given set of production resources and constraints. Such planning usually involves one product or a family of similar products with such small differences from a production viewpoint that considering the problem from an aggregate stand is justified. Production is assumed to be constant and the demand data are assumed to be known with certainty; however, provisions for forecast error may be incorporated. The goal is to meet the forecasted demand in a cost-effective manner. Typical costs related to aggregate planning include payroll, hiring/layoffs, overtime/undertime, inventory shortage and backordering, and the like.

Numerous aggregate planning models with varying degrees of sophistication have been introduced in the last four decades. Despite their methodological elegance, most of these models had little impact on the business environment due to their theoretical thrusts, complexities, rigid assumptions and unpracticable solutions. Without doubt, there exists a gap between the contributions made by researchers and the expectations of managers responsible for implementing aggregate production plans. The intent of the research presented in this paper is to bridge this gap by introducing a multiple criteria mixed integer linear programming (MCMILP) model that is a realistic, practical alternative to more sophisticated, complex APP models. The practicability of the model is demonstrated by applying it to solve an actual APP problem of a Portuguese firm that produces construction materials. In addition, we developed a computerized decision support system (DSS) that will aid managers to solve similar APP problems without having to familiarize themselves with the theoretical complexities of the model. The proposed DSS, based on the *learning-search oriented* concept, systematically searches for the best solution, if one exists. The operational features of the system have been designed to generate, evaluate, and compare different solutions through a series of interactive steps.

The paper is organized as follows. Section 2 presents a brief overview of APP models. The theoretical background of multiple criteria approaches is provided Section 3, followed by a description of the MCMILP model in Section 4. The main features of the interactive DSS are explained in Section 5. Section 6 includes an illustrative example of the DSS and discusses the results. Finally, conclusions and future research directions are presented in Section 7.

2. A brief overview of Aggregate Production Planning models

Despite the several models proposed to solve APP, only a few of these are called "classic". The rest are extensions of the classic models, with modifications both in the assumptions and the cost structures used. In this section we will describe the most common models existing in literature, with an emphasis on the mathematical programming models since this study falls in the category of these normative models. The following are six well-known models in literature:

1. The Linear Decision Rule (LDR)

Holt *et al.* (1955) developed one of the first APP models, commonly known as the HMMS or LDR model. The objective was to develop a production plan to minimize the total cost of an actual paint factory. Using quadratic approximations to the operational costs of the firm (regular payroll, hiring, layoff, overtime, and inventory costs), a mathematical expression was formulated to generate the paint production plan over a 12-month planning horizon given the product demand and subject to certain inventory restrictions.

In spite of the favorable results yielded by the quadratic cost structure models, these techniques have not been found useful by practicing managers as much as expected. In today's business environment where workers' participation and team work have been widely adopted, it is not desirable to use a layoff and hiring strategy to cope with the demand variability because of its negative impact on job security and worker's morale. Silva *et al.* (2000), based on the LDR model, derived a decision rule that considers a constant level of employment during the entire planning period. In this modified model the work force cannot be downsized or laid off, and therefore, it is more realistic for companies facing strong union opposition or government regulations that restrict firing.

2. Linear programming

Hanssman and Hess (1960) developed a model using a linear cost structure of the decision variables. They argue that in practice the cost structure of the firm can be assumed linear within the range of interest of the decision variables under consideration, and convex (concave) separable functions can be treated with piecewise linear approximations. These approximations result in linear models whenever a convex (concave) cost function is minimized (maximized).

The major advantage of linear cost models over nonlinear models is that the former can be solved easily and quickly, even for a model with a large number of decision variables. Besides, linear programming models permit parametric analysis, which can be helpful to managers making aggregate production planning decisions.

Several extensions and approaches to the Hanssman and Hess model have been proposed. Haehling (1970) developed a model for multiproduct, multistage production systems in which optimal disaggregation decisions can be made under capacity constraints. Assuming linear cost structures for the different cost functions, the model determines the amount of demand for each product that should be satisfied, backlogged, or remain unfilled. In addition, Bowman (1956) introduced another version of the linear programming model using the transportation algorithm.

3. Goal programming (GP)

Goodman (1974) developed a GP model, which approximated the original nonlinear cost terms of the LDR model by linear terms and solved it using a variant of the simplex method. The objective was to attain simultaneously satisfactory work force, production, and inventory levels instead of minimizing a single objective such as cost.

Rakes *et al.* (1984) criticized the previous attempts of goal programming applications because of the lack of realism, especially the use of deterministic demand in most of the studies. They suggested a chance-constrained GP approach to production scheduling which allows the decision maker (DM) to specify probabilistic product demands. The chance-constrained approach is a special case of stochastic programming in which some or all of the cost parameters of the model are random variables. The solution is a random variable that allows managers to use the confidence levels previously assigned to each goal, along with a sensitivity analysis, to reformulate and revise their priorities they wish to optimize.

4. Regression models

The management coefficient model presented by Bowman (1963) generates a firm's aggregate production decisions by capturing and replicating its historical decisions with a multiple regression model. The purpose of the model is to develop coefficients for the key variables, such as initial inventory, forecasted demand, and current employment levels that were considered by management in making production decisions in the past. The model seeks to develop a framework for improving the consistency of aggregate production planning decisions.

The primary advantage of Bowman's model is that it is easy to implement. The coefficients of the regression model can be easily calculated using the historical data of a production system. The model is intended to provide managers with some guidelines for future decisions.

5. Heuristic models

Computer simulation has been used as an important tool to deal with situations where analytical models are difficult to develop. Vergin (1966) developed a simulation model that starts with an initial schedule based on the past data or current practice of the firm. Then, using logical "rules of thumb" the effects of modified employment levels, overtime, inventory, and the like are evaluated and the corresponding costs computed. This iterative process continues until no further reduction in total variable costs is possible. The results yielded by the simulation model were reported to be much better than the existing production schedule and the solutions provided by the LDR model.

Several other search-based models have been developed since the publication of Vergin's study. Taubert (1968) proposed a search decision rule (SDR) that does not guarantee optimality of the solution, but provides a procedure to generate acceptable. It includes an objective function that describes the cost structure of the firm with no restrictions on the mathematical form of the objec-

tive function. Two decisions, the production (P) and work force (W) levels, are made in each period. Given the demand forecast, the search procedure finds the values of P and W for each period that minimizes the planning cost over the planning horizon.

Using a simulation technique, Jones (1967) formulated the Parametric Production Planning (PPP) model with an objective function that describes the cost structure (payroll, hiring and layoff, overtime, inventory) of a firm. Allowing no restriction on the mathematical form of the objective function, two decision rules, one for the work force level and the other for the production level, are formulated and applied in each period. Given the demand forecast, a search procedure is employed to find the weights (parameters) that minimize the cost objective over the planning horizon.

Mellichamp and Love (1978) recognized the gap between the research on aggregate production planning and its applications to real-world situations because most of the models yield production and work force decisions that are difficult to implement. Consequently, they proposed a model to determine the aggregate production decisions that is simple to use and consistent with actual management practices.

