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# A conceptual framework for a normative theory of "decision-aid".

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# A CONCEPTUAL FRAMEWORK FOR A NORMATIVE THEORY OF "DECISION-AID". *

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## Abstract

The intent of this paper is to develop a conceptual framework for a normative theory of "decision-aid". Here the word normative does not apply to the decision-maker, for whom aid is provided, but to the scientist and to his work of analysis and modelling. This framework is appropriate to problems with multiple conflicting objectives. The traditional optimization on a fixed set of alternatives (mutually exclusive actions) is treated as a particular case. More generally, cases dealing with either fixed or evolutive sets of potential actions not necessarily pairwise incompatible are considered.

Amongst others, the concept of a consistent family of criteria together with those of true-criterion, precriterion, semi-criterion, pseudo-criterion are introduced. Reasons for which multicriteria decision-aid may not fit in with the assessment of a unique true-criterion are briefly discussed. Several situations calling for a modelling of global preferences so as to "extract" good actions from a given set, otherwise than by optimizing a value function, are considered. Lastly, a new interactive procedure called "evolutive target procedure" leading to compromises, in the presence of n conflicting criteria and flexible constraints, is proposed.

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# 0 - INTRODUCTION

Decision-aid refers to the activity of a scientist who tries, by means of more or less formalised models, to help a decision-maker, so as to improve his control (this word having its cybernetical connotation) of the decision-making process. To improve in this context signifies to increase the coherence between the evolution of the process and the different objectives intervening in it. This presumes amongst others things to elicit the objectives, to clarify their antagonisms and to find implementable solutions which exceed them.

In this perspective, modelling has firstly a passive role in helping to comprehend, by mastering the various possible actions and by the reflections it gives to pre-existing preferences, and secondly an active role in the sense that the model contributes to the formation and evolution of the preferences of the different actors on stage so as to make acceptable or uncover possibilities which were previously refuted or not considered.

In this vein I would like to analyse the "hinge" phase of modelling and propose a conceptual framework, useful to the scientist's work. This I intend to do by considering, successively in sections I, II, III, the three stages in modelling shown in table 1.

TABLE 1

THE THREE STAGES IN MODELLING

| | |
|---|---|
| STAGE I | SUBJECT MATTER OF THE DECISION<br>- Formal definition of the set A of POTENTIAL ACTIONS : case globalised, fragmented, fixed, evolutive (cf. table 2)<br>- Choice of PROBLEM FORMULATION : "one", "all", "some" (cf. table 4). |
| STAGE II | ELEMENTARY CONSEQUENCES<br>- Formal description of the cloud of consequences $\gamma(a)$ : SCALES, VALUATION on each dimension, THRESHOLDS, .. (cf. tables 5, 6, 7).<br>- Choice of a CONSISTENT FAMILY OF CRITERIA, $g_1$, .... $g_n$ adapted to the discriminating power and to mesurability or graduability on each scale (cf. table 8 and question Q1). |
| STAGE III | GLOBAL PREFERENCE<br>- Formal definition of BEST, WORST, GOOD and BAD : comparability, independance properties, trade-offs, sub-aggregation, ..... (cf. table 9 and question Q2).<br>- Choice of an OPERATIONAL ATTITUDE adapted to the degrees of complexity, fuzziness and uncertainty of the "aggregation logic" and decision process (cf. table 10). |

The retroaction shown on the left side of this table points out that, having reached stage III, the scientist is frequently led to modify work previously accomplished in stage I and stage II, either because his first deductions incite him to do so or because he places himself at a different level of insertion into the decision process.

# I - POTENTIAL ACTIONS AND PROBLEM FORMULATION.

## I. 1 - Potential actions

Before we can even begin to talk about an optimum we must first make reference to :

- alternatives conceived as mutually exclusive, each one representing a global action a including in a extensive way every aspect of the decision.
- a set A embracing all imaginable global actions but only those which are implementable (usually called feasible alternatives), this a priori delimitation being based on the existence of a rigid objective frontier separating the admissible and inadmissible.

However, there may be problems for which it is futile or simply maladroit to use such a set as a starting basis for decision-aid. Firstly, the frontier between the acceptable and the unacceptable is often fuzzy. Sometimes this depends on the nature of the boundaries : a factory's maximum production capacity is affected by recourse to overtime or sub-contactors, the possibility of loading certain machines beyond their normal capacity or increasing their potential by annexing other equipment. In other cases (job planning with delivery date, portfolio composition, diet for an overweight person, ...) it is the diagnosis of acceptability in its globality which will create the problems, due to the complex arrangement of the diverse fragments constituting the envisaged actions. Secondly, insufficient performance brought to light by a preliminary calculation, the clash of ideas between the principal actors in the decision-making process, or simply the impossibility of imagining, a priori, all possible actions, are all circumstances which lead to the evolution of set A (cf. table 2).

Moreover, let us point out that analysis of the subject matter of the decision often brings out the artificial and uselessly complicated character of a conception which necessitates definition of mutually exclusive actions. Many false problems are born from this conception. Let us consider, for example, the decision regularly taken in a bank in relation to requests for credit, or in a private firm in relation to the remuneration of personnel, by a panel in connection with a diploma, by an individual with regard to his meal in the factory cafeteria, ...

When the actions are not naturally mutually exclusive, we may seek to determine those configurations of fragments which are. Thus we are led to substitute for the natural set A of elementary compatible actions, a sub-set of $\mathcal{P}(A)$ (set of all sub-sets of A), the elements of this sub-set appearing as global actions pairwise incompatible. By doing this we are taking the risk of encountering difficulties (occasionally insurmontable) in delimiting the sub-set of $\mathcal{P}(A)$ of acceptable configurations (feasible alternatives).

Table 2 characterises the 4 cases which it seems natural to add as a conclusion to the foregoing brief discussion. Table 3 contains a list of examples illustrating each of these 4 cases and to which we will refer in preference. The references cited will help the reader with his reflection on the interest of the 4 cases even though the models described in them may not be exactly the same as those referred to .

TABLE 2

FOUR CASES FOR THE MODELLING OF THE SET A OF POTENTIAL ACTIONS.

| The set A is a priori defined in a strict and exhaustive manner (by a rigid frontier, a non ambiguous test of membership, ...) | The elements of A are mutually exclusive: YES | The elements of A are mutually exclusive: NO |
|---|---|---|
| YES | case globalised and fixed | case fragmented and fixed |
| NO | case globalised and evolutive (or flexible) | case fragmented and evolutive (or flexible) |

The important factor, in connection with the set A , is that the potential actions (1) within the context of a given stage in decision-aid are clearly identified. This by no means signifies that the actions are mutually exclusive or independent, but that they may be considered separately from one another, without becoming devoid of meaning. This does not mean either that they are all immediately acceptable, there is nothing binding and some may be considered unacceptable in a subsequent stage.

Depending on the problem studied, A may be defined (modelled) :

- by a list which very precisely identifies each potential action (e.g. 5, 11, 17, 23 in table 3) ;
- by a "generator" enabling systematic generation (at least in theory) of all potential actions (e.g. 3, 4, 19, 20 in table 3) ;
- as the solution set of a series of conditions or constraints, expressed mathematically, on the characteristics of the potential actions (e.g. 4, 12, 13, 15 in table 3).

I.2 Problem formulation

In conjunction with this option as to the conception of the set A, the scientist must take another, just as fundamental. It concerns the choice of the type of problem formulation, account being taken of the level of intervention of the model and its present stage of development.

It is often thought that the problem formulation $\alpha$ in table 4 is the only natural one. The unique quality of the final decision in the case of A globalised has come to reinforce this belief. By experience we know how difficult it is for a scientist to convince the principal actors in the decision-making process that the optimal solution in keeping with his model is the one which should be adopted. Moreover, this particular problem formulation $\alpha$ ceases to be self-evident when A is evolutive and/or fragmented. The scientist may then consider either problem formulations $\beta$ or $\gamma$. Let us give a brief presentation of each one.

(1) I feel that the terms "action" and "potential" are better respectively, than "alternative" and "feasible or admissible" which seemed to be too strongly related to case A globalised for the former and to case A fixed, for the latter.

