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A COMPARISON OF DESCRIPTIVE APPROACHES TO MULTIPLE-CRITERIA DECISION MAKING PROBLEMS (*)

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UNE COMPARAISON D'APPROCHES DESCRIPTIVES DES PROBLEMES DE DECISION MULTICRITERE

RESUME

Etant donné un ensemble de M actions réalisables évaluées sur N critères, nous envisageons le problème suivant : choisir une action k, la préférée à toute autre action de par son évaluation X_{kj}, $j = 1, 2, \ldots, N$.

Stewart (1979) a traité ce problème à l'aide de deux techniques descriptives : l'analyse factorielle et l'analyse des correspondances, qui ont été appliquées à deux cas pratiques concernant le choix de la localisation d'une installation et le choix d'une voiture.

Le but de ce cahier est de montrer comment on peut utiliser l'analyse des concordances et l'analyse des proximités pour résoudre ce problème. Nous procédons à une comparaison des résultats obtenus par les différentes méthodes en réutilisant les données de Stewart.

La méthode utilisant l'analyse des concordances et des proximités, parce qu'elle permet d'attribuer des poids aux différents critères, est montrée comme étant d'une grande souplesse. De plus, il est possible d'introduire un seuil lorsque l'on procède à des comparaisons par paires sur chaque critère. Ce seuil doit être dépassé pour qu'une action puisse être considérée comme préférable à une autre. Des indices permettant de mesurer la qualité de l'ajustement du classement des actions sont également proposés.

Enfin, le classement est présenté de façon à faire apparaître l'ampleur de la différence entre chaque action et deux actions fictives qui ont respectivement la meilleure et la pire des évaluations possibles.

A COMPARISON OF DESCRIPTIVE APPROACHES TO MULTIPLE-CRITERIA DECISION MAKING PROBLEMS

ABSTRACT

Given a set of M feasible alternatives and an evaluation of each alternative on a set of N criteria, the following problem can be defined : select an alternative k, with a collection of criteria values (X_{kj}, j = 1, 2, ..., N), which is preferred to any other alternative.

Stewart (1979) has applied two descriptive techniques, factor analysis and correspondence analysis, to this problem. The techniques have been applied to two practical problems, first, a facility location problem and second, a car selection problem.

The purpose of this paper is to show how concordance analysis and multi-dimension scaling can be used to tackle the problem. Using the same data sets as Stewart a comparison of the results is given. It is suggested that the method involving concordance analysis and factor analysis offers greatest flexibility, because weights can be assigned to criteria. Also, a threshold can be introduced when pairs of alternatives are compared with respect to each criterion. Third, indices to measure the goodness of fit of the classification of the alternatives are provided. Finally, the classification is provided in such a way as to show the magnitude of the difference between each alternative and hypothetical best or worst alternatives.

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1. INTRODUCTION

Consider the problem of selecting a best alternative from a set of M feasible alternatives given an evaluation of each alternative on a set of N criteria. The basic information for the problem is presented as an impact matrix of size (M x N). The value in each cell, x_{ij}, is the assessment of alternative i using criterion j. Interval or ordinal assessments can be used. Stewart (1979, p. 4) summarizes the problem succinctly:

> "The problem facing the decision maker is thus to select an alternative k such that the collection of criterion values (x_{kj}, j = 1, 2, ... N) is preferable to that of any other alternative."

Stewart (1979)* has applied two descriptive techniques, factor analysis and correspondence analysis, to this problem. He provides brief comments on these techniques and they are applied to two sets of data. The first considers a facility location problem with 9 alternatives and 16 criteria, the second addresses a car selection problem of 4 alternatives and 24 criteria. The data for the two problems are shown on Tables 1 and 2. In order to interpret the patterns produced by the two techniques an ideal alternative is incorporated into each matrix. The techniques do not use any information on the relative importance of criteria, as expressed by a weighting scheme or a preference structure.

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* A shorter version appears in Stewart (1981a).