6. *Integrated models*

There exist a few studies in the literature that integrate the interactive aspects of different functional areas of a company. Damon and Schram (1972) analysed the interrelationship between short run production rates and the roles of functional areas such as finance and marketing using an integrated model. The model provided optimal choices for the work force, average hours worked, material purchases, advertising expenditures, product price, investment in marketable securities, and short-term borrowing. These decisions indirectly determined production, inventory levels, demand and cash holding for the different periods of the planning horizon. The objective was to maximize the "cash equivalent" position of the firm at the end of the planning periods, where cash equivalent was defined as the sum of cash, marketable securities, accounts receivable, and inventory excluding short term debt. The production constraints were based on the LDR model, but instead of production quantities, the average number of hours worked and the amount of material ordered in each period were considered as decision variables. In this integrative model, the contribution from the marketing sector was the estimation of the demand, which was a function of the firm's advertising and pricing policy. The input from the financial sector was incorporated through the profits on investment (or disinvestments) in marketable securities and short-term debt operations.

Pega *et al.* (2000) presented an integrated approach to address the aggregate planning problem. The technique, based on the LDR model and is applied to a Portuguese mining firm, generated the cash budget, which is then compared to its actual cash flow. The results of this study tend to underscore the need for integrating the production and finance subsystems of a firm when devising the production plan.

Excellent surveys of aggregate production planning models can be found in Nam and Logendran (1992), and Pan and Kleiner (1995). The former study identified 140 articles and 14 books, and classified the APP models in two categories: (1) models that produce mathematically optimal solutions and (2) techniques that provide near optimal solutions.

The popularity of *multiple criteria mathematical models* among managers has remained less than sufficient despite many attempts by researchers to enhance their application in practice. The approach presented herein addresses this issue and is more suitable for modeling many real-world situations. Masud and Hwang (1980) presented a multiple criteria linear programming model with four criteria (profit maximization, changes in work force level minimization, inventory investment minimization, and backorder minimization) subject to a set of constraints related to work force, product balance, and production capacity balance. The model, however, does not consider different worker types, workers training period, subcontracted products, legal constraints on firing, and an upper limit on storage capacity. These aspects are included in our model.

A review of literature indicates that the prior approaches on multiple criteria mathematical modelling have been less than adequate. In general, one cannot determine a solution that optimizes all the important criteria simultaneously (see Vincke 1992). Given this limitation and the associated computational difficulty, an alternative approach would include the generation of all the nondomi-

nated solutions, followed by the selection of the “best” solution through an interactive approach. In such a scenario, the decision maker can gradually insert his/her preferences sequentially through a series of choices in order to select the “best” solution, if it exists. In this study, we included a friendly interactive DSS to help the DM choose the “best” solution. The major advantage of the DSS is that it does not require the DM to understand the complexities of the multiple criteria algorithm to solve the problem. In other words, the DSS guides the DM “to solve” the problem in a way that requires the least amount of cognitive effort from the user.

3. Theoretical background

In this section we present the general form of a multiple criteria mixed integer linear programming (MCMILP) model as well as some related definitions (see Steuer 1986).

3.1 Multiple criteria mixed integer linear programming definitions

A multiple criteria mixed integer linear programming model can be formulated as follows:

P1:

$$\begin{aligned} \max z_1 &= f_1(x) \\ \max z_2 &= f_2(x) \\ &\dots \\ \max z_p &= f_p(x) \\ \text{s.t.:} \\ x &\in S \end{aligned}$$

where, $z_i = f_i(x)$ ($i=1, \dots, p$) is a linear criterion function to be maximized, x is the decision variables, $S = \{x \in R^n : Ax = b \in R^m, x \geq 0, x_i \in N(i \in I \subset \{1, \dots, n\})\}$ is the set of feasible solutions, A is an $(m \times n)$ matrix, and b is the right-hand-side vector for the constraint $Ax = b$.

Definition 1 [Dominance]. Let $\bar{z} = (\bar{z}_1, \bar{z}_2, \dots, \bar{z}_p)$ and $z = (z_1, z_2, \dots, z_p)$ be two feasible solutions. z dominates $\bar{z}$ iff $z \geq \bar{z}$ and $z \neq \bar{z}$ (where $z \geq \bar{z}$ and $z \neq \bar{z}$ means that $z_i \geq \bar{z}_i$ for all $i=1, \dots, p$ with at least one strict inequality).

A feasible solution $\bar{z} = (\bar{z}_1, \bar{z}_2, \dots, \bar{z}_p)$ is said to be *non dominated* if it is not dominated by any other feasible solution.

Note that due to the integer nature of certain decision variables, we might distinguish between two different kinds of non dominated solutions: the *supported non dominated solutions* and the *unsupported non dominated solutions*. The supported non dominated solutions are those that are placed on the frontier of the convex hull of the set of feasible vectors in the criteria space. The rest are *unsupported non dominated solutions*. A *supported non dominated* solution can be *extreme* or *non-extreme*. It is *extreme* if it is an extreme point of the convex hull. Otherwise, it is *non-extreme* (see Figure 1).

Definition 2 [Weak Efficiency]. The feasible solution $\bar{x} \in S$ is a *weak efficient solution* iff there does not exist another feasible solution $x \in S$ such that $z > \bar{z}$ (where $z = (f_1(x), \dots, f_p(x))$ and $\bar{z} = (f_1(\bar{x}), \dots, f_p(\bar{x}))$).

The efficiency concept is used in the decision space (S) and corresponds to the non dominated concept in the criterion space. Therefore, weak efficient solutions become weak non dominated solutions in the criterion space.

There exist some weak non dominated solutions that are of no interest to the DM because it is possible to improve the performances of at least one criterion without degrading the performance of the remaining criteria. Suppose that z and $\bar{z}$ are weak non dominated solutions and $\bar{z} \leq z$ and $\bar{z} \neq z$, then $\bar{z}$ is "inferior" to z .

Non dominated (not weak non dominated) solutions can be obtained for a particular problem using the Tchebycheff metric, the so-called L_∞ weighted and augmented metric. The distance between two points $u (u_1, \dots, u_p)$ and $v (v_1, \dots, v_p)$ according to the Tchebycheff metric is defined as follows:

$$\max\{\lambda_1|u_1 - v_1|, \dots, \lambda_p|u_p - v_p|\} + \rho \sum_{i=1}^p |u_i - v_i|$$

where, $\lambda_i \geq 0$ for $i=1, \dots, p$ are the weights of the vector components and ρ is a very small positive value.

The problem to obtain a non dominated solution, z , minimizes the weighted and augmented Tchebycheff distance to a reference point, z^{**} , and can be stated as follows:

$$\min\{\max\{\lambda_1|z_1^{**} - z_1|, \dots, \lambda_p|z_p^{**} - z_p|\} + \rho \sum_{i=1}^p |z_i^{**} - z_i|\}.$$

It is well known that this *MinMax* problem can be solved as a linear programming model.

P2:

$$\begin{aligned} & \min \left\{ \alpha + \rho \sum_{i=1}^p (z_i^{**} - z_i) \right\} \\ & \text{s.t. :} \\ & \alpha \geq \lambda_i (z_i^{**} - z_i) \quad \forall i = 1, \dots, p \\ & z_i = f_i(x) \quad \forall i = 1, \dots, p \\ & x \in S \\ & \alpha \geq 0 \end{aligned}$$

where $z_i^{**} = z_i^* + \varepsilon_i$, which represents the reference value to the objective function z_i , z_i^* is the optimum of z_i and ε_i is a small positive value. λ_i represents the weight of the objective function z_i , the i^{th} component of the vector λ which belongs to set Λ , defined as follows:

$$\Lambda = \left\{ \lambda \in R^p : \lambda_i > 0 \text{ and } \sum_{i=1}^p \lambda_i = 1 \right\}.$$

Problem P2, also called the scalarizing problem, has the capability to generate extreme and non-extreme non dominated solutions as well as unsupported non dominated solutions.

Figure 1a shows a feasible region in the criteria space (solid vertical lines) of a mixed integer problem. The contour of the L_∞ weighted and augmented metric is identified in Figure 1b, and the non dominated solutions are presented in Figure 1c. Both supported and unsupported non dominated solutions can be obtained by solving P2. Furthermore, z^l is a non dominated supported non-extreme solution obtained by solving P2: the contour of L_∞ weighted and perturbed metrics tangent to the admissible region at point z^l .

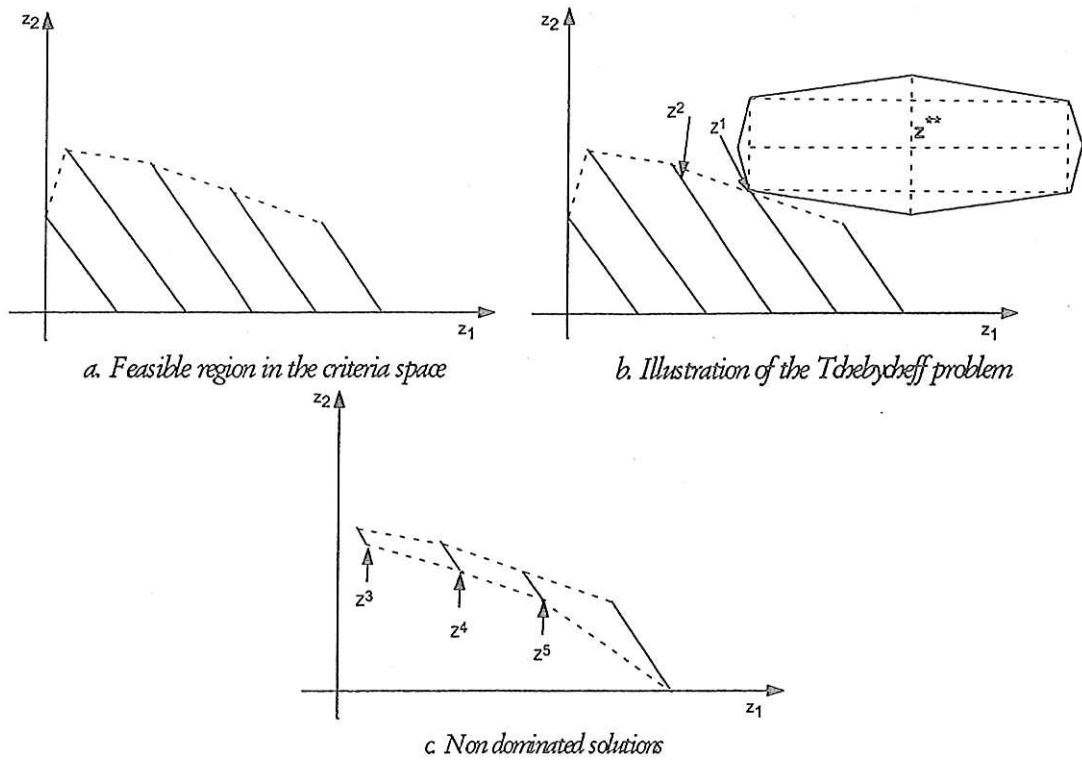


Figure 1: Illustration of the feasible region, Tchebycheff problem and non dominated solutions

The points z^2 and z^1 are the unsupported non dominated and supported non dominated solutions, respectively, while z^3 , z^4 and z^5 represent the weak non dominated solutions of the problem.

The DSS presented in this study uses a generalization of P2 in order to find the non dominated solutions, where α can be negative if at least one of the reference points is less than the corresponding ideal point component.

3.2 Multiple criteria approaches

The following is a list of approaches to deal with multiple criteria real-world problems (Vincke 1992):

1. *a priori methods*: In these methods the DM's preferences are integrated *a priori* to construct a single objective function, which, in almost all real-world situations, combines all the dimensions of a problem into a single criterion. This utility or value function represents the structure of preferences of the DM in the decision process.
2. *a posteriori methods*: In these methods the DM must evaluate all or a large set of non dominated solutions that are previously obtained. These methods require considerable computational efforts and generate many solutions. Therefore it becomes difficult for the DM to identify the "best" solution(s).
3. *Interactive methods*: Multiple criteria interactive procedures, which are quite appropriate for many real-world situations, require cooperation between a user (DM) and a system (computer DSS). An interactive procedure utilizing the DM's preferences can be built using the following concepts (Figueira *et al.* 1997a).
 - *The search oriented concepts*: These procedures help the user to guide the search towards a satisfactory point using his/her utility or value function. These methods converge in a mathematical sense because if the user rejects any solution at the beginning of the interactive process, he or she cannot accept that solution as a definitive solution later.

Therefore, these methods guide the user in the search of the satisfying point because the responses are consistent with his/her structure of preferences (Benayoun *et al.* 1971; Geoffrion *et al.* 1972).

- *The learning oriented concepts:* These procedures help the user build his or her preference structures during the interactive process. They do not require any assumptions on the user's utility or value functions (Roy 1976; Vincke 1976).
- *The learning-search oriented concepts:* Based on the work of Vanderpooten (1990), these procedures allow for free as well as guided exploration of the set of feasible solutions. Such an exploration favors the learning of the user's preference structure.
- *The learning-search oriented concepts with a test to analyze the adequacy of the model:* These procedures are similar to a learning-search approach; however, when a satisfying solution is reached, an *a posteriori* analysis can be made to determine the robustness of the solution. The procedures also allow a change in the model formulation during the interactive process by adding and/or removing some criteria, constraints, or parameters (see Figueira *et al.* 1997b).

The decision support system (DSS) presented in this study is based on a learning-search oriented concept.

4. The Multiple Criteria Mixed Integer Linear Programming model

We developed the multiple criteria model considering three issues: company profit, client satisfaction and labor environment. Profit maximization instead of cost minimization criterion was used because the company is interested in evaluating the market aspects of each product family, which is important, when product demand may not be fulfilled. The number of late orders is used to measure the client's level of satisfaction. The change at the work force level, measured by the number of workers hired and fired, reflects the labor environment.

The decision variables are disaggregated by period and by product families or by period and by worker type. The model also considers other production aspects such as the length of training periods, legal constraints on downsizing workers, production subcontracting, and storage capacity.

4.1 Notation

1. Indices:

- m - number of worker types ($j=1, \dots, m$);
- q - number of product families ($i=1, \dots, q$);
- n - number of periods (months) in the planning horizon ($t=1, \dots, n$).

2. Input parameters:

- v_{it}^p - selling price per unit of product family i in period t ;
- c_{it}^{late} - late order delivery cost per unit of product family i in period t ;
- c_{jt}^{sal} - salary per worker for worker type j in period t ;
- c_{it}^{inv} - inventory holding cost per unit of product family i in period t ;
- c_{jkt}^{red} - downsizing cost per worker for worker type j hired in period k and downsized in period t ;
- c_{jt}^{tr} - training cost per worker for worker type j in period t ;
- c_{jt}^{over} - overtime production cost per worker for worker type j in period t ;
- c_{it}^{sub} - subcontracting production cost per unit of product family i in period t ;
- c_{it}^p - production cost per unit of product family i in period t ;
- H_{ji} - number of working hours needed per worker for worker type j to produce one unit of product family i ;

$h_{jt}^{o-\max}$	- maximum number of allowed overtime hours per worker for worker type j in period t ;
$h_{jt}^{r-\max}$	- maximum number of allowed regular hours per worker for worker type j in period t ;
w_{j0}	- initial number of workers for worker type j ;
$w_{jt}^{\max}$	- maximum number of workers allowed for worker type j in period t ;
$w^{\max}$	- overall maximum number of workers allowed in each period (same for all the periods);
p_{i0}^{over}	- initial overdue level concerning product family i ;
p_{it}^{dem}	- demand level for product family i in period t ;
p_{i0}^{inv}	- initial inventory level for product family i ;
$p_{it}^{\max}$	- maximum number of units that can be produced in period t for product family i ;
$p^{\max}$	- overall maximum number of units to be produced (same for all the periods);
p_i^{space}	- storage space filled per unit of product family i ;
$p_{it}^{s-\max}$	- maximum number of units of product family i that can be subcontracted in period t ;
p^{space}	- storage capacity level;
μ	- number of periods a worker must work before he/she could be downsized;
α_j	- required number of training periods per worker for worker type j ;
β_j^{reg}	- percentage of the regular working hours spent by each worker in training for worker type j ;
β_j^{over}	- percentage of the overtime hours spent by each worker in training for worker type j ;
ξ_{jk}	- percentage of worker type j hired in period k that could be downsized;

3. Decision variables,

H_{jt}^{reg}	- number of regular hours by worker type j in period t ;
H_{jt}^{over}	- number of overtime hours by worker type j in period t ;
W_{jt}	- overall number of type j workers employed in period t ;
W_{jt}^{tr}	- number of type j workers in training in period t ;
W_{jt}^{hire}	- number of type j workers hired in period t ;
W_{jkt}^{down}	- number of type j workers hired in period k and downsized in period t ;
P_{it}^{late}	- units of product family i backordered in period t ;
P_{it}	- units of product family i produced in period t ;
P_{it}^{sto}	- units of product family i stored in period t ;
P_{it}^{sub}	- units of product family i subcontracted in period t .

The variables W_{jt} , W_{jt}^{tr} , W_{jt}^{hire} and W_{jkt}^{down} are restricted to integers.

4.2 Performance criteria

The model includes three performance criteria, *profit maximization*, *late orders minimization* and *minimization of the changes in the work force level*, as follows:

1. *Profit maximization*: This objective function, shown below, includes two main components: sales and costs. The cost component includes inventory holding costs, late order delivery costs, training costs,

overtime costs, workers salaries, workforce reduction costs, subcontracting costs, and production costs.

$$\begin{aligned}
 Max\ z_1 = & \underbrace{\sum_{i=1}^q \sum_{t=1}^n v_{it}^p (p_{it}^{dem} - P_{it}^{late} + P_{it-1}^{late})}_{\text{sales value}} - \underbrace{\sum_{i=1}^q \sum_{t=1}^n c_{it}^{inv} P_{it}^{sto}}_{\text{storage costs}} - \\
 & \underbrace{\sum_{i=1}^q \sum_{t=1}^n c_{it}^{late} P_{it}^{late}}_{\text{late deliveries costs}} - \underbrace{\sum_{j=1}^m \sum_{t=1}^n c_{jt}^{tr} W_{jt}^{tr}}_{\text{training costs}} - \underbrace{\sum_{j=1}^m \sum_{t=1}^n c_{jt}^{over} H_{jt}^{over}}_{\text{overtime costs}} - \underbrace{\sum_{i=1}^q \sum_{t=1}^n c_{it}^{sub} P_{it}^{sub}}_{\text{subcontracting costs}} - \\
 & \underbrace{\sum_{j=1}^m \sum_{t=1}^n c_{jt}^{sal} W_{jt}}_{\text{salary costs}} - \underbrace{\sum_{j=1}^m \sum_{k=0}^n \sum_{t=1}^n c_{jkt}^{red} W_{jkt}^{down}}_{\text{work force reducing costs}} - \underbrace{\sum_{i=1}^q \sum_{t=1}^n c_{it}^p P_{it}}_{\text{other production costs}}
 \end{aligned} \tag{1}$$

2. *Late orders minimization:* Promised orders not delivered on time have a negative impact on the reputation of the company. This criterion minimizes the number of late orders during the planning period.

$$Min\ z_2 = \sum_{t=1}^n \sum_{i=1}^q P_{it}^{late} \tag{2}$$

late deliveries

3. *Minimization of the changes in the work force level.* Frequent changes in the number of workers adversely influence the work force stability. This criterion minimizes the number of workers downsized or hired during the planning period.

$$Min\ z_3 = \underbrace{\sum_{t=1}^n \sum_{i=1}^m W_{it}^{hire}}_{\text{hired workers}} + \underbrace{\sum_{j=1}^m \sum_{k=0}^n \sum_{t=1}^n W_{jkt}^{down}}_{\text{downsized workers}} \tag{3}$$

4.3 Constraints

The model includes three groups of constraints: workforce, production and inventory level.

- *Work force:* This group of constraints limits the number of workers employed globally and by worker type. They are required due to legal, technical, and/or operational constraints.

1. *Definition of the number of workers for each worker type in each period.* This set of constraints states that the number of workers (for each worker type) in the current period must be equal to that in the last period plus the number of workers hired minus that downsized in the current period.

$$W_{jt} = W_{jt-1} + W_{jt}^{hire} - \sum_{k=0}^t W_{jkt}^{down} \quad j = 1, \dots, m; t = 1, \dots, n \tag{4}$$

2. *Legal limitations to downsizing workers:* These constraints guarantee that workers hired in the current period will not be downsized in the next μ periods.

$$\sum_{\substack{w=k \\ 1 \leq w \leq n}}^{k+\mu-1} W_{jkt}^{down} = 0 \quad j = 1, \dots, m; k = 0, \dots, n \tag{5}$$

3. *Firing constraints:* These constraints indicate that the total number of workers (for a given worker type) fired in the future periods cannot exceed a percentage of those hired in the current periods. With respect to the initial period, the model restricts the number of workers downsized within a certain percentage of the initial work force.

$$\sum_{\substack{w=1 \\ w \geq k}}^n W_{jkw}^{down} \leq \xi_{jk} W_{jk}^{hire} \quad j = 1, \dots, m; k = 0, \dots, n \quad (6)$$

4. *Upper limit for each worker type:* Due to operational and logistic reasons, the number of workers of a given type has an upper limit. This limit depends on the worker type and could vary from period to period.

$$W_{jt} \leq w_{jt}^{\max} \quad j = 1, \dots, m; t = 1, \dots, n \quad (7)$$

5. *Upper limit for the total number of workers:* This constraint reflects the global operational and logistic capacities of the company with regard to the number of workers. It is assumed to be constant during the planning periods because it can be changed only in the long term.

$$\sum_{j=1}^m W_{jt} \leq w^{\max} \quad t = 1, \dots, n \quad (8)$$

6. *Workers in training:* This set of constraints defines the number of workers for each worker type currently engaged in training. Thus, for each period, the number of workers in training is equal to the total number of workers hired in the preceding α periods minus the number of workers hired and downsized in the same periods.

