Laboratoire d'Analyses et Modélisation de Systèmes pour l'Aide à la Décision

UMR 7243

CAHIER DU LAMSADE

399

Revised version June 2022

Integrated Laycan and Berth Allocation Problem with ship stability and conveyor routing constraints in bulk ports

Hamza Bouzekri, Gülgün Alpan and Vincent Giard

Problem with ship stability and conveyor routing constraints in bulk ports

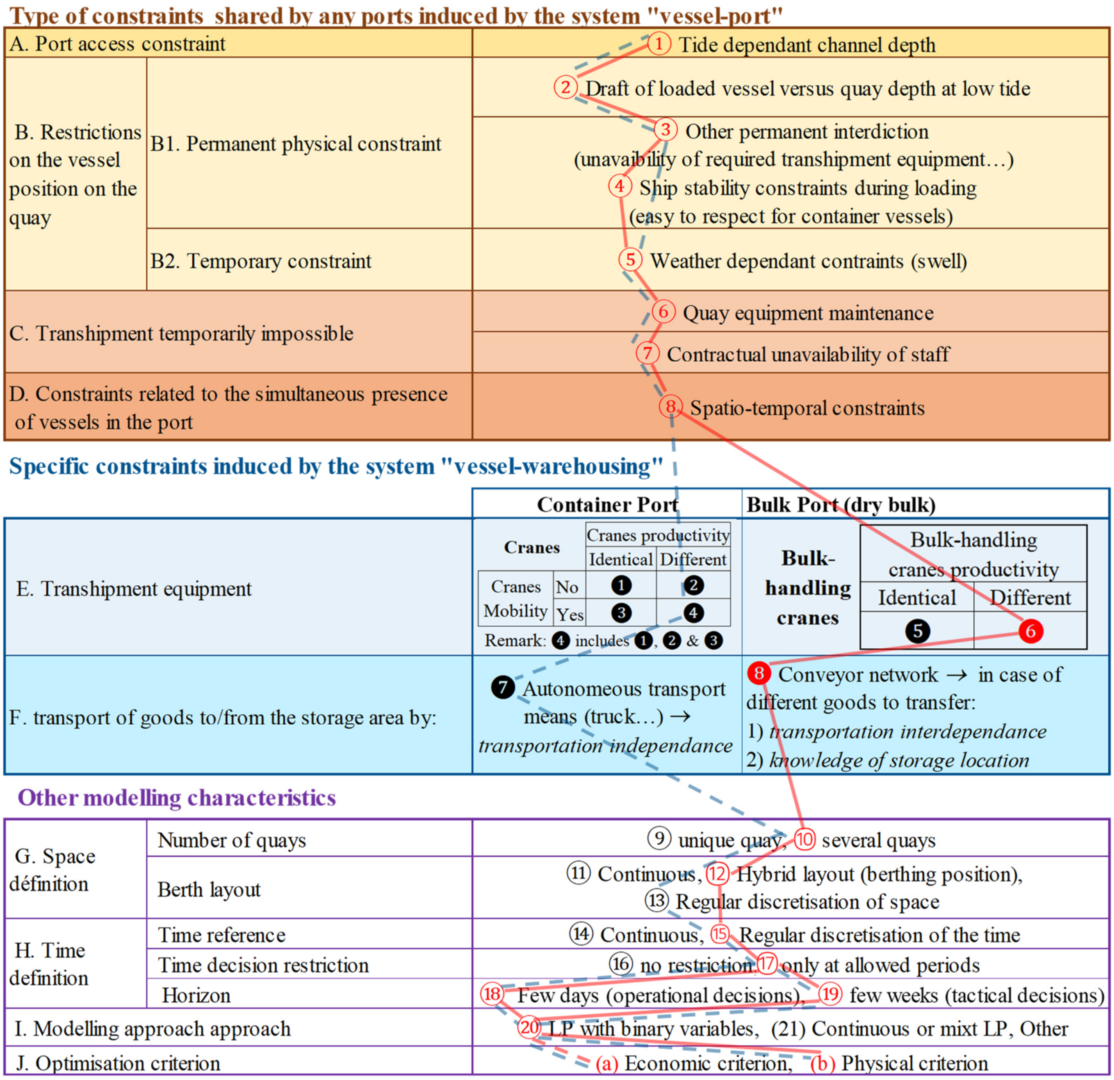
In this paper, we study the integrated Laycan and Berth Allocation Problem (LBAP) in the context of bulk ports, which considers two problems in an integrated way: the tactical Laycan Allocation Problem and the dynamic hybrid case of the operational Berth Allocation Problem. To make the LBAP closer to reality, we consider tidal bulk ports with conveyor routing constraints between storage hangars and berthing positions, preventive maintenance activities, multiple quays with different water depths and fixed heterogeneous bulk-handling cranes, navigation channel restrictions, vessels with multiple cargo types, charter party clauses, non-working periods, and ship stability considerations during loading operations. The proposed integer programming model aims to define an efficient schedule for berthing chartered vessels and optimal laycans for new vessels to charter. The model is formulated with predicates that guarantee maximum flexibility in the implementation and greatly improve the computational performance. Finally, the model is tested and validated through a small set of relevant case studies inspired by the operations of OCP Group at the bulk port of Jorf Lasfar in Morocco in very reasonable computational time using commercial Software.

Keywords: laycan allocation; berth allocation; tidal bulk ports; ship stability; conveyors; preventive maintenance; integer programming

1. Introduction:

With an estimated 80 per cent of the volume of world merchandise trade by sea, international shipping and ports provide crucial linkages in global supply chains and are essential to enable all countries to access global markets UNCTAD (2019). Although containerization has revolutionized the shipping industry, bulk cargoes are still the fundamental and enduring trades that support the dynamism of maritime transport. Five cargo types can be distinguished: container cargo, liquid bulk, dry bulk, break-bulk cargo, and ro-ro. In container terminals, all cargo is packed into standard containers, and thus there is no need for any specialized equipment to handle any particular type of cargo. In contrast, cargo is not packaged in bulk ports, and a wide variety of loading/unloading equipment and means of transport is used depending on the vessel requirements and cargo properties. For example, dry bulk goods are handled using fixed bulk-handling cranes and are transferred using conveyors between storage hangars and bulkers considering conveyor routing constraints, while liquid bulk goods need pipelines to be handled and transferred between storage tanks and tankers. On the other hand, containers are handled using mobile cranes and are transferred between storage areas and container ships using internal vehicles. Despite their importance in maritime logistics, bulk ports have received less attention than container terminals in the scientific literature.

Container and bulk port management share many common characteristics, but some specificities prevent applying RO models treating containerships to bulkers. To show the originality of our approach, we must point out the shared and distinctive characteristics of these two classes of problems, mainly since our paper uses several proposals on the treatment of common characteristics proposed in (Bouzekri et al. 2021). Figure 1 compares the characteristics of container and bulk port models by focusing mainly on the relationship between vessels and the port and the relationship between vessels and warehousing. A third axis treats general characteristics of time and space referencing as well as the used optimization criterion; these shared characteristics cannot be included in the sets defining the two-precedent axis. Characteristics are numbered, and a solid line links the characteristics retained in our modelling, whereas a dashed line links the characteristics considered in the model previously proposed for container vessels. Many of the common characteristics were never considered by models dealing with bulk ports; there is no reason not to introduce them in our modelling, which has its own scientific originality.



As shown in Figure 1, the main difference between container and bulk port problems deals with the interaction between vessel and warehousing, underlined by the characteristic ❽, essential to consider in case of a significant variety of goods to load (if this variety does not exist, the bulk problem is not very different from the container problem). A second difference is the consideration of ship stability during the loading (characteristic ④). In the case of bulkers, cargoes are loaded in several holds, each hold receiving a unique kind of goods to keep product integrity. Loading operations are processed from a conveyor along the vessel, considering the possibility of transporting batches of different goods to store in different holds. This detailed scheduling problem, close to real-time, is correctly treated by commercial Software which does not consider conveyor constraints. At the studied operational level, our concern is the expedition of batches from hangars to vessels compatible with loading constraints considered in detail by commercial Software.

The used economic criterion (characteristic (a)) conciliates two important decision problems in port management: the tactical Laycan Allocation Problem (LAP) and the operational Berth Allocation Problem (BAP). The LAP assigns berthing time windows to new vessels to charter within a medium-term planning horizon (three to four weeks), considering the availability of cargo and port resources (berthing positions, handling equipment, etc.). Hence, the LAP has a clear interaction with one of the most important operational problems in the seaside area of ports: the BAP. The latter assigns berthing positions and times to every vessel projected to be served within a short-term planning horizon (one to two weeks) such that a given objective function is optimized. To easily manage the integration between these two problems that have different decision levels, we consider a modular decision time-interval inside the planning horizon (characteristic ⑰). This approach was first proposed in Bouzekri et al. (2021) to integrate the LAP with the integrated Berth Allocation and Quay Crane Assignment Problem (BACAP) in the context of container terminals. To the best of our knowledge, the current paper is the first research to integrate the LAP and the BAP in the context of bulk ports, considering ship stability and conveyor routing constraints with preventive maintenance activities between storage hangars and berthing positions.

This study also considers all common constraints of port management listed above (Figure 1), never considered altogether in bulk ports modeling. Finally, we use predicates in the proposed integer linear programming model to define the feasibility zone of decision variables. This approach permits reducing the number of variables and constraints and hence makes it possible to solve real size problems using commercial Software. This type of formulation has successfully been used in Bouzekri et al. (2021). The current paper confirms its efficiency.

The remainder of this paper is organized as follows. In Section 2, we present a literature review of the BAP and the LAP in the context of bulk ports. Section 3 is dedicated to the description of the LBAP, while Section 4 is dedicated to its mathematical formulation. In Section 5, we present a case study with an illustrative example, then we discuss the results. Finally, in Section 6, we draw some conclusions and indicate future research directions.

2. Literature review:

In this section, we review the academic literature on the BAP and the LAP in the context of bulk ports.

2.1.BAP Literature:

The BAP in bulk ports has received little attention in Operations Research literature compared to container terminals until recently. A list of papers that propose new models for the BAP in the context of bulk ports, as an individual problem or using an integrated approach, is described below.

The berth layout can be either discrete, continuous, or hybrid. Barros et al. (2011) propose an integer linear programming model for the discrete BAP considering homogeneous berthing positions with tide and stock level constraints, prioritizing vessels related to the most critical mineral stock level. The authors then propose a Simulated Annealing-based algorithm as a valid alternative to the commercial solver to find good and fast solutions for hard instances. Ribeiro et al. (2016) also solve the discrete BAP by proposing a mixed-integer linear programming model considering maintenance activities. The authors model each maintenance activity as a dummy vessel which must be handled at a precise time by a specific berthing position, which means that this berthing position cannot receive vessels during that time. They then develop an adaptive large neighborhood search heuristic that finds good solutions within low computational times on all instances.

Ernst et al. (2017) solve the continuous BAP with tidal constraints that limit the departure of fully loaded vessels from dry bulk terminals using a commercial solver. The authors propose two new mixed-integer linear programming models, and then they provide several valid inequalities for both models, which improve both their solution quality and run time. To solve efficiently medium to large-sized instances of Ernst et al. (2017), Cheimanoff et al. (2020) develop a metaheuristic approach based on the Reduced Variable Neighborhood Search. The authors also develop a machine learning algorithm to tune the metaheuristic's hyper parameters.

Umang et al. (2013) study the hybrid BAP by proposing two exact methods based on mixed-integer programming and generalized set partitioning and a heuristic method based on squeaky wheel optimization. The authors consider the fixed equipment facilities, such as conveyors and pipelines, which are installed at only certain sections along the quay, the cargo type on the vessel and its draft. They also consider the time taken to transfer cargo between its location on the yard and the berthing position of the vessel. de León et al. (2017) propose a Machine Learning-based system to select the best algorithm for solving the BAP model proposed by Umang et al. (2013) in each particular case. The latter depends on factors such as the percentage of vessels that need specialized handling equipment, and the congestion level, which is influenced by the distribution of the estimated time of arrival of vessels and their workload.

Most authors consider dynamic vessel arrivals, while Tang et al. (2016) consider static vessel arrivals. The authors implement a multi-phase particle swarm optimization algorithm to minimize the total service time of vessels or their makespan.

Since the operational problems observed in port terminals are often interrelated, some authors study the BAP using an integrated approach. Indeed, Robenek et al. (2014) extend the work of Umang et al. (2013) by integrating berth allocation and yard assignment problems. The authors propose an exact solution algorithm based on a branch and price framework and a metaheuristic approach based on a critical-shaking neighborhood to solve this integrated problem. Al-Hammadi and Diabat (2017) apply the model proposed by Robenek et al. (2014) for Mina Zayed Port in Abu Dhabi in order to test different scenarios as a means of sensitivity analysis, with respect to certain factors such as the congestion level, in terms of the relative arrival time of vessels, the unavailability of certain resources and the addition of new resources.

In the same logic of integrating problems, Unsal and Oguz (2019) propose an exact solution procedure for an integrated problem that consists of three operations: berth allocation, reclaimer (a large machine used to recover bulk material from a stockpile) scheduling and stockyard allocation, considering tide and reclaimers non-crossing constraints. The authors develop a novel logic-based Benders decomposition algorithm in which a master problem and a subproblem are modeled using mixed-integer programming and constraint programming, respectively. The subproblem's role is either to find a feasible schedule for reclaimer schedules and yard allocations given mooring and departure times of vessels or to prove that the problem instance is infeasible.