Alternative	Criterion															
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	3	4	8	10	7	6	1	6	6	10	10	2	2	8	3	5
2	3	2	1	10	7	7	4	6	6	10	10	2	7	5	3	5
3	5	2	2	10	6	8	3	7	3	8	7	2	6	4	2	4
4	7	2	2	10	6	8	1	7	3	10	7	4	6	4	1	4
5	6	6	5	10	8	8	2	10	4	10	10	4	8	6	3	3
6	7	5	4	10	8	8	2	10	3	10	10	4	8	6	4	4
7	7	5	3	10	5	6	5	5	7	10	10	4	7	7	2	5
8	6	6	5	10	8	8	3	10	4	10	10	4	8	6	4	5
9	1	6	6	6	7	7	3	10	4	10	10	6	6	7	7	7
Ideal	7	6	8	10	8	8	5	10	7	10	10	6	8	8	7	7

TABLE 1: Facility location problem: criterion ratings

(Source: Stewart, 1979.)

Alternative	Criterion						
	Age		Make	Mileage		Bonus	
	Yrs.	Norma= lized	Rating	Thousand Miles	Norma= lized	Cash	Norma= lized
1	0	0	0.4	0	0.00	400	0.33
2	4	0.4	0.4	15	0.36	800	0.67
3	3	0.3	0.4	10	0.24	400	0.33
4	3	0.3	0.6	30	0.71	300	0.25
5	4	0.4	0.8	39	0.93	400	0.33
6	3	0.3	0.6	32	0.76	400	0.33
7	1	0.1	0.6	7	0.17	300	0.25
8	4	0.4	1.0	30	0.71	0	0.00
9	3	0.3	0.8	16	0.38	300	0.25
10	5	0.5	0.4	36	0.86	1 200	1.00
11	6	0.6	1.0	42	1.00	800	0.67
12	4	0.4	0.6	29	0.69	600	0.50
13	4	0.4	0.6	30	0.71	1 000	0.83
14	2	0.2	0.6	15	0.36	100	0.08
15	5	0.5	0.8	39	0.93	1 000	0.83
16	5	0.5	1.0	37	0.88	600	0.50
17	6	0.6	0.4	42	1.00	1 000	0.83
18	6	0.6	0.4	42	1.00	900	0.75
19	2	0.2	0.4	15	0.36	1 000	0.83
20	3	0.3	1.0	23	0.55	300	0.25
21	3	0.3	0.4	23	0.55	900	0.75
22	10	1.0	1.0	32	0.76	800	0.67
23	6	0.6	1.0	42	1.00	900	0.75
24	5	0.5	1.0	39	0.93	600	0.50
Ideal	0	0.0	1.0	0	0.00	1 200	1.00

TABLE 2: Car selection problem: criterion values

(Source: Stewart, 1979)

It is implicit that all criteria are equally important.

2. CONCORDANCE ANALYSIS VERSUS STATISTICAL APPROACHES

The purpose of this paper is to show how concordance analysis and multi-dimension scaling can be used to classify a set of alternatives, given that each alternative is characterised by scores on a set of criteria. Further, we will offer a method for carrying out the classification so that each alternative can be compared to a hypothetical ideal alternative. It is suggested that the closer an alternative is to this ideal, in the multi-dimension space, the more preferred is the alternative. Details of concordance analysis and multi-dimension scaling are given in Massam (1980) and summarized in Massam (1981) and they will not be repeated here. Data used by Stewart (1979) will be used to test our method and the results will be compared to those provided by Stewart (1979). We suggest that the use of concordance analysis and multi-dimension scaling as a procedure for tackling the problem as defined by Stewart are more flexible than the two statistical methods he used for the following reasons.

First, the method allows for the criteria to be weighted. By changing the relative importance of selected criteria, that is by assigning heavier weights to some criteria or a single criterion, it is possible to study the effect on the classification. This can be considered as a sensitivity test. With respect to this point Stewart (1980) notes "the main advantage of correspondence analysis is that various runs with different weights are not needed in order to judge their effect. In the correspondence analysis 'map', alternatives differ from the 'ideal' (or any other hypothetical alternative if desired) along various directions which can be related to criteria or combinations of them. Thus by inspection it is possible to see

a) if one alternative is further from ideal than another along a similar direction, then no change in criterion weights can alter this fact;

b) if one alternative is further from ideal than another along substantially different directions then sufficient weighting of criteria associated with the direction of the former will alter the relative distance."