$$W_{jt}^{tr} = \sum_{\substack{k=t-\alpha_j+1 \\ k \geq 1}}^t W_{jk}^{hire} - \sum_{\substack{w=t-\alpha_j+1 \\ w \geq 1}}^t \sum_{\substack{k=t-\alpha_j+1 \\ k \geq 1}}^t W_{jkw}^{down} \quad j = 1, \dots, m; t = 1, \dots, n \quad (9)$$

7. *Limitation of regular working hours:* These constraints state that for each worker type in a given period, the number of regular working hours cannot exceed its upper limit. Since some workers may be in training, a percentage of the regular work executed by these workers is added to the right hand side of each constraint.

$$H_{jt}^{reg} \leq h_{jt}^{r-\max} (W_{jt} - W_{jt}^{tr}) + \beta_j^{reg} h_{jt}^{r-\max} W_{jt}^{tr} \quad j = 1, \dots, m; t = 1, \dots, n \quad (10)$$

8. *Limitation of overtime hours:* The interpretation of these constraints is similar to those in the previous set.

$$H_{jt}^{over} \leq h_{jt}^{o-\max} (W_{jt} - W_{jt}^{tr}) + \beta_j^{over} h_{jt}^{o-\max} W_{jt}^{tr} \quad j = 1, \dots, m; t = 1, \dots, n \quad (11)$$

- *Production:* To produce a given product family, it is necessary to deploy different types of workers. In this group of constraints we restrict the upper limit on the number of subcontracted units.

9. *Production balance:* These constraints require that the amount of products produced directly by the firm plus those that are subcontracted and the number of late orders in the current period must be equal to the product demand plus the outstanding late orders from the preceding period and the difference in inventory between the current and preceding periods.

$$P_{it} + P_{it}^{sto} + P_{it}^{late} = P_{it}^{dem} + P_{it-1}^{late} + P_{it}^{sto} - P_{it-1}^{sto} \quad i = 1, \dots, q; t = 1, \dots, n \quad (12)$$

10. *Upper limit of the production capacity:* These constraints require that the planned production in each period cannot exceed the production capacity.

$$\sum_{i=1}^q P_{it} \leq p^{\max} \quad t = 1, \dots, n \quad (13)$$

11. *Upper limit of the production capacity by product:* These constraints limit the maximum production of each product family in each period. The limits depend on the period and the product family.

$$P_{it} \leq p_{it}^{\max} \quad i = 1, \dots, q; t = 1, \dots, n \quad (14)$$

12. *Upper limit on subcontracting:* The maximum quantity of product for each product family that can be subcontracted is limited by the production capacity of the subcontracted company.

$$P_{it}^{sub} \leq p_{it}^{s-\max} \quad i = 1, \dots, q; t = 1, \dots, n \quad (15)$$

13. *Production limitation for a worker in a period*: These constraints state that the production output by each worker type is also constrained by the number of available regular and overtime working hours.

$$\sum_{i=1}^q h_{ji} P_{it} = H_{jt}^{reg} + H_{jt}^{over} \quad j = 1, \dots, m; t = 1, \dots, n \quad (16)$$

- *Inventory level*: This set of restrictions assumes that the storage capacity will remain the same for all the periods.
- 14. *Upper limit on storage capacity*. These constraints state that the quantity of products that can be stored is limited by the available space.

$$\sum_{i=1}^q p_i^{space} P_{it}^{sto} \leq p^{space} \quad t = 1, \dots, n$$

The solution by the model will include the following for each period in the planning horizon: the number of workers of each type to be hired, fired and trained, and the total number of worker(17) required; the number of regular and overtime working hours for each worker type; and the amount of product to be produced, subcontracted, stored and late (backordered?) for each product family.

5. An Interactive Decision Support System

We propose an interactive DSS based on the *learning-search oriented* concept. The procedure includes two phases (see Appendix for details):

1. In the first phase, the *learning-oriented phase*, the DM becomes familiar with different non dominated solutions.
2. In the second phase, the *search-oriented phase*, the DM learns more about the previous solutions and can be guided by the procedure to find the "best" solution, if it exists.

In essence, the procedure generates a set of possible definitive solutions from which successive expansions and contractions can occur.

In the *learning-oriented phase* the DM searches for fix reference points or evaluates the weights on each objective function without any guidance from the system. These reference points and/or weights are then used to obtain new solutions. The DM can also reduce the admissible region by adding some constraints at the objective function levels. The aim of these constraints is both to preclude solutions that might be unacceptable and to enable a what-if analysis. Upon obtaining a solution, the DM has to decide whether the achievements at the objective function levels are acceptable or not. If they are, the solution is added to a Generator Set (GS), increasing the cardinality of the GS. The circumstances under which a given solution becomes acceptable do not necessarily represent a stopping condition. This learning oriented phase is repeated while the DM considers whether the information obtained is sufficient to build this preference structure. During this stage the DM could accept or reject any given solution.

The *search-oriented phase* begins when the DM decides to undertake a complete evaluation of the GS. The goals in this phase are:

1. to provide the DM more information about the solutions retained from the previous phase,
2. to help the DM understand the underlying tradeoffs between the objectives, and
3. to generate new potential solutions from the current GS.

Thus, the DM could choose from the following possible interactive options:

- *Evaluating solutions*: The DM chooses a solution from the GS he or she wants to evaluate. This happens when the DM is highly satisfied with some levels of the objective functions but not with the others. Consequently, the DM specifies the solutions that have satisfactory as well as unsatisfactory levels of objectives. For the former, the DM identifies the maximum amount to renounce in order to improve the level of at least one of the remaining criteria. Regarding the latter, the DM sets the minimum level of each objective that must be satisfied. These constraints

- *Neighborhood search:* The DM selects a solution by using a set of weights and a reference point, and the system then searches some neighboring solutions. Of all the neighboring solutions, the ones that are found satisfactory by the DM are included in the GS. The DM may select more than one neighboring solution if he or she desires.
- *Comparing Solutions:* The DM selects two solutions to compare. The choice of the solutions could be based on some apparently irrelevant aspects, when the achievement levels in the objective functions are somewhat similar. The DM could eliminate one of these solutions because it is indifferent from or inferior to the other.
- *Removing solutions:* The DM decides to directly remove a solution from the GS. Based on the knowledge accumulated during the interactive process, he or she considers that the solution is not good enough for implementation.
- *Returning to the learning-oriented phase:* The DM decides to return to the first phase (*learning-oriented phase*) because he or she wants to improve his or her knowledge about the non dominated region. This could result if the DS eliminates all of the solutions from the GS or is interested in finding new solutions not available in the current GS.

If the GS contains only one solution and the DM does not want to continue with any more interaction, then that would be the "best" compromise solution. Otherwise, using the unique element of GS, the DM may choose to find the "best" solution through the process of discovering, testing and evaluating various alternatives.

We developed a DSS, written in Borland Delphi 4.0, to help the DM implement the MCMILP model presented in this paper. The DSS extensively uses graphs to represent and process information, enhancing the interaction with the DM. In addition, it is efficiently interfaced with EXCEL to retrieve the parameter values off the model and send back the planning maps and respective graphs containing the decision variable values. The structure of the DSS is presented in the Appendix.

The following section illustrates the application of the model and the use of the DSS to solve an APP problem.

6. An illustrative example

In order to demonstrate the application of the model, we developed a case study using the data collected from the Portuguese firm. The firm, located near Oporto, makes polished granite for pavements and other products for the construction industry. The company has 36 workers with an annual sales of nearly 500 million escudos (Portuguese currency). The products belong to three product families and there are three worker types. For the firm, an adequate production plan for a 12-month period is essential in order to cope with fierce competition as well as to provide low cost products in a timely manner. We simulated an *a posteriori* decision making process with a fictitious DM. Note that the actual operational data could not be revealed for the sake of confidentiality.

The model includes three objective functions and $n(8m+3+3q)+2m$ constraints. In addition, there are $n(n(5+n)+4q)$ decision variables, of which $m(4+n)$ are integers and $n(m+4q)$ are continuous. In this case study, as mentioned above, we have a planning horizon of 12 months ($n = 12$), 3 worker types ($m = 3$) and 3 product families ($q = 3$). Therefore, the problem includes 792 variables (576 integers and 216 continuous) and 438 constraints. The DSS to solve this problem utilized the Linear Programming solver LINGO 5.0 and was run on COMPAQ computer with a Pentium 133 MHz processor.

At the beginning of the interactive protocol, the DM only knows the optimal solutions for each criterion. This information provides the optimization limits for profits, late orders, levels of change in the regular work force, and the relative position of the remaining parameter values. As shown in Figure 2, the first matrix includes the optimal value for each criterion, obtained separately by optimizing each objective function. The second matrix presents the relative values (or percentages)

determined as follows: the element ixj of the matrix is equal to $\frac{100}{z_i^* - z_i^{nad}}(z_{ij} - z_i^{nad})$, where z_i^*

represents the optimal value of the objective i , z_i^{nad} corresponds to the worst value of line i of ma-

trix one, and z_{ij} is the element ij of the first matrix. For example, the element 3×2 is equal to 20 ($\frac{100}{0-10}(8-10) = 20$).

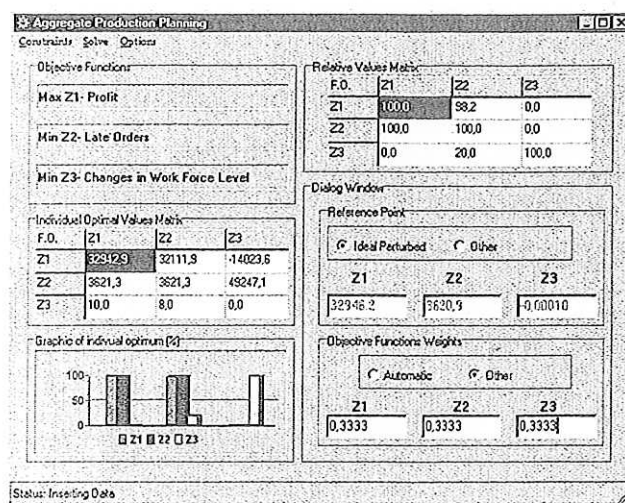


Figure 2: Presentation menu

6.1 First phase of the interactive method

During the first phase, the DM searches for new solutions by introducing new constraints in the criterion space by assigning weights to each criterion and by specifying reference points. This is presented in the menu in Figure 2.

- *First interaction.* The information provided by the DSS at the beginning of the interactive protocol is the matrix of the individual optimal solutions and the corresponding percentage in the non dominated region. Unfamiliar with the mechanics of obtaining non dominated solutions, the DM accepted the default reference point. The defined vector of weights was $(0.333, 0.333, 0.333)$, and the problem is solved. The solution obtained is $S_1 = (15663.0702, 18428.7785, 3)$, which corresponds to $(63.208\%, 67.546\%, 70\%)$. The DM included this solution in the GS while reserving the option of choosing an alternative option later. We therefore have $GS = \{S_1\}$.
- *Second interaction.* The *Second interaction* has to do with the possibility of introducing constraints in the criteria space, which makes the interactive protocol easier. Next, a constraint was introduced to prevent profits from being less than 20 million escudos, equivalent to a requirement of 72.442% of the optimal profit. These requirements are introduced in the menu in Figure 3, which follows from the Figure 2 menu. The vector of the weights was fixed at the previous level $(0.333, 0.333, 0.333)$ so the weights did not "favor" any spacing from the reference point corresponding to the ideal perturbed point. The resolution of the problem leads to the solution $S_2 = (22640.617, 11074.970, 4)$, which, in relative terms, corresponds to $(78.065\%, 83.663\%, 60\%)$. This solution was added to GS, which is now composed of $\{S_1, S_2\}$.

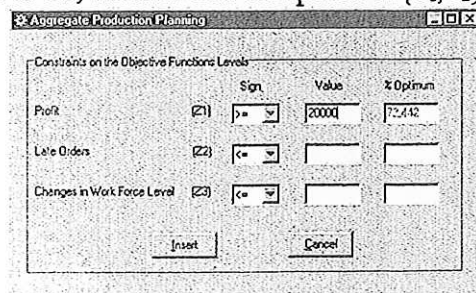


Figure 3: Including restrictions

- *Third interaction.* The value of late orders in S_2 was not as low as the DM expected. For this criterion function, the DM anticipated a value no higher than 8000 square meters (m^2) (90.403%). The vector of weights and the reference point remained unchanged. The solution obtained was $S_3 = (27547.777, 6740.097, 5)$ which corresponds to (88.513%, 93.164%, 50%). The DM realized that the solution was obtained at the expense of changes in the employee level (from 4 to 5). The profit increased and the number of late orders changed from the previous reference point. The DM decided to add S_3 to GS, which now makes $GS = \{S_1, S_2, S_3\}$.
- *Fourth interaction.* The DM continued to emphasize the number of late orders. The DM observed in S_3 that the number of late order increased along with the instability in work force. Thus two additional restrictions were added: late orders volume must be no higher than 5000 m^2 (96.978%) and the change in the work force must be no more than 6 workers (40%). The weights and the reference points were not modified. However, these conditions made the problem impossible to solve.
- *Fifth interaction.* In this interaction the DM decreased the magnitude on the change in the work force level, allowing a flotation of 7 workers, the limit value (30%), and keeping the number of late orders at 5000 m^2 . The solution obtained was $S_4 = (31390.459, 3992.445, 7)$, which is equivalent to (96.695%, 99.187%, 30%) in relative terms. Thus, with an increase of one worker in the "changes in the work force" criterion function, it was possible to obtain a value for the late orders better than what the DM had wanted. The profit level also improved. This solution is included in the GS, which now stands as $GS = \{S_1, S_2, S_3, S_4\}$.
- *Sixth interaction.* The previous solutions showed relatively low degree of satisfaction for the changes in the work force level. The DM considered whether it would be possible to obtain a solution with a profit of at least 25 millions of escudos and a change in the work force level not exceeding 3 employees. These restrictions correspond to an 83.088% and 70% of the profit and work force, respectively. The procedure indicates that the problem is impossible to solve with this requirement.
- *Seventh interaction.* The DM changed the weights by setting a vector where profit assumed a stronger emphasis than the number of late orders and changes in the work force (0.7, 0.2, 0.1). The reference point was the perturbed ideal point and did not restrict the criteria space. The solution obtained was $S_5 = (29166.441; 5550.202; 6)$, equivalent to (91.959%, 95.772%, 40%). It was accepted for evaluation and included in the GS. We now have $GS = \{S_1, S_2, S_3, S_4, S_5\}$.
- *Eighth interaction.* Here the DM altered the weights on the late orders criterion and his or her profit preferences. The vector of weights is now equal to (0.2, 0.7, 0.1). The solution obtained was once again S_3 , leaving the GS unchanged.
- *Ninth interaction.* In this stage the DM again modified the weights, giving preference to the proximity of the best value for the changes in the work force level. The vector of weights is (0.1, 0.2, 0.7), and no new restrictions are introduced. The solution obtained was $S_7 = (4710.002; 29741.965; 2)$, corresponding to (39.887%, 42.75%, 80%). This solution was rejected because the DM already had globally superior solutions.
- *Tenth interaction.* In this interaction the DM did not introduce any restrictions in the criteria space and the weights were (0.333, 0.333, 0.333). Based on the prior solutions, the DM specified 30 million escudos for profit, 5000 m^2 for the number of late orders, and a maximum of 3 workers for the changes in the work force level as the reference values. The solution obtained was once again S_2 .