Note that few papers consider tide with navigation channel restrictions, such as the maximum number of vessels to pass simultaneously through the navigation channel and the vessels' incapability to pass in opposing directions. These restrictions are considered by Zhen et al. (2017) and Corry and Bierwirth (2019) in the context of container terminals. In the context of bulk ports, Pratap et al. (2017) develop a decision support system to solve the integrated problem of berth and ship unloader allocation, under the condition that the channel allows only one vessel to pass at a time, using metaheuristics. The authors consider two different approaches: either solving the problem sequentially as a two-phase optimization model, berth allocation and ship unloader allocation or integrating the two phases in a single-phase problem. The integrated approach gives a better result than the sequential approach, but the latter is useful for the port authorities to revise their contract with their clients. Liu et al. (2021) propose a mixed-integer linear programming model for integrated planning of berth allocation and vessel sequencing in tidal seaports with one-way navigation channel, which obliges vessels to queue up to enter or leave the port alternately. The authors also develop a tailored adaptive large neighborhood search algorithm to solve the integrated problem within a reasonable time.

Krimi et al. (2019, 2020) study the integrated Berth allocation and Quay Crane Assignment Problem in tidal bulk ports with multiple quays, vessels with multiple cargo types, and unavailability constraints due to preventive maintenance of quay cranes and bad weather conditions. The authors develop a general variable neighborhood search-based approach to solve instances the commercial solver failed to solve optimally.

We also note that we found no papers that consider the BAP with conveyor routing constraints between storage hangars and berthing positions. Some authors consider conveyor routing constraints in other problems. For example, Menezes et al. (2017) study the production planning and scheduling problem in bulk ports which defines the amount and destination of each product and simultaneously establishes a set of feasible routes from storage subareas to vessels, where there is no conflict regarding equipment allocation. However, the authors consider the berthing positions of vessels as inputs in the problem.

The journal papers cited above are summarized in Table 1, in which the following information is presented.

- Port type: either import or/and export ports.
- Spatial attribute: it concerns the berth layout (either discrete, continuous or hybrid), the number of quays (either a single quay or multiple quays), and it specifies if the BAP considers the draft of berthing positions and the restrictions of the navigation channel and the conveyor system when deciding on a vessel's berthing position.
- Temporal attribute: it describes the arrival process of vessels (either static or dynamic), and it specifies if the BAP considers tide constraints and non-working periods (e.g., non-working days and maintenance activities) when deciding on a vessel's berthing time.

- Handling time attribute: it describes the productivity of handling equipment (either homogeneous or heterogeneous). In the case of homogeneous handling equipment, handling time of vessels is fixed, while it is variable in the case of heterogeneous handling equipment. The attribute also specifies if the BAP considers the distance between berthing positions and storage locations when calculating a vessel's handling time.
- Performance measure attribute: it specifies the optimization criteria used in the objective function (either efficiency or effectiveness). Most models consider minimizing various times or costs.
- Vessel attributes: it specifies the number of cargo types (if a vessel carries only a single type or multiple types of cargo).
- Problems integrated with the BAP: it specifies the problems that are integrated with the BAP when the latter is studied using an integrated approach.
- Resolution approach: either exact methods, heuristics, or/and metaheuristics.
- Modeling choices: it specifies if the conditions of the BAP (berthing time and space, draft, tide, navigation channel, non-working periods) are modeled as either constraints or predicates using binary variables.

For detailed reviews of the BAP literature in the context of container terminals, we refer readers to Bierwirth and Meisel (2010, 2015).

2.2.LAP Literature:

While BAP literature is abundant, only two papers were found that deal with the LAP.

Lorenzoni et al. (2006) develop a mathematical model, based on a multi-mode resource-constrained scheduling problem improving the attendance of vessels. The proposed model determines laycans in a way that avoids simultaneous or nearly simultaneous arrivals of vessels competing with the same port resources (berthing positions, handling equipment, etc.), under the first come first served regime of attendance. However, the authors consider only time windows for resources' availability without considering spatial constraints such as vessel and berth lengths.

Bouzekri et al. (2021) study the integrated Laycan and Berth Allocation and time-invariant Quay Crane Assignment Problem in tidal ports with multiple quays. Then, they extend the integrated problem to the Specific Quay Crane Assignment, which includes the assignment of a set of specific quay cranes to each vessel. This research is more suitable for container terminals since it does not specify the cargo type, which is an important point in bulk ports.

As a conclusion to this section, we highlight that in this paper, we propose a new integer linear programming model for a new integrated problem: the LBAP in the context of bulk ports. Moreover, we consider numerous conditions related to port management in the definition of the LBAP, which reduces the gap between the abstract representation of the studied problem and its applicability in real situations. Indeed, our model considers tidal bulk ports that have conveyor routing constraints between storage hangars and berthing positions with preventive maintenance activities, a navigation channel, multiple quays with different water depths and heterogeneous loading equipment, vessels with multiple cargo types, charter party clauses, non-working periods, and ship stability constraints.

Table 1. BAP literature in the context of bulk ports.

Authors	Port type		Spatial attribute								Temporal attribute					Handling time attribute			Performance measure attribute		Vessel cargo types		Problems integrated with the BAP					Resolution approach		Modeling choices				
			number of quays		berth layout						vessel arrivals					handling equipment																		
	import	export	mono	multi	discrete	continuous	hybrid	draft	navigation channel	conveyor system	static	dynamic	tide	maintenance	non-working periods	homogeneous	heterogeneous	distance	efficiency	effectiveness	mono	multi	laycan allocation	quay equipment assignment	yard equipment scheduling	yard assignment	vessel sequencing	exact methods	heuristics/metaheuristics	berthing time	berthing space	draft	tide	non-working periods
Barros et al. (2011)	x	x	x		x							x	x			x			V		x								x	C	C		C	
Umang et al. (2013)	x	x	x				x	x				x				x		x		S	x							x	x	C	C	C		
Robenek et al. (2014)	x	x	x				x	x				x				x		x		S	x					x		x	x	C	C	C		
Ribeiro et al. (2016)	x	x	x		x							x		x			x		V		x								x	C	C			C
Tang et al. (2016)	x	x	x		x						x						x			S, M	x								x	C	C			
Pratap et al. (2017)	x		x		x				x			x				x				W, H, R	x			x					x	C	C			
Al-Hammadi and Diabat (2017)	x	x	x				x	x				x				x		x		S	x					x		x		C	C	C		
Ernst et al. (2017)		x	x			x						x	x			x				D	x							x		C	C		C	
Unsal and Oguz (2019)		x	x		x							x	x			x		x		D	x				x	x		x		C	C		C	
Krimi et al. (2019, 2020)		x		x		x						x	x	x			x			T		x		x					x	C	C		C	C
Cheimanoff et al. (2020)		x	x			x						x	x			x				D	x								x	C	C		C	
Liu et al. (2021)	x	x	x		x			x	x			x	x			x				S	x						x		x	C	C	P	C	
Our paper		x		x			x	x	x	x		x	x	x	x		x		V	D		x	x					x		P	P	P	P	P

Performance measure attribute: D: Departure times; T: Tardiness; W: Waiting times; H: Handling times; S: Total service times; M: Makespan; R: Priority deviation; V: Demurrage vs despatch
Modelling choices: P: Predicate; C: Constraint

3. Problem description:

In this section, we present all the constraints considered in the problem modelling. These constraints include both general port constraints and specific bulk port constraints. Then we present some possible optimization criteria related either to efficiency or effectiveness.

3.1. General port constraints:

We consider a tidal port with multiple quays. Each quay has a hybrid layout where large vessels may occupy more than one berthing position; however, small vessels cannot share a berthing position. In Figure 2, the vessel 3 occupies the berthing position 5 that is the union of berthing positions 3 and 4, however, the vessels 2 and 2′ cannot share the berthing position 2.

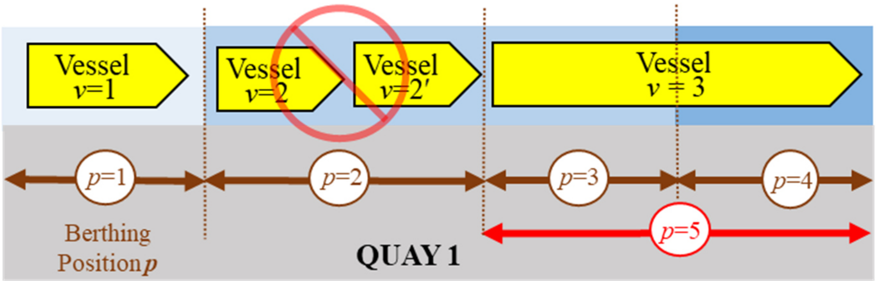


Figure 2. Hybrid berth layout.

Each berthing position is characterized by a length and a minimum water depth. All the berthing positions of a quay can have the same water depth, or the water depth increases seaward by berthing positions, as in Figure 3. The indexation of berthing positions is independent of the quays.

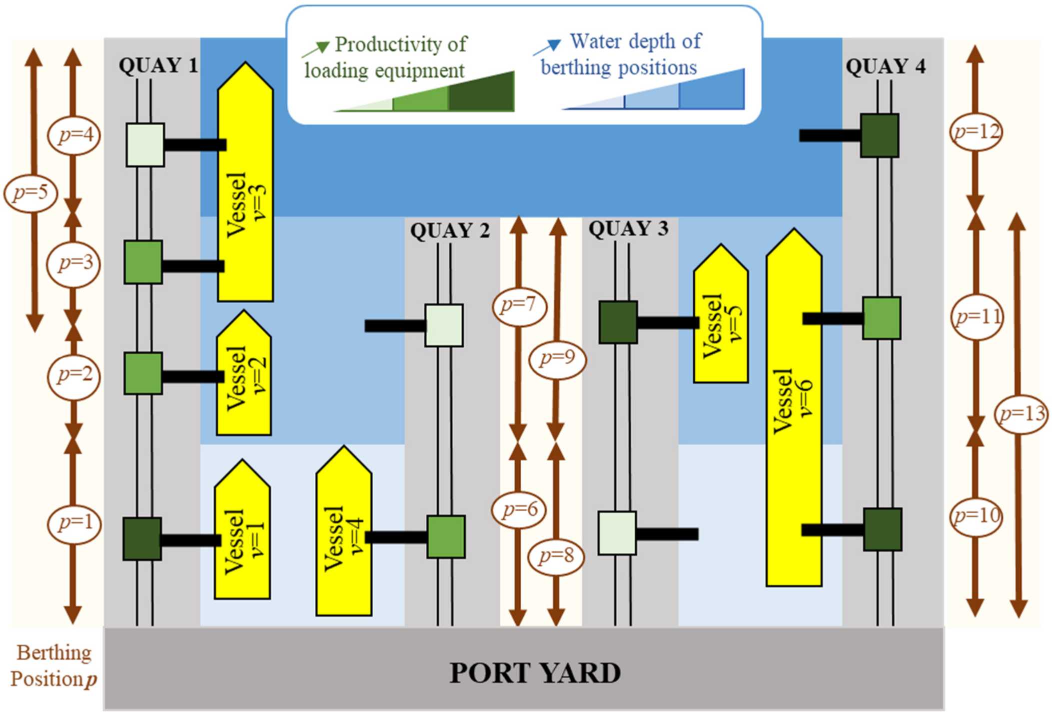


Figure 3. Example of a bulk port.

We consider three types of vessels:

- Already berthed vessels: these vessels have residual handling time and a predetermined berthing time and position.
- Chartered vessels: the charter party of these vessels is already signed. Consequently, their expected arrival time is fixed. The decisions remaining to take are when and where to berth.
- New vessels to charter: the charter party of these vessels is under negotiation. Consequently, their laycan is not yet fixed. The decisions to take are the first layday and where to berth.

We assume dynamic vessel arrivals, which means that expected arrival times are given for chartered vessels. Each vessel is characterized by a length and a draft. A maximum waiting time in the harbor per vessel is also introduced to circumvent solutions with very high waiting times. The port manager can fix this parameter based on what he judges acceptable. Besides this practical relevance, it also plays an interesting role in the computational performance by limiting the search space. We also consider the technical constraints of vessels that prohibit their berthing at some berthing positions or oblige them to berth at a specific berthing position.

In tidal ports, the use of the navigation channel is impacted by the tide cycle. We assume that large loaded vessels with deep drafts cannot pass through the navigation channel while leaving the port during low tides and thus have to wait for high tide cycles where the sea level is superior to their drafts (Figure 4). The detailed calculation of the tide parameters used in the case study is shown in the appendix (Table 11). Moreover, a maximum number of vessels must not be exceeded in a given period of time while passing simultaneously through the navigation channel.

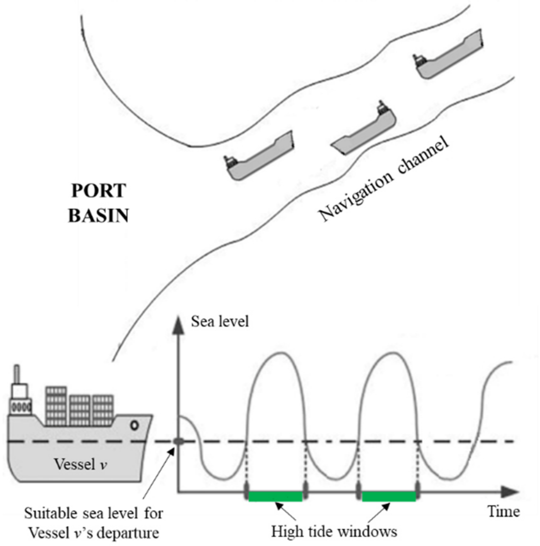


Figure 4. Navigation channel restrictions.

We also consider non-working periods, which can either be included or excluded in the counting of the laytime. For example, in SSHEX (Saturdays/Sundays/Holidays Excluded), the time lost in port on Saturdays, Sundays and Holidays does not count as laytime (from 5 pm on Friday until 8 am on Monday, and on holidays from 5 pm of the day preceding a holiday until 8 am of the next working day), while in SSHINC (Saturdays/Sundays/Holidays Included), no exception periods are in effect and the laytime will count seven days a week as well as during holidays. The used restriction of the case study is given in appendix (Table 12).