The method discussed in this paper avoids the use of subjective interpretations implicit in inspections and the determination of similarity of direction.

Second, the method is based on pairwise comparisons of alternatives for each criterion, and in this step in the algorithm, we can define a threshold size which has to be exceeded in order for one alternative to be rated as different (superior or inferior) to another. If the impact values x_{ij} are not subject to any measurement errors, and if any difference between x_{ij} and $x_{i'j}$, where x_{ij} is the value for alternative i for criterion j, and $x_{i'j}$ is the value for alternative i' for criterion j, then the parameter defined as the just noticeable difference (J.N.D.) is set to 100. If the difference has to be at least five percent in order for one alternative to be preferred to another, the J.N.D.

for this particular criterion is set to 95. Each criterion has an associated value for the J.N.D.

Third, the level of agreement between the classification of the alternatives and the information from which it was derived is measured by two indices, first, an index of agreement (A) and second, Kruskal's stress coefficient (K). The use of these indices provides an objective measure of the goodness of fit of the classification. The indices should not be interpreted as having statistical significance, rather as objective functions. The higher the value of A (minimum = 0; maximum = 1.0) the better the fit, and the lower the value of K (minimum = 0; maximum = 1.0) the better the fit. The A index is used to describe the ordinal scale, whereas the K index is used to describe the interval scale. This latter scale may be in one, two or more dimensions.* In the examples discussed here a maximum of two-dimensions is used for the classification. It should also be noted that "a level of agreement is also available from the sum of eigenvalues in correspondence analysis". (Stewart, 1980)

Further, a classification of all alternatives is provided in such a way as to show the magnitude of the difference between each alternative and the hypothetical alternatives. These hypothetical alternatives can be defined in a variety of ways, for example, as

* Full details of these indices are included in Massam (1980), Chapter 6.

the ideal best, the worst, the second best, or the second worst. An elaboration on the reasons for selecting these hypothetical alternatives will be given later. A comparison of the classification with and without hypothetical alternatives is needed. Details of this exercise are included later. Stewart notes that:

"A magnitude of difference between each alternative and any hypothetical alternatives chosen is directly obtainable in correspondence analysis ..." (Stewart, 1980).

In the next section of the paper a summary of Stewart's results will be given, and the results using the concordance analysis and multi-dimension scaling.

It should be recognized that for each problem Stewart has attempted to identify the best small set of alternatives for presentation to decision makers. No formal objective routine is offered to identify the size or the members of this set.

Other multicriteria approaches include outranking methods as developed by Roy (1976), utility theory (Keeney and Raiffa, (1976)) and ordinal regression methods (Siskos, (1982)). The use of an interactive approach using principles of statistical inference has been presented by Stewart (1981b). A discussion on the use of a pairwise comparison method developed by Saaty (1980) is provided by Massam and Askew (1982). It is argued that this method would be used to determine a set of weights for inclusion in concordance analysis.

3. THE FACILITY LOCATION PROBLEM

According to Stewart's analyses of the nine alternatives, it is suggested that 7, 8 and 9 should be presented as the best ones. Alternative 9 is included only if criterion "O" is exceptionally important.

The following five experiments shown on Table 3 were defined for study using the concordance analysis and multi-dimensional scaling.

FACILITY LOCATION PROBLEM-EXPERIMENTS — TABLE 3

EXPERIMENTS	HYPOTHETICAL ALTERNATIVES	WEIGHTS	J.N.D.
1	NIL	All Equal	100
2	IDEAL (Stewart's)	All Equal	100
3	IDEAL (Stewart's)	"O" → 0.25; OTHER → 0.05	100
4	IDEAL and WORST	All Equal	100
5	IDEAL and WORST	"O" → 0.25; OTHER → 0.05	100

For each of the five experiments the ordinal and the interval scales are shown on Table 4. The values for the indices A and K are also given. As these values are very close to the theoretical best values, we can conclude that the scales correspond closely to the information from which they were derived.

A slight improvement is provided in the K values when two-dimension maps are produced. These maps are shown as Figure 1. For each map the K value is less than 0.05, these correspond to an almost perfect fit in each case.

FACILITY LOCATION PROBLEM
ROW SUM SCALES: ORDINAL VALUES

TABLE 4

	EXPERIMENTS				
	1	2	3	4	5
BEST		10	10	10	10
	8	8	8	8	8
	9	9	9	5	9
	5	5	6	6	6
	6	6	5	9	5
	7	7	7	7	7
	1	1	1	1	1
	2	2	2	2	2
	4	4	4	4	3
	3	3	3	3	4
WORST				11	11
A INDEX	0.97	0.98	0.90	0.94	0.94

FACILITY LOCATION PROBLEM
ONE-DIMENSION: INTERVAL VALUES

TABLE 4 cont'd...

EXPERIMENTS									
1		2		3		4		5	
8	-100.0	10	100.0	10	100.0	10	100.0	10	-100.0
6	- 69.2	6	53.4	8	42.0	8	56.4	8	-60.5
5	- 63.2	8	51.4	6	31.4	6	41.8	6	-49.5
9	- 46.4	5	44.8	9	16.3	5	38.8	9	-39.7
7	- 27.3	9	10.5	5	4.6	9	19.8	5	-31.5
1	7.9	7	1.4	7	-20.3	7	21.4	7	-16.1
2	15.5	1	-38.5	1	-32.4	1	5.8	1	-4.0
4	71.6	2	-46.2	2	-38.3	2	0.5	2	-3.9
3	100.0	4	-82.4	4	-71.7	4	-15.8	4	24.0
		3	-100.0	3	-100.0	3	-33.4	3	40.8
						11	-100.0	11	100.0
K_1	0.10		0.09		0.10		0.06		0.09
K_2	0.01		0.04		0.03		0.03		0.04

K_1: Kruskal's stress coefficient: one-dimension.

K_2: Kruskal's stress coefficient: two-dimensions.

If we consider experiments 2 and 4 (all criteria weighted equally) for both the ordinal and interval scales, and examine the top three alternatives it is observed that four alternatives, 8, 5, 6 and 9, occur with frequencies 4, 4, 3 and 1. If we consider experiments 3 and 5 (criterion 0 weighted heavily) then alternatives 8, 9, and 6 occur with frequencies 4, 4, and 4. From this information we would suggest that alternatives 8, 6, 9 and 5 be presented to the decision makers.

4. CAR SELECTION PROBLEM

On the basis of the correspondence analysis Stewart (1979) suggests that alternatives 2, 3, 7, 9 and 19 lie closest to the ideal. As such these should be presented to the decision maker for final selection. Interpretation of the factor analysis persuades Stewart (1979) to recommend alternatives 2, 19 and 13 as the best ones.

Initially three experiments were designed for testing using concordance analysis and multi-dimension scaling. The details of each experiment are provided on Table 5.

The results of the ordinal scale and the one-dimension interval scale are shown on Tables 5 and 6. The two-dimension maps are shown on Figure 2. These results show that alternatives 19, 13, 7 and 20 occur with frequency values of 5 each when the top five alternatives for each of the scales shown on Tables 6 and

and 7 are examined. The next most frequently occurring alternative is 1, with a frequency value of 3.

CAR SELECTION EXPERIMENTS

TABLE 5

EXPERIMENT	WEIGHTS	J.N.D.	HYPOTHETICAL ALTERNATIVES
6	All Equal	100	NIL
7	All Equal	100	+ideal (after Stewart)
8	All Equal	100	+ideal (after Stewart) + worst

CAR SELECTION PROBLEM ORDINAL SCALE

TABLE 6

	EXPERIMENTS (6)	(7)	(8)
		25	25
	19	19	19
	7	1	7
	1	7	1
	20	20	20
	13	13	13
	21	21	9
	9	9	21
	14	14	14
	3	3	3
	2	2	2
	15	15	15
	12	12	12
	16	16	16
	6	6	6
	22	22	22
	8	8	8
	4	24	24
	24	4	4
	10	10	10
	23	23	23
	5	5	5
	11	11	11
	17	17	17
	18	18	18
			26
A INDEX	0.77	0.79	0.80

CAR SELECTION PROBLEM INTERVAL ONE-DIMENSION SCALE

TABLE 7

EXPERIMENTS					
(6)		(7)		(8)	
		25	100.0	25	100.0
19	100.0	17	-82.7	10	-13.9
10	78.9	19	-73.1	13	-9.2
8	68.8	20	-62.6	20	-8.4
13	56.2	7	-62.6	7	-6.8
5	42.0	2	-60.3	16	-5.3
15	39.6	9	-60.1	15	-5.0
4	36.3	11	-59.3	1	-4.7
14	35.6	23	-56.3	24	-3.5
12	16.4	22	-50.5	12	-2.4
24	14.0	3	-49.8	6	0.2
3	10.4	14	-49.1	8	1.1
6	9.2	21	-48.0	9	1.8
16	8.3	6	-47.4	5	2.0
1	6.1	24	-39.7	22	2.4
22	-16.2	12	-39.6	4	2.5
11	-19.7	4	-39.4	11	5.0
7	-20.9	16	-38.9	14	5.1
23	-21.2	15	-38.3	3	5.7
9	-27.7	5	-37.7	23	6.3
2	-28.7	1	-36.5	21	8.2
21	-53.4	13	-27.4	2	8.7
20	-54.8	8	-26.3	19	17.8
17	-87.2	10	-17.9	17	26.7
18	-100.0	18	-100.0	18	35.9
				26	100.0
K_1 =	0.28		0.24		0.19
K_2 =	0.18		0.16		0.15

When the two-dimension maps for experiments 7 and 8 are examined (Figure 2) we note that the solutions tend to be degenerate. The hypothetical alternatives are different from all the existing alternatives and their addition causes a radical change to the pattern shown for experiment 6, (Figure 2) on this map alternatives 19, 13, 7 and 20 do not cluster.

Four more experiments were conducted using new definitions for the hypothetical alternatives in order to try to avoid degenerate solutions. Details of these experiments are given on Table 8.

CAR SELECTION PROBLEM EXPERIMENTS — TABLE 8

EXPERIMENT	CRITERIA (1)	VALUES * (2)	(3)	(4)	HYPOTHETICAL
9	1	0.8	7	1000	HIGH
10	1	1.0	7	1000	HIGH
11	1	0.8	7	1000	HIGH
	6	0.6	39	100	LOW
12	1	1.0	7	1000	HIGH
	6	0.4	42	100	LOW

J.N.D. - 100 in all cases
All criteria equal weights

* These values are derived from values shown on Table 2.

The results for the ordinal scale are shown on Table 9, and the interval one-dimension scales are shown on Table 10. Among the top five alternatives for each of experiments 4, 5, 6 and 7, we can identify alternatives 13, 7, 20, 19 and 1 as the closest to the ideal. These alternatives occur with frequencies 8, 7, 6, 6 and 6. These results confirm that of the initial set of twenty-four, the best ones appear to be 19, 13, 7 and 20.

The results of the two-dimension maps are shown on Figure 3. The degree of clustering of alternatives 19, 13, 7 and 20 appears to be greater on these than for the initial set of three experiments.

5. CONCLUSION

The experiments reported in this paper suggest that concordance analysis and multi-dimension scaling can be applied to problems of selecting a sub-set of alternatives from a larger set using impacts on multiple criteria.

The major characteristics and advantages of the concordance analysis and multi-dimension scaling approach have been given earlier in this paper and they are elaborated in Massam (1980, Chapter 6). With respect to the two empirical problems, a comparison of the results as presented by Stewart (1979) and derived in this paper is given on Table 11. There is a high degree of correspondence among the results.