TABLE 5 : EXAMPLES

| A | GLOBALISED | (1) | (2) | (3) | FRAGMENTED | (1) | (2) | (3) |
|---|---|---|---|---|---|---|---|---|
| FIXED | 1. Management of a blood bank | α | 1 | [36] | 7. A regional investment policy (hydro-agriculture) | α | 1 | [42] [46] |
| | 2. Admission to or graduation from a university, college, .., on the basis of an examination | α | 1 | [43] | 8. Educational qualification based on an examination | β | 1 | [13] [2] |
| | 3. Sequencing of operations with rules of priority | α | 1 , 2 | [51] [20] | 9. References cited by a documentary centre in reply to a question | β | 1 , 3 | [16] [39] |
| | 4. Product mix ( rubber<br>( detergent | α<br>β | 3<br>2 | [9]<br>[7] | 10. Admission to or graduation from a university, college, .., on the basis of an examination | γ | 1 | [62] |
| | 5. Location of (an airport<br>(a factory | α<br>γ | 1<br>1 , 2 | [23] [44]<br>[30] | 11. Press support for an advertising campaign | γ | 2 | [12] [57] |
| | 6. A regional investment policy (hydro-agriculture) | β<br>γ | 2<br>2 | [18] [15] | 12. Distribution of warehouses in an allocation network | γ | 3 | [31] [24] |
| EVOLUTIVE | 13. Allocation of limited resources in project management | α<br>γ | 1<br>3 | [51] [58] | 19. Assignment of engines to pull trains | α | 3 | [51] |
| | 14. Buying a car | α | 2 , 3 | [52] | 20. Request for bank financing | β | 1 , 2 | [43] [56] |
| | 15. Diet taking into account personality of the patient | α | 3 | [4] [19] | 21. Promotion (in the sense of remuneration) in an hierarchical system | β | 1 , 3 | [8] |
| | 16. Industrial production program (chemistry, ...) | β | 2 | [45] | 22. Discussion between sales, production and finance departments, within an organisation of sale conditions (quantity, delay, price,..) | β | 3 | [48] |
| | 17. The working of a decision panel (urban planning, architecture) | β | 3 | [66] [40] | 23. R & D projects selection in a firm | γ | 1 , 3 | [37] [61] |
| | 18. Mapping out the route of a proposed highway | γ | 2 , 3 | [11] [14] | 24. Composition of his menu by a customer in a restaurant | γ | 2 | [53] |

(1) Possible problem formulations considering the modelling of A chosen (cf. table 4)
(2) Operational attitudes implied, or those most natural according to anterior options (cf. table 10)
(3) Bibliographical references

TABLE 4

TYPE OF PROBLEM FORMULATION ON A.

| | | |
|---|---|---|
| the objective of problem formulation is to select | $\alpha$ "one" | one and only one action considered the "best" |
| | $\beta$ "all" | all those actions which seem "good" amongst those studied |
| | $\gamma$ "some" | several actions amongst the "best" studied |

When the actions of A imply an examination with, for example, a view to entry into an educational establishment, the awarding of a diploma, the allotment of credit, a grant or a subsidy, the acceptance of a minor or exploratory research and development project, etc..., the scientist may envisage the problem in the following terms : accept all the "sufficiently" good actions, reject all those "far too" bad and ask for a complementary examination of the others. He is then led for instance to use a procedure using a trichotomy of the set A :

$$A = A_1 \cup A_2 \cup A_3 \qquad A_h \cap A_k = \emptyset \quad \text{for } h \neq k$$

an action $a \in A$ being :

- in $A_1$ if it calls for acceptance without the intervention of the decision-maker,
- in $A_3$ if it calls for rejection without the intervention of the decision-maker,
- in $A_2$ if it calls for a complementary examination ( requesting supplementary information, discussion, decision-maker's judgment, ....).

There are many cases in which the decision-maker, without being constrained to accept only a single action of A , knows a priori that he must give up the idea of accepting all of the good ones. This may be the case when the actions concern, for example, important research and development operations for a firm, regional development projects, equipment conceived to carry out certain functions, candidates destined to fill a series of similar posts, press supports before starting an advertising campaign, stocks or bonds entering into a portfolio, ... In these examples the idea of competition prevails. The elements of A can for instance be regrouped into equivalence classes, as small as possible, these classes being ordered so as to define a weak order on A . It is this weak order which will be used to establish the final decision : the demarcation line (acceptance/rejection) may either be a subject for the decision-maker's judgement,

who will judge its level of acceptability (financial, physical, psychological, ....) or else a subject for negotiation, or even a "local" study.

This double option (cf. tables 2 and 4) leads to 4 x 3 = 12 cases, each corresponding to a real situation. A superficial analysis may leave us with the impression that the globalised cases imply the $\alpha$ type problem formulation. In actual fact, they do not, because decision aid has for objective to present all of the efficient actions (optimal in the Pareto sense) with respect to $n(\geq 2)$ criteria (problem formulation $\beta$ - cf. 4 and 16, table 3) or even the actions forming the kernel of an outranking relation (problem formulation $\gamma$, cf. 6 and 18) : the fragmented cases do not exclude problem formulation $\alpha$ since decision-aid can then proceed by successive iterations, each consisting of a selection of a "best fragment" (cf. 7 and 19).

These examples anticipate, to a slight extent, section III but in so doing they illustrate the retroaction from the 3rd to the 1st stage in the modelling : the choice of operational attitude is at the same time cause and consequence of the choice of problem formulation, in truth successive problem formulations adapted to the progressive development of the model.

In my view it is the quality of the insertion in the decision-making process (and not the facility in resolution) which is determinant in the fixing of :

- the nature of the potential actions backing up the reasoning and the structure and nature of the data ;
- the problem formulation, guideline for deduction and discussion, widely responsible for the adoption (or total rejection) of the model : problem formulation unacceptable, unrealistic, incomprehensible to the principal actors.

0
0 0

Now, having thus clarified the subject matter of the decision, the scientist has to bear in mind that a decision is very rarely the reflection of preferences of a single person or even of a well-identified group of people. The decision is an important moment in the evolution of a process involving many actors and providing decision-aid means to take part in this process. This implies the identification of the one among those actors who play a determinant role in the achievement of the process for whom or in whose name decision-aid is provided.

This means that decision-aid is very rarely conceivable without the scientist's acceptance (provisionally) to "play the game" for a certain decision-maker. The scientist can do this by treating the global preference modelling problem not only for the decision-maker, but successively, for several of the actors in the decision-making process. In fact, the notions of best, worst, good and bad have, only exceptionally, an absolute sense and it is unrealistic, I believe to talk about preferences without specifying the actor who expresses them and wants to have them accepted in the decision-making process.

Nevertheless, the analysis of the elementary consequences of the diverse potential actions can generally go ahead independently of the chosen decision-maker. Then the scientist will have a scientific attitude since he will clearly dissociate :

- the formal description of all the elementary consequences that one at least of the actors may wish to be considered ; he will be able to try to synthesise them by using a consistent family of criteria which is acceptable and comprehensible to all (this will be the subject of the second section) ;

- the modelling of global preference taking into account the decision-maker's personality ; he will be able to try to do this according to the operationa attitude which seems to him to be the most effective in the decision-making process (this will be the subject of the third section).

## II. FROM CONSEQUENCES TO THE CONSISTENT FAMILY OF CRITERIA.

Even relative to a clearly identifiable decision-maker (a manager, a selection committee, a community), the consequences of a potential action a , on which this action is supposed to be judged (with a view to eventually comparing it with others) will appear at first sight imprecise, badly differentiated, multiple and confused. For this reason we call this complex reality the cloud of consequences of the action a , and denote it by $\nu(a)$.

### II. 1 - Primary concepts

The scientist must therefore devote himself to analysing and modelling in order to construct an abstract representation of $\nu(a)$ integrating all the relevant consequences needed for the assessment of global preferences. The elaboration of such a model is generally based on several primary concepts. I have attempted to define these concepts in table 5 so that they underly a coherent methodology which is as general as possible (cf. examples in table 3). Let us illustrate these definitions by an example.

In a problem ( manufacturing of windshields, printing of a magazine, ....) involving the choice of priority rules designed to establish sequencing of operations in a workshop and to determine the conditions under which recourse to exceptional means are necessary (overtime, sub-contractors, ...) three elementary consequences can be identified :

a) operatingcosts (energy, manpower, the fixed assets),
b) customer satisfaction in relation to delivery dates,
c) complexity of management in relation to planned adaptation in order to cope with habitual problems (breakdowns, illness, unforeseen jobs).

With regard to the elementary consequence a) there is a financial aspect and a state indicator to appraise the average annual outlay using a certain priority rule. A single point evaluation will be judged satisfactory even if it is approximate (it is important that it be unbiassed).

A delay dimension is convenient to fix the possible states of elementary consequence b) . Here the concept of average delay may be judged too rough to satisfactorily compare two priority rules on this single dimension. If the scientist does not wish to lay himself open to prematurely prejudging the way in which this consequence influences global preferences, he need only introduce a non single point state indicator completed by a modulation indicator (cf. table 6). The evaluation of the rule a on this dimension i can then be constituted by :

- the set $\gamma_i(a)$ of possible delays for an order, expressed by the number of working days,
- the distribution $\delta_i(a)$ indicating the degreee of importance of each dealy $e \in \gamma_i(a)$, on the basis of the number of orders (per year) having a delay equal to e or of a theoretical probability of such a delay for an order.

TABLE 5 : FROM ELEMENTARY CONSEQUENCES TO EVALUATIONS IN THE ANALYSIS OF $\nu(a)$

| CONCEPTS AND NOTATIONS | DEFINITIONS |
|---|---|
| Elementary Consequence | Aspect or attribute of $\nu(a)$ considered pertinent to the problem and for which the possible states can be evaluated on a dimension by a state indicator. |
| Dimension | Common feature of the set of states associated with an elementary consequence determining a complete order and destined to justify their comparison in relation with preferences |
| State Indicator $\gamma_i(a)$ | Process which is operational and homogeneous from the point of view of comparison used by the scientist to identify, in relation to the dimension i, the state or states, to which the potential action a may lead.<br>Formally we set : $\gamma_i(a) \subset E_i$ (scale associated with the dimension i ) |
| Scale $E_i$<br>Grade e<br>$<^i$ | Completely ordered set taken as the formal representation of the state set associated with dimension i<br>Element of a scale representing a state relative to the corresponding dimension<br>Order relation translating on $E_i$ the inherent order in the dimension i |
| Set $\nu = \{1, \ldots \bar{n}\}$ of dimensions | List of dimensions considered necessary and sufficient (for the problem) to make the description of $\nu(a)$ realistic and complete on the basis of corresponding state indicators, possibly completed by modulation indicators. |
| Modulation Indicator $\delta_i(a)$ | Complementary information about $\gamma_i(a)$ when the valuation is not single point on the dimension i (see table 6) |
| Evaluation of a on the dimension i | $\gamma_i(a)$ if $\gamma_i(a) \in E_i$ the valuation is then said to be single point<br>$[\gamma_i(a), \delta_i(a)]$ otherwise the valuation is then said to be non single point |

With regard to the last elementary consequence we associate a dimension "degree of flexibility" relecting the capacity of a priority rule to absorb incidents encountered in normal management (without provoking excessive tensions, damaging disorder, ...). The associated scale can only be qualitative (without spending a great deal of time on its definition and corresponding valuations). It is not easy to code a priority rule on such a scale, although the head of the organisation and methods bureau often attaches a great deal of importance to the global and subjective idea he has of this degree of flexibility. Therefore a realistic way to go about this might be to ask an expert to classify the different rules by means of this dimension, without seeking to code the states and their modulations ; i.e. only the relation established by the judge is important. This is an example of a relational case (cf. table 6).

In certain cases the scientist may procede in a different way. Let us suppose that an analysis of past activities of the shop reveals a small number of typical situations :

- normal situation,
- conjuncture producting a sudden increase in the workload,
- functioning below capacity because of the unavailability of a key machine or a specialised labour force,
- .....

easily discriminant with respect to operating costs, customer satisfaction and the adaptation possibilities for each rule. Let $\mathcal{E}$ denote the set of elements, called events, characterising each of the considered situations. For each dimension $i$, the set of states, $\gamma_i^{\mathcal{E}}(a)$ to which a specified event $\mathcal{E}$ may lead, can generally be reduced to a proper subset of $\gamma_i(a)$. On certain dimensions, $\gamma_i^{\mathcal{E}}(a)$ may be systematically reduced to a single element of $E_i$ : this is (for a class of exclusive events) the case "single indexed event" of table 6. As for the dimensions for which this is not the case, a modulation indicator must specify (in a distributional or relational way) the relative importance of the states of $\gamma_i^{\mathcal{E}}(a)$ : this is the case "complex indexed event" of table 6.

We will leave it to the reader to reflect on this by reconsidering the preceding elementary consequences in this context. Anyway, the act of clarifying the class $\mathcal{E}$ has, when the discriminant influence stands out clearly on at least two dimensions, the merit of bringing to light a causal liaison that global preference modelling can not ignore.

Within or outside of "indexed event" cases, there may exist other relations between the modelling indicators which can subsequently supply useful information and it is in the scientist'interest to diagnose them at this stage of the analysis. These relations are laid down at the end of table 7.

In order to illustrate the remainder of table 7, let us return to the three dimensions, finance, delay and flexibility, introduced above. Note that for each of the three corresponding scales, the objective is to achieve (even if unobtainable) one of the two extreme grades (cost nil, no delays, maximum degree of flexibility). It could quite well be otherwise. Let us suppose that deliveries are a source of problems for customers. Each grade will then represent an algebraic difference between the actual and contractual delivery dates and the goal "no delay" will no longer be at the end of the scale. If two grades of the same sign are still directly comparable in terms of preferences, a further step in the study of preferences must be taken so as to compare two grades with different signs.

TABLE 6 : NATURE OF THE MODULATION INDICATORS USED IN NON SINGLE POINT VALUATIONS.

| MOST USUAL CASES | $\delta_i(a)$ is a : |
|---|---|
| Distributional (1) | Distribution defined on $\gamma_i(a)$ used by the scientist to quantify the relative importance of the different states (according to the size of the population concerned, to a degree of fuzziness of membership, in a probabilistic sense, with respect to dates, ...). |
| Relational (2) | Number or vectors associated with $\gamma_i(a)$ used by the scientist to establish a relation (preference, indifference, incomparability) between $\gamma_i(a)$ and $\gamma_i(a')$ for each potential action a' (based on a modulation of importance, of likelihood, ...., between the states of each of these two sub-sets) |
| Single "indexed event" (3) | Mapping from $\mathcal{C}$ (set of exclusive events) onto $\gamma_i(a)$ used by the scientist to characterise the state $\gamma_i^{\varepsilon}(a)$ to which event $\varepsilon \in \mathcal{C}$ leads; to each event of the class $\mathcal{C}$, $\delta_i(a)$ then associates a single point valuation. If $\mathcal{C}$ is provided with a probability distribution, $\delta_i(a)$ then defines a probability distribution on $\gamma_i(a)$. |
| Complex "indexed event" (4) | Mapping from $\mathcal{C}$ (set of events) into $\mathcal{P}[\gamma_i(a)]$ (set of parts of $\gamma_i(a)$) used by the scientist to characterise those states to which each of the events of the class $\mathcal{C}$ may lead, it may be added to by a distribution or relational modulation when the image in $\mathcal{P}[\gamma_i(a)]$ is a sub-set containing at least two elements of $\gamma_i(a)$. |

(1) Von Neumann-Morgenstern utility theory
(2) Arrow social welfare theory
(3) Game theory
(4) Scenarios method.

WITH A VIEW TO A MORE SYNTHETIC MODELLING OF $\nu(a)$

| CONCEPTS AND NOTATIONS | DEFINITIONS |
|---|---|
| Goal $o_i$ | Grade of $E_i$ such that $\forall e, e' \in E_i$ $e'$ is preferred to or is indifferent from $e$ when $e \prec^i e' \precsim^i o_i$ or $o_i \precsim^i e' \prec^i e$ |
| Threshold of indiscrimination | Maximum interval between 2 grades (of the same scale) indiscriminated by the actors who do not consider them as really distinct ; it may vary along the scale ; |
| Threshold of imprecision | Maximum interval between 2 grades (of the same scale) for which the imprecision of the state indicator does not allow the two to be separated for certain ; it may vary along the scale ; |
| Threshold of unforeseeableness | Maximum interval between 2 grades (of the same scale) for which the quality of the previsions does not allow the two to be distinguished for certain ; it may vary along the scale ; |
| Threshold of presumed preference<br>$s_i^+(e)$ | Generic term used for one, or a mixture of the preceding thresholds such that :<br>$e'$ is strictly preferred to $e$ when :<br>$e \prec^i e' \precsim^i o_i$ or $o_i \precsim^i e' \prec^i e$<br>and that the interval $[e, e']$ (defined by reference to a measure : number of grades, ...) is strictly greater than (in absolute value) $s_i^+(e)$ ; for a smaller interval there is a situation of presumed preference which includes those of indifference and large preference (cf. table 9) |
| Threshold of indifference $q_i^+(e)$<br>$q_i^+(e) \leqslant s_i^+(e)$ | Maximum interval between 2 grades of $E_i$ such that :<br>$e'$ is indifferent from $e$ when :<br>$e \prec^i e' \precsim^i o_i$ or $o_i \precsim^i e' \prec e$<br>and that the interval $[e, e']$ (defined by reference to a measure) remains strictly less than (in absolute value) $q_i^+(e)$ ; for a larger interval, there is large or strict preference |
| Modular informational Relation from the dimension $h$ to the dimension $i$ | The precision of a state $e_h \in \gamma_h(a)$ $E_h$ leading to a modification of the modulation $\delta_i(a)$ relative to $\gamma_i(a) \subset E_i$ |
| $\delta_i(a, e_h)$ | Modulation indicator on the dimension $i$ conditionned by $e_h$ |
| $\delta(a)$ | Global indicator integrating all the modular informational relations |

The personality of the actors, the nature of the potential actions and scales, the mode of elaboration of the state and modulation indicators are the basis of different types of frequently intermingled thresholds. To admit that the different actors are indifferent to two priority rules leading to the same valuations, except regarding average operating costs for which valuations are $e$ and $e - \eta$, may very well raise different explanations :

- $\eta$ is a negligible sum in comparison with the sums in play elsewhere and with the sensitiveness of each actor in this dimension ;

- $\eta$ is too low a sum in view of the techniques implemented for data collection ;

- $\eta$ is a non-significant sum in view of the risks that the mode of calculation sets aside.