The evolution of the successive retained solutions during the decision making process is shown in Figure 4. Let us assume that the DM considers the solutions included in GS are representative of the whole non dominated solution set and decides to use it as a generator set for new solutions. The DM now moves to the second phase with the *search* option.

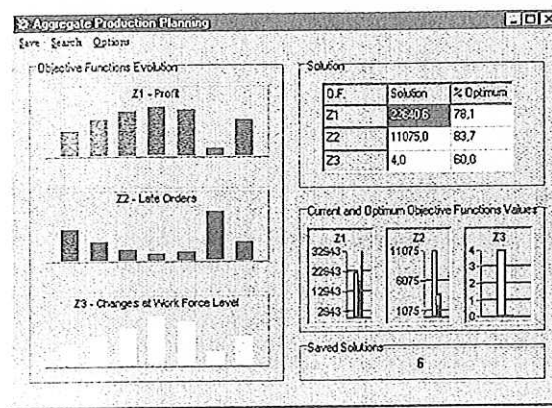


Figure 4: First phase results

6.2 Second phase of the interactive method

In this phase the DM uses GS from phase one to do a more detailed search through the non dominated solutions space. The DM thus increases his/her knowledge about the solutions in the GS. In this phase, five interaction options are available: evaluating, comparing, removing and searching neighbouring solutions, and returning to the learning oriented phase.

At the beginning of the second phase, a menu is presented with all the retained solutions from the previous phase and the five interaction options.

Suppose the DM started by evaluating solution S_1 and considered the profit level unsatisfactory. Therefore, the minimum profit level was raised to 25 millions of escudos and the number of employees in the regular work force increased by one. Since the DM did not set the limit for late orders (Figure 5), the problem therefore became impossible to solve. The DM consequently increased the number of employees by one more, which yielded solution S_3 . Based on the results, the DM concluded that it would not be possible to obtain the desired profit level by changing the late orders alone. The alteration in the number of employees in the work force was necessary to improve the profit level as well as to reduce the number of late orders.

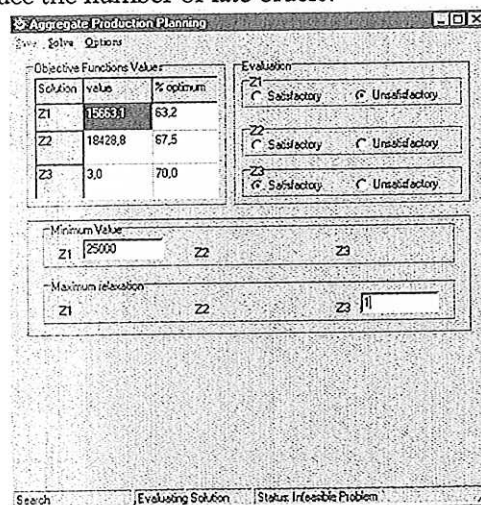


Figure 5: Evaluation menu

The solution S_1 shows nearly the same percentage levels in the three criteria. The DM searched for a new solution in its neighborhood, and it was again S_2 again (Figure 6). The search for another solution adjacent to S_1 produced S_6 . The DM observed that S_1 and S_6 are nearly identical with respect to the number of late orders and changes in the number of employees in the regular work force, and therefore excluded S_6 from the GS.

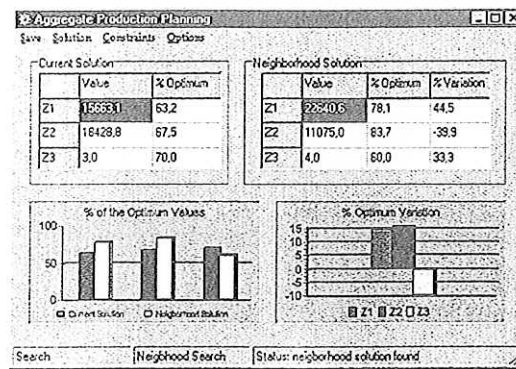


Figure 6: Neighborhood solution search

Upon comparing S_1 and S_2 (Figure 7), the DM decided that the decrease in the work force performance criterion was acceptable because the profit level increased while the number of late orders decreased. The DM therefore eliminated S_1 . We now have $GS = \{S_2, S_3, S_4, S_5\}$.

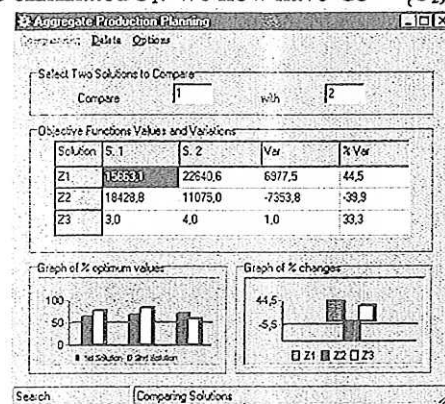


Figure 7: Comparison menu

The DM now observes that solution S_4 is inferior to the other solutions with regard to the change in the work force level criterion. Since the GS contains solutions with a better result in this criterion along with comparable levels for the others, the DM decides to drop S_4 . The GS now includes only S_2, S_3 , and S_5 .

The DM next proceeds to eliminate S_2 from the GS because the volume of late orders is unsatisfactory since it exceeds 8000 m^2 . The current GS contains S_3 and S_5 . The similarity of the change in the work force criterion for S_3 and S_5 necessitates a more detailed analysis of these two solutions. It appears that the increase in the profit level, along with the decrease in the number of late orders, outweighs the deterioration in the work force criterion. Therefore, S_3 is eliminated. The DM concludes that S_5 is the definitive solution of the interactive process.

Information on the decision variables for this solution is given in the menus in Figure 8. It provides the DM with the necessary information to formulate the company's aggregate production plan for a 12-month horizon. Additional information may be obtained by analysing the graphs in Figure 8.

Figure 8 shows the results in which the sales and costs values are disaggregated by product family and by month, sales and production by month, and the sales and production accumulated in the planning horizon. It also shows the production level by product family and by month, late orders by month, and the overtime hours for each worker type by month.

