# CAHIER DU LAMSADE

Laboratoire d'Analyse et Modélisation de Systèmes pour l'Aide à la Décision

(Université de Paris-Dauphine)

Unité Associée au C.N.R.S. nº 825

MAIN SOURCES OF INACCURATE DETERMINATION, UNCERTAINTY AND IMPRECISION IN DECISION MODELS (\*)

CAHIER Nº 75 février 1987

B. ROY

<sup>(\*)</sup> Conférence à paraître dans MUNIER B., SHAKUN M.F. (eds.) (1987) : Compromise, Negotiation and Group Decision, Reidel, Dordrecht-Boston-Tokyo.

# **CONTENTS**

		Pages
RES ABS	<u>UME</u> TRACT	I I
0.	INTRODUCTION  0.1 General framework  0.2 The traditional mono-criterion approach  0.3 The subject matter of the paper	1 1 1 2
1.	THE MAP IS NOT THE TERRITORY  1.1 Territories and maps  1.2 Omissions, simplifications, aggregations  1.3 The object is to compare territories, not maps	3 3 4 4
2.	THE FUTURE IS NOT A PRESENT TO COME  2.1 Uncertainty often masks inaccurate determination  2.2 Probability distribution and "punctualization" techniques  2.3 The object is to compare in a significant, not in an unambiguous, way	6 6 7 8
3.	THE DATA ARE NOT THE RESULT OF EXACT MEASUREMENT  3.1 Type I and Type II data  a) Type I: Data required to describe the attitudes, consequences or outcomes of a  b) Type II: Data required to reduce the complexity of an intermediate-stage map by aggregation procedures	9 9 10
	and punctualization techniques 3.2 Imprecision often masks inaccurate determination 3.3 The object is to consider crucial objective data individually, not to amalgamate them	10 11 13
4.	A MODEL IS NOT THE DESCRIPTION OF A REAL ENTITY INDEPENDENT  OF THAT MODEL  4.1 Descriptive and constructionist points of view  4.2 Conflictive and ambiguous situations  a) Situations devoid of hesitation or ambiguity b) Conflictive or ambiguous situations  4.3 Impact of modelling on the subject matter of the model	14 14 14 15 15
REF	ERENCES	18

# PRINCIPALES SOURCES D'IMPRECISION, D'INCERTITUDE ET D'INDETERMINATION DANS LES METHODES D'AIDE A LA DECISION

#### RESUME

Nous nous intéressons, dans cet article, à des modèles habituellement proposés pour l'aide à la décision ou à la négociation. Généralement, ceuxci mettent en jeu des relations de préférence entre des objets ou des actions en faisant intervenir un ou plusieurs critères. En pratique, on éprouve souvent de grandes difficultés pour fonder et/ou expliciter, de façon significative, de telles relations de préférence en raison des multiples sources d'imprécision, d'incertitude et d'indétermination. L'objet de cet article est d'approfondir les principales causes de ces difficultés et de parvenir à comprendre pourquoi le fait de considérer plusieurs critères peut aider à surmonter ces difficultés.

# MAIN SOURCES OF INACCURATE DETERMINATION, UNCERTAINTY AND IMPRECISION IN DECISION MODELS

#### ABSTRACT

In this paper, we consider models which are commonly proposed for decision-aid or negotiation-aid. By means of one or several criteria, they always bring preference relations between objects or actions into play. In practice, it is generally difficult to assess, in a significant way, such preference relations because there are many kinds of imprecision, uncertainty and inaccurate determination. The purpose of this paper is to analyse the main sources of such difficulties and to arrive at an understanding of why the consideration of several criteria may help alleviate these difficulties.

#### O. INTRODUCTION

### 0.1 General framework

In this paper A will designate a set of elements a, each a being viewed as an alternative, an action, or an object taken into account in a decision or a negotiation process. In the context of such processes, we will suppose that attributes, outcomes or consequences are defined so as to characterize, in an appropriate way, those alternatives, actions or objects.

We will focus attention on comparisons of elements from A when these comparisons are made in order to argue for, <u>a posteriori</u>, or clarify, <u>a priori</u>, certain decisions or compromises.