By definition (cf. table 7), $s_i^+(e)$ is the maximum value of $\eta$ (on the financial dimension $i$ considered) such that $e - \eta$ is not recognised as significantly better than $e$. For $\eta \leqq s_i^+(e)$, the scientist may consider, either $e - \eta$ as presumed preference to $e$, or (more simply) $e - \eta$ as indifferent from $e$ ($s_i^+(e) = q_i^+(e)$ ).

To avoid giving a discriminating role to differences of little significance, the scientist will sometimes be led to pay particular attention to such thresholds in the neighbourhood of the goal, and for example define an interval $[s_i', s_i'']$ (containing $o_i$) for which all the states will be judged as "good" as they are sufficiently close to the goal.

II. 2 - <u>The concept of a consistent family of criteria and the underlying nature of the criteria</u>.

When $|\nu| = \bar{n} > 1$ or when the unique valuation is a non single point one, the description of $\nu(a)$ obtained :

$$[\gamma_i(a), \delta_i(a)] \qquad \forall i \in \nu$$

merits a slight transformation so as to be more manageable as much for global preferences modelling as for decision aid.

This transformation consists in the elaboration of what I will call a consistent family $F$ of criteria (see tables 8 and 9) (1).

Let us point out that, if $f(x)$ is any increasing function, the substitution in $F$ of $f[g_k(a)]$ for $g_k(a)$ gives a new consitent family for the problem.

It is important to note that the correspondence between criteria of the family and dimensions retained in the analysis is only simple (see in particular table 8 "single point equivalent on the dimension $i$ ") in the case where the criterion $g_k$ only brings into play indicators relative to a single dimension. When this is the case the passage from thresholds relative to the scales, to those of the same nature relative to the criterion $k$ (denoted by $s_i^+(x)$ and $q_i^+(x)$ respectively) presents no major difficulties. In the case of sub-aggregates, this passage may be a little more complex.

(1) The notation $F$ designates the family of criteria as well as the set $\{1, 2, \ldots, n\}$ of indices.

According to the discriminative power recognised for the criterion $k$, I suggest to call (see the end of table 8) :

- pseudo-criterion (most general case) : a criterion for which an indifference threshold $q_k^+(x)$ and a threshold of presumed preference $s_k^+(x)$, $s_k^+(x) \geqq q_k^+(x)$ are defined ;
- semi-criterion : a pseudo-criterion for which $q_k^+(x) = s_k^+(x)$
- precriterion : a pseudo-criterion for which $q_k^+(x) = 0$ or is not defined ;
- true-criterion : a pseudo-criterion for which $q_k^+(x) = s_k^+(x) = 0$ (every difference is significant).

These thresholds can not be any function of $x$, as is indicated in table 8 for a threshold of presumed preference, but remains true for an indifference threshold. This is quite simply explained by the fact that, for $y > x$ there can not exist (without inconsistencies) a value $z$ of $g_k$ such that :

$$y + s_k^+(y) < z \leqq x + s_k^+(x) .$$

On studying the underlying structure of each one of these types of criterion, it is easy to deduce that we are concerned with :

- a complete order for a true-criterion ;
- a semi-order for a semi-criterion ;
- what appears to be an orientated semi-order for a precriterion ;
- a more complex structure for a pseudo-criterion, to which I shall refer under the name of pseudo-order.

---

In what follows a semi-order is characterised by the definition of two relations $I$ and $P$ such that :

a) $I$ is reflexive and symmetric (which corresponds to indifference) ;
b) $P$ is an anti-symmetric "complement" of $I$ in the sense that one and only one of the following three possibilities holds $\forall x, y$ :

$$x\ I\ y, \quad x\ P\ y, \quad y\ P\ x$$

(which corresponds to strict preference) ;
c) $P\ I\ P \subset P$ (implies the transitivity of $P$) ;
d) $P^2 \cap I^2 = \emptyset$

(for further details, see FISHBURN (22) or JACQUET-LAGREZE (34)).

In order to complete an elucidation of the use that the scientist may make of the criterion $k$, he must investigate the relations which connect any two intervals of the type :

$$w_k = [x_k, x_k + \omega_k] \quad \text{and} \quad w'_k = [x'_k, x'_k + \omega'_k]$$

defined by two ordered-pairs $(a,b)$ and $(a',b')$ of potential actions such that :

$$g_j(a) = g_j(b) = g_j(a') = g_j(b')\ \forall j \neq k,\ g_k(a) = x_k,\ g_k(a') = x'_k$$

$$g_k(b) - g_k(a) = \omega_k \geqq 0,\ g_k(b') - g_k(a') = \omega'_k \geqq 0$$

Do such intervals sufficiently reveal an underlying reality to assess, in a significant and operational way , a comparison between the superiority of b over a and that of b' over a' ? The objective of this comparison is to lead the scientist to opt (as in table 9) in favour of one of the following fundamental mutually exclusive situations :

- superiority identical : $(x_k + \omega_k)$ differs from $x_k$ exactly as $(x'_k + \omega'_k)$ differs from $x'_k$ ;
- superiority strictly greater : $(x_k + \omega_k)$ differs from $x_k$ strictly more than $(x'_k + \omega'_k)$ differs from $x'_k$ ;
- superiority weakly greater : $(x_k + \omega_k)$ differs from $x_k$ at least as much as $(x'_k + \omega'_k)$ differs from $x'_k$ but it is impossible to say if it is strictly or exactly;
- superiority incomparable : neither of the three former situations dominate .

These considerations lead me to formulate the question :

$Q_1$ { How to discriminate the preceding four fundamental mutually exclusive situations so as to compare the "importance" of any two intervals of the type $w_k$ $w'_k$ ?

When one of the two intervals is null, the reply to this question has already been given through the concepts of thresholds (which may depend on the abscissa $x_k$ of the non null interval). Thus it is with the other cases that we are concerned.

It is rare that the scientist stops here in the sense that he implicitly admits that the reply is given (independently of $x_k$ and $x'_k$) by a simple examination of the sign of $\omega_k - \omega'_k$ . Frequently he goes as far as treating the ratio $\omega_k / \omega'_k$ as a measure of the superiority of b over a when that of b' over a' is taken as unit. It is clear that here very strong hypotheses make the criterion $g_k$ appear as a high precision instrument. Not only, $g_k$ is a true-criterion, but it is self imposing up to a positive linear transformation, i.e. $g_k$ is a measure (with reference to the set of intervals) : only the unit and origin are arbitrary.

If the scientist can define a function $f(x)$ (non decreasing) for which $f[g_k]$ (substituted for $g_k$ in F ) is a measure, then I will say $g_k$ is a measurable criterion in F . So that such a property holds, $g_k$ must satisfy the axioms 1 through 4 written below. Before let us introduce a new definition.

By discriminating interval I will refer to an interval of the type $w_k = [x_k, x_k + \omega_k]$ such that b is strictly preferred to a .

AXIOM 1 : any non null interval is a discriminating interval ( $g_k$ is a true-criterion).

AXIOM 2 : the reply to $Q_1$ is independent of the values $g_j(a) = g_j(a') = g_j(b) = g_j(b')$ $j \neq k$ , whatever be the pairs a b and a' b' .

AXIOM 3 : the reply to $Q_1$ excludes superiority incomparable.

AXIOM 4 : on the subset of discriminating intervals, the reply to question $Q_1$ excludes superiority weakly great and leads to transitive answers compatible with inclusion and union of the intervals, for superiority identical as well as for superiority strictly greater.

