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THE SEARCH FOR THE BEST ROUTE : AN APPLICATION OF A FORMAL METHOD USING MULTIPLE CRITERIA (*)

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RECHERCHE D'UN MEILLEUR CHEMIN : APPLICATION A UN PROBLEME MULTICRITERE

RESUME

Dans un cahier récent, Marchet et Siskos (1979) ont utilisé un ensemble de méthodes multicritères pour comparer les différents itinéraires entre deux villes et identifier le meilleur de ces itinéraires. La région que traverse l'itinéraire est composée d'une série de cinquante huis zones. On suppose que chaque zone est homogène en ce qui concerne l'impact de l'itinéraire sur quatre critères.

Le but de ce cahier est de montrer comment on peut utiliser une autre méthode de création et de comparaison d'itinéraires pour résoudre le problème du choix d'un itinéraire. La méthode utilise une variante de l'indicateur de concordance pour construire une matrice de dissimilarité. Cette matrice est analysée à l'aide d'un algorithme d'analyse de proximités et classe les cinquante huit zones par rapport à une zone "idéale" supposée existante. On utilise un algorithme de recherche du plus court chemin pour trouver un ensemble de zones qui donne un chemin non interrompu au coût le plus bas. Le coût est déterminé par les distances des zones par rapport à la zone "idéale".

Dix expériences ont été faites avec cinq pondérations différentes (quatre sont extraites du cahier de Marchet et Siskos, la cinquième donne un poids égal à tous les critères) et deux méthodes différentes pour définir la zone "idéale". La méthode tient compte de la taille de chacune des cinquante huit zones.

Chacune des dix expériences donne le même itinéraire. La solution ne paraît pas être sensible aux changements de poids pour les quatre critères. On a comparé deux des itinéraires classés par Marchet et Siskos (1979) parmi les meilleurs avec l'itinéraire le meilleur selon notre approche.

THE SEARCH FOR THE BEST ROUTE : AN APPLICATION OF A FORMAL METHOD USING MULTIPLE CRITERIA

ABSTRACT

In a recent paper by Marchet and Siskos (1979) a set of multicriteria procedures was used to compare alternate routes to link two towns, and to identify the best one. The study area through which the route passes is composed of a series of fifty-eight zones. Each zone is assumed to be homogeneous with respect to the impact of a route on four criteria.

The purpose of this paper is to show how an alternate method for generating and comparing routes can be used to tackle the route selection problem. The method uses a modification of the concordance index to derive a dissimilarity matrix. This matrix is analyzed using a multidimension scaling algorithm to give a classification of the fifty-eight zones with respect to a hypothetical "best" zone. A shortest path algorithm is used to find the combination of a subset of zones to produce a continuous route which has the least cost. The cost is determined by the distances of the zones from the hypothetical "best" zone.

A series of ten experiments is conducted using five weighting schemes (four from the paper by Marchet and Siskos, and one scheme in which all criteria are equally important) and two alternate methods for defining the hypothetical "best". The method considers the size of each of the fifty-eight cells.

The same route is identified as the best for each of the ten experiments. The solution appears to be insensitive to changes in weighting for the four criteria. A comparison is presented of two of the routes identified by Marchet and Siskos (1979) as among the best with the route identified in this paper.

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INTRODUCTION

In a recent paper by Marchet and Siskos (1979) the following problem was posed. Find the best route between two towns A and B, passing through a series of zones (troncons) as defined on Figure 1. For the purposes of the exercise it was assumed that each zone was homogeneous in terms of the impact of the route on a set of criteria. Four criteria were used in the study, details of each criterion are given in the original paper. An impact matrix for the 58 zones and the 4 criteria is shown on Table 1. The scores shown on Table 1 range from 1 to 6. A value of 1 indicates high damage will occur if a route traverses the zone, a value of 6 indicates that damage is minimum. In this matrix no account is taken of the size of each zone or the length of a route passing through a zone.

Three types of multicriteria methods were used by Marchet and Siskos (1979) to evaluate alternate routes linking towns A and B. The first method is an interactive Additive Utility Model -AU- (two versions of this were used, AU_1 and AU_2), the second refers to the method of Keeney and Raiffa (1976) and the third uses a recent version of ELECTRE, a method originally developed by Roy (1977).