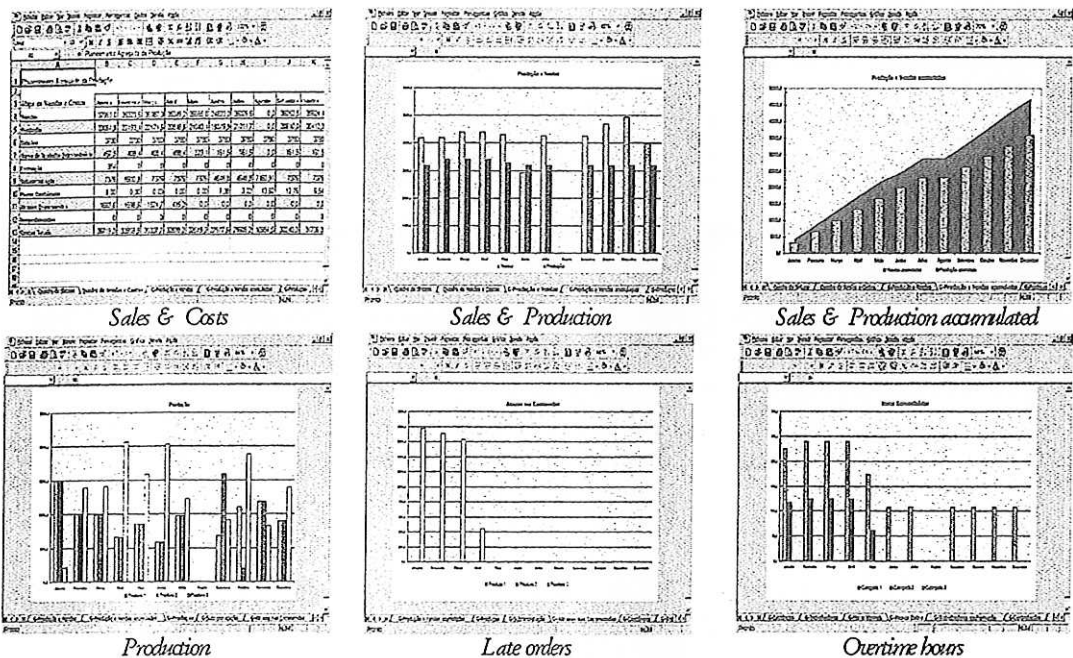


Figure 8: Excel graphs associated with S_1

7. Conclusions and future research

This paper presents a multiple criteria mixed integer linear programming model to solve aggregate production planning problems. The model, which includes three performance criteria and a set of constraints, was applied to solve an actual aggregate planning problem faced by a Portuguese firm that produces construction products. In addition, we developed a computerized decision support system that searches for and provides "the best solution," if it exists, for this type of problems. The operational features of the decision support system, based on the *learning-search oriented* concept, have also been demonstrated in detail through a series of interactive steps between the system and a hypothetical user.

Over the past few decades, researchers have proposed many aggregate planning models. However, many of these models have experienced limited industrial applications because of their methodological complexities and inflexible assumptions. The research is an attempt to bridge this gap between the theory and practice of aggregate planning models. The multi-criteria model presented in this study allows managers to explicitly consider different aspects of a typical aggregate planning problem, such as profit, labor environment and the clients' satisfaction level. In spite of the complexity of the model, the DSS does not require the user (manager) to have a high knowledge level on the mathematical aspects of the model. The user can generate, evaluate, and compare different solutions provided by the DSS through a helpful, user-friendly and interactive process. Furthermore, the DSS is flexible enough in that various operational restrictions of a production system can be easily incorporated into the model as constraints.

Future research might consider the development of an interactive DSS that will help managers cope with uncertain environments in the contexts of uncertainty and/or inaccuracy of forecast data. Another worthwhile avenue for extending this research would be finding ways to include new constraints and objective functions in the model in flexible and efficient manners. Including modules to the current DSS that will allow the user to analyze the sensitivity of different operational and cost parameters is another logical extension to this research.

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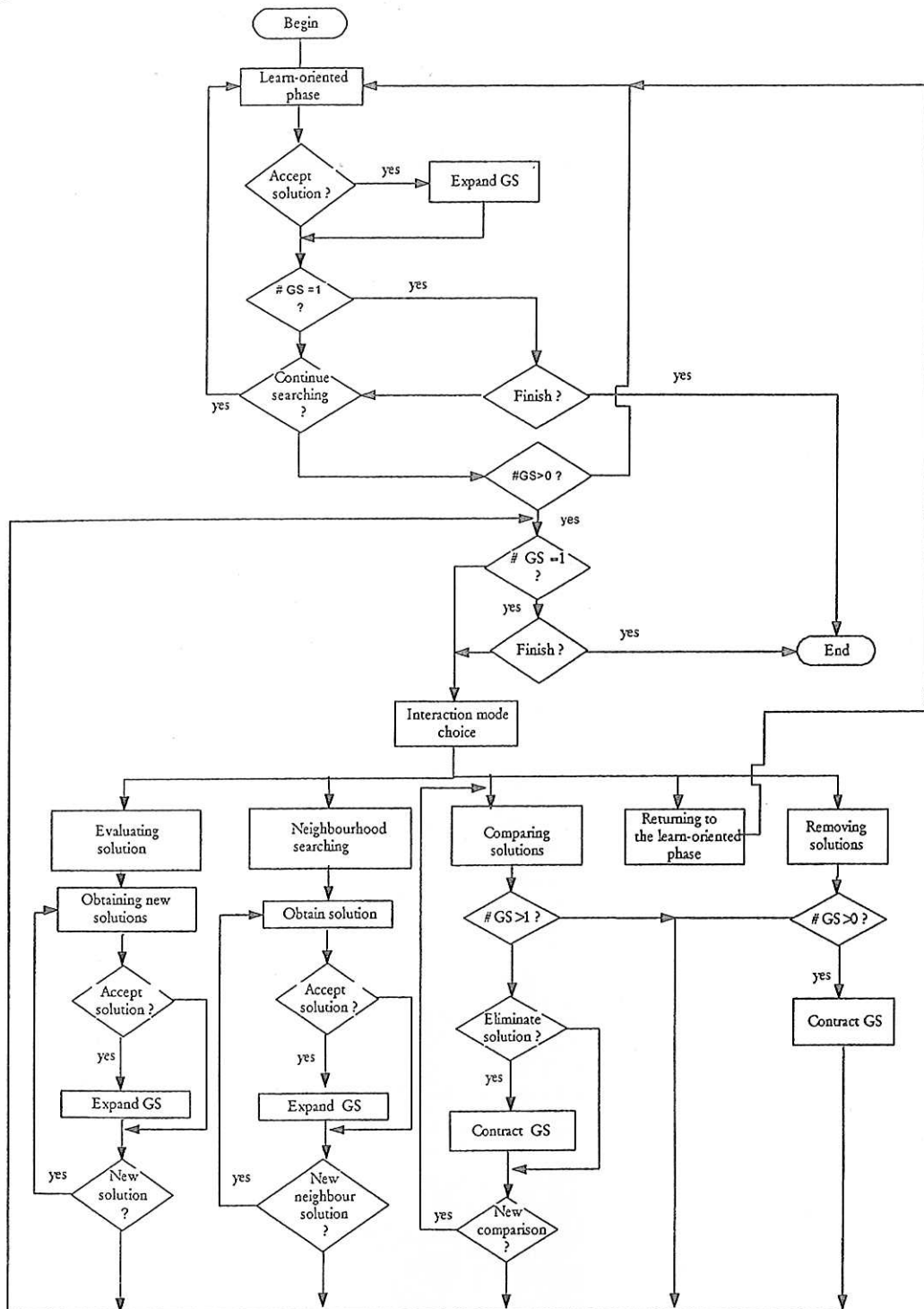
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Appendix:



Structure of the interactive method