TABLE 8 : MODELLING $\nu(a)$ ON THE BASIS OF A CONSISTENT FAMILY OF CRITERIA

| CONCEPTS AND NOTATIONS | DEFINITIONS |
|---|---|
| Criteria vector<br>$g(a) = [g_1(a), \ldots, g_n(a)] \in R^n$ | Exhaustive résumé (for the problem) of $\nu(a)$ given by the $n$ values assumed, with regard to action $a$, by the criteria $g_j$ $j = 1, \ldots, n$ forming a consistent family relative to the $\bar{n}$ dimensions considered. |
| Consistent family $F$ of criteria<br>$g_j$ $j = 1, \ldots, n$<br>or $j \in F$ | A family of $n$ functions (called criteria) subject to three conditions :<br>1 - Exhaustivity condition : the $g_j(a)$ are real-valued functions, argument of which are the state and modulation indicators constituting the valuations of $a$ on the $\bar{n}$ dimensions ; they are defined in such a way that :<br>$g_j(a) = g_j(a')$ $j = 1, \ldots n$ $\Longleftrightarrow$ $a$ is indifferent from $a'$ ;<br>2 - Non incomparability condition : if $a$ and $a'$ are two actions such that :<br>$g_j(a) = g_j(a')$ $\forall j \neq k$ and $g_k(a') > g_k(a)$<br>then $a'$ is preferred to or is indifferent from $a$ (see table 9) ;<br>3 - Non redundancy condition : dropping any one of the $g_j$ of the family invalidates one or more of the preceding conditions (for a pair of actions $(a, a')$ real or fictitious) |
| $g_j(a) = \gamma_j(a)$<br><br>Single point equivalent on the dimension $i$ | This implies :<br>- the valuation on the dimension $j$ is single point $\forall a \in A$<br>- $o_j$ is the highest grade of $E_j$ which grades are identified by figures<br><br>The valuation on the dimension $i$ is not single point (at least for one action) but it occurs only in a single criterion, which is justified by the existence of a single point equivalent (average value, certainty equivalent, realised value, ...) substitutable for the couple state indicator, modulation indicator. |
| $g_j$ : sub-aggregate of dimension $i_1, \ldots, i_p$ | $g_j$ appears as a résumé of the valuations relative to the dimensions $i_1, \ldots, i_p$ |
| Explosion of the dimension $i$ | The valuation on the dimension $i$ intervenes, by at least one of its indicators, in more than one criterion. |

TABLE 8 : END

| | |
|---|---|
| $g_k$ is a <u>true criterion</u> | if the condition 2 above implies strict preference (indifference and presumed preference being excluded) |
| $g_k$ is a <u>precriterion</u> | if the condition 2 above involves, besides strict preference, presumed preference situations which can be characterized as follows by a threshold function $s_k^+(x)$ :<br>- presumed preference if : $g_k(a) = x < g_k(a') \leqq x + s_k^+(x)$<br>- strict preference otherwise<br>So as not to lead to inconsistencies, $s_k^+(x)$ must also satisfy :<br>$\frac{s_k^+(x) - s_k^+(y)}{x - y} \geqq -1$ where x, y are possible values for the precriterion $g_k$ |
| $g_k$ is a <u>semi-criterion</u> | if the condition 2 above involves, besides strict preference, indifference situations which can be characterized as follows by a threshold function $q_k^+(x)$ :<br>- indifference if : $g_k(a) = x < g_k(a') \leqq x + q_k^+(x)$<br>- strict preference otherwise (large preference here is excluded)<br>so as not to lead to inconsistencies $q_k^+(x)$ must also satisfy the same condition as that above relative to a precriterion |
| $g_k$ is a <u>pseudo-criterion</u> | if it is a precriterion for which presumed preference involves besides large preference (see table 9) indifference situations which can be characterised as follows by a threshold function $q_k^+(x) \leqq s_k^+(x)$ ;<br>- indifference if $g_k(a) = x < g_k(a') \leqq x + g_k^+(x)$<br>- large preference if $x + q_k^+(x) < g_k(a') \leqq x + s_k^+(x)$<br>here $q_k^+(x)$ must satisfy the same condition as $s_k^+(x)$ |

These four axioms do not imply measurability as the reader may verify by reconsidering the example of II.1 relative to the elementary consequence c) : adaptation possibilities. Suppose that the scale $E_i$ adopted is the following :

$e_0$ : rule very rigid,

$e_1$ : rule involving non negligible rigidity factors,

$e_2$ : rule normally flexible,

$e_3$ : rule exceptionally flexible,

and to which corresponds a single point valuation $\gamma_i(a)$ giving rise to a true-criterion $g_k$ : $g_k(a)$ = index of the grade $\gamma_i(a) \in E_i$.

The scientist may then reply to question $Q_1$ (in agreeement with axioms 2, 3, 4) by defining on the set of non null intervals, a weak order (transitive and complete binary relation) such as the following :

$(0, 1) \prec (1, 2) \prec (2, 3) \prec (0, 2) \prec (1, 3) \prec (0, 3)$

The table below shows that there are various transformations of the criterion $g_k$ which render the lengths of the intervals compatible with the above classification but which lead to very different values for the ratios $\omega_k / \omega'_k$ .

| $E_1$ | $e_o$ | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|---|
| $g_k$ | 0 | 1 | 2 | 3 |
| $f^1[g_k]$ | 1 | 6 | 12 | 19 |
| $f^2[g_k]$ | 2 | 4 | 10 | 17 |
| $f^3[g_k]$ | 2 | 6 | 11 | 19 |

A set of actions necessary and sufficient (1) to imply measurability may be obtained by completing axioms 1, 2, 3, 4 by the following :

AXIOM 5 : for all three values of $g_k$ : $x \prec x' \prec y'$, $\exists$ a unique value $y$ of $g_k$ such that the superiority of $y$ over $x$ is identical to that of $y'$ over $x'$.

Past experience has shown that in a good many real world problem, it is impossible for the scientist to justify the exact values which axiom 5 postulates (either because there is more than one single value or because none is acceptable to all th actors who intervene in the decision-making process). The reliance on the idea of approximation to justify the arbitrary part included in the selected values (and in order to remain within the framework of the set of axioms) is not always a realistic attitude.

(1) The author thanks Ph. VINCKE for his contribution to the verification of the exactness of this assertion.

In fact this difficulty arises usually because axiom 1 is not operational. It follows that axiom 4 has also to be reconsidered.

I suggest to call the criteria which satisfy axioms 2 and 3 graduable and to use the term of graduation to designate those (graduable) for which the values allow a direct comparison of the intervals (in the above table $g_k$ is graduable but only the last three criteria are graduations).

The structure intoduced (by an axiom similar to axiom 4) on the set of intervals of the type $w_k$ by replies to question $Q_1$, is intimately related to the techniques used to assess these replies. In the case of graduable criteria we can characterise the criteria :

- truly-graduable : weak order structure (see above) ;
- semi-graduable : semi-order structure (see what follows) ;
- pregraduable : oriented semi-order structure ;
- pseudo-graduable : pseudo-order structure.

In the example of II.1 let us consider a semi-criterion g associated with the dimension "delay" defined as follows :

$$\begin{cases} g(a) = x \qquad q^+(x) = \text{Min}(q.x,\ 100 - x),\ 0 < q < 1. \\ x \in \{0,\ 1,\ 2,\ \ldots,\ 100\} \end{cases}$$

expresses the percentage of orders for which the delay exceeds a week.