On Figure 2 a selection of alternate routes are shown from the set of thirteen best routes identified by AU_1. The results on Table 2 indicate the ranking of these routes on methods AU_2, Keeney and Raiffa, and ELECTRE III. It appears that routes 10 and 11 are consistently among the top three routes for each method. These routes will be compared to the best route identified using the method presented in this paper.

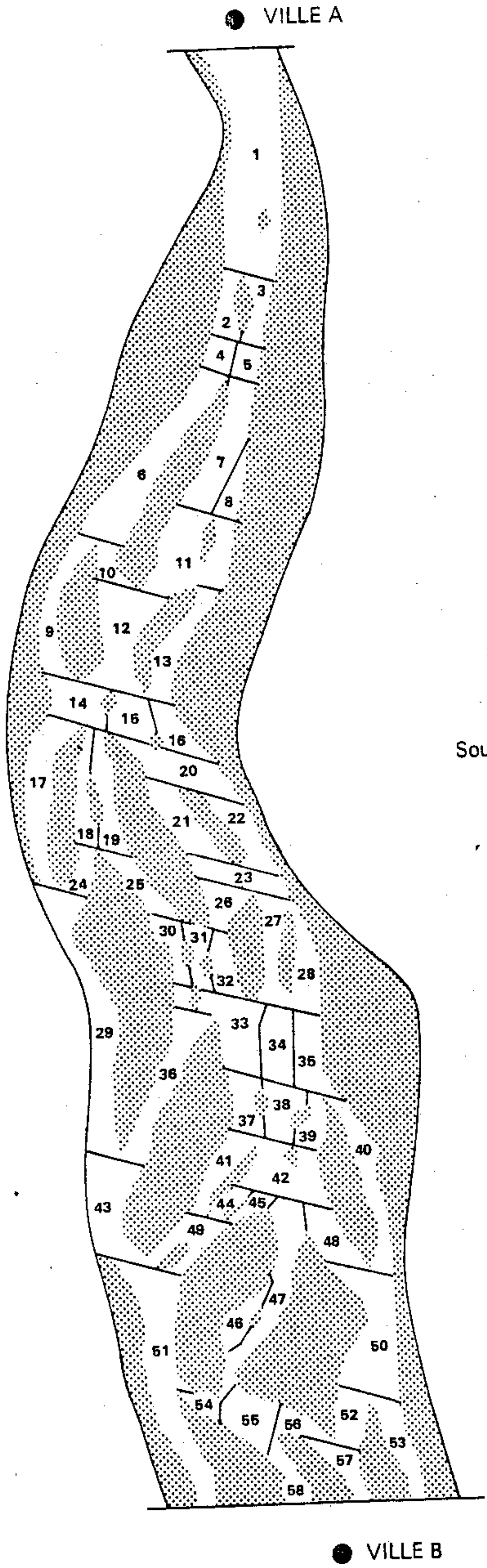


Figure 1
STUDY AREA

Ilot de contrainte

3 Numero du troncon

Source: Marchet and Siskos, 1979

Multicriteria evaluations TABLE 1

Evaluation multicritère / N° du tronçon	A CRITERE HUMAIN	B CRITERE PROTECTION	C CRITERE PRODUCTION	D CRITERE PAYSAGE
1	5	6	1	2
2	4	6	1	3
3	4	4.5	1	5
4	6	2	2	3
5	6	6	2	3
6	4	2	3	3
7	4.5	3	2	2
8	4.5	1	2	2
9	3	2	2	3
10	4	2	1	2
11	5	4	2	3
12	4	3	2	3
13	3.5	1.5	2	1
14	4	5	1	4
15	4	3	1	3
16	4	3	1	1
17	4	3	2	2
18	4	4	1	2
19	4	4	3	5
20	4.5	5	2	1
21	5	4	2.5	2
22	6	6	2	5
23	6	6	3	2
24	4	1	3	5
25	4	2	2.5	4
26	4	6	2.5	5
27	4.5	4.5	4	1.5
28	4.5	6	3.5	2
29	1	2	2	1.5
30	3	4.5	4	3
31	3	4.5	4	2
32	3	4.5	4	2
33	4	4	5	3
34	4	6	1	3
35	4	6	3	3
36	1.5	5	4.5	2
37	4.5	5	5	3
38	4.5	2.5	5	3
39	4.5	3	5	3
40	4.5	3.5	3	1
41	5	4	2.5	4
42	5	4.5	4	2
43	1	3	2	4
44	4	6	4	3.5
45	4	4	4	3.5
46	3	2.5	4.5	2
47	3	2.5	2	2
48	4	2	4	1
49	3	6	3.5	4
50	2.5	4	2	2
51	3	4	5	2
52	4	3	5	3
53	2	2	5	1
54	4	5	5	2
55	3	3.5	5	3
56	3.5	2	5	1
57	4	4.5	5	1
58	4	3	3.5	2