Further work is needed to study the effects on the two-dimension maps of adding hypothetical alternatives into the initial matrix. For an illustration of these effects we can compare the plot for experiment 1 with plots for experiments 2, 3, 4 and 5 all on Figure 1, and the plot for experiment 6 with plots for experiments 7 and 8, on Figure 2 and experiments 9-12 on Figure 3. The addition of ideal alternatives serves to aid in the interpretation of the patterns and as such is a benefit, but what, if any, are the costs in terms of distortion and loss of information.

Finally, there remains the critical tests, namely, can these descriptive methods improve intuition and judgement to allow decision-makers to do their job more effectively, can we identify the advantages of these methods and persuade practitioners to employ them. There is need for clear understanding of the theoretical principles and implicit and explicit assumptions. Perhaps with the increase in usage of multicriteria methods as illustrated by Voogd (1980), Marchet and Siskos (1979), Marchet (1980) and Stewart (1979) for tackling real, non-contrived problems, the future augers well for the field.

CAR SELECTION PROBLEM ORDINAL SCALE

TABLE 9

	EXPERIMENTS			
	(9)	(10)	(11)	(12)
	25	25	25	25
	19	19	19	19
	1	1	7	7
	7	7	1	1
	20	20	20	20
	13	13	13	13
	9	21	9	9
	21	9	21	21
	14	14	14	14
	3	3	3	3
	15	2	15	15
	2	15	2	2
	12	12	12	12
	16	16	16	16
	22	6	6	6
	6	22	22	22
	8	8	8	24
	24	24	24	4
	4	4	4	8
	10	10	10	10
	23	23	23	23
	5	5	5	5
	11	11	11	11
	17	17	17	17
	18	18	18	18
			26	26
A INDEX	0.79	0.79	0.80	0.80

CAR SELECTION PROBLEM INTERVAL ONE-DIMENSION SCALE TABLE 10

EXPERIMENTS							
(9)		(10)		(11)		(12)	
25	-100.0	25	-100.0	25	-100.0	25	100.0
10	-34.9	19	-18.0	20	-20.0	19	31.3
8	-17.8	10	-2.3	7	-17.9	10	29.0
13	-12.7	13	7.1	10	-17.0	13	18.7
7	-6.4	1	11.6	13	-9.9	7	17.2
5	1.6	15	16.4	15	-9.4	1	14.7
15	1.7	7	16.9	16	-7.4	15	14.6
21	3.1	8	21.6	24	-6.5	16	8.7
24	4.5	16	28.2	1	-4.9	20	5.5
1	5.0	12	29.3	9	-2.1	24	2.9
16	6.7	24	30.7	5	5.1	5	2.0
12	18.2	5	31.5	12	8.2	4	0.7
6	22.3	6	38.6	22	8.7	12	0.4
4	24.5	4	38.9	4	9.5	6	-1.1
22	25.3	22	44.2	8	11.5	14	-5.3
3	26.8	14	46.9	6	11.6	3	-7.7
11	29.1	11	47.2	14	19.8	22	-7.8
23	34.2	3	47.3	3	21.1	9	-11.1
14	42.0	23	50.8	23	21.4	2	-11.4
2	43.0	2	54.8	11	22.0	23	-11.8
9	43.1	9	56.2	2	24.4	11	-12.5
20	44.4	20	67.4	19	24.5	8	-23.2
19	63.4	21	68.1	21	39.2	21	-24.4
17	80.5	17	80.3	17	52.9	17	-38.1
18	100.0	18	100.0	18	70.3	18	-51.4
				26	100.0	26	-100.0
K_1	0.29		0.25		0.28		0.23
K_2	0.17		0.16		0.18		0.16