## TABLE 9 : SITUATIONS TO WHICH MAY LEAD THE COMPARISON OF TWO POTENTIAL ACTIONS a, a'

| | SITUATIONS | DEFINITIONS |
|---|---|---|
| Four fundamental mutually exclusive situations | indifference | The two actions are indifferent in the sense that there exist clear and positive reasons to choose equivalence<br>example : $g_j(a) = g_j(a')\ \forall_j$ some of those equalities not being necessarily rigorous but only approximate. |
| | strict preference | One of the two actions (which one being known) is strictly preferred to the other<br>example : $g_j(a) = g_j(a')\ \forall j \neq k$, $g_k(a') - g_k(a)$ a significant difference |
| | large preference | One of the two actions (which one being known) is not strictly preferred to the other, but it is impossible to say if the other is strictly preferred to or indifferent from the first one because neither of the two former situations dominates<br>example : $g_j(a) = g_j(a')\ \forall j \neq k$, $g_k(a') = g_k(a)$ neither sufficiently small to justify indifference nor sufficiently large to justify strict preference |
| | incomparability | The two actions are not comparable in the sense that neither of the three former situations dominates<br>example : $g_j(a) > g_j(a')$ for $j = 1, \ldots, p$, $g_j(a') > g_j(a)$<br>for $j = p + 1, \ldots, n$ the majority of the differences being significant |
| Two important re-groupings | presumed preference | This excludes the strict preference and incomparability situations, and consequently embraces indifference and large preference situations be they separated or only assumed separable<br>example : see tables 7 and 8 (prescription concept) |
| | preference | This covers the two situations of strict and large preference separated or assumed separable and consequently excludes indifference and incomparability<br>example : see table 7 and 8 (condition 2 and the quasi-criterion concept) |

The scientist may envisage appreciating the importance (in terms of quality of service) of an interval $w = [x, x + \omega]$ (percentage increased from $x$ to $x + \omega$) by an indicator such as :

$$v(w) = \frac{\omega}{100 - (x + \omega)}$$

More precisely he may assess the comparison of the intervals $w$ and $w'$ on those of the "degrees of importance" $v(w)$ and $v(w')$ . He may admit, for example, that (for $v(w) \geqq v(w')$ ) :

- the importance of $w$ is strictly greater than that of $w'$ if :

$$v(w) \geqq v(w') + \frac{\text{Min}(q.x,\ 100 - x)}{100 - [x + \text{Min}(q.x,\ 100 - x)]}$$

- the importance of $w$ is identical to that of $w'$ otherwise

Then $g$ will appear as a semi-criteria semi-graduatable.

$$\begin{matrix} & 0 & \\ 0 & & 0 \end{matrix}$$

Although table 9 has a genaral bearing, the comparison of two potential actions has only been studied up until now, in the particular case when $g_j(a) = g_j(a')\ \forall j \neq K$ and by prejudging as little as possible the personality of the decision-maker. This together with the fact that the consistent family of criteria must be, as far as possible, comprehensible and acceptable by all the actors in the decision-making process, implies proscribing all premature sub-aggregates, source of confusion and contestation. The comparison according to a single criterion $k$ consequently only resumes what the analysis has objectively shown on a dimension, or on several, which are very homogeneous or highly correlated. Now we must pass to a higher level : that of global preferences (cf. table 1, stage III).

## III. FROM GLOBAL PREFERENCES TO OPERATIONAL ATTITUDE.

### III. 1 - Operational limits to global preferences modelling.

Apart from the relatively exceptional case when the synthetic description of $\nu(a)$ naturally leads to a consistent family reduced to a single criterion, complementary information and reflections are necessary in order to determine the conditions which, on the basis of associated criteria vectors, will allow to state : such an action is good or bad, better or worse than another in the eyes of an identified decision-maker.

Let us note that the question of knowing whether an action $a$ is good or bad can always be posed in terms of comparison : comparison of $a$ with reference actions (real or fictitious) acting as norms (cf. for example [43] or [54] ).

So, the scientist has now to consider the following question :

$Q_2$ { How to discriminate the four fundamental mutually exclusive situations of table 9 when : $g_j(a) \neq g_j(a')$ $j \in J \subset F$ and $|J| > 1$ ?

Such is the fundamental problem of global preference modelling. In fact it proves to be intimately related as much to the underlying nature of the criteria (cf. II.2), as to the operational attitude adopted by the scientist concerning the way in which decision-aid is to be provided.

Let us point out that if the reply to question $Q_2$ is, irrespective of the pair $a\ a'$ of potential actions considered, independent of the values

$$g_j(a) = g_j(a') \qquad j \in F - J \text{ (supposed non-empty)},$$

then the sub-family $J$ of criteria can be said preferentially independent of the complementary sub-family $F - J$. Preferential independence so introduced generalises this classical concept (cf. (35) and (64) ) to cases in which none of the four fundamental situations of table 9 is a priori excluded; comparisons being established on the basis of a consistent family non exclusively made of true-criteria.

It is by looking for such independence properties, trying to specify the relative importance of the $n$ criteria ... in connection with question $Q_2$ that the scientist will progress in global preferences modelling. But how far has he to go in such a modelling ?

Most often the scientist seeks a complete and explicite reply to $Q_2$ which a priori conforms with the following axiom :

AXIOM OF COMPLETE TRANSITIVE COMPARABILITY :

a) indifference situations define on $A$ a binary relation $\sim$, symmetric and transitive

b) preference situations define on $A$ a binary relation $\succ$, antisymmetric and transitive

c) large preference and incomparability situations do not exist.

Let us call $\succcurlyeq$ the relation defined by :

$$a \succcurlyeq a' \iff a \succ a' \text{ or } a \sim a'$$

If the three conditions of the axiom are satisfied, then $\succcurlyeq$ defines a weak order on $A$ (and vice-versa). In real world problems, it is always possible [1] (and in an infinite number of ways) to characterise such a weak order by means of a function.

$$V_A(a) = V_A\,[g_1(a), \ldots g_n(a)] \in R$$

such that

$$V_A(a) = V_A(a') \iff a \sim a'$$

$$V_A(a) > V_A(a') \iff a \succ a'$$

Such a function, then appears like a true-criterion aggregating the $n$ criteria of the family $F$.

Making this function explicit is an attitude which has proved its efficacity (see table 10) particularly when the scientist adopts the problem formulation $\alpha$ on a globalised and fixed set $A$ (cf. ex. 1, 2, 3, 5 table 3) and

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(1) See FISHBURN (22) theorem 3.1.

TABLE 10 : THREE TYPES OF OPERATIONAL ATTITUDE

allowing us, by combining them, to rediscover the quasi-totality of those observed

1. To wish to exclude incomparability and completely express the preferences by a unique criterion

This attitude leads to an aggregation, in a direct and complete way, of the n criteria forming the consistent family so as to determine what is called, according to the context, the economic function, ordinal function, value function, utility function, which is deemed to represent the global preference of the decision maker.

The principal methodologies are those of aggregation of votes, "déclassements comparés", goal programming, economic calculus (cost-benefit), multi-attribute utility theory (cf. [5] , [21] [22] , [23] , [25] , [34] , [38] , [45] , [64] , [73] ).

2. To accept incomparability and to only partially model the preferences through the intermediary of an outranking relation (fuzzy or not)

Here the scientist contents himself with modelling only those preferences he is capable of establishing objectively and with sufficient reliability, generally having recourse to the concept of outranking : i.e. a binary relation possibly fuzzy, defined on A , not necessarily complete or transitive (cf. [54] and [56] ).

For the principal methodologies (notably ELECTRE I and II) see [1] , [2] , [11] , [13] , [29] , [32] , [34] , [48] , [43] , [48] , [50] , [66] .

3. To attempt to eleborate in a interactive way one or more compromises based on local preferences

Here the scientist seeks to submit to the decision maker or his representatives (eventually to the demander) a certain action (or actions), real or fictitious, for his reactions and so doing, regathering information relative to his local preferences. Guided by this information, the scientist seeks one (or more) action presumed better than its precedent, which he in turn presents to the decision maker or his representatives for their reaction. When improvement is no longer necessary, or when it becomes impossible, we will say that we have reached a compromise, which appears, more or less clearly according to the followed procedure, as a local optimum relative to an implicit criterion.

Diverse methodologies have been proposed, notably in [6] , [8] , [10] , [17] , [26] , [40] , [41] , [60] , [67] , [68] , [69] , [70] , [72] , [73]

this, to such a point that for many problems it is the only envisageable attitude. This attitude is nevertheless a source of encumbrance for the scientist who may notably :

a) for certain pairs of actions, not know how to, not want to, not be able to compare them (cf. B. ROY ( 53) ) ;

b) for rough, qualitative, random valuations, or expressed in heterogeneous units (francs, minutes, number of inhabitants, degree of similarity) be in no position to extract a common dimension (cf. ex. 12 and 22, table 3) ;

c) under criteria which are more or less correlated, non measurable, counterbalancing within a complex imprecise logic, not know how to synthesise them in unique criterion (cf. ex. 11 and 18, table 3) ;

d) for an a priori delimited set of potential actions with frontiers almost artificial in their clarity, not feel capable of appreciating, a priori and in all their aspects, the structural transformations to be integrated in the definition of a unique criterion acceptable within A , so as to extend it to the frontier and a little beyond (cf. ex. 15 and 17 table 3) ;

e) for an evolutive set A and/or consisting of non-exclusive potential actions and/or problem formulation for which the objective is not to directly select a unique action, not judge, this an appropriate attitude (cf. ex. 19 and 24, table 3).

For these reasons or for others, he may renounce this first attitude, or wish to make it more flexible, or even to defer it.