Echelle : 1 (secteur très sensible, traversée très dommageable) à 6 (secteur non sensible)

(Source: Marchet and Siskos, 1979)

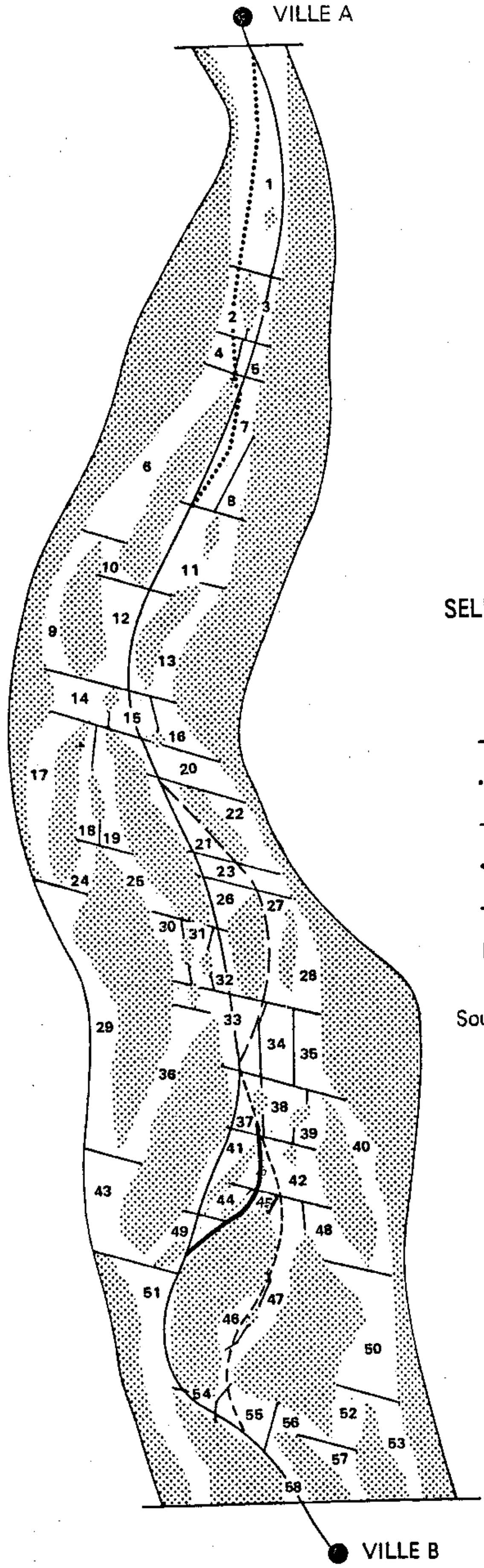


Figure 2
SELECTED ALTERNATE ROUTES

Ilot de contrainte

3 Numero du troncon

Trace numero 1

Trace numero 2

Trace numero 3

Trace numero 4

Trace numero 5

Note: Segments of routes 2, 3, 4 and 5 follow route 1.