### 0.2 The traditional mono-criterion approach

All the models proposed (in economics, management science, operations research, game theory, ...) for decision-aid or negotiation-aid were, until recently, founded on a single criterion used to think about, elicit or justify the comparison between elements from A. More precisely, this traditional mono-criterion approach can be characterized as follows:

- a single cycle (value, utility, efficiency, entropy,  $\dots$ ) g(a) is attached to each a in A with the following properties :
  - g(a') = g(a) reflects or proves an indifference between a' and a, g(a') > g(a) reflects or proves a preference in favor of a' with respect to a;
- for the computation of the figure g(a), in other words in the definition of the criterion g, the alternative, action, or object a only intervenes through a description of it given by the attributes, outcomes or consequences chosen.

This means, in particular, that if  $g(a') - g(a) \neq 0$  and even if such a difference is very small, then it reflects or proves a well-defined pre-

ference. In order that conceptually and practically, such figures can play the role we have just assigned to them, it is important for them not to appear arbitrary with respect to diverse sources of inaccurate determination, uncertainty or imprecision. Let us imagine that, due to such sources, if we change even slightly the way a and a' are described or the value assigned to certain coefficients involved in the definition of g, we may modify the relations of equality or inequality between g(a') and g(a). If such an occurence is possible, the model's capacity for describing reality or arguing one way or another in a decision or negotiation context is considerably weakened.

The doubts and criticisms leveled against many models based on the traditional mono-criterion approach stem from the fact that the comparison between a' and a which is founded, as defined above, on a single comparison of two figures, g(a') and g(a), does not inspire confidence.

## 0.3 The subject matter of the paper

We propose to reach a deeper understanding of why, in practice, it proves so difficult and questionable to have indifferences and/or preferences founded on the traditional mono-criterion approach. As emphasized above, this is due to the fact that a great number of elements which appear to be ill-determined, too uncertain or imprecise have an important impact on the single figure g(a) used to characterize a. In our opinion, such elements come essentially from four sources. The first three are derived from the following observations:

- 1) The map is not the territory.
- 2) The future is not a present to come.
- 3) The data are not the result of exact measurements.

These three observations are related to what is currently analyzed in terms of "quality" of "data" which, at different levels, are used to calculate the value of each g(a) figure. The last of these four sources is derived from another observation:

4) The model is not the only description of a real entity independent of the model.

This observation is thus related to the very object of the model and to the complexity of its interactions with the modelling process.

The observations supply four sources of arbitrariness which limit the practical scope of any model used in the behavioral sciences. Nonetheless, as we shall endeavor to demonstrate below, the attendent problems posed for decision-aid or negotiation-aid could be more easily overcome if the traditional mono-criterion approach were modified so as to encompass one or two thresholds or, better yet, if we did not set ourselves the goal of founding comparisons on the elicitation of a single criterion.

### THE MAP IS NOT THE TERRITORY

# 1.1 Territories and maps (cf. BATESON (1975))

In order to compare complex real entities, i.e. territories, the model-ling process substitutes for each of them a more or less abstract synthesis, i.e. a map. With the traditional mono-criterion approach, the map which corresponds to the territory a, is the single figure g(a).

For example, let a be an alternative for the lay-out of a high-voltage line. To compare a and a' (two such complex real alternatives), it is necessary to take into account attributes and consequences dealing with finance, technology, economics, ecology, noise, aesthetics, etc. That is why the maps g(a) and g(a') must synthesize all of these heterogeneous figures (here the alternatives a and a') which confer upon each territory its own identity.

We would like to consider another example. Suppose that all of a country's main postal sorting centers should be equipped with similar parcelsorting machines. A set A of different types of proposed machines is

considered. Then the territory may bring into play the different kinds of costs, efficiency (out-flow per hour), a number of economic aspects, the risk of inaccurate sorting or of break-downs and accidents, the technical performance capacity (number of sorting destinations, the capacity of each computer), the time for building the machines and perhaps, too, the confidence in the manufacturer.