Recent results in multi-attribute utility theory have slightly moved forward limits of this attitude but always within the same framework : complete transitive comparability axiom, F exclusively made of true-criteria.

Let us now briefly examine the two others attitudes of table 10.

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With the second attitude the scientist is more prudent. He limits his ambition as far as modelling is concerned, but tries to increase the reliability and the acceptability of his model. It is precisely this which was at the origin of the notion of outranking which I briefly recall (cf. B. ROY (53) or (54)) below.

By definition an outranking relation is a binary relation $S_A$, defined on A such that $\forall a, a \in A$ :

a) the relation (i.e. outranking) holds in both senses, a outranks a' and a' outranks a , if and only if the scientist estimates that, for the decision-maker, the pair (a, a') corresponds to an indifference situation ;

b) the relation holds only in one sense, for example a outranks a' if and only if the scientist estimates that for the decision-maker the pair (a, a') corresponds to a preference situation (strict or large preference) ;

c) the relation holds in neither of the two senses when the scientist can not, does not want to, does not know how to choose between one or other of the two preceding cases.

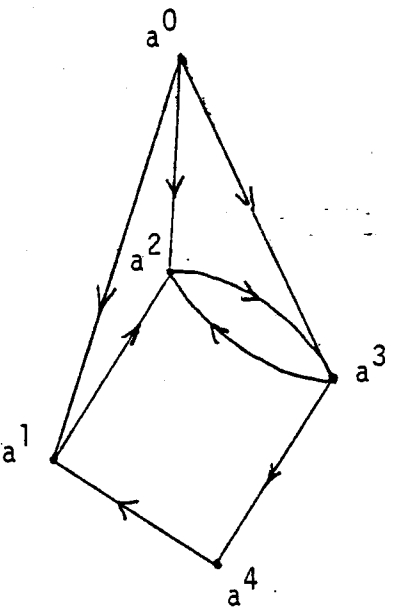
The outranking relation does not pretend to be the exact reflection of all the decision-maker's preferences, and in general it does not lead to a weak-order, and probably not even a partial weak-order, in that intransitivities may occur.

As an illustration let us return to the example of II.1 comparing 5 rules of priority $a^0$, $a^1$, $a^2$, $a^3$, $a^4$ globaly according to :

TABLE 11

COMPARISON ON 2 CRITERIA OF 5 RULES OF PRIORITY

| | vis-à-vis | COMPARISONS |
|---|---|---|
| $a^0$ | $a^1$ | $g_1$ : presumed slightly more flexible<br>$g_2$ : perceptibly equivalent |
| | $a^2$ | $g_1$ : equivalent<br>$g_2$ : 2 to 3 % more economical |
| | $a^3$ | $g_1$ : equivalent<br>$g_2$ : 2 to 3 % more economical |
| | $a^4$ | $g_1$ : presumed slightly less flexible<br>$g_2$ : 4 to 6 % more economical |
| $a^1$ | $a^2$ | $g_1$ : equivalent<br>$g_2$ : 2 to 3 % more economical |
| | $a^3$ | $g_1$ : presumed slightly less flexible<br>$g_2$ : 2 to 3 % more economical |
| | $a^4$ | $g_1$ : certainly less flexible<br>$g_2$ : 4 to 6 % more economical |
| $a^2$ | $a^3$ | $g_1$ : equivalent<br>$g_2$ : perceptibly equivalent |
| | $a^4$ | $g_1$ : presumed slightly less flexible<br>$g_2$ : 2 to 3 % more economical |
| $a^3$ | $a^4$ | $g_1$ : equivalent<br>$g_2$ : 2 to 3 % more economical |




indifference cf. a) above

preference cf. b) above

incomparability cf. c) above

- the precriterion $g_1$ : adaptability to perturbations
- the semi-criterion $g_2$ : direct functioning costs.

Table 11 gives the estimates of each of the 5 rules according to each criterion. If the scientist considers that for the decision-maker, a significant difference in adaptability can not be compensated by a saving inferior to 10 %, he is naturally led to the following outranking relation shown in table 11.

Under such a relation and within the context of problem formulation $\alpha$ , the scientist may advise the decision-maker to choose between $a^0$ and $a^4$ because the subset $N = \{a^0, a^4\}$ is a kernel for the relation that is to say :

- each $a^h \notin N$ is outranked by at least one element of $N$ ;
- elements of $N$ are strongly contrasted as far as they are incomparable.

There are well defined techniques for constructing outranking relations and numerous ways in which they can be used : details can be found in B. ROY (54) and (56). Let us only remark here that :

- the scientist may be more or less insistant that the outranking is accepted. It is for this reason that he frequently introduces several interrelated relations for the same problem (cf. (1), (14), (18), (66), (43) ). It is clear that the richer an outranking relation is the more risky it is, risk of mistaking the decision-maker's real preference and consequently of giving him bad advice. The scientist's conviction to opt for an outranking, may in certain cases be formalised through a degree of credibility in the outranking, which leads to the concept of fuzzy outranking (cf. (56).
- outranking relations are evidently only an intermediary ; they are often incomplete models of the decision-maker's preference but in many real-world problems it will suffice in the framework of decision-aid. In some cases complementary work by the scientist will be required.

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We come finally to the third operational attitude (see table 10). It can be characterized in that it refers to three types of mechanisms, more than to a particular global preferences modelling, which one remains widely implicit. It is the use of these three types of mechanisms (described below) in an appropriate iterative sequence (cf. for example III.2) which leads to the interactive elaboration of one or more compromises.

a) Research mechanism : by this the scientist exploits the data regathered as a result of the previous reaction (cf. c) in order to make headway in the elaboration of compromises. It is concerned with :

- the analysis and comparison of the new data with the old ;
- research (taking into account the results of the analysis of compromise projects and/or of certain of their characteristics (maximum performances, shadow prices, ....).

b) Reinitialisation mechanism : it is this which creates, having taken into account the results of a) , the new conditions under which the next reaction must be performed. It concerns :

- translation, into a language comprehensible by the decision-maker or his representative, of those results or situations (real or fictitious) they have to understand ;
- obtaining the necessary conditions (understanding certain results, reflection on antagonisms between certain criteria, ....) so that the regathered data during the reaction will be as significant as possible.

c) Reaction mechanism : it is this which, within the framework of a previously developed condition (cf. b), leads to regathering information on the local preferences of the decision-maker. It may take extremely different forms :

- discussion as unbiassed as possible ;
- discussion based on a pre-prepared questionnaire ;
- reactions to insertion in a business-game ;
- methodology of agreement ;
- voting procedure ;
- survey ;
- ....

In practice, even if these two latter mechanisms have no reason to be dissociated, it is important in spite of all, I think, to ensure that the function they are destined to fulfil are effectively fulfilled in the course of the interactive process.

This third attitude may, on condition that the appropriate procedures are used, be made operational, even when the decision-maker's preference are not completely formed or susceptible to be progressively influenced as new information or new results appear. Although the scientist traditionally seeks to determine hidden preferences which he postulates the pre-existence and coherence, the reality is often quite different (cf. JACQUET-LAGREZE [33] , TERNY [63], YU [70] or the examples 17, 21, 22 of table 3).

To reveal these preferences, especially within the framework of an intereactive procedure, is not neutral : their emergence is hardly ever conceivable without a certain adaptation because of other actors preferences, a certain reajustment because of impossibilities. For this reason care must be taken that this interactive elaboration of compromises does not become, all things considered, synonymous with compromising.

Be that as it may, we are in the right to question ourselves on the significance of the compromises resulting from this attitude which certain people may judge too pragmatic. In fact an "acceptable procedure" must guarantee that there does not exist any potential action which we can prove, having recourse only to regathered data, to be strictly preferable to any of the final compro-

mises. To verify that this is so may well uncover difficult theoretical problems.

III. 2 - The evolutive target procedure (E.T.P.)

Most of the actually proposed interactive procedures do not tackle the problem in the way described above. They are concerned in a mathematical programming framework, with reference to strongly coherent answers provided by the decision-maker so that the final compromise can appear, under a more or less realistic hypothesis as an optimum. To achieve this, or for other different reasons, they neglect the two latter types of mechanisms and concern themselves with the former. Their practical application because of this is often limited. Nevertheless, they appear as very useful starting point.

By conceiving a new procedure (see table 12) I tried to pay equal attention to each of the three above types of mechanisms.

The word "evolutive" in the denomination E.T.P. not only applies to the modelling of A (cf. I.1.) but also to the "capturing" of global preferences which are assumed capable of evolution (apprenticeship of the decision-maker or his representatives during the process). The idea of the "target", as well as its role in the elaboration of a compromise, was designed in such a way as to be operational using the most general coherent families, therefore not exclusively composed of true-criteria.