3.2.Specific bulk port constraints:

The following list of specific bulk port constraints is considered in our problem modelling.

Each berthing position in the port is characterized by one (or two) fixed bulk-handling crane(s) also characterized by a productivity (see Figure 3). Berthing positions are linked to storage hangars by a conveyor system, which can be divided into sections composed of identical parallel conveyors.

Berthing positions are linked to storage hangars by a conveyor system, as shown in Figure 5. The conveyor system is a set of identical parallel conveyors (represented by horizontal bars) connected by switches (represented by dots) and identical feeding/transfer conveyors (represented by vertical bars). The black box represents a flexible transfer system that connects the upstream conveyors linked to the hangars to the downstream conveyors linked to each berthing position on the quays. A route is a collection of interconnected horizontal and vertical conveyors that links a hangar to a berthing position. The number of possible routes is quite high (e.g., more than 1.3 million combinations in the example of Figure 5). It must be noted that all potential routes cannot be operational at the same time since a conveyor cannot transport two different bulk products at a time. This particularity is used to circumvent the combinatorial nature of the problem by defining compatible routes Menezes et al. (2017). Two routes are said to be compatible if and only if they do not share a conveyor (or a switch). This reduces considerably the number of routes to consider in assigning conveyors. However, it is a delicate task to list all compatible routes without errors.

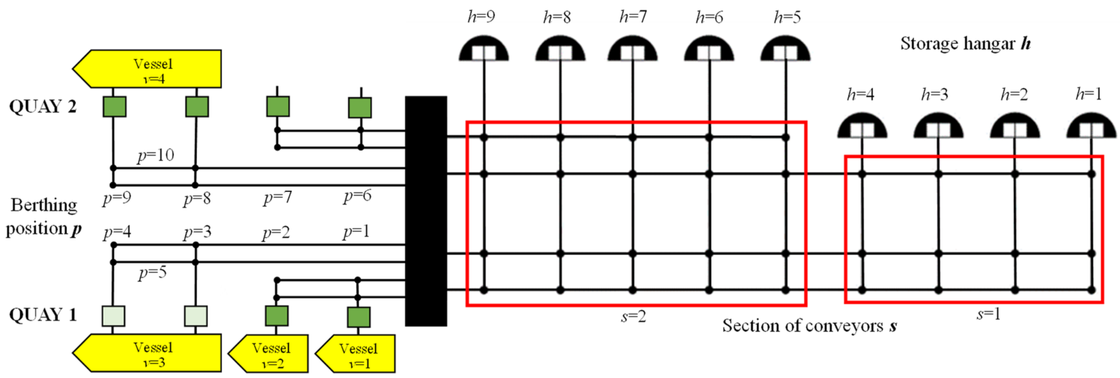


Figure 5. Sections of the port conveyor system.

Therefore, in this paper, we adopt another approach. Instead of considering a conveyor assignment problem, we consider conveyor capacity allocation. To this end, we divide the port conveyor system into sections (for example, two sections, s=1 and 2, in Figure 5). We define two parameters: the first one gives the number of identical parallel conveyors in each section that expresses the maximum number of bulk products that can be transported simultaneously in the given section (e.g., in Figure 5, maximum 3 in section 1 and 4 in section 2). The second one is a Boolean parameter with three indices (storage hangars, berthing positions and sections) that specifies which section is needed to transfer a product from a given hangar to a given berthing position (e.g., in Figure 5, the conveyors of any route between berthing position 1 and storage hangar 3 belong to sections 1 and 2). This allows replacing a list of compatible routes that is hard to build free of errors, by a simple constraint that limits the number of identical parallel conveyors to use simultaneously in a section, at a given time period. The solution to our model will provide which bulk product to be transported from which hangar to which berthing position at a time, respecting this capacity constraint. Given this solution, the allocation of conveyors and the maintenance of the model in case of infrastructural changes in the conveyor system can be done easily a posteriori.

We also consider scheduled preventive maintenance activities to be performed at the conveyors and berthing positions over a period of time or at a fixed date. Maintenance activities at conveyors in the same section can overlap with each other, while they are disjoint at a berthing position. Some berthing positions can also be discarded to some vessels, either permanently due to some of their characteristics or, for the next few days, due to weather conditions (tempest, equinoctial tide…). We note that the conveyor system is the bottleneck of the port since it limits the number of vessels that can be handled simultaneously (e.g., in Figure 5, only four out of ten vessels can be handled simultaneously).

Handling times of vessels depend on loading equipment's productivity in the berthing positions, and a vessel can be served by more than one loading equipment depending on its length. Each vessel is also characterized by a number of cargo types with different amounts to load on it. These amounts of cargo types can be expressed as batches. Each batch is characterized by an availability date and a storage hangar. It has to be noted that the batches to load on a single vessel can be stored in the same hangar or different hangars. We assume that only one batch at most can leave a storage hangar at a time and that two (or more) batches cannot be loaded at the same time on a vessel, but they can be loaded in any order without downtime. This assumption favors the waiting of vessels in the harbor until their continuous loading is guaranteed to minimize their berthing time in the port.

In Figure 6, vessel 3 is a chartered vessel that is berthed in berthing position 5. The vessel is represented by a large rectangle placed in the area of berthing position 5 starting from its berthing time with a length equal to its handling time. Small rectangles inside the big rectangle represent the batches to load on the vessel according to a loading sequence chosen by the optimal solution. Each small rectangle starts from the loading start time of the batch with a length equal to its handling time. Vertical bars represent high tide windows.

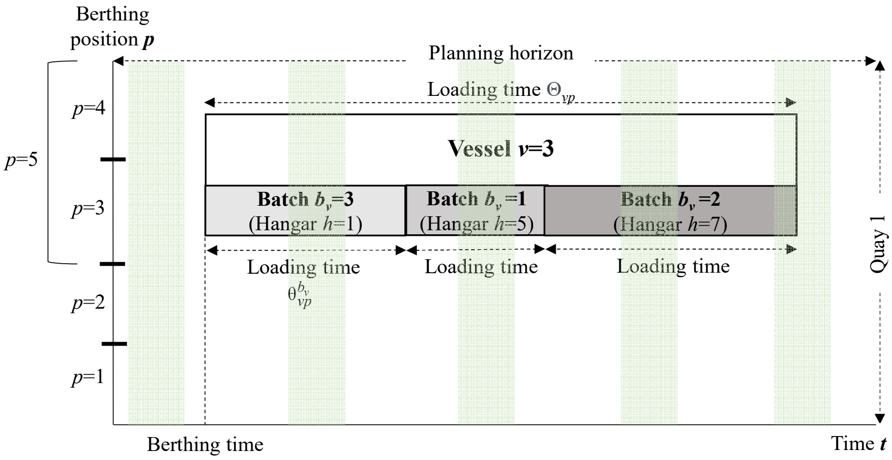


Figure 6. Representation of a chartered vessel and the batches to load on it.

That list of specific constraints would not be completed without considering stability issues while loading bulkers. Ship stability can be defined "as its characteristic or tendency to return to its original state or upright state, when an external force is applied on or removed from the ship" (Karan, 2021), involving that the center of gravity remains in the same position. Specific rules must be followed during loading or unloading operations (Directive 2001/96/EC), and several commercial Software treats that issue in a real-time perspective with a granularity that is too fine for our modelling. In addition, they do not consider conveyor constraints which must be considered when more than one type of bulk is to load, each one into specific holds. The split of large cargoes of the same product into several reasonably sized batches makes it possible to define a loading sequence compatible with the use of this kind of Software. Many predefined loading sequences compatible with the stability search can be defined. However, the use of one of them in our model is discarded because enforcing the vessel's sequence in the problem formulation can make it very difficult, if not impossible, to obtain a solution respecting conveyor routing constraints. The retained modelling approach, compatible with the stabilization of the gravity center, defines the sequence to use as a result of the optimization. Stability is considered through constraints inspired by project scheduling and illustrated in Figure 7. The batches are of homogeneous composition, numbered starting from the bow towards the stern and assigned to a hatch that can receive a unique product. The loading sequence starts from the vessel extremities and progresses towards the vessel center. In that example, sequencing starts with two batches (1 and 8) which can be placed in any order, before sequencing two batches (2 and 7) placed in any order, also before sequencing two batches (3 and 6) placed in any order, and ends by sequencing two batches (4 and 5) also placed in any order. The retained sequence is a solution given by optimization.

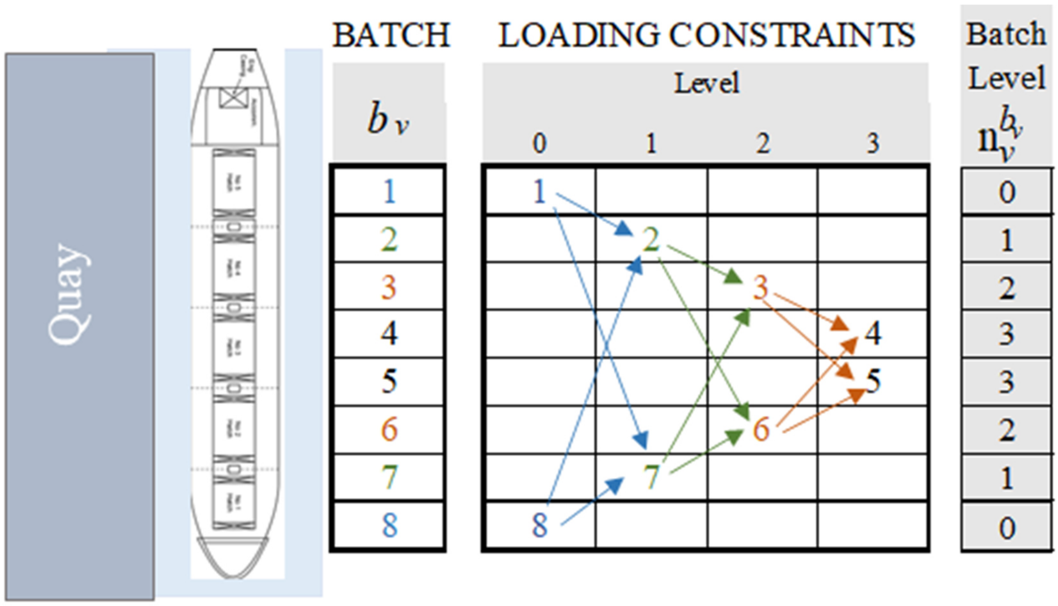


Figure 7. Batch loading constraints used in modelling.

3.3. Optimization criteria:

3.3.1. Efficiency criteria:

The LAP defines some major contractual terms that are found in a maritime contract between a *shipowner* and a *charterer* for the hire of a vessel (called *charter party*). Some of these contractual terms are the following: *laycan, laydays, laytime, demurrage,* and *despatch.* All these chartering terms are shown in Figure 8 and will be used for expressing the objective function and some decision variables in the mathematical model proposed in this paper.

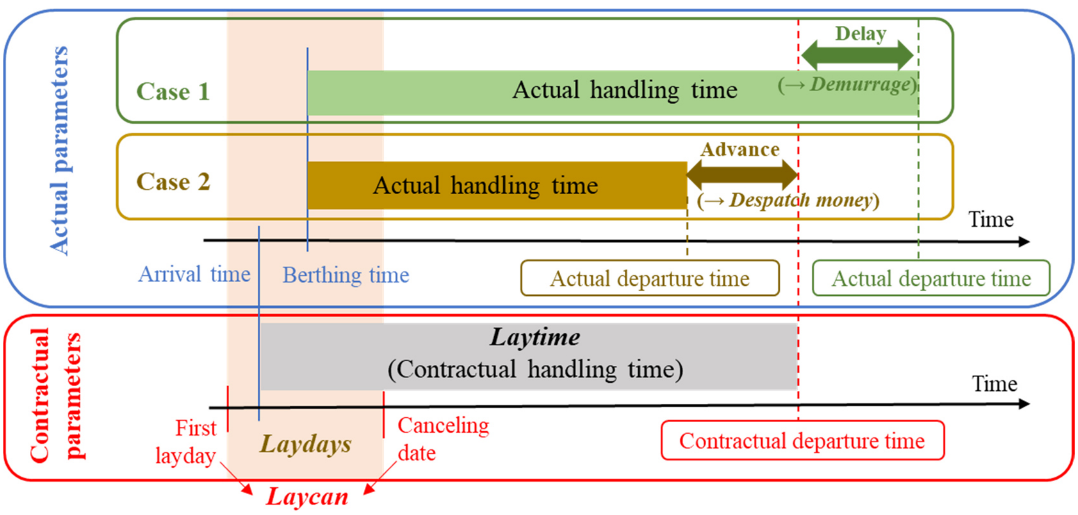


Figure 8. Contractual vs actual parameters of vessels (adapted from Bouzekri et al. (2021).

One of the efficiency criteria that could be applied is to find an efficient schedule for berthing chartered vessels that maximizes the sum of the difference between the despatch money and the demurrage charges for each vessel (i.e., minimize the demurrage charges and maximize the despatch money) while proposing optimal laycans for new vessels to charter considering all the characteristics and constraints described above.

3.3.2. Effectiveness criteria:

We note that other objective functions based on physical criteria could be considered, such as minimizing the sum of expected vessel departure times or vessel stay times. These sums can be weighted to consider vessel priority. The effectiveness point of view does not require any change in the model, except in the objective function's formula.

4. Model formulation:

4.1.Notation:

The sets are represented by calligraphic letters, the parameters by Greek letters or capital Latin letters, the variables by italic letters, and the indices by italic lowercase letters. The latter are always written as subscripts, except for the indices b_v, m_p and m_s, that are related to the indices *v*, *p* and *s*, which are always written as superscripts.

Index	*Description*
t	Index of time periods $\mathcal{T} = \{1,...,\mathrm{T}\}$.
v	Index of vessels $\mathcal{V} = \{1,...,\mathrm{V}\}$ with $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2 \cup \mathcal{V}_3$ and $\mathrm{V} = \mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3$, where: • $\mathcal{V}_1 = \{1,...,\mathrm{V}_1\}$ is the set of already berthed vessels. • $\mathcal{V}_2 = \{\mathrm{V}_1 + 1,...,\mathrm{V}_1 + \mathrm{V}_2\}$ is the set of chartered vessels. • $\mathcal{V}_3 = \{\mathrm{V}_1 + \mathrm{V}_2 + 1,...,\mathrm{V}\}$ is the set of new vessels to charter.
b_v	Index of batches to load on vessel v $\mathcal{B}_v = \{1,...,\mathrm{B}_v\}$.
$\mathrm{n}_v^{b_v}$	Batch level of batch b_v (see Figure 7) used in the batch sequencing in vessel

	v loading to maintain ship stability
p	Index of berthing positions $\mathscr{P} = \{1,...,\mathrm{P}\}$.
m_p	Index of maintenance activities to be performed at berthing position p $\mathscr{M}_p = \{1,...,\mathrm{M}_p\}$.
s	Index of sections composed of identical parallel conveyors $\mathscr{S} = \{1,...,\mathrm{S}\}$.
m_s	Index of maintenance activities to be performed at a conveyor in section s $\mathscr{M}_s = \{1,...,\mathrm{M}_s\}$.
h	Index of storage hangars $\mathscr{H} = \{1,...,\mathrm{H}\}$.

Parameter	*Description*
Navigation channel	
M	Maximum number of vessels allowed to pass simultaneously through the navigation channel.
Time decision restriction	
K_t	Boolean parameter that equals 1 if a decision of berthing vessels can be taken during time period *t*, 0 otherwise (see Section 4.3).
Tide cycle	
O_t	Boolean parameter that equals 1 if time period *t* is within a high tide cycle, 0 otherwise.
Berthing positions	
Q_p	Length of berthing position *p*.
W_p	Minimum water depth of berthing position *p*.
ρ_p	Productivity of berthing position *p*
$\mathrm{E}_p^{p'}$	Boolean parameter that equals 1 if berthing positions *p* and p' share a berthing position, 0 otherwise (e.g., in Figure 2, berthing positions 3 and 5 share berthing position 3). When $p = p'$, $\mathrm{E}_p^{p'} = 1$.
Sections	
U_s	Number of identical parallel conveyors in section *s*.
F_{sh}	Boolean parameter that equals 1 if one of the conveyors belonging to the route that links a berthing position to storage hangar *h* belongs to section *s*, 0 otherwise.

Preventive maintenance activities

$R_p^{m_p}$ Duration of maintenance m_p to be performed at berthing position *p*.

$\underline{R}_p^{m_p}$ Earliest time to perform maintenance m_p at berthing position *p*.

$\overline{R}_p^{m_p}$ Latest time to perform maintenance m_p at berthing position *p*.

$R_s^{m_s}$ Duration of maintenance m_s to be performed at a conveyor in section *s*.

$\underline{R}_s^{m_s}$ Earliest time to perform maintenance m_s at a conveyor in section *s*.

$\overline{R}_s^{m_s}$ Latest time to perform maintenance m_s at a conveyor in section *s*.

Vessels

A_v Expected arrival time of chartered vessel *v* and earliest time a new vessel to charter *v* can arrive to the port.

J_v Contractual handling time of vessel *v*, calculated with the contractual productivity ρ: $J_v = \sum_{b_v \in \mathcal{B}_v} \varphi_v^{b_v} / \rho, \forall v \in \mathcal{V}, \forall p \in \mathcal{P}$; $\varphi_v^{b_v}$ is the weight of batch b_v.

δ_v Contractual finishing time of vessel *v*, $\delta_v = A_v + J_v - 1, \forall v \in \mathcal{V}$.

Θ_{vp} Loading time of vessel *v* when the latter is berthed at berthing position *p*, which equals the sum of loading times of all the batches to load on this vessel, in any order without downtime: $\Theta_{vp} = \sum_{b_v \in \mathcal{B}_v} \theta_{vp}^{b_v}, \forall v \in \mathcal{V}, \forall p \in \mathcal{P}$; $\theta_{vp}^{b_v}$ is the loading time of batch b_v.

λ_v Length of vessel *v*.

D_v Draft of vessel *v* when it is fully loaded.

I_v Maximum waiting time in the harbor of vessel *v*.

ω_v Boolean parameter that equals 1 if vessel *v* is tide-dependent, 0 otherwise.

γ_v Boolean parameter that equals 1 if the handling of vessel *v* is restricted to working periods, 0 otherwise.

L_v Laydays of vessel *v*.

η_v Contractual demurrage by hour of vessel *v*. $\eta_v = 0, \forall v \in \mathcal{V}_1$ and $\eta_v = 1, \forall v \in \mathcal{V}_3$.

β_v	Contractual despatch by hour of vessel *v*. $\beta_v = 0, \forall v \in \mathscr{V}_1$ and $\beta_v = 1, \forall v \in \mathscr{V}_3$. $I_v = 48, \forall v$
G_{vp}	1 if vessel *v* can berth at berthing position *p*, 0 otherwise.
Batches	
$H_v^{b_v}$	Hangars where batch b_v to load on vessel *v* is stored
$C_v^{b_v}$	Date of availability of batch b_v to load on vessel *v*.
$\varphi_v^{b_v}$	Weight of batch b_v
$n_v^{b_v}$	Batch level of batch b_v (see Figure 7) used in the batch sequencing in vessel *v* loading to maintain ship stability
$\theta_{vp}^{b_v}$	Loading time of batch b_v on vessel *v* when the latter is berthed at berthing position *p*: $\theta_{vp}^{b_v} = \varphi_v^{b_v} / \rho_p, \forall v \in \mathscr{V}, \forall p \in \mathscr{P}, \forall b_v \in \mathscr{B}_v$.
Time framework	
ψ_{vt}	Boolean parameter that equals 1 if the handling of vessel *v* should not be carried out during time period *t*, 0 otherwise (see Section 4.2).
Γ_{vt}	Relative period of the absolute time period *t* of vessel *v* considering non-working periods (see Section 4.2).
Υ_{vt}	Absolute period of the relative time period *t* of vessel *v* considering non-working periods (see Section 4.2).

Decision variable	*Description*
x_{vpt}	1 if vessel *v* starts berthing at berthing position *p* in time period *t*, 0 otherwise.
$y_{vpth}^{b_v}$	1 if batch b_v stored in hangar *h* starts loading on vessel *v* at berthing position *p* in time period *t*, 0 otherwise.
$z_{pt}^{m_p}$	1 if maintenance m_p starts performing at berthing position *p* in time period *t*, 0 otherwise.
$z_{st}^{m_s}$	1 if maintenance m_s starts performing at a conveyor in section *s* in time period *t*, 0 otherwise.
u_v	Delay of vessel *v*, which is the number of time periods exceeding its laytime,

	$u_v \in \mathbb{Z}^+$ (since the planning horizon is divided into equal-sized time periods).
w_v	Advance of vessel *v*, which is the number of time periods saved in its laytime, $w_v \in \mathbb{Z}^+$.

Intermediary variable	*Description*
μ_v, $\tau_v^{b_v}$	Berthing position of vessel *v* in the decision variables x_{vpt} and $y_{vpth}^{b_v}$ respectively.
ε_v	Berthing time of vessel *v*.
π_v	Finishing time of vessel *v*.
$\rho_v^{b_v}$	Loading start time of batch b_v.
$\sigma_v^{b_v}$	Loading finishing time of batch b_v.

4.2. Representation of time

Port operations might be unavailable at some periods for some vessels (e.g. non-working days). If such periods coincide with the berthing period of the related vessel, they must be considered to estimate the ending time of berthing for this vessel. This requires adjusting index *t* for this vessel in the mathematical model. We use the approach proposed by Bouzekri et al. (2021) to this end.

We define four parameters; γ_v to indicate if the handling of vessel *v* is restricted to some working periods $(\gamma_v = 1)$ or not $(\gamma_v = 0)$. The second one is, ψ_{vt}, which is used to indicate the non-working periods for the vessels for which $\gamma_v = 1$. ψ_{vt} is equal to 1 in this case and will be equal to zero for all other situations. The third parameter Γ_{vt} is a relative time scale that keeps track of the non-working periods and accounts for them in advancing time. It is written as follows when $\gamma_v = 1$: $\Gamma_{vt|\psi_{vt}=0} = t - \sum_{t'=1}^{t'=t} \psi_{vt'}, \forall t \in \mathscr{T}, \forall v \in \mathscr{V}$ and $\Gamma_{vt|\psi_{vt}=1} = \Gamma_{vt'}, \forall t \in \mathscr{T}, \forall v \in \mathscr{V}$, where t' is the first working period after *t*, as it is the first $t' > t \,|\, \psi_{vt'} = 0$. Obviously,

$\Gamma_{vt} = t, \forall t \in \mathcal{T}, \forall v \in \mathcal{V}$, when $\gamma_v = 0$. Finally, the fourth parameter Υ_{vt} records the calendar time *t* for vessel *v*.

An example of these parameters is shown in Table 2, where each time period is one hour long and non-working periods last two hours. A realistic assignment of two day-long non-working periods is provided in Appendix (Table 10).

Table 2. Calculation of non-working periods parameters.

	Ψ_{vt}		Γ_{vt}		Υ_{vt}	
t/γ_v	0	1	0	1	0	1
1	0	0	1	1	1	1
2	0	0	2	2	2	2
3	0	1	3	3	3	5
4	0	1	4	3	4	6
5	0	0	5	3	5	7
6	0	0	6	4	6	8
7	0	0	7	5	7	9
8	0	0	8	6	8	12
9	0	0	9	7	9	13
10	0	1	10	8	10	14
11	0	1	11	8	11	15
12	0	0	12	8	12	16
13	0	0	13	9	13	19
14	0	0	14	10	14	20
15	0	0	15	11	15	21
16	0	0	16	12	16	22
17	0	1	17	13	17	23
18	0	1	18	13	18	0
19	0	0	19	13	19	0
20	0	0	20	14	20	0
21	0	0	21	15	21	0
22	0	0	22	16	22	0
23	0	0	23	17	23	0

To consider non-working periods in the calculation of the contractual finishing time of vessel *v*, $\delta_v = \mathrm{A}_v + \mathrm{J}_v - 1$ becomes $\delta_v = \Upsilon_{v\left(\Gamma_{v\mathrm{A}_v} + \mathrm{J}_v - 1\right)}$. Indeed, $\Gamma_{v\mathrm{A}_v}$ gives the relative period of the absolute expected arrival time of vessel *v* considering non-working periods, then $\Upsilon_{v\left(\Gamma_{v\mathrm{A}_v} + \mathrm{J}_v - 1\right)}$ gives the absolute period of the relative contractual finishing time of vessel *v* also considering non-working periods. If vessel *v* is not restricted to working periods, $\Upsilon_{v\left(\Gamma_{v\mathrm{A}_v} + \mathrm{J}_v - 1\right)} = \mathrm{A}_v + \mathrm{J}_v - 1$, therefore $\delta_v = \Upsilon_{v\left(\Gamma_{v\mathrm{A}_v} + \mathrm{J}_v - 1\right)}$ is used in both cases.

4.3. Decision time-interval:

To reduce the computational complexity and consider the increasing uncertainty of inputs as the length of the planning horizon increases, we follow the approach proposed by Bouzekri et al. (2021), which modulates decision time-interval through the planning horizon. So, we define a Boolean parameter K_t, that equals 1 if vessels can berth during time period *t* (without considering other constraints). Thanks to this parameter, we are able to restrict berthing decision periods inside the planning horizon and hence change the decision time interval.

The user of the model is free to define the values of K_t. For example, during the first week, chartered vessels can berth every hour « 1 », hence $\mathrm{K}_t = 1, \forall t$; during the second week, every four hours « 0 – 0 – 0 – 1», hence $\mathrm{K}_t = 1, \forall t \mid (t)_{\mathrm{mod}\,4} = 0$ and $\mathrm{K}_t = 0$ otherwise; during the third week, every eight hours « 0 – 0 – 0 – 0 – 0 – 0 – 0 – 1», hence $\mathrm{K}_t = 1, \forall t \mid (t)_{\mathrm{mod}\,8} = 0$ and $\mathrm{K}_t = 0$ otherwise. New vessels to charter can be planned during the second and third week, providing for them an estimated position in the schedule. Then, as we advance in the planning horizon, the schedule is refined: some chartered vessels $(\mathscr{V}_2)$ will become berthed vessels $(\mathscr{V}_1)$, and some new vessels to charter $(\mathscr{V}_3)$ will become chartered vessels $(\mathscr{V}_2)$, and hence their laydays will be replaced by an expected arrival time. The decisions related to the loading of batches are constrained by the decisions related to the berthing of vessels, as they must be done during the vessel's stay, which starts from the berthing time but can be taken at any period without considering the parameter K_t.

Modulating time intervals in this manner helps integrating short-term decisions (BAP) and medium-term decisions (LAP) in a single model. As the time approaches to present, the decisions are taken in a finer granularity (every hour), while for decisions that concern the planning in a few weeks from now, a rough decision is taken (every 8 hours). Besides facilitating the integration of LAP and BAP, this approach also helps control the number of variables (i.e., the number of variables is lower for medium-term decisions).