### 1.2 Omissions, simplifications, aggregations

The more complex or rich the territory is, the more difficult it is to build the map. In passing from territory to map, significant impoverishment occurs due to a large number of omissions or simplifications and to the necessity of aggregating many heterogeneous features. Such impoverishment inevitably introduces arbitrariness. This comes from the fact that there are several different ways, each equally valid, of proceeding to those omissions, simplifications and aggregations. It is impossible to claim that they all lead to the same system of equalities and inequalities when the map is a single figure g(a). Certain changes in the nature of omissions or simplifications, certain modifications in the aggregation procedures would result in different figures and for some pairs (a, a'). Indifference would then be transformed into preference, and preference in favor of a turned into preference in favor of a'.

#### 1.3 The object is to compare territories, not maps

Even when the territory is much simpler than those mentioned in the two examples above (see § 1.1), a map, even when it is not reduced to a single figure, cannot be identical to a given territory. Moreover, in passing from territory to map, it is impossible to avoid arbitrariness. Indeed, such arbitrariness increses as the number of figures constituting the map decreases. It follows that a map based on the elicitation of several criteria is often better suited to a comparison of two or more territories than one arrived at through the traditional mono-criterion approach.

Let us now consider that a designates an alternative, such as the route to be followed by a high-voltage line, or an object to be built, such as a parcel-sorting machine. Everything leads us to believe (cf. GRASSIN (1986), RENARD (1986)) than in either instance a is more readily discernible if it is represented by a vector, each component of which describes a's performance according to a criterion which involves only one well defined category of attributes or consequences of the same type, than if it is represented by a single number, obtained by somewhat obscure means, whose meaning is necessarily less clear.

In general, in order to arrive at a map which is a more faithful representation of the territory and is more intelligible than one derived from the traditional mono-criterion approach, we must look for support to axes of meaning (aesthetics, safety, ergonomics, deadlines, price paid, ...). Each of these should be homogeneous, comparatively well identified, familiar to the actors' way of thinking (i.e., occurring naturally within the process of decision-making or negotiation) in order to avoid aggregates which might require opting in favor of any single of the several value systems involved. We can thus (for more details, see ROY (1985)) characterize a by various figures (which here we would call performances) which indicate preferences limited to the axes of meaning chosen.

Too many theoretical works, presented as efficient tools for decision-aid or negotiation-aid, are more oriented towards the comparison of maps than towards the comparison of territories by virtue of the maps. However, anyone who endeavors to use a model for decision-aid or negotiation-aid should first of all ask what — is possible to infer from the comparison of maps, linked by the models to different elements within — A, concerning the comparison of territories which in actuality constitute — A. Seen in this light, the model's value is closely connected to the fact that the procedure for passing from territories to maps appears both intelligible and significant. This emphasizes one of the advantages of a multi-criteria over a mono-criterion approach.

### 2. THE FUTURE IS NOT A PRESENT TO COME

# 2.1 Uncertainty often masks inaccurate determination

As we emphasized in our introduction, any comparison of elements from A should be based on what the consequences of implementing each action a from A would be. In other words, the territories we should compare are those which exist after the high-voltage wire has been hung, aften the machine chosen in the different sorting centers has been built and installed. Yet the future almost always conceals something unpredictable or indeterminable.

When an investment is undertaken, it is usually impossible to say exactly how much it will cost. When a car is first produced, its market position is unknown except within a broad range. To explain why we cannot stand and wait for such a future (final cost of the investment, total number of cars sold), since deterministic knowledge of what it will bring is not conceivable, two types of reasons are commonly cited:

- the behavior of others ;
- environmental changes.

There is yet another reason for unpredictability which is usually forgotten. The action, alternative or object a under consideration is not and cannot be perfectly well-defined. In other words, a (the investment, the new car, ...) designates something which is not determined in a comprehensive way. So, the future often consists of defining more and more precisely what the real entity a will finally be.

Let us come back to the problem concerning how high-voltage lines should be hung. The exact route of a line a can be fully known only after it has been built. Yet we have to compare a to other lines well before it is built, while it is still in the planning stage. Some of the uncertainty concerning the consequences of a line a results from the fact that certain elements in the project are ill-determined at the time when the project must be compared with others. In other words, this inaccurate deter-

mination is inherent in the nature of a : this is also true of the uncertainty surrounding the resulting consequences of a.

Thus, in addition to the external factors responsible for uncertainty (mentioned at the beginning of the paragraph), there are internal factors of a given action a which result from carrying out the action. The impact of the internal factors is, in general, not distinguishable from the impact of the external factors. Decision-aid and negotiation-aid models must therefore take both types of factors into account.

# 2.2 Probability distribution and "punctualization" techniques

Whatever its origin, the uncertainty about the future is frequently taken into account through one or more probability distributions (see FISH-BURN (1970), KEENEY, RAIFFA (1976), SAVAGE (1954)). In practice, such probability distributions are nearly always defined in a rather arbitrary way. This is mainly due to the complexity of the phenomena which cause the external and internal factors mentioned above to come into play. Substituting fuzzy numbers for probabilistic considerations has not surmounted this fundamental difficulty (see DUBOIS (1983), LOOTSMA et al. (1986), NIJKAMP et al. (1985), PONSARD (1984), SKALA (1984)).

As an illustration, let us consider again the cost C(a) of an investment a. This is sometimes represented by a Gaussian random variable (see for instance KEENEY, NAIR (1976)). Even if some well-known theretical properties present arguments in favor of such a distribution, there may be good practical reasons for preferring a disymetric distribution. Moreover, other factors of arbitrariness appear in the computational rules, leading, for each a, to the expected cost m(a) and the standard deviation of the cost  $\sigma(a)$  on the basis of different characteristics of a.

Possible futures, whether or not they are described in probabilistic terms, are generally taken into account by a criterion g(a) by means of a single figure. This figure results from what we call a "punctualization technique", the object of which is precisely to substitute this single figure for a set of dispersed values which will be more or less credible

according to the possible future to which they refer. Any punctualization technique may be viewed as a procedure for aggregating, with respect to a given consequence (the cost, for example), the values which can occur in different hypothetical futures. With a large number of models which do not treat the future simply as the attaining of a time-to-come which can be read and foreseen in the present, we observe the use of punctualization techniques. Yet not one technique of this type can be singled out as the valid technique to use. Consequently, any choice at all in this area introduces some element, however slight, of arbitrariness.

We must always bear in mind that the axiomatic foundations of the utility theory (cf. FISHBURN (1970), KEENEY, RAIFFA (1976), VON NEUMANN, MORGENSTERN (1954)) (\*) give an especially privileged position to the expected value of the utility used as a punctualization technique. Nonetheless, we should not forget that:

- the axiomatic foundations can only legitimate the use of this technique within the context of a descriptive attitude (cf. BOUYSSOU (1984), ROY (1985b)) (\*\*);
- the assessment of the utility functions on which the computations of the expected value are based pose serious problems in practice, if not in theory (cf. COHEN, JAFFRAY (1987), FISCHER et al. (1986), de NEUFVILLE, DELQUIE (1987)).

# 2.3 The object is to compare in a significant, not in an unambiguous, way

Taking into account the uncertainty the future holds (primarily because the content of any a is ill-determined) by means of the expected value of a utility thus necessitates;

- first of all, a probabilistic description of various eventualities;
- then giving a numerical value, in terms of their utility, to each of these eventualities.

(\*\*) See also § 4.1.

<sup>(\*)</sup> Here we are interested only in instance where dispersion is due to the fact that consequences have been described in a probabilistic manner. The same type of problem occurs when consequences are dispersed in time or in space. Similar punctualization techniques might, therefore, be used in both instances (cf. ROY (1985) and GRASSIN (1986)).

Each of these two phases leads to difficulties which can only be surmounted by paying the price of introducing a not inconsiderable measure of arbitrariness. Under these conditions, the desire to construct a single criterion g which will allow us to compare any action a to any other action a', in the absence of any ambiguity according to the rules set forth in § 1.2, could, at least in some cases, seem to be an unreasonable goal. Should we not rather set ourselves the goal of establishing significant comparisons? We would thus emphasize the fact that the way in which a and a' are compared should be influenced as little as possible by the elements of arbitrariness just mentioned even if to achieve this we must give up the hoped-for absence of ambiguity in our mode of comparison.

Firstly, this would mean not automatically reducing (punctualizing) the data to a single figure. In other words, as we observe in the work of certain researchers, it is possible to take into account a risk criterion (risk of ruin) alongside an expected utility criterion or, again, a criterion which would reflect the greater or lesser reliability of the utility computation.

Secondly, an indifference threshold and/or a preference threshold (see BOