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Abstract. Mass customization, a major trend in modern economy, relies on components assembly in discrete production, while in continuous production, it consists in a sequence of batches of different products. This limits achievable diversity and multiplies transportation issues. This situation is addressed by the fertilizer industry which, to enable sustainable agriculture, must deliver fertilizers tailored to soil and crops characteristics. We are now proposing a new approach to handle this situation, with a solution called *reverse blending*, which involves delayed differentiation performed near end-users. This consists in defining the components for a very limited number of inputs whose blend exactly matches the characteristic of the required fertilizers. In this solution, the problem is modelled by a quadratic program used to define input optimal composition, respect fertilizer components constraints and cater to any type of demand. Reverse blending may have a major impact on supply chain organization. A short case study of this new approach is provided.

Keywords: Reverse Blending, delayed differentiation, mass customization, fertilizer, quadratic programming.

1. Introduction

With constantly growing global population, feeding humanity is a crucial challenge that needs to be addressed urgently. Every country must rise to this challenge and encourage farmers to adopt sustainable agriculture by both increasing agricultural yield and preserving soil fertility. Of all the short-term factors that can rapidly increase agricultural production, using chemical fertilizers is the most efficient in achieving the highest yields and delivering the best return on investment (Pratt, 1965). However, to preserve future soil fertility, the fertilizers must be used selectively by using specifically adapted formulas based on the actual crop and parcel of arable land, to match the precise nutrient needs.

OCP, one of the world phosphate market leaders, is aware of this challenge and committed to developing proactive commercial strategies based on knowing customers' needs so as to deliver customized fertilizer formulas. In practice, a customized fertilizer is defined by a formula whose nutrients and portions differ according to the pedological characteristics and the desired crops. For a fertilizer producer, this constraint amounts to developing complex fertilizer compounds, manufactured through raw material chemical reactions upstream of granulation. This involves managing a wide variety of distribution flows. An alternative is to produce fertilizers by blending simple or compound fertilizers that are already granulated or compacted, in facilities that are remote from original production sites. This alternative solution simplifies logistical problems somewhat but requires substantial investments, without actually meeting final demand. The objective therefore, is to find a good compromise between producing a large variety of fertilizers and reducing logistics costs.

To this end, rather than blending ready-made fertilizers of known nutritional composition, this paper proposes a new approach based on the chemical identification of a limited of products (inputs). These are not directly usable fertilizers, but their blending enables production of a wide variety of custom-made fertilizers (outputs). The nutritional composition of these inputs, therefore is to be determined. This new approach, which we call *Reverse Blending*, aims, through a parameterized quadratic program, to define the optimal specifications of a number N , which we seek to keep down to a minimum, of primary inputs whose blending enables production of the required diversity of outputs (fertilizer formulas). According to this drastically new approach, we will be able to rely on small capacity blending units located close to actual end-use areas and fed by massive flows. This approach forms part of delayed differentiation because we aim to perform the blending as close as possible to local market. The resulting flow consolidation simplifies both the production and transportation management.

This paper differs from the traditional structure in that it does not begin with a literature review, since our proposed approach is entirely new and therefore is not accounted for in the literature. In section 2, we explain the need for fertilizer customization to achieve reasoned and sustainable agriculture and show that the nutrient needs are highly diversified. In section 3, we describe our reverse blending model before illustrating it with a simplified real example in section 4.

2. Customized fertilizers: a prerequisite of reasoned agriculture

In order to feed a global population of 9.1 billion people by 2050, food production will have to increase by about 70% by 2050 from its 2005 level (FAO, 2009). Rising to this challenge demands rational fertilization to provide plants with needed nutrients in the most appropriate way. However, as a result of low fertilizer utilization and high nutrient extraction rates, soils are often unable to provide these nutrients without recourse to supplementary ingredients (Fixen *et al.*, 2015). These ingredients must contain the appropriate proportions of a number of nutrients, the most important of which are Nitrogen (N), Phosphorus (P) and Potassium (K).

- Nitrogen plays a key role in plant growth and crop yield (Hirel *et al.*, 2007; Krapp *et al.*, 2014; Ruffel *et al.*, 2014; Vidal *et al.*, 2014; Wang *et al.*, 2012). Its availability and internal concentration affect the distribution of biomass between roots and shoots (Bown *et al.*, 2010) as well as metabolism, physiology and plant development (O'Brien *et al.*, 2016).

- Phosphorus is an essential nutrient for root development and nutrient availability (Jin *et al.*, 2005). It is essential for cell division, reproduction and metabolism of plants and allows to store energy and regulate its use (Epstein and Bloom, 2004).

- Potassium plays a major role in regulating the opening and closing of stomata which is necessary for photosynthesis, the transport of water and nutrients and the cooling of plants (Kalavati and Modi, 2012).

Recommending fertilizer blends of these three elements, requires a good knowledge of the different aspects of fertilization including objective yield, crop nutrient need and nutrient supply by the soil (Cottenie, 1978). To quantify this supply, farmers must perform soil tests to manage nutrients and avoid long-term nutritional and health problems (Watson *et al.*, 2007). These tests are required at least every three years (Warncke *et al.*, 2000) as soil properties vary over time in response to changes in land management practices and inherent soil characteristics (Jenny, 1941). As a matter of fact, soils are undergo multiple processes: biological, physical, chemical and human. Thus, not a single hectare of cultivated soil is completely homogenous from a pedological standpoint. This results in a very large variety of fertilizer needs, hence the need to produce customized fertilizer formulas.

Such customization can encourage farmer loyalty and therefore increase market share, provided it is affordable, which is practically impossible where multiple fertilizers are to be produced and transported. OCP is already using the principle of delayed differentiation by seeking to satisfy needs through blending of ready-made fertilizers in facilities that are relatively close to its customers. This approach, already discussed by several authors (Ashayeri, 1994, Bassam Aldeseit1, 2014, Lima, Severino *et al.*, 2011, Loh, Cheong *et al.*, 2015), does not meet precisely all requests because it often delivers either insufficient or excessive levels of one or more nutrients. The "Reverse Blending" approach addresses this problem of effective and efficient delayed differentiation in a radically different way. It aims to produce a very large number of fertilizers (outputs) by combining a very limited number of inputs that are not ready-made ones fertilizers but inputs the optimal composition of which we are attempting to define.

3. The Reverse Blending Model

In the traditional formulation of a blending problem, a set of N possible inputs ($i=1..N$) is available. Any input i is characterized by a set of C components ($c=1..C$) (N, P, K...). The relative weight of component c in input i is α_{ci} and they satisfy relation (1) which is a constraint that parameters must comply with.

$$\sum_c \alpha_{ci} = 1, \forall i \quad (1)$$

Blending aims at defining the optimal mixture of selected inputs taken in quantities x_{ij} (order variables) to obtain quantity D_j , requested quantity of output j ($j=1..J$), which complies with constraint (2).

$$D_j = \sum_i x_{ij}, \forall j \quad (2)$$

The relative weight of component c in output j obtained by blending $\beta_{cj} = \sum_i \alpha_{ci} \cdot x_{ij} / D_j$ may have to belong to a range of values flowing from constraints (3).

$$\beta_{cj}^{\text{Min}} \leq \sum_i \alpha_{ci} \cdot x_{ij} / D_j \leq \beta_{cj}^{\text{Max}} \quad (3)$$

Input i has an acquisition cost γ_i and the problem of traditional blending is to identify the blends that minimize acquisition cost (relation (4)).

$$\text{Max}_{i,j} \left[\sum_i \gamma_i \sum_j x_{ij} \right]. \quad (4)$$

This problem, defined by relations (2) to (4), is linear. Since the problem variables are continuous, there is an infinite number of possible solutions or none, if the problem is insoluble.

The reverse blending problem is an extension of traditional blending where one starts from output demand to define the characteristics α_{ci} of N inputs (justifying the name of reverse blending given to it). In this approach, characteristics α_{ci} become decision variables, relation (1) becomes a constraint and, due to relation (3), the problem becomes quadratic. As we seek to keep the number N of inputs down to a minimum, reverse blending becomes a **parametric quadratic problem** where one looks for the solution for the lowest possible value of N , starting with 3 and adding to it until a solution is found. Among the possible solutions with the lowest N , the best ones are those where the weight of a very limited number of inputs represent the highest percentage of total inputs needed for output production. To this end, criterion (4) is replaced by (5) which maximizes use of input $j=1$.

$$\text{Max} \left[\sum_j x_{1j} \right]. \quad (5)$$

4. Case study

We set forth below a simplified case study based on actual data. We begin by defining the demand to be satisfied (§4.1) before presenting the results obtained after optimizing our reverse blending model (§4.2).

4.1 Characteristics of the custom-made fertilizer demand

To help Moroccan farmers identify their exact needs in N, P and K, OCP, in collaboration with the Moroccan Ministry of Agriculture and Maritime Fisheries, has developed a solution "*Fertimap*". This tool was designed by leading Moroccan agronomy experts. This software recommends the appropriate quantities, in kg / hectare, to be applied to a pre-defined parcel of land, according to the relevant soil fertility indicators, the desired crop and the yield objective. We used this tool to deduce the calculation formulas of nutrient needs so that we could use them for the determination of custom fertilizers. Rather than aiming to serve actual farmers and achieve specific yields, our purpose is to use these formulas to identify nutrient needs for optimal yields of crops whose ecological requirements are compatible with the pre-selected soil area. These formulas were deduced from linear regression using reverse engineering of the *Fertimap* approach. This regression was based on a sample of about 30 records taken from the *Fertimap* platform for each of the three nutrients. This allowed, for a particular crop, to extrapolate the functional relationships linking the explained variables (N, P and K needs) to the explanatory variables (soil fertility indicators and target yield). The optimal doses of these nutrients are determined independently since in addition to the target yield the need for N, P and K depends on organic material, available phosphorus (P_2O_5) and exchangeable potassium (K_2O). Figure 1 shows the N and K requirement curves for three different yields relevant for wheat.

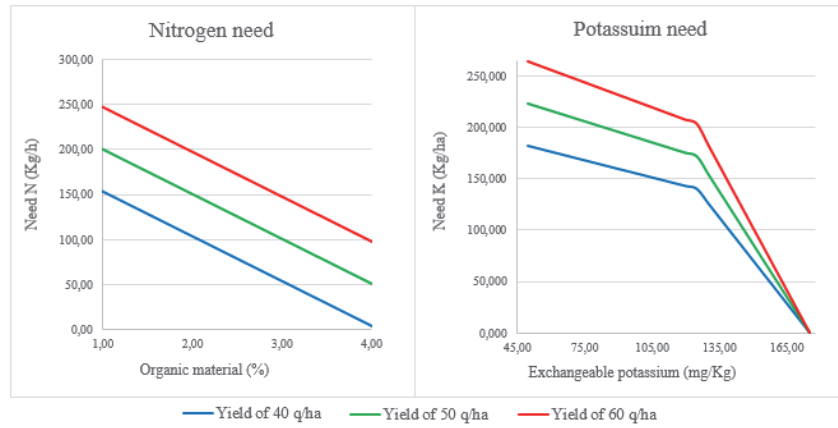


Figure 1: N and K requirement behavior versus soil indicators for three different yields

We document the customization needs calculated by these linear formulas for five different crops (olive, almond, lentil, wheat and orange citrus). The selected geographical areas are those whose soil properties are compatible with the ecological requirements for these crops. Table 1 represents the $J=40$ fertilizer formulas inferred from *Fertimap* quantitative recommendations. The customized fertilizer formulas being the outputs j of the model developed in (§3).

Output	Province	Crop	N	P	K	Filler	Demand (tons)	Output	Province	Crop	N	P	K	Filler	Demand (tons)
$j=1$	El Hajeb	Wheat	56.00%	11.00%	0.00%	33.00%	2123	$j=21$	K.sraghna	Orange	55.54%	11.46%	0.00%	33.00%	33
$j=2$	Meknès	Wheat	67.00%	0.00%	0.00%	33.00%	1443	$j=22$	Berkane	Orange	47.07%	19.93%	0.00%	33.00%	1626
$j=3$	S.Bennour	Wheat	54.35%	13.00%	0.00%	33.00%	12421	$j=23$	Khénifra	Almond	37.47%	29.53%	0.00%	33.00%	34
$j=4$	Khémisset	Lentil	33.00%	34.00%	0.00%	33.00%	637	$j=24$	Khémisset	Almond	35.88%	31.12%	0.00%	33.00%	188
$j=5$	Sidi Kacem	Lentil	18.00%	49.00%	0.00%	33.00%	9	$j=25$	Meknès	Almond	44.02%	22.98%	0.00%	33.00%	746
$j=6$	Sidi Slimane	Lentil	20.00%	47.00%	0.00%	33.00%	73	$j=26$	Taza	Olive	32.95%	20.57%	13.48%	33.00%	26397
$j=7$	Larache	Orange	34.00%	11.00%	22.00%	33.00%	244	$j=27$	Taounat	Olive	45.92%	21.08%	0.00%	33.00%	36966
$j=8$	Fes	Wheat	34.00%	13.00%	20.00%	33.00%	3669	$j=28$	Meknès	Olive	59.14%	7.86%	0.00%	33.00%	7350
$j=9$	El Hajeb	Almond	34.00%	33.00%	0.00%	33.00%	1092	$j=29$	Khémisset	Olive	26.05%	11.40%	29.55%	33.00%	8480
$j=10$	Ifranc	Almond	54.00%	13.00%	0.00%	33.00%	9	$j=30$	F. Ben salah	Olive	43.63%	23.37%	0.00%	33.00%	1907
$j=11$	M.Yacoub	Almond	41.00%	26.00%	0.00%	33.00%	115	$j=31$	Béni Mellal	Wheat	37.00%	20.00%	10.00%	33.00%	500
$j=12$	Béni Mellal	Olive	55.00%	12.00%	0.00%	33.00%	1981	$j=32$	Berrechid	Wheat	37.00%	30.00%	0.00%	33.00%	670
$j=13$	Benslimane	Wheat	38.00%	29.00%	0.00%	33.00%	6791	$j=33$	El Hajeb	Lentil	30.00%	21.00%	16.00%	33.00%	230
$j=14$	Taounat	Wheat	53.00%	14.00%	0.00%	33.00%	13689	$j=34$	Berrechid	Olive	49.00%	18.00%	0.00%	33.00%	1560
$j=15$	Meknès	Lentil	48.00%	19.00%	0.00%	33.00%	48	$j=35$	Benslimane	Lentil	17.00%	22.00%	28.00%	33.00%	1000
$j=16$	Berrechid	Lentil	37.00%	30.00%	0.00%	33.00%	250	$j=36$	Taourirt	Orange	58.00%	9.00%	0.00%	33.00%	286
$j=17$	Settat	Lentil	0.00%	67.00%	0.00%	33.00%	1120	$j=37$	Taourirt	Olive	26.00%	41.00%	0.00%	33.00%	2156
$j=18$	F.Ben salah	Orange	23.00%	44.00%	0.00%	33.00%	804	$j=38$	Figuig	Almond	31.00%	0.00%	36.00%	33.00%	3021
$j=19$	Fes	Orange	50.00%	17.00%	0.00%	33.00%	226	$j=39$	Nador	Orange	29.00%	19.00%	19.00%	33.00%	800
$j=20$	Béni Mellal	Orange	44.77%	22.23%	0.00%	33.00%	1378	$j=40$	Sale	Almond	16.00%	18.00%	33.00%	33.00%	420

Table 1: 40 custom-made fertilizer formulas and demand types

Fertilizers are sold in the form of fertilizer formulas for which the percentage of N, P and K is known. However, besides these nutrients, a fertilizer has an additional component that has no impact on yield, called the filler. The filler is used for chemical (stabilization of granule composition) and practical reasons (excess fertilizer concentration would burn the soil). In our example, when converting recommended quantities (Kg/ha) into percentages, we opted for 33% filler in fertilizer composition.

The optimal formulas are characterized by a tolerance resulting in ranges of values for these percentages. We do not know this tolerance in advance, but in our model it is translated by a parameterized variable and we set it at $\pm 1\%$ in the following calculations. Table 2 shows the target demand structure.

Globally, Moroccan soils are rich in potassium which is why recommended K levels are almost always of zero (see table 1). In fact, there is no level at which potassium becomes toxic to plants. Nevertheless, when plants get too much potassium, the absorption of some other nutrients (Nitrogen, Magnesium and Manganese) is inhibited. Therefore, even though the tolerance was fixed at 1%, if the percentage of K in the fertilizer formula is zero, it is best to keep it that way, as a first step.

		Component											Component								
		e=1			e=2			e=3					e=1			e=2			e=3		
		Minimum structure	Exact structure	Maximum structure	Minimum structure	Exact structure	Maximum structure	Minimum structure	Exact structure	Maximum structure			Minimum structure	Exact structure	Maximum structure	Minimum structure	Exact structure	Maximum structure	Minimum structure	Exact structure	Maximum structure
		(≥)	(=)	(≤)	(≥)	(=)	(≤)	(≥)	(=)	(≤)			(≥)	(=)	(≤)	(≥)	(=)	(≤)	(≥)	(=)	(≤)
Output	j=1	55,00%		57,00%	10,00%		12,00%		0,00%		Output	j=21	54,54%		56,54%	10,46%		12,46%		0%	
	j=2	66,00%		68,00%	0,00%		1,00%		0,00%			j=22	46,07%		48,07%	18,93%		20,93%		0%	
	j=3	53,35%		55,35%	12,00%		14,00%		0,00%			j=23	36,47%		38,47%	28,53%		30,53%		0%	
	j=4	32,00%		34,00%	33,00%		35,00%		0,00%			j=24	34,88%		36,88%	30,12%		32,12%		0%	
	j=5	17,00%		19,00%	68,00%		50,00%		0,00%			j=25	43,02%		45,02%	21,98%		23,98%		0%	
	j=6	19,00%		21,00%	46,00%		48,00%		0,00%			j=26	31,95%		33,95%	19,57%		21,57%	12,48%		14,48%
	j=7	33,00%		35,00%	10,00%		12,00%	21,00%		23,00%		j=27	44,92%		46,92%	20,08%		22,08%		0%	
	j=8	33,00%		35,00%	17,00%		14,00%	19,00%		21,00%		j=28	58,14%		60,14%	6,86%		8,86%		0%	
	j=9	33,00%		35,00%	32,00%		34,00%		0,00%			j=29	25,05%		27,05%	10,40%		12,40%	28,55%		30,55%
	j=10	53,00%		55,00%	12,00%		14,00%		0,00%			j=30	42,63%		44,63%	22,37%		24,37%		0%	
	j=11	40,00%		42,00%	25,00%		27,00%		0,00%			j=31	36,00%		38,00%	19,00%		21,00%	9,00%		11,00%
	j=12	54,00%		56,00%	11,00%		13,00%		0,00%			j=32	36,00%		38,00%	29,00%		31,00%		0%	
	j=13	37,00%		39,00%	28,00%		30,00%		0,00%			j=33	29,00%		31,00%	20,00%		22,00%	15,00%		17,00%
	j=14	52,00%		54,00%	13,00%		15,00%		0,00%			j=34	48,00%		50,00%	17,00%		19,00%		0%	
	j=15	47,00%		49,00%	18,00%		20,00%		0,00%			j=35	16,00%		18,00%	21,00%		23,00%	27,00%		29,00%
	j=16	36,00%		38,00%	29,00%		31,00%		0,00%			j=36	57,00%		59,00%	8,00%		10,00%		0%	
	j=17	0,00%		1,00%	66,00%		68,00%		0,00%			j=37	25,00%		27,00%	40,00%		42,00%		0%	
	j=18	22,00%		24,00%	43,00%		45,00%		0,00%			j=38	30,00%		32,00%	0,00%		1,00%	35,00%		37,00%
	j=19	49,00%		51,00%	16,00%		18,00%		0,00%			j=39	28,00%		30,00%	18,00%		20,00%	18,00%		20,00%
	j=20	43,77%		45,77%	21,23%		23,23%		0,00%			j=40	15,00%		17,00%	17,00%		19,00%	32,00%		34,00%

Table 2: Constraints on the nutrient composition of fertilizer formulas

Let's now see how to satisfy these $J = 40$ outputs with $N = 3$ inputs (the lowest value found in the parametric quadratic program) whose optimal composition is to be determined.

4.2 Case Study Results

In this example, we have 129 variables and 280 constraints of which 240 are quadratic non convex. The other 40 constraints are those related to the requested quantities of fertilizers. These quantities depend on the surface of the cultivated areas as well as on the rate of fertilizer use. (The quantities shown in table 1 were found by assuming that the rate of fertilizer use is 100%). The solution of this problem is presented in Table 3.

Since the search for the optimal solution is guided by the maximization of the first input concentration in the final blends, the solver has managed to consume about 93726.30 tons from Input 1 (65.78 % of total demand). The solver (Xpress-Non Linear solver of Xpress-IVE of Fico) yields no solution by setting the number of inputs to two, and gives an infinity of solutions beyond three inputs. In this way, reverse blending was able to find the minimum number of inputs ($N=3$) that allowed us to produce, under the same economic conditions, forty formulas of fertilizer.

Although these formulas were established for optimal agriculture, the requested quantities are not fixed as the cultivated areas and the fertilizer use rate may change from one season to the next. This wouldn't be a problem though since these quantities will surely impact the proportion of inputs needed to meet demand but not their number and composition. To demonstrate this, we will first solve the model by taking identical demands ($D_j = 100, \forall j$) (see table 4). Subsequently, we will start from the optimal inputs proposed by this last model, but this time we take different demand types (those shown in table 1), to see how this change impacts input concentrations in total demand (see table 5).

Optimal composition of the inputs (α_{ci})					Optimal quantities to be taken from the inputs (X_{ij})									
Input	$i=1$	$i=2$	$i=3$	Filler	Component				Input				Demand	
					c=1	c=2	c=3	Filler	$i=1$	$i=2$	$i=3$	D (j)		
$i=1$	66.0%	1.00%	0.00%	33.00%										
$i=2$	0.0%	0.51%	67.99%	31.50%										
$i=3$	0.0%	69.03%	0.00%	30.97%										

Concentrations of inputs in total demand					Optimal quantities to be taken from the inputs (X_{ij})									
Input	$i=1$	$i=2$	$i=3$	Total demand	Component				Input				Demand	
					c=1	c=2	c=3	Filler	$i=1$	$i=2$	$i=3$	D (j)		
$i=1$	93726.30	65.78%		142492 Tons										
$i=2$	12088.28	8.48%												
$i=3$	36677.44	25.74%												

Concentrations of inputs in total demand					Optimal quantities to be taken from the inputs (X_{ij})									
Input	$i=1$	$i=2$	$i=3$	Total demand	Component				Input				Demand	
					c=1	c=2	c=3	Filler	$i=1$	$i=2$	$i=3$	D (j)		
$i=1$	93726.30	65.78%		142492 Tons										
$i=2$	12088.28	8.48%												
$i=3$	36677.44	25.74%												

Table 3: Optimal solution for potential demand types

Optimal composition of the inputs (α_{ci})					Optimal quantities to be taken from the inputs (X_{ij})									
Input	$i=1$	$i=2$	$i=3$	Filler	Component				Input				Demand	
					c=1	c=2	c=3	Filler	$i=1$	$i=2$	$i=3$	D (j)		
$i=1$	66.00%	1.00%	0.00%	33.00%										
$i=2$	7.25%	1.00%	57.55%	34.20%										
$i=3$	0.00%	69.03%	0.00%	30.97%										

Concentrations of inputs in total demand					Optimal quantities to be taken from the inputs (X_{ij})									
Input	$i=1$	$i=2$	$i=3$	Total demand	Component				Input				Demand	
					c=1	c=2	c=3	Filler	$i=1$	$i=2$	$i=3$	D (j)		
$i=1$	2355.83	58.90%		4000 Tons										
$i=2$	400.00	10.00%												
$i=3$	1244.17	31.10%												

Concentrations of inputs in total demand					Optimal quantities to be taken from the inputs (X_{ij})									
Input	$i=1$	$i=2$	$i=3$	Total demand	Component				Input				Demand	
					c=1	c=2	c=3	Filler	$i=1$	$i=2$	$i=3$	D (j)		
$i=1$	2355.83	58.90%		4000 Tons										
$i=2$	400.00	10.00%												
$i=3$	1244.17	31.10%												

Table 4: Optimal solution for identical demand types

By setting demand at 100 tons by default, total demand for the 40 fertilizers is 4000 tons of which 58.9% is from the first input, 10% from the second and 31.10% from the third one.

The taken quantities from inputs i

Optimal composition of the inputs (α_{ci})					The taken quantities from inputs i				
Component					Input				Demand
					$i = 1$	$i = 2$	$i = 3$		D (j)
Input	c=1	c=2	c=3	Filler	j=1	j=2	j=3	j=4	j=5
	i=1	66.00%	1.00%	0.00%	33.00%	1833.50	0.00	289.50	2123
	i=2	7.25%	1.00%	57.55%	34.20%	1443.00	0.00	0.00	1443
	i=3	0.00%	69.03%	0.00%	30.97%	10412.62	0.00	2008.38	12421
Concentrations of inputs in total demand					Input				Demand
					$i = 1$	$i = 2$	$i = 3$		D (j)
Input	Total quantit	Inputs share	Total demand		j=21	j=22	j=23	j=24	j=25
	i=1	91702	64.36%	142492 Tons	28.25	1185.01	0.00	4.75	33
	i=2	15274	10.72%		440.99	19.83	0.00	14.17	1626
	i=3	35516	24.92%		82.89	105.11	0.00	237.36	746
					j=26	j=27	j=28	j=29	j=30
					12705.03	26268.26	0.00	10697.74	36966
					6692.95	6692.95	0.00	657.05	7350
					2912.64	4395.66	1171.71	8480	
					1288.67	0.00	618.33	1907	
					275.68	92.03	132.29	500	
					385.76	0.00	284.24	670	
					99.44	66.33	64.24	230	
					1181.82	0.00	378.18	1560	
					211.88	494.14	293.99	1000	
					255.67	0.00	30.33	286	
					882.00	0.00	1274.00	2156	
					1180.69	1840.31	0.00	3021	
					328.73	271.36	199.91	800	
					78.23	242.99	98.78	420	

Table 5: Input consumption when combining optimal composition obtained for identical demand types and different demand types of table 1

We note that starting from the same inputs and having changed demand structure, the proportion of the inputs in total production (142492.08 tons) is no longer the same (64.36% of input 1, 10.72% of input 2 and 24.92% of input 3). The concentration flows, therefore, depend on demand structure: changing it may imply a diversity of flows. An analysis of the results of the reverse blending model shown in Table 2, revealed that the share of the first input in total demand increased slightly (65.78% of input 1, 8.48% of input 2 and 24.92% of input 3). This means that when defining new inputs based on a new demand structure, the structure of input flows may vary slightly.

Basically, reverse blending is a new, valuable, approach for fertilizer producers: instead of responding to a wide variety of fertilizer requirements by blending a multitude of them, they can now do so by blending a very limited number of components none of which are fertilizers. To show the benefits of this new approach compared with the existing way of producing fertilizers from a blend of fertilizers using the traditional blending model (§7), we have tested this approach to find the minimum number S of fertilizers (inputs) that will be able to produce the 40 fertilizer formulas (outputs). With 32 inputs (from the 40 potential ones) one is able to produce the 8 other outputs (see table 6). Under $S=32$ inputs, some outputs are impossible to produce.

		Share of the 3 selected inputs out of 32			Demand D_j
		$i = 1$	$i = 17$	$i = 40$	
Unused inputs ($N-S = 8$ inputs)	$j = 3$	98.21%	1.79%	0%	100%
	$j = 12$	100%	0%	0%	100%
	$j = 14$	96.43%	3.57%	0%	100%
	$j = 15$	87.50%	12.50%	0%	100%
	$j = 18$	42.86%	57.14%	0%	100%
	$j = 30$	79.70%	20.30%	0%	100%
	$j = 34$	89.29%	10.71%	0%	100%
	$j = 35$	8.77%	9.42%	81.82%	100%

Table 6: The selected inputs required for the production of the other 8 outputs

5. Conclusion

Reverse blending is a very efficient way to deliver customized fertilizers that exactly satisfy the needs of sustainable agriculture while reducing production and transportation issues. The data used, deduced from *Fertimap* by linear regression may not be the right ones but they do not put the approach into question. The theoretical feasibility of this innovative approach has now definitely been demonstrated. For it to be operational, several complementary studies are required: first, the rules for establishing fertilizer needs, which have been determined empirically must be validated by agronomists. In addition, the census of these needs for an entire country, that depends on the “soil : crop” couple is indispensable and involves cooperating with agronomists. Similarly, chemists shall be required to assess the constraints related to chemical feasibility of the inputs. Finally, we will have to work on large instances and study the impact of this transformation of production processes on OCP’s supply chain and eventually design new distribution schemes. These will be based on several scenarios describing the impact of huge transportation flow consolidation as well as the nature, location and sizing of post-manufacturing blending facilities to be implemented in Africa, one of OCP’s largest markets.

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7. Formulation of the blending model

Indices

- Input $i=1..N$
- Output $j=1..J$
- Component $c=1..C$

Parameters

- α_{ci} : Input structure (component weight c included in 1 ton of input i).
- Structure of the outputs :
 - β_{cj}^{Min} : Minimum weight of component c included in 1 ton of output j .
 - β_{cj}^{Max} : Maximum weight of component c included in 1 ton of output j .
- D_j : Ordered quantity of output j .

Command variables

- x_{ij} : Quantity of input i integrated in output j .
- w_j : Binary variable such as $w_j=1$ if output j is produced.
- v_i : Binary variable such as $v_i=1$ if input i is used.

Constraints of the problem

- Production of outputs j
$$\sum_i x_{ij} = D_j \cdot w_j, \forall j$$
- Matching demand structure for output j .
$$\sum_i \alpha_{ci} \cdot x_{ij} \cdot v_i > \beta_{cj}^{\text{Min}} \cdot D_j \cdot w_j, \forall j, c$$

$$\sum_i \alpha_{ci} \cdot x_{ij} \cdot v_i < \beta_{cj}^{\text{Max}} \cdot D_j \cdot w_j, \forall j, c$$
- Fixing the number of inputs S to be used (Starting from $S=N$, this number is to be decremented until finding the minimum number of inputs that enables producing all the outputs).
$$\sum_i v_i = S$$
- Determination of the value taken by v_i where M is an upper bound.
$$\sum_j x_{ij} < M \cdot v_i, \forall i$$

Optimization criteria

- Maximizing the number of produced outputs.
$$\text{Max}(\sum_j w_j)$$