These are the most original aspects of E.T.P. By many aspects, this procedure is close to those elaborated by BENAYOUN (cf. STEP and STEM Methods : (10) and ZELENY (cf. Method of Displaced Ideal : Linear Multiobjective Programming. Springer Verlag, 1974) ; it also interferes with YU's ideas on persuasion and negociation (cf. 70).

Each box of table 12 concerns a part of one (but only one) of the mechanisms a) b) c) which occurs in an iterative sequence as it is shown.

At each iteration (pass through box $\alpha$) the set A is redefined to take into consideration :

- the extension of A authorized by the data regathered, according to the case, in box $\delta$ or in box $\gamma'$ (relaxing certain constraints, integrating supplementary variables, introduction of structural transformations, ....) ;

- the priority coordinate $i^o$ when $\alpha$ follows $\gamma'$ ; this being done by adding the constraint

$$g_{i^o}(a) \geqq g^o_{i^o} + \Delta_{i^o} ,$$

$\Delta_{i^o}$ denotes the minimum improvement which remains significant (it may be associated with threshold estimates by the scientist).

By definition, the target associated with A , is a point $\underline{g}^*$ of the criteria space having as coordinates :

$$g_i^* = \underset{a \in A}{\text{Max}}\ g_i(a) \qquad i = 1, \ldots, n$$

i.e. the best value that a potential action may give to the criterion i (concerning the $i^{th}$ coordinate).

Generally, this target is out of reach ; it is this the scientist has to make understood by the decision-maker or his representatives in $\beta$ : for ex. by using as a basis those actions which determine the values $g_i^*$. Then the decision-maker or his representatives, being familiar with fixing this target, it is again used as a "revealer" of global preferences in this region of the criteria space. The mechanism in box $\beta$ has to be conceived in view of the switching between $\gamma$ or $\delta$ so as to prepare the actors to the corresponding reactions.

When none of the $g_i^*$ appear no longer capable of being lowered, or when, first of all, a little flexibility or quite simply imagination relative to A seems necessary, we seek (cf. $\delta$ ) to obtain, from the decision-maker or his representatives, all the interesting possibilities taking into account the various aspects (constraints, value of parameters, structural features, ...) which seem responsible for the gap between $\underline{g}^*$ and the $\underline{g}(a)$'s for $a \in A$.

The box $\delta$ and the path leading either to the box $\alpha$ or to a cul-de-sac are self explanatory.

As for the box $\gamma$ it deals with obtaining additionnal information on preferences in a small region surrounding the target. The objective is to specify the coordinates $\tilde{g}_i \leqq g_i^*$ of a point $\tilde{g}$ significantly different from $g^*$ which, if accessible, would appear to be a "best" compromise in the neighbourhood of $g^*$. The reaction may be provoked for example, by a set of "minimum losses" :

$p_i = g_i^* - \tilde{g}_i \quad i = 1, \ldots, n$ which must be adjusted

- render the $p_i \neq 0$ approximately equivalent whilst remaining significant of a real difference (account taken of the different thresholds) ;
- conserve $p_i = 0$ each time that a significant difference creates a loss heavier than that realised by the $p_i \neq 0$.

The couple $\Delta = (g^*, \tilde{g})$ thus defined serves in $\alpha'$ to point the investigation towards a potential action as "near" as possible to the target. This is done by means of a proximity indicator $d_\Delta [g^*, g(a)]$, numerical characteristics of which depend on $\Delta$ so as to reflect the relative influence of the various criteria in the region considered. For the remainder it may be derived, for example, from a Lp norm (a weighted min-max norm such as the one used in BENAYOUN [10] seems very appropriate).

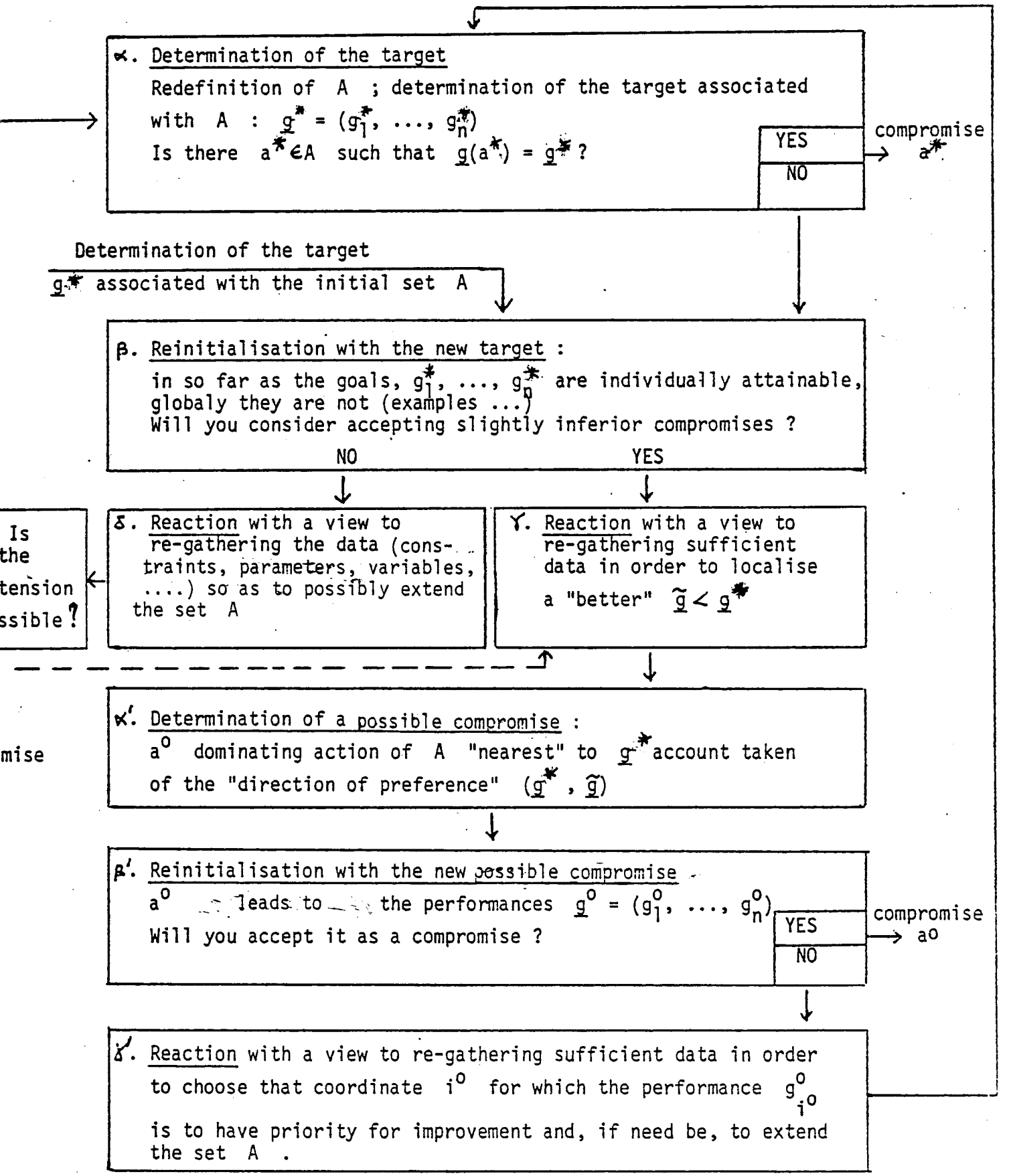
Box $\alpha'$ finally leads to select the potential action (or actions) minimizing $d_\Delta [g^*, g(a)]$ for the present definition of A.

The action $a^0$ thus selected appears, at this stage as the best compromise possible to obtain. This must be explained to the decision-maker or his representatives in $\beta'$. It may be interesting for that, to compare $g(a^0)$ with $\tilde{g}$ and to examine individual performances on each criterion.

When, because of this comparison or of the absolute performances $a^0$ is refused, addional or revised information are required to progress. They concern (cf. box $\gamma'$) :

- the choice of the coordinate $i^0$ which appears as the most responsible for the refusal and to which priority must be given in order to improve the performance ;
- the definition of A which can be reconsidered mainly in order to try to reduce the gap between $g^0$ and $\tilde{g}$ ; moreover in certain cases, it is desirable to test the opportunity of conserving the constraints which the coordinates selected in $\gamma'$, during the preceding iterations, have led to the addition of (cf. $\alpha$).

TABLE 12 : EVOLUTIVE TARGET PROCEDURE



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