K_1 and K_2 are defined on Table 3

COMPARISON OF RESULTS: STEWART - MASSAM

TABLE 11

	STEWART	MASSAM
FACILITY PROBLEM	9, 8, 7.	9, 8, 6, 5
CAR SELECTION PROBLEM	19, 13, 2.	19, 13, 7, 20

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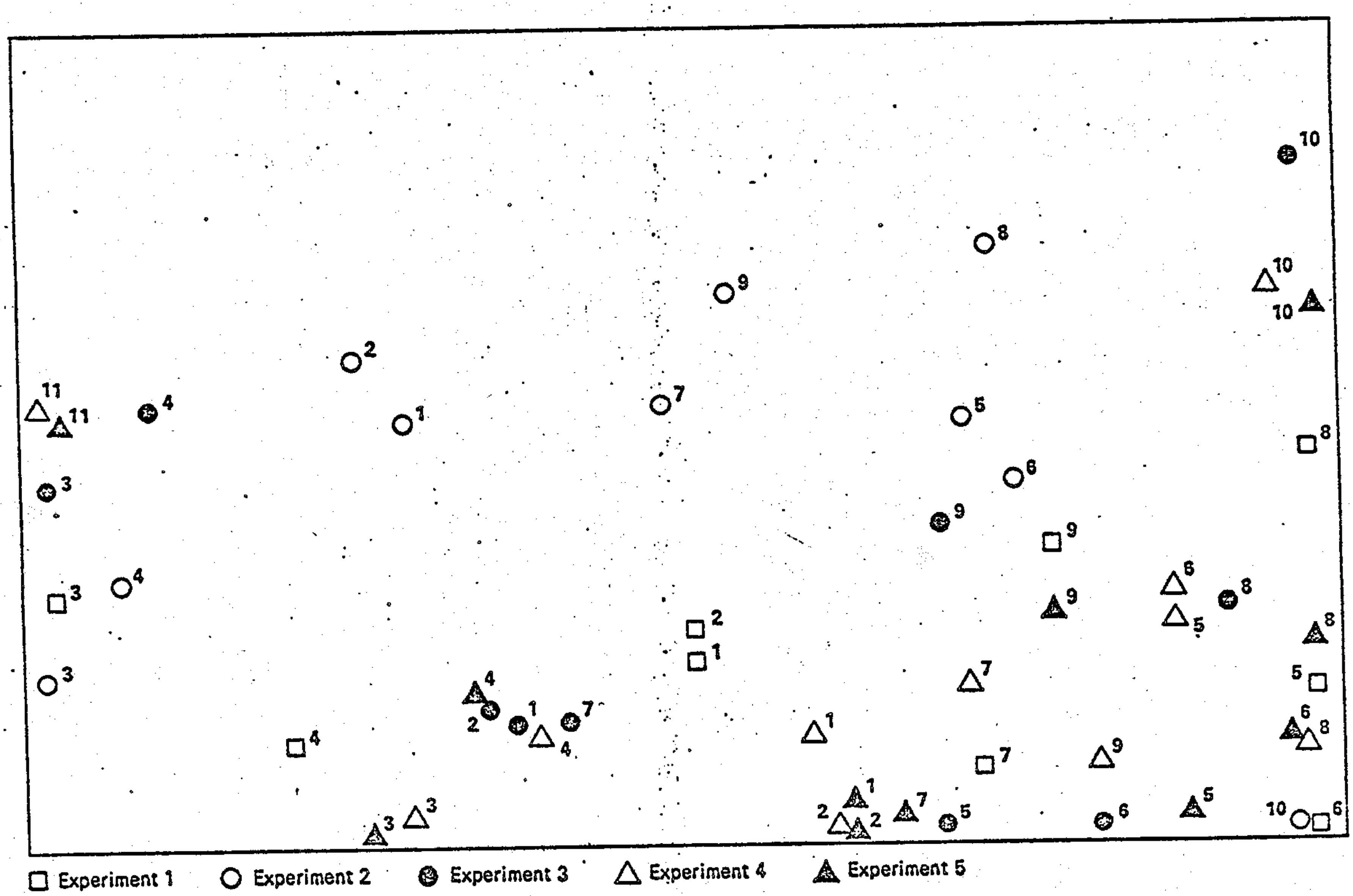
APPENDIX: FIGURES 1, 2, 3

Figure 1
FACILITY LOCATION PROBLEM
(Two dimension maps, experiments 1 to 5)



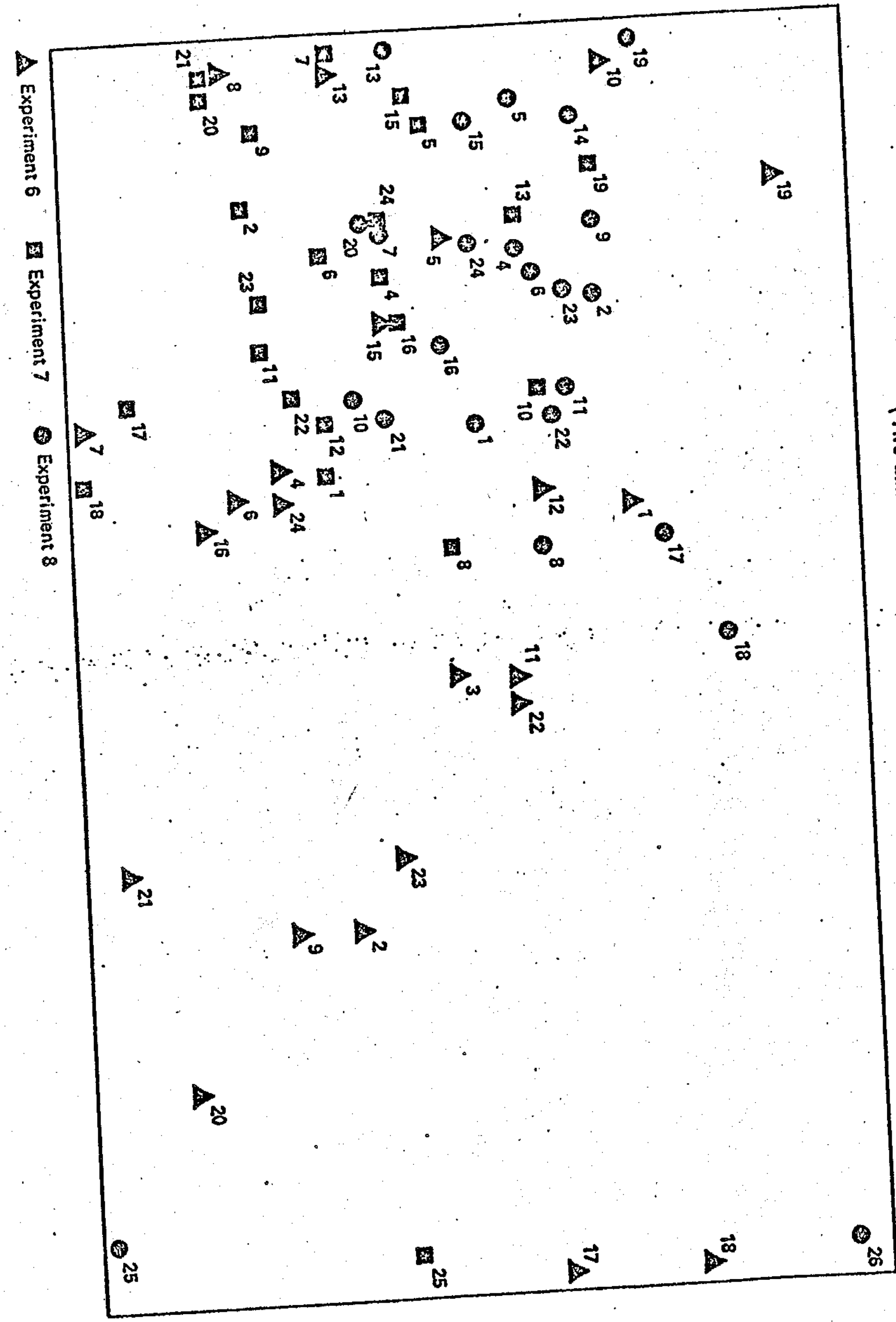


Figure 2
CAR SELECTION PROBLEM
(Two dimension maps, experiments 6, 7 and 8)

Figure 3
CAR SELECTION PROBLEM
(Two dimension maps, experiments 9 to 12)