4.4.Predicates:

A mathematical program is made of a set of variables and a set of constraints made of a linear or non-linear combination of these variables, one of them being an objective function to optimize. The variables' validity domain is usually restrained by constraints, each defined for a set of variables through a universal quantifier. The variables' validity domain may be narrowed by using an Algebraic Modelling Language (AML, available in some software like Xpress (used here) or GAMS…; Fourer (2013)), which rests on the separation of a generic description of the model, and data to use; after what, an instance of the model combining the generic model and data, which can be submitted to a solver. AML allows the usage of predicates to drive the creation of an instance of the problem. A predicate is a logical statement that returns either a value of "True" or "False", based on the parameter values used in the statement, which in turn binds the existence of a variable, depending on the parameters' values. Predicates can be used to restrain

- the number of expanded constraints in relation using a universal quantifier.
- the validity domain of some variable without using a constraint, decreasing the number of constraints in a model; for example, the use of the predicate $\mathrm{D}_v \le \mathrm{W}_p$ (which enforces the draft of vessel v to not exceed the water depth of berthing position) in the definition of the validity domain of variable x_{vpt} avoids creating the constraint $\mathrm{D}_v \cdot x_{vpt} \le \mathrm{W}_p, \forall p,t$. The use of this kind of predicates presents the advantage of preventing the introduction of additional constraints in modelling a complex problem. It also avoids unnecessary calculations in the optimization search, as the predicate used in the problem expansion guarantees the respect of that (unintroduced) constraint.

The extensive use of predicates in the proposed model acts like a pre-treatment based on the problem data reducing the number of binary variables and constraints. Consequently, problems of practical sizes can be solved in a reasonable time using off-the-shelf commercial Software

We will use these logical statements to describe the validity domain of decision variables. In our model, a decision variable exists only when the associated set of predicates returns "True". In this section, we present how predicates are implemented in our mathematical model.

The decision variable x_{vpt} determines for each chartered vessel $(v \in \mathscr{V}_2)$, the berthing time *t* and berthing position *p*. Each already berthed vessel $(v \in \mathscr{V}_1)$ has a residual handling time and a predetermined berthing position. For simplicity, we assume the berthing time $t = 1$ for these latter vessels.

The decision variable $y_{vpth}^{b_v}$ determines for each chartered vessel $(v \in \mathscr{V}_2)$ berthed in berthing position *p*, the loading start time *t* of batch b_v, stored in hanger *h*. The first batch to load on each already berthed vessel $(v \in \mathscr{V}_1)$ has a residual handling time with a loading start time assumed at $t = 1$.

For each new vessel to charter $(v \in \mathscr{V}_3)$, the berthing position will be reserved from its latest berthing time decreased by its laydays, $t - \mathrm{L}_v + 1$, until its latest finish date, $\Upsilon_{v(\Gamma_{vt}+\Theta_{vp}-1)}$ (Figure 9). Similarly, the conveyors used to transport each batch b_v from storage hangar *h* to berthing position *p* will be reserved from the latest loading start time of the batch decreased by the vessel's laydays, $t - \mathrm{L}_v + 1$, until the latest finish date of the batch, $\Upsilon_{v\left(\Gamma_{vt}+\theta_{vp}^{b_v}-1\right)}$, depending on the loading sequence chosen by the optimal solution. This assures that, any time during their laydays, new vessels to charter can be berthed at the reserved berthing position, and thus all the batches can be loaded, in any order without downtime, using the reserved conveyors.

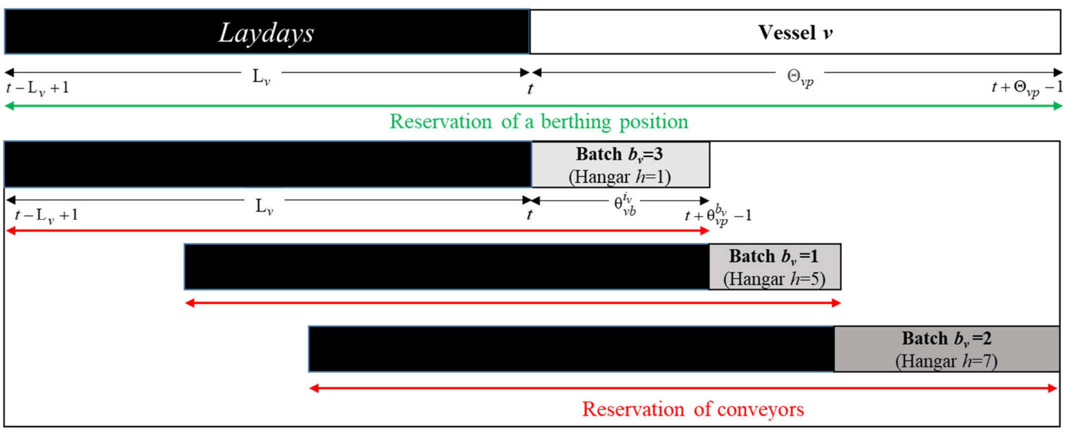


Figure 9. Reservation of port resources for new vessels to charter.

The existence of the decision variable x_{vpt} is subject to seven conditions:

(1) Vessel *v* must be able to berth at berthing position *p*: $\mathrm{G}_{vp} = 1$.

(2) The length of vessel v must not exceed the length of berthing position p: $\lambda_v \leq \mathrm{Q}_p$.

(3) The draft of vessel v must not exceed the water depth of berthing position p: $\mathrm{D}_v \leq \mathrm{W}_p$.

(4) Vessel v can berth only after its expected arrival time without exceeding its maximum waiting time in the harbor: $\mathrm{A}_v \leq t \leq \mathrm{A}_v + \mathrm{I}_v$. To allow new vessels to charter $(v \in \mathscr{V}_3)$ to berth at their first layday, t is replaced by $t - \mathrm{L}_v + 1$, then condition 4 becomes $\mathrm{A}_v \leq t - \mathrm{L}_v + 1 \leq \mathrm{A}_v + \mathrm{I}_v$. Already berthed and chartered vessels have fictitious laydays equal to one hour $(\mathrm{L}_v = 1)$ since they have fixed expected arrival times, and hence they are not concerned by the decision of fixing laycans as in the case of new vessels to charter. Hence, the new condition is valid for all types of vessels. The same applies to the following conditions. This modelling approach allows for the merging of LAP and BAP decisions.

(5) Vessel v can berth only during time periods where a decision of berthing vessels can be taken: $\mathrm{K}_{(t-\mathrm{L}_v+1)} = 1$.

(6) If the handling of vessel v is restricted to working periods $(\gamma_v = 1)$, it can enter the port only during working periods: $\psi_{v(t-\mathrm{L}_v+1)} = 0$.

(7) If vessel v is tide-dependent $(\omega_v = 1)$, it can leave the port at the time period $t = \Upsilon_{v(\Gamma_{vt}+\Theta_{vp}-1)}$ if the latter is within a high tide $(\mathrm{O}_t = 1)$: $(1\text{-}\omega_v) + \omega_v \cdot \mathrm{O}_{\Upsilon_{v(\Gamma_{vt}+\Theta_{vp}-1)}} = 1$.

Similarly, the existence of the decision variable $y_{vpth}^{b_v}$ is subject to seven conditions:

(1) Conditions 1, 2 and 3 of the existence of the decision variable x_{vpt}:

$\mathrm{G}_{vp} = 1 \wedge \lambda_v \leq \mathrm{Q}_p \wedge \mathrm{D}_v \leq \mathrm{W}_p$.

(2) Batch b_v can be loaded on vessel v between the expected arrival time of this vessel and its finishing time as it reaches its maximum waiting time in the

harbor, minus the loading time of this batch:

$$\mathrm{A}_v \le t - \mathrm{L}_v + 1 \le \Upsilon_{v\left(\Gamma_{v(\mathrm{A}_v+\mathrm{I}_v)}+\Theta_{vp}-\theta_{vp}^{b_v}\right)}$$

(3) Batch b_v can be loaded on vessel *v* only after its date of availability:

$t - \mathrm{L}_v + 1 \ge \mathrm{C}_v^{b_v}$.

(4) If the handling of vessel *v* is restricted to working periods $(\gamma_v = 1)$, batches can start loading only during working periods: $\psi_{v(t-\mathrm{L}_v+1)} = 0$.

(5) Batch b_v is loaded on vessel *v* from its storage hangar *h*: $h = \mathrm{H}_v^{b_v}$.

Regarding the preventive maintenance activities, the decision variables $z_{pt}^{m_p}$ and $z_{st}^{m_s}$ determine the starting time *t* of performing maintenance m_p at berthing position *p* and maintenance m_s at a conveyor in section *s*, respectively. The existence of these two decision variables is subject to only one condition, each which states that each maintenance must be performed between its earliest and latest time: $\underline{\mathrm{R}}_p^{m_p} \le t \le \overline{\mathrm{R}}_p^{m_p}$ for berthing positions and $\underline{\mathrm{R}}_s^{m_s} \le t \le \overline{\mathrm{R}}_s^{m_s}$ for sections. When the maintenance has a fixed date, the indices of the decision variables are predetermined.

To facilitate the readability of the mathematical model, we represent each predicate by a simplified notation given in Table 3.

Table 3. Notation of predicates.

Predicate	*Notation*
$\mathrm{G}_{vp} = 1 \wedge \lambda_v \le \mathrm{Q}_p \wedge \mathrm{D}_v \le \mathrm{W}_p$	P_{vp}
$\mathrm{A}_v \le t - \mathrm{L}_v + 1 \le \mathrm{A}_v + \mathrm{I}_v \wedge \mathrm{K}_{(t-\mathrm{L}_v+1)} = 1 \wedge \psi_{v(t-\mathrm{L}_v+1)} = 0 \wedge$ $(1\text{-}\omega_v) + \omega_v \cdot \mathrm{O}_{\Upsilon_{v\left(\Gamma_{vt}+\Theta_{vp}-1\right)}} = 1$	P_{vpt}
$\mathrm{A}_v \le t - \mathrm{L}_v + 1 \le \Upsilon_{v\left(\Gamma_{v(\mathrm{A}_v+\mathrm{I}_v)}+\Theta_{vp}-\theta_{vp}^{b_v}\right)} \wedge t - \mathrm{L}_v + 1 \ge \mathrm{C}_v^{b_v} \wedge \psi_{v(t-\mathrm{L}_v+1)} = 0$	$\mathrm{P}_{vpt}^{b_v}$
$h = \mathrm{H}_v^{b_v}$	$\mathrm{P}_{vh}^{b_v}$
$\underline{\mathrm{R}}_p^{m_p} \le t \le \overline{\mathrm{R}}_p^{m_p}$	$\mathrm{P}_{pt}^{m_p}$

$\underline{\mathrm{R}}_s^{m_s} \le t \le \overline{\mathrm{R}}_s^{m_s}$	$\mathrm{P}_{st}^{m_s}$

The logical conditions of the existence of the decision variables x_{vpt}, $y_{vpth}^{b_v}$, $z_{pt}^{m_p}$ and $z_{st}^{m_s}$ are the following ones. For instance, the variable x_{vpt} exists only when the predicates P_{vp} and P_{vpt} are both "True".

$$x_{vpt} \in \{0,1\}, \forall v \in \mathscr{V}, \forall p \in \mathscr{P} \Big| \mathrm{P}_{vp}, \forall t \in \mathscr{T} \Big| \mathrm{P}_{vpt}$$

$$y_{vpth}^{b_v} \in \{0,1\}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v, \forall p \in \mathscr{P} \Big| \mathrm{P}_{vp}, \forall t \in \mathscr{T} \Big| \mathrm{P}_{vpt}^{b_v}, \forall h \in \mathscr{H} \Big| \mathrm{P}_{vh}^{b_v}$$

$$z_{pt}^{m_p} \in \{0,1\}, \forall p \in \mathscr{P}, \forall m_p \in \mathscr{M}_p, \forall t \in \mathscr{T} \Big| \mathrm{P}_{pt}^{m_p}$$

$$z_{st}^{m_s} \in \{0,1\}, \forall s \in \mathscr{S}, \forall m_s \in \mathscr{M}_s, \forall t \in \mathscr{T} \Big| \mathrm{P}_{st}^{m_s}$$

4.5.Mathematical model:

First, we define the intermediary variables μ_v and $\tau_v^{b_v}$, which give for each vessel *v* the berthing position in the decision variables x_{vpt} and $y_{vpth}^{b_v}$, respectively.

$$\mu_v = \sum\nolimits_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum\nolimits_{t \in \mathscr{T} | \mathrm{P}_{vpt}} p \cdot x_{vpt}, \forall v \in \mathscr{V}$$

$$\tau_v^{b_v} = \sum\nolimits_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum\nolimits_{t \in \mathscr{T} | \mathrm{P}_{vpt}^{b_v}} \sum\nolimits_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} p \cdot y_{vpth}^{b_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$$

$$\varepsilon_v = \sum\nolimits_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum\nolimits_{t \in \mathscr{T} | \mathrm{P}_{vpt}} t \cdot x_{vpt}, \forall v \in \mathscr{V}$$

$$\rho_v^{b_v} = \sum\nolimits_{p \in \mathscr{P} | \mathrm{P}_{vp}} \sum\nolimits_{t \in \mathscr{T} | \mathrm{P}_{vpt}^{b_v}} \sum\nolimits_{h \in \mathscr{H} | \mathrm{P}_{vh}^{b_v}} t \cdot y_{vpth}^{b_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$$

Similarly, we define for each vessel *v* the berthing and finishing times ε_v and π_v, by replacing $b \cdot x_{vpt}$ in μ_v by $t \cdot x_{vpt}$ and $\Upsilon_{v\left(\Gamma_{v(t-\mathrm{L}_v+1)}+\Theta_{vp}-1\right)} \cdot x_{vpt}$, respectively. Likewise, we define for each batch b_v to load on vessel *v*, the loading start and finishing times $\rho_v^{b_v}$ and $\sigma_v^{b_v}$ by replacing $p \cdot y_{vpth}^{b_v}$ in $\tau_v^{b_v}$, respectively, by $t \cdot y_{vpth}^{b_v}$ and $\Upsilon_{v\left(\Gamma_{v(t-\mathrm{L}_v+1)}+\theta_{vp}^{b_v}-1\right)} \cdot y_{vpth}^{b_v}$.

The mathematical model can be formulated as follows:

$$Max \sum_{v\in\mathscr{V}} (\beta_v \cdot w_v - \eta_v \cdot u_v) \quad (1) \qquad \text{or } Min \sum_{v\in\mathscr{V}} \pi_v \quad (1')$$

$$\sum_{p\in\mathscr{P}|\mathrm{P}_{vp}} \sum_{t\in\mathscr{T}|\mathrm{P}_{vpt}} x_{vpt} = 1, \forall v \in \mathscr{V} \quad (2)$$

$$\sum_{p\in\mathscr{P}|\mathrm{P}_{vp}} \sum_{t\in\mathscr{T}|\mathrm{P}_{vpt}^{b_v}} \sum_{h\in\mathscr{H}|\mathrm{P}_{vh}^{b_v}} y_{vpth}^{b_v} = 1, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \quad (3)$$

$$\sum_{t\in\mathscr{T}|\mathrm{P}_{pt}^{m_p}} z_{pt}^{m_p} = 1, \forall p \in \mathscr{P}, \forall m_p \in \mathscr{M}_p \quad (4)$$

$$\sum_{t\in\mathscr{T}|\mathrm{P}_{st}^{m_s}} z_{st}^{m_s} = 1, \forall s \in \mathscr{S}, \forall m_s \in \mathscr{M}_s \quad (5)$$

$$\mu_v = \tau_v^{b_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \quad (6)$$

$$\begin{cases} \rho_v^{b_v} \geq \varepsilon_v, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \middle| \mathrm{n}_v^{b_v} = 0 \\ \rho_v^{b_v} > \rho_v^{b'_v}, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \middle| \mathrm{n}_v^{b_v} > 0, \forall b'_v \in \mathscr{B}_v \middle| \mathrm{n}_v^{b'_v} = \mathrm{n}_v^{b_v} - 1 \end{cases} \quad (7)$$

$$\sigma_v^{b_v} \leq \pi_v, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v \quad (8)$$

$$\sum_{b_v\in\mathscr{B}_v} \sum_{p\in\mathscr{P}|\mathrm{P}_{vp}} \sum_{t'\in\mathscr{T}\left|\mathrm{P}_{vpt'}^{b_v} \wedge \Gamma_{vt'}+\theta_{vp}^{b_v}-1\leq\mathrm{T} \wedge t'\leq t\leq\Upsilon_{v\left(\Gamma_{vt'}+\theta_{vp}^{b_v}-1\right)}\right.} \sum_{h\in\mathscr{H}|\mathrm{P}_{vh}^{b_v}} y_{vpt'h}^{b_v} \leq 1, \quad (9)$$
$$\forall t \in \mathscr{T}, \forall v \in \mathscr{V}$$

$$\sum_{v\in\mathscr{V}} \sum_{b_v\in\mathscr{B}_v|\mathbf{P}_{vh}^{b_v}} \sum_{p\in\mathscr{P}|\mathbf{P}_{vp}} \sum_{t'\in\mathscr{T}\left|\mathbf{P}_{vpt'}^{b_v} \wedge \Gamma_{vt'}+\theta_{vp}^{b_v}-1\leq\mathrm{T} \wedge t'-\mathrm{L}_v+1\leq t\leq\Upsilon_{v\left(\Gamma_{vt'}+\theta_{vp}^{b_v}-1\right)}\right.} y_{vpt'h}^{b_v} \leq 1, \quad (10)$$
$$\forall t \in \mathscr{T}, \forall h \in \mathscr{H}$$

$$\sum_{v\in\mathscr{V}} \sum_{b_v\in\mathscr{B}_v} \sum_{p\in\mathscr{P}|\mathbf{P}_{vp}} \sum_{t'\in\mathscr{T}\left|\mathbf{P}_{vpt'}^{b_v} \wedge \Gamma_{vt'}+\theta_{vp}^{b_v}-1\leq\mathrm{T} \wedge t'-\mathrm{L}_v+1\leq t\leq\Upsilon_{v\left(\Gamma_{vt'}+\theta_{vp}^{b_v}-1\right)}\right.}$$
$$\sum_{h\in\mathscr{H}|\mathrm{F}_{sh}=1\wedge\mathbf{P}_{vh}^{b_v}} y_{npt'h}^{b_v} \leq \mathrm{U}_s - \sum_{m_s\in\mathscr{M}_s} \sum_{t'\in\mathscr{T}\left|t-\mathrm{R}_s^{m_s}+1\leq t'\leq t\wedge\mathbf{P}_{st}^{m_s}\right.} z_{st'}^{m_s}, \forall t \in \mathscr{T}, \forall s \in \mathscr{S} \quad (11)$$

$$\sum_{v\in\mathscr{V}} \sum_{p'\in\mathscr{P}|\mathrm{E}_p^{p'}=1\wedge\mathbf{P}_{vp'}} \sum_{t'\in\mathscr{T}\left|\mathbf{P}_{vp't'} \wedge \Gamma_{vt'}+\Theta_{vp'}-1\leq\mathrm{T} \wedge t'-\mathrm{L}_v+1\leq t\leq\Upsilon_{v\left(\Gamma_{vt'}+\Theta_{vp'}-1\right)}\right.} x_{vp't'} \leq$$
$$1 - \sum_{p'\in\mathscr{P}|\mathrm{E}_p^{p'}=1} \sum_{m_{p'}\in\mathscr{M}_{p'}} \sum_{t'\in\mathscr{T}\left|t-\mathrm{R}_{p'}^{m_{p'}}+1\leq t'\leq t\wedge\mathbf{P}_{p't}^{m_{p'}}\right.} z_{p't'}^{m_{p'}}, \forall t \in \mathscr{T}, \forall p \in \mathscr{P} \quad (12)$$

$$\left[\sum_{v\in\mathscr{V}_2\cup\mathscr{V}_3}\sum_{p\in\mathscr{P}|\mathrm{P}_{vp}}\sum_{t'\in\mathscr{T}|\mathrm{P}_{vpt'}\wedge t'=t+\mathrm{L}_v-1} x_{vpt'}\right]+$$
$$\left[\sum_{v\in\mathscr{V}}\sum_{p\in\mathscr{P}|\mathrm{P}_{vp}}\sum_{t'\in\mathscr{T}|\mathrm{P}_{vpt'}\wedge\Gamma_{vt}-\Theta_{vp}\geq 0\wedge t'=\Upsilon_{v\left(\Gamma_{vt}-\Theta_{vp}+1\right)}} x_{vpt'}\right]\leq \mathrm{M},\forall t\in\mathscr{T} \tag{13}$$

$$u_v \geq \pi_v - \delta_v, \forall v\in\mathscr{V} \tag{14}$$

$$w_v \geq \delta_v - \pi_v, \forall v\in\mathscr{V} \tag{15}$$

$$u_v - w_v = \pi_v - \delta_v, \forall v\in\mathscr{V} \tag{16}$$

$$u_v, w_v \geq 0, \forall v\in\mathscr{V} \tag{17}$$

Objective function (1) is based on an efficiency criteria that maximizes the difference between the despatch money and the demurrage charges of each vessel *v* while objective function ($1'$) is based on an effectiveness criteria that minimizes the finishing time (departure time) of each vessel *v*. Equation (2) ensures that each vessel *v* starts berthing at a unique berthing position *p*, and in a unique time period *t*. Equation (3) ensures that each batch b_v starts loading in a unique vessel *v* at a unique berthing position *p*, in a unique time period *t*, and is stored in a unique hangar *h*. Equation (4) ensures that maintenance m_p to be performed at a berthing position *p* has a unique start time. Similarly, equation (5) ensures that maintenance m_s to be performed at a conveyor in section *s* has a unique start time. Equation (6) ensures that berthing position *p* is the same in both decision variables x_{vtp} and $y_{vtph}^{b_v}$. Equations (7) ensures that the loading of batch b_v can only begin once vessel *v* has been berthed and all the batches that must precede b_v in the loading sequence have been loaded. Equation (8) ensures that each vessel *v* can only leave the port when all batches have been loaded. Equation (9) ensures that at most one batch can be loaded at the same time on each vessel *v*. Equation (10) ensures that only one batch at most can leave at a time from each storage hangar *h*. Equation (11) limits the number of identical parallel conveyors used simultaneously in a section *s* during the loading time of each batch due to the limit or/and the maintenance of conveyors. Equation (12) avoids the overlapping of vessels in each berthing position *p*, the simultaneous use of berthing positions that share a space of the quay since the berth layout of each quay is hybrid and the use of berthing positions where maintenance activities are performed (e.g., in Figure 2, berthing positions 3 and 5 share berthing position 3, consequently, they cannot be used simultaneously. Moreover, if maintenance is performed at berthing position 3, it will also be formed at berthing position 5. The opposite is also true.). Equation (13) limits the number of incoming and outgoing vessels to pass simultaneously through the navigation channel. Equations (14-17) determine the delay and the advance of each vessel.

5. Case study:

In this section, we describe one test instance of the case study and report the computational results. An example schedule obtained using the model is also given to illustrate a typical output (from another test instance). The formulations are written on Mosel and implemented in Xpress IVE Version 1.24.24, with 64 bits. All the tests are run on a server with an Intel® Xeon® Gold 6138 processor (8 cores) of 2.00 GHz processing speed and 32 GB of memory using the Xpress Optimizer Version 33.01.05 with the default options.

5.1. Test instances

Six test instances, based on data obtained from OCP group, a world leader in the phosphate industry, operating six quays in the bulk port of Jorf Lasfar in Morocco, recognized as the largest bulk port in Africa. Each set corresponds to the actual observation of vessels' expected arrivals during four weeks of different months (expedition range of 250,000t to 660,000t by 13 to 24 boats), and all their characteristics. The first set is used as a case study; the other ones will be presented in an extended version which is about to be submitted to a scientific journal. As the port management problem depends on the hangar configuration, conveyor network and production, the classical robustness test approach based on factors under control is problematic and must lay on arbitrary assumptions on upstream characteristics. The used samples share the same configuration, and the observed dispersion provides a kind of robustness proof. For all instances:

- We assume a 4-week planning horizon discretized into 1-hour intervals, hence T=672. We vary the decision time interval inside this planning horizon to handle term and medium-term decisions as explained in Section 4.3. For the short-term planning (e.g. during the first week in our instances) the accuracy of the planning is set to every hour. The decision time interval is set to four, eight and twelve hours for weeks two, three and four, respectively. Characteristics related to time periods (decision time-intervals, high-tide cycles and non-working periods) can be found in Table 10.

- We consider a navigation channel in which the maximum number of vessels allowed to pass simultaneously is limited to three vessels. We also consider two quays with hybrid berth layout, partitioned into five berthing positions each. Table 4 gives respectively, for each berthing position, the length (m), the minimum water depth (m), the productivity (t/h), the incompatibilities and the number of maintenance activities with the duration (h), the earliest and latest time (h) to perform each one.

Table 4. Characteristics of berthing positions.

	p	Q_p	W_p	Productivity ρ_p	$E_p^{p'}$										M_p	$R_p^{m_p}$		$\underline{R}_p^{m_p}$		$\overline{R}_p^{m_p}$	
					$p'=1$	$p'=2$	$p'=3$	$p'=4$	$p'=5$	$p'=6$	$p'=7$	$p'=8$	$p'=9$	$p'=10$		$m_p=1$	$m_p=2$	$m_p=1$	$m_p=2$	$m_p=1$	$m_p=2$
Quay 1	1	180	13.5	2000	1	0	0	0	0	0	0	0	0	0	2	12	10	24	328	48	328
	2	255	14.5	2000	0	1	0	0	0	0	0	0	0	0	1	7	0	405	0	450	0
	3	150	15.6	1000	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	4	150	15.6	1000	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	5 (3 ∪ 4)	300	15.6	2000	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
Quay 2	6	180	13.5	2000	0	0	0	0	0	1	0	0	0	0	2	29	14	266	320	266	320
	7	235	14.5	2000	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	8	125	14.5	2000	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
	9	125	15.6	2000	0	0	0	0	0	0	0	0	1	1	2	10	6	10	360	53	382
	10 (8 ∪ 9)	250	14.5	4000	0	0	0	0	0	0	0	1	1	1	1	23	0	300	0	320	0

- We consider nine storage hangars which are linked to all the berthing positions via a conveyor system (Figure 5). The latter is divided into two sections composed of different numbers of identical parallel conveyors. Table 5 gives for each section of the port conveyor system the number of identical parallel conveyors, the conveyors that belong to it and the number of maintenance activities with the duration ($R_s^{m_s}$), the earliest ($\underline{R}_s^{m_s}$) and latest time ($\overline{R}_s^{m_s}$) to perform each one.

Table 5. Characteristics of sections of the port conveyor system.

s	F_{sh}									U_s	M_s	$R_s^{m_s}$			$\underline{R}_s^{m_s}$			$\overline{R}_s^{m_s}$		
	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	$h=7$	$h=8$	$h=9$			$m_s=1$	$m_s=2$	$m_s=3$	$m_s=1$	$m_s=2$	$m_s=3$	$m_s=1$	$m_s=2$	$m_s=3$
1	1	1	1	1	0	0	0	0	0	3	2	12	8	0	282	284	0	282	286	0
2	1	1	1	1	1	1	1	1	1	4	3	6	10	9	213	290	362	213	290	390

- The case study data on vessels and cargos are given in tables 6 and 7. In the studied sets, the length of vessels varies between 100 and 300 meters, their draft varies between 5 and 15 meters; vessels having a draft over 14 meters are tide-dependent while leaving the port; the number of different fertilizers references to load on each vessel varies between one and three; the tonnage of fertilizers to load in a boat varies between 2.000 and 55.000 tones; 10% of vessels have a

SSHEX clause; the laydays are set arbitrarily at 48 hours for new vessels to charter, while already berthed and chartered vessels have fictitious laydays equal to one hour.

- For chartered vessels, the contractual demurrage is chosen randomly from a Uniform distribution between 50 and 150. The contractual dispatch is assumed half the demurrage. For new vessels to charter, we assume negligible demurrage and dispatch per hour. Note that these are fictitious values and are only used in order not to impact the economic results of already chartered vessels. Finally, for already berthed vessels, hourly demurrage and dispatch rates are assumed zero since no decisions need to be made for this group of vessels; they are already berthed.

- The maximum waiting time in the harbor is set arbitrarily at 72 hours for all the chartered vessels and one week for all the new vessels to charter. The latter have high maximum waiting times in the harbor in order not to affect the economic results of chartered vessels.
- The availability date $C_v^{b_v}$ of batch b_v to load on vessel v is lower or equal to the date of the expected arrival time A_v of vessel v.

Thanks to these conventions, all the vessels are dealt with together since there is no need to define specific constraints for each type of vessels (already berthed vessels, chartered vessels, and new vessels to charter).

5.2. Case study

In this section, we present a typical schedule that the port manager can obtain by using the proposed model.

It uses the characteristics of time periods, berthing positions, conveyor sections and vessels presented in the previous section. Table 6 gives the characteristics of 15 chartered vessels and 2 new vessels to charter. Table 7 gives the characteristics of batches to load, with their storage location and the concerned fertilizer (which is interesting to know to see the solution impact on conveyor use).

Table 6. Characteristics of vessels.

vessel	Expected Arrival Time	Vessel Length	Vessel Draft	Tide Dependent	Working Restriction	Laydays	Contractual handling time	Demurrage rate	Despatch rate	N_{vp} vessel v can berth at position p p =									
v	A_v	λ_v	D_v	ω_v	γ_v	L_v	J_v	η_v	β_v	1	2	3	4	5	6	7	8	9	10
1	1	194	8	0	0	1	9	101	50.5	1	1	1	1	0	1	1	1	1	0
2	12	197	9	0	0	1	17	59	29.5	1	1	1	1	0	1	1	1	1	0
3	22	188	8	1	0	1	17	111	55.5	1	1	1	1	0	1	1	1	1	0
4	55	190	10	0	1	1	11	105	52.5	1	1	1	1	0	1	1	1	1	0
5	107	182	9	0	0	1	14	97	48.5	1	1	1	1	0	1	1	1	1	0
6	171	128	7	0	0	1	3	101	50.5	1	1	1	1	0	1	1	1	1	0
7	174	129	5	1	0	1	2	64	32.0	1	1	1	1	0	1	1	1	1	0
8	190	191	9	0	0	1	11	94	47.0	1	1	1	1	0	1	1	1	1	0
9	203	131	5	0	1	1	2	61	30.5	1	1	1	1	0	1	1	1	1	0
10	299	201	11	0	0	1	25	143	71.5	1	1	1	1	0	1	1	1	1	1
11	440	207	11	0	0	1	23	111	55.5	1	1	1	1	0	1	1	1	1	1
12	480	158	9	0	1	1	20	130	65	1	1	1	1	0	1	1	1	1	0
13	514	231	13	0	0	1	25	147	73.5	1	1	1	1	0	1	1	1	1	1
14	541	112	5	0	0	1	2	131	65.5	1	1	1	1	0	1	1	1	1	0
15	400	122	7	1	0	48	2	93	46.5	1	1	1	1	0	1	1	1	1	0
16	430	174	10	0	0	48	11	56	28	1	1	1	1	0	1	1	1	1	0

Maximum waiting time $I_v = 48, \forall v$

Table 7 characteristics of batches

Vessel v	Batch b_v	Fertilizer	Quantity $\varphi_v^{b_v}$	Date of batch availability $C_v^{b_v}$	Storage hangar of batch b_v $H_v^{b_v}$	Batch level used in the loading batch sequencing $n_v^{b_v}$
1	1	6	2750	1	1	0
	2		2750	1	1	1
	3	3	3667	1	4	2
	4		3667	1	8	1
	5		3667	1	9	0
2	1	11	4125	1	9	0
	2		4125	1	8	1
	3		4125	1	4	2
	4		4125	1	5	3
	5		4125	1	4	3
	6		4125	1	9	2
	7		4125	1	9	1
	8		4125	1	9	0
3	1	11	4450	1	7	0
	2		4400	1	9	1
	3	6	3867	1	1	2
	4		3867	1	3	3
	5		3867	1	2	3
	6	3	3844	1	8	2
	7		3844	1	4	1
	8		3844	1	6	0
4	1	3	2750	1	10	0
	2		2750	1	12	1
	3	6	4074	1	8	2
	4		4074	1	8	2
	5		4074	1	6	1
	6		4074	1	8	0
5	1	2	4375	1	9	0
	2		4375	31	5	1
	3		4375	1	5	2
	4		4375	31	7	2
	5		4375	1	4	1
	6		4375	31	4	0
6	7	2	2750	1	3	0
	8		2750	22	2	0
7	1	2	3245	28	3	0
8	1	9	3767	1	12	0
	2		3767	1	12	1
	3		3767	1	11	2
	4	6	3413	1	1	2
	5		3413	1	3	1
	6		3413	15	2	0
9	1	3	3500	299	5	0
10	1	6	4415	341	2	0
	2		4415	369	1	1
	3		4415	395	1	2
	4		4415	204	1	3
	5		4415	232	3	4
	6		4415	258	2	4
	7		4415	122	2	4
	8		4415	150	3	3
	9		4415	176	2	2
	10		4415	286	2	1
	11	8	4450	313	3	0
11	1	8	4500	372	4	0
	2		4500	413	8	1
	3		4500	124	9	2
	4		4500	166	9	3
	5		4500	206	7	4
	6		4500	248	8	4
	7		4500	290	7	3
	8		4500	330	9	2
	9		4500	455	5	1
	10		4500	496	4	0
12	1	3	3967	393	5	0
	2		3967	430	4	0
	3		3967	466	5	0
	4		3967	503	5	1
	5		3967	540	8	2
	6		3967	576	6	3
	7	2	3828	600	12	3
	8		3828	600	10	3
	9		3828	600	10	2
	10		3828	600	12	1
13	1	1	4400	326	8	0
	2		4400	367	8	0
	3		4400	407	7	1
	4		4400	448	4	2
	5		4400	489	7	3
	6		4400	529	9	4
	7		4400	570	6	4
	8		4400	610	4	4
	9		4400	204	8	3
	10		4400	245	5	2
	11		4400	285	9	1
14	1	8	4400	537	4	0
	2		4400	578	4	0
	3		4400	618	7	0
	4		4400	659	8	1
	5		4400	699	6	2
15	1	1	3300	548	4	1
16	1	1	2350	637	2	0

$$C_v^{b_v} = A_v, \forall v \in \mathscr{V}, \forall b_v \in \mathscr{B}_v$$

Figure 9 shows an example of Gantt chart of vessel and batch schedule, drawn from the solution of another dataset. At its bottom, there is a timeline in hours and a one-week time frame (from Friday of week 2 to Monday of week 3) with high tide hours. The decision time-intervals are highlighted every four hours for week 2 and every eight hours for week 3. All constraints are respected by the solution given by the LBAP model for the case study of Figure 9.

- All vessels are berthed at restricted time periods for which a decision can be made and do not occupy berthing positions where maintenance is performed.
- The batches of each vessel are loaded in any order without downtime after their date of availability, and only one batch at most leaves at a time a storage hangar.

- Vessels 2 and 3 have high drafts and thereby are tide-dependent, so they occupy berthing sections with high water depth and leave the port during high tides.
- The handling of vessel 3 stops during non-working hours because it has a SSHEX clause.
- The handling time of vessel 2 is shorter than its contractual handling time since it occupies a berthing position with high productive loading equipment.
- The number of vessels passing through the navigation channel and used parallel conveyors per section do not exceed their limits. The number of allowed parallel conveyors per section can decrease due to maintenance.
- The optimal laycan proposed for the new vessel to charter 7 is [276, 323]. This vessel can berth at any time period during its laydays. We note that, a precise berthing time will be assigned to this vessel as its status changes from new vessel to chartered one and as the time progresses from week two towards week one.
- Table 8 details the vessel berthing schedule (time and location). Table 9 details the batch loading schedule. Figure 10 gives the batch loading schedule for vessel 3 to ensure ship stability; it can be noticed that the diversity of fertilizers to load implies switching fertilizers in conveyor transportation. Table 10 details the maintenance schedule of quay positions and conveyor sections, which is compatible with the berthing schedule.

Thanks to the integration of the LAP and the BAP, the port managers can propose laycans for the new vessels to charter considering the allocation of berthing positions to already chartered vessels and conveyors to batches, thereby avoiding the payment of demurrage charges, and knowing when to accept or refuse a new vessel to charter.

Table 8. Vessel berthing schedule

	Position	Vessel	START	END
Quay 1	2	3	22	39
	2	4	55	70
	2	5	107	124
	2	7	176	177
	2	8	212	223
	1	9	324	325
	2	11	564	593
	2	13	636	668
	1	15	576	577

	Position	Vessel	START	END
Quay 2	7	1	1	10
	7	2	12	35
	7	6	176	179
	10	10	424	445
	6	12	624	643
	7	14	744	758
	7	16	672	673

Table 9. Batch loading schedule

	Vessel v	Batch b_v	Start	End
Quay 1	3	1	24	26
		2	27	29
		3	34	35
		4	36	37
		5	38	39
		6	32	33
		7	30	31
		8	22	23
	4	1	55	56
		2	63	64
		3	68	70
		4	65	67
		5	60	62
		6	57	59
	5	1	110	112
		2	113	115
		3	119	121
		4	122	124
		5	116	118
		6	107	109
	7	1	176	177
	8	1	214	215
		2	218	219
		3	222	223
		4	220	221
		5	216	217
		6	212	213
	9	1	324	325
	11	1	567	569
		2	570	572
		3	579	581
		4	585	587
		5	591	593
		6	588	590
		7	582	584
		8	576	578
		9	573	575
		10	564	566

	Vessel v	Batch b_v	Start	End
Quay 1	13	1	636	638
		2	642	644
		3	651	653
		4	654	656
		5	663	665
		6	666	668
		7	660	662
		8	657	659
		9	648	650
		10	645	647
		11	639	641
	15 to charter	1	[576, 623]	[624, 577]
Quay 2	1 berthed	1	3	4
		2	7	8
		3	9	10
		4	5	6
		5	1	2
	2	1	12	14
		2	18	20
		3	27	29
		4	30	32
		5	33	35
		6	24	26
		7	21	23
		8	15	17
	6	1	178	179
		2	176	177

	Vessel v	Batch b_v	Start	End
Quay 2	10	1	424	425
		2	430	431
		3	434	435
		4	438	439
		5	442	443
		6	444	445
		7	440	441
		8	436	437
		9	432	433
		10	428	429
		11	426	427
	12	1	624	625
		2	628	629
		3	632	633
		4	636	637
		5	640	641
		6	642	643
		7	638	639
		8	634	635
		9	630	631
		10	626	627
	14	1	744	746
		2	750	752
		3	756	758
		4	753	755
		5	747	749
	16 to charter	1	[672, 719]	[673, 720]

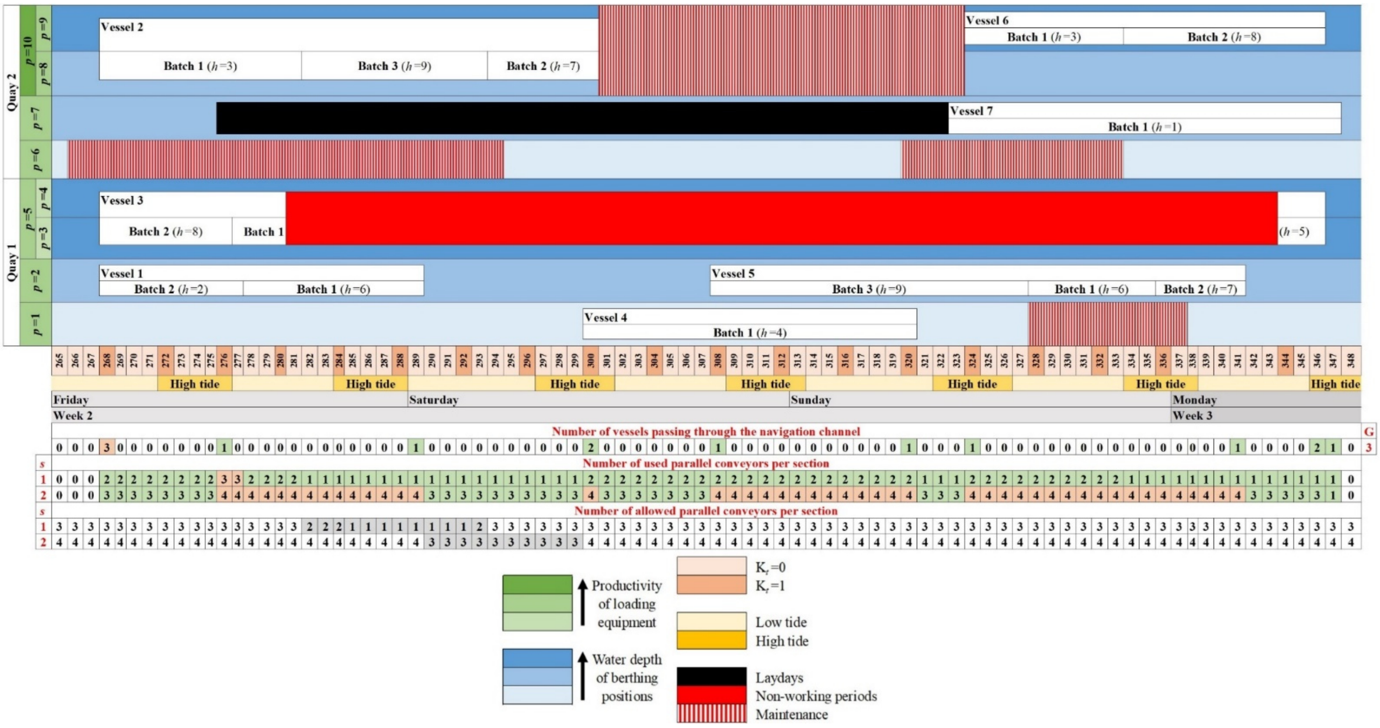


Figure 9. Example of Gantt chart of vessel berthing and loading.