Source: Marchet and Siskos, 1979

TABLE 2

CLASSIFICATION OF 13 BEST ROUTES USING UTA I

Résultats / N° de l'action	(AU I) UTA I Rang du classement à partir du surclassement flou	(AU II) UTA II Rang du classement par l'utilité obtenue	Rang du classement par la méthode de Keeney-Raïffa	Rang du classement par ELECTRE III (pondération ***)
10	1	2	2	14
97	2	3	14	25
11	3	1	1	1
178	4	18	12	15
173	4	20	12	15
86	6	7	4	20
174	7	15	6	4
179	7	15	6	4
1	9	5	5	17
3	10	8	17	27
87	11	4	3	12
4	12	9	11	25
2	13	13	20	42

(Source: Marchet and Siskos, 1979)

The basic purpose of this paper is to show how an alternate method for generating and comparing routes can be used to tackle this route selection problem. For the purposes of this problem the data in the matrix shown on Table 1 and the map on Figure 1 will be used.

THE METHOD

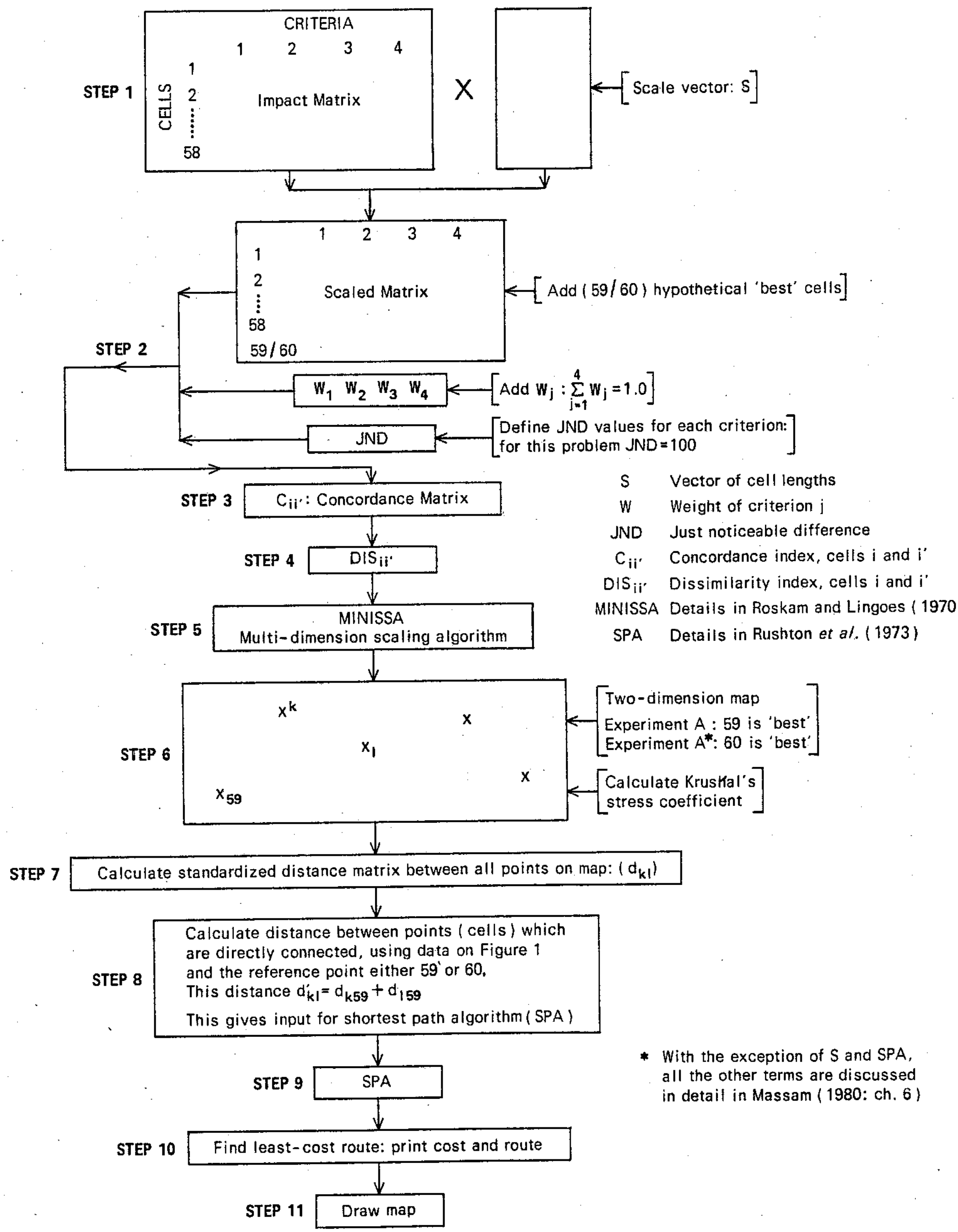
The method is a modification of the one developed by Massam (1980) for comparing alternate plans. It incorporates the concordance index of Roy (1968) with a scaling procedure and the use of hypothetical "best" zones. Full details of the general method are given in Massam (1980) and will not be repeated here, we will only provide details of the modifications necessary to treat this particular problem. Thus far the method has been used to clarify a set of alternate plans or routes and to allow a best alternative to be identified. In this problem modifications have been made in order to generate a best route from a combination of zones (troncons), after all the zones have been classified and compared to a hypothetical best zone. The purpose is to find the combination of zones which will produce a least cost path from A to B.

The procedure used to tackle this route selection problem is summarized on the schematic flow diagram shown on Figure 3.

APPLICATION OF METHOD

A series of ten experiments were conducted. The details of each

Figure 3
SCHEMATIC FLOW DIAGRAM



are shown on Table 3. For the set A, of five experiments the "best" cell was defined as having the scores which tied with the best values from the set of 58 existing cell values shown in Table 1. The second set, A^*, of five experiments used a "best" cell as defined by the second-highest values from Table 1. Details regarding the rationale for selecting a tied "best", and a second-highest value are provided in Massam (1980: chapter 6). In some cases the use of a tied "best" results in a degenerate solution where all the 58 cells are so different from this hypothetical best that it is not possible to distinguish among the set of 58.

As the impact scores shown on Table 1 should be treated as ordinal values we have restricted the J.N.D. values for each criterion to 100. If the scores are interval values then it is possible to introduce values for J.N.D.$\leqslant$100. A set of experiments using J.N.D.=95 and J.N.D.=90 were run. The final selection of the least-cost route using J.N.D.=100 is the same as that selected when J.N.D.=95 or J.N.D.=90.

The two-dimension maps for each experiment corresponded closely with the values in the dissimilarity matrices. The goodness of fit was measured by Kruskal's stress coefficient (S). In each case the value was less than .130. If the value for S is zero, the fit is perfect. It should be noted that this coefficient can not be used in an inferential way, it has no statistical significance.

The route that was selected as representing the least-cost path from cells 1 to 58, for each experiment is shown on Figure 4. Least-cost for this procedure refers to the combination of distance values

DETAILS OF EXPERIMENTS

TABLE 3

SCHEME		J.N.D.	CRITERIA WEIGHTS* A	B	C	D	BEST
A	1	100 :	.2500	.2500	.2500	.2500	59
	2	100 :	.4150	.3330	.1110	.1410	59
	3	100 :	.2730	.5430	0	.1820	59
	4	100 :	.1430	.5150	.0600	.2820	59
	5	100 :	.2820	.3180	.1850	.2150	59
A*	1	100 :	.2500	.2500	.2500	.2500	60
	2	100 :	.4150	.3330	.1110	.1410	60
	3	100 :	.2730	.5430	0	.1820	60
	4	100 :	.1430	.5150	.0600	.2820	60
	5	100 :	.2820	.3180	.1850	.2150	60

Weights for schemes 2, 3, 4 and 5 were those used in Marchet and Siskos (1979).

Weights for scheme 1 are provided for references purposes, each criterion has the same weight.

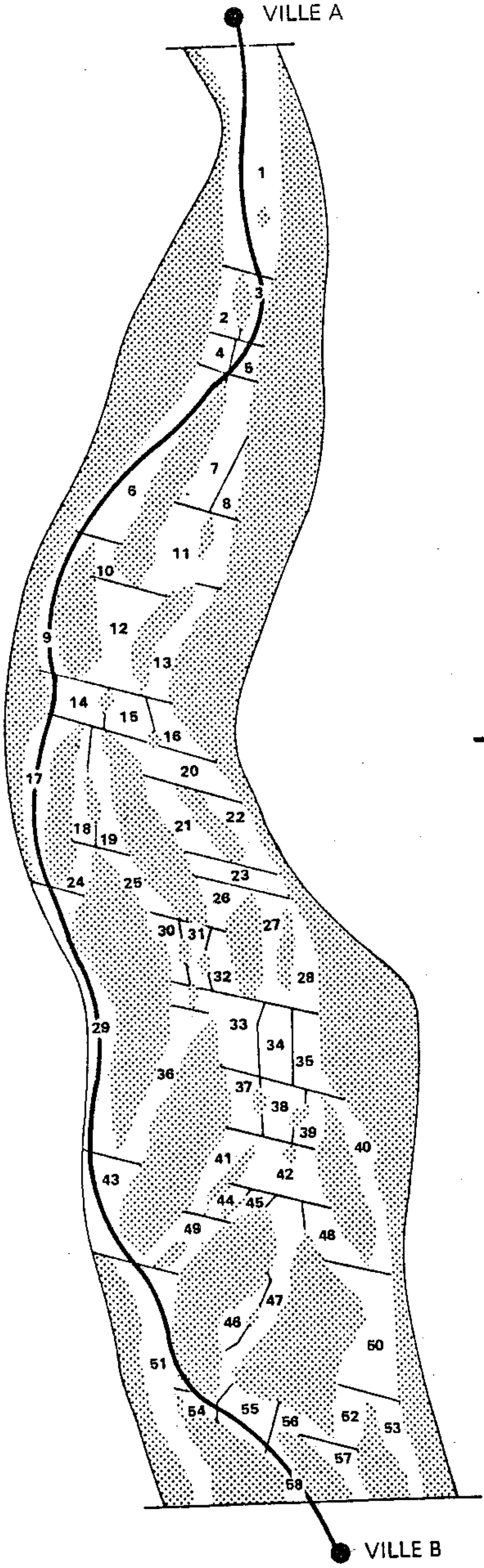


Figure 4
LEAST-COST ROUTE

Ilot de contrainte

3 Numero du troncon

Least-cost route

derived in step 8 on Figure 3, under the constraints of the connectivity matrix (8A) on Figure 3. The costs associated with the route for each experiment are shown on Table 4. The units are standard distance units derived in step 7 of the algorithm.

The costs associated with routes 10 and 11, shown on Figure 2 and Table 3 are consistently about 10 per cent higher than the values given on Table 4 for each experiment.

COMMENTS

The method discussed in this paper has traditionally been used to compare a small set of given alternate plans, and to classify the plans with respect to defined benchmark plans. These latter could be either an ideal "best", a "worst", or a combination of "best" and "worst". The modifications presented here allow the method to be applied to a spatial problem of generating the alternate route which combines together the zones which are as similar as possible to the hypothetical best zone.

With respect to the empirical problem addressed we find that under a wide variety of weighting schemes for the criteria, one route is consistently chosen as superior to any other. Associated costs in standardized distance units for this route, for each of the ten experiments, have been presented. Using the same basic set of distances the associated costs for two routes suggested by Marchet and Siskos (1979) as among the best have also been calculated.

COSTS FOR EACH EXPERIMENT

TABLE 4

SCHEME		J.N.D.	COSTS FROM NODE 1 to 58
A	1	100	2077
	2	100	2481
	3	100	4069
	4	100	2679
	5	100	2507
A^*	1	100	2631
	2	100	2704
	3	100	2725
	4	100	2691
	5	100	2738

One of the major differences between the method presented in this paper and those discussed by Marchet and Siskos (1979) is that the values in the impact matrix shown on Table 1 have been weighted according to the length of each cell.

Further work is needed to make formal comparisons of multi-criteria methods in order to identify the most appropriate one(s), not only from the point of view of the internal logic of the method, but also in terms of data requirements and comprehensibility to potential user and flexibility to undertake sensitivity analyses and to be programmed in a mode which will allow a decision-maker to work interactively with the procedure.

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