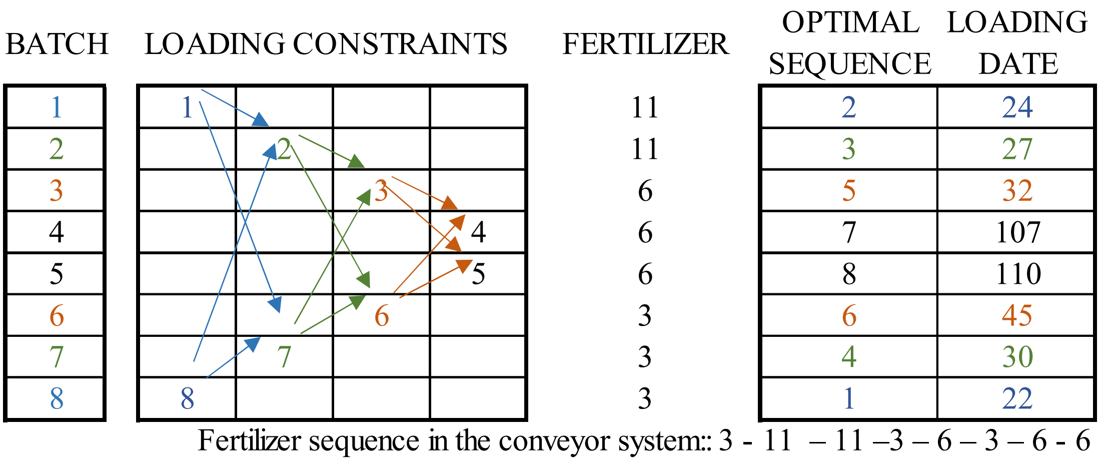


Figure 10. Batch loading schedule for Vessel 3.

Table 10. Maintenance scheduling.

	Quay Position	Maintenance Start	Maintenance End
Quay1	1	36	47
		328	337
	2	417	423
Quay 2	6	266	294
		320	333
	9	365	374
		20	25
	10	305	327

Conveyors Section	Maintenance Start	Maintenance End
1	282	293
	284	291
2	213	218
	290	299
	374	382

This instance and the five other ones are solved to optimality in computation times not exceeding one hour. Hence, we can assume that the integer linear programming model proposed for the LBAP can easily be used in bulk ports where such decisions need to be made frequently, with only commercial Software. Thus, developing a heuristic for the problem is not necessary.

6. Conclusions:

In this paper, we integrate the Laycan Allocation Problem and the dynamic hybrid Berth Allocation Problem in the context of tidal bulk ports with multiple quays and a conveyor system between storage hangars and berthing positions. While laycans concern only vessels for export, a symmetric approach can be applied for berthing decisions in the context of import ports. Our research is motivated by the bulk port of Jorf Lasfar, but it is also valid for any other bulk port. A new integer linear programming model is proposed to solve this integrated problem. The latter integrates two problems with different decision levels (tactical and operational) thanks to the modulation of the time-interval between decisions and the introduction of fictitious laydays for already berthed and chartered vessels.

Several characteristics are addressed simultaneously in the definition of the LBAP to make it closer to reality, such as the multiplicity of quays, navigation channel restrictions, conveyor routing constraints with preventive maintenance activities, the variation of water depth, vessel tide-dependency, the productivity of bulk-handling cranes, the multiplicity of cargo types on the same vessel, charter party clauses and non-working periods. Instead of expressing these characteristics by a set of constraints in the model, we used predicates to formulate them. These characteristics were already present in a previous model (Bouzekri et al., 2021), and this model also includes two new characteristics that must be considered for ports that deal with a variety of bulk to load: conveyor constraints modelled in a more general way and constraints of boat stability during loading. This approach is quite handy predicates are easy to modify in the model. Furthermore, they reduce the number of variables and constraints in the model and improve the computational performance. Moreover, the port conveyor system is modeled in a new way that does not list each route of conveyors between storage hangars and berthing positions, which makes easier the formulation of the problem.

This model assumes that the cargo to load on vessels is always available in the hangars; this assumption is verified in the used data set, but, in practice, it depends on the upstream supply chain that can work in a pull mode (production-to-stock) or a push mode (production-to-order) and on the variety of products to manage as the management difficulty increases with the product variety. An extension to this study could be searching for an integrated approach of the port, hangars and production management from a supply chain perspective.

References

Al-Hammadi, Jasem, and Ali Diabat. 2017. "An Integrated Berth Allocation and Yard Assignment Problem for Bulk Ports: Formulation and Case Study." *RAIRO - Operations Research* 51 (1): 267–84. https://doi.org/10.1051/ro/2015048

Barros, Victor Hugo, Tarcísio Souza Costa, Alexandre C.M. Oliveira, and Luiz A.N. Lorena. 2011. "Model and Heuristic for Berth Allocation in Tidal Bulk Ports with Stock Level Constraints." *Computers & Industrial Engineering* 60 (4): 606–13. https://doi.org/10.1016/j.cie.2010.12.018

Bierwirth, Christian, and Frank Meisel. 2010. "A Survey of Berth Allocation and Quay Crane Scheduling Problems in Container Terminals." *European Journal of Operational Research* 202 (3): 615–27. https://doi.org/10.1016/j.ejor.2009.05.031

Bierwirth, Christian, and Frank Meisel. 2015. "A Follow-up Survey of Berth Allocation and Quay Crane Scheduling Problems in Container Terminals." *European Journal of Operational Research* 244 (3): 675–89. https://doi.org/10.1016/j.ejor.2014.12.030

Bouzekri, Hamza, Gülgün Alpan, and Vincent Giard. 2019. "Integrated Laycan and Berth Allocation Problem." In 2019 *International Conference on Industrial Engineering and Systems Management* (IESM), 1–6. https://ieeexplore.ieee.org/document/8948110/

Bouzekri, Hamza, Gülgün Alpan, and Vincent Giard. 2021. "Integrated Laycan and Berth Allocation and Time-Invariant Quay Crane Assignment Problem in Tidal Ports with Multiple Quays." *European Journal of Operational Research* 293 (3): 892–909. https://doi.org/10.1016/j.ejor.2020.12.056

Burdett, Robert, Corry, Paul , Eustace, Colin and Smith Simon, "Scheduling pre-emptible tasks with flexible resourcing options and auxiliary resource requirements". *Computers & Industrial Engineering*, Volume 151, 2021, 106939, https://doi.org/10.1016/j.cie.2020.106939

Cheimanoff, Nicolas, Frédéric Fontane, Mohamed Nour Kitri, and Nikolay Tchernev. 2021. "A Reduced VNS Based Approach for the Dynamic Continuous Berth Allocation Problem in Bulk Terminals with Tidal Constraints." *Expert Systems with Applications* 168 (April): 114215. https://doi.org/10.1016/j.eswa.2020.114215

Corry, Paul, and Christian Bierwirth. 2019. "The Berth Allocation Problem with Channel Restrictions." *Transportation Science*, April, trsc.2018.0865. https://doi.org/10.1287/trsc.2018.0865

Directive 2001/96/EC of the European Parliament and of the Council of 4 December 2001 establishing harmonized requirements and procedures for the safe loading and unloading of bulk carriers. https://eur-lex.europa.eu/eli/dir/2001/96/oj

Ernst, Andreas T., Ceyda Oğuz, Gaurav Singh, and Gita Taherkhani. 2017. "Mathematical Models for the Berth Allocation Problem in Dry Bulk Terminals." *Journal of Scheduling* 20 (5): 459–73. https://doi.org/10.1007/s10951-017-0510-8

Fourer R. (2013) Algebraic Modeling Languages for Optimization. In: Gass S.I., Fu M.C. (eds) *Encyclopedia of Operations Research and Management Science*. Springer, Boston, MA. https://doi.org/10.1007/978-1-4419-1153-7_25

Karan, Chopra, Understanding Intact & Damage Stability of Ships, in *Naval Architecture, Marine Insight*, 2021, https://www.marineinsight.com/naval-architecture/understanding-intact-damage-stability-of-ships/

Krimi, Issam, Rachid Benmansour, Abdessamad Ait El Cadi, Laurent Deshayes, David Duvivier, and Nizar Elhachemi. 2019. "A Rolling Horizon Approach for the Integrated Multi-Quays Berth Allocation and Crane Assignment Problem for Bulk Ports." *International Journal of Industrial Engineering Computations* 10 (4), 577–91. https://doi.org/10.5267/j.ijiec.2019.4.003

Krimi, Issam, Raca Todosijević, Rachid Benmansour, Mustapha Ratli, Abdessamad Ait El Cadi, and Afaf Aloullal. 2020. "Modelling and Solving the Multi-Quays Berth Allocation and Crane Assignment Problem with Availability Constraints."

Journal of Global Optimization 78 (2): 349–73. https://doi.org/10.1007/s10898-020-00884-1

León, Alan Dávila de, Eduardo Lalla-Ruiz, Belén Melián-Batista, and J. Marcos Moreno-Vega. 2017. "A Machine Learning-Based System for Berth Scheduling at Bulk Terminals." *Expert Systems with Applications* 87 (November): 170–82. https://doi.org/10.1016/j.eswa.2017.06.010

Liu, Baoli, Zhi-Chun Li, Dian Sheng, and Yadong Wang. 2021. "Integrated Planning of Berth Allocation and Vessel Sequencing in a Seaport with One-Way Navigation Channel." *Transportation Research Part B: Methodological* 143 (January): 23–47. https://doi.org/10.1016/j.trb.2020.10.010

Lorenzoni, Luciano Lessa, Hannu Ahonen, and Arlindo Gomes de Alvarenga. 2006. "A Multi-Mode Resource-Constrained Scheduling Problem in the Context of Port Operations." *Computers & Industrial Engineering* 50 (1–2): 55–65. https://doi.org/10.1016/j.cie.2005.11.001

Menezes, Gustavo Campos, Geraldo Robson Mateus, and Martín Gómez Ravetti. 2017. "A Branch and Price Algorithm to Solve the Integrated Production Planning and Scheduling in Bulk Ports." *European Journal of Operational Research* 258 (3): 926–37. https://doi.org/10.1016/j.ejor.2016.08.073

Pratap, Saurabh, Ashutosh Nayak, Akhilesh Kumar, Naoufel Cheikhrouhou, and Manoj Kumar Tiwari. 2017. "An Integrated Decision Support System for Berth and Ship Unloader Allocation in Bulk Material Handling Port." *Computers & Industrial Engineering* 106 (April): 386–99. https://doi.org/10.1016/j.cie.2016.12.009

Ribeiro, Glaydston Mattos, Geraldo Regis Mauri, Saulo de Castro Beluco, Luiz Antonio Nogueira Lorena, and Gilbert Laporte. 2016. "Berth Allocation in an Ore Terminal with Demurrage, Despatch and Maintenance." *Computers & Industrial Engineering* 96 (June): 8–15. https://doi.org/10.1016/j.cie.2016.03.005

Robenek, Tomáš, Nitish Umang, Michel Bierlaire, and Stefan Ropke. 2014. "A Branch-and-Price Algorithm to Solve the Integrated Berth Allocation and Yard Assignment Problem in Bulk Ports." *European Journal of Operational Research* 235 (2): 399–411. https://doi.org/10.1016/j.ejor.2013.08.015

Tang, M., D. Gong, S. Liu, and H. Zhang. 2016. "Applying Multi-Phase Particle Swarm Optimization to Solve Bulk Cargo Port Scheduling Problem." *Advances in Production Engineering & Management* 11 (4): 299–310. https://doi.org/10.14743/apem2016.4.228

Umang, Nitish, Michel Bierlaire, and Ilaria Vacca. 2013. "Exact and Heuristic Methods to Solve the Berth Allocation Problem in Bulk Ports." *Transportation Research Part E: Logistics and Transportation Review* 54 (August): 14–31. https://doi.org/10.1016/j.tre.2013.03.003

UNCTAD. 2019. Review of Maritime Transport. United Nations Conference on Trade and Development. https://unctad.org/system/files/officialdocument/rmt2019_en.pdf

Unsal, Ozgur, and Ceyda Oguz. 2019. "An Exact Algorithm for Integrated Planning of Operations in Dry Bulk Terminals." *Transportation Research Part E: Logistics and Transportation Review* 126 (June): 103–21. https://doi.org/10.1016/j.tre.2019.03.018

Zhen, Lu, Zhe Liang, Dan Zhuge, Loo Hay Lee, and Ek Peng Chew. 2017. "Daily Berth Planning in a Tidal Port with Channel Flow Control." *Transportation Research Part B: Methodological* 106 (December): 193–217. https://doi.org/10.1016/j.trb.2017.10.008

Appendix

Table 11. High tide periods.

O_t HIGH TIDE PERIOD for vessels 3, 7 & 15 (ω_v =1 for v =3,7 & 15)

	DAY																																	
HOUR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
6	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0
7	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0
8	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0
9	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
10	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1
11	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1
12	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1
13	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1
14	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
19	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0
20	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0
21	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
22	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1
23	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1
24	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1

Table 12. Non-working periods.

ψ_{vt} NON-WORKING PERIODS for Vessels 4, 9 & 12 (γ_v =1 for v = 4,9,12)

		DAY																																	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
HOUR	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	2	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	3	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	4	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	5	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	6	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	7	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1
	8	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	9	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	10	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	11	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	12	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	13	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	14	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	15	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	16	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1
	17	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	18	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	19	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	20	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	21	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	22	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	23	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1
	24	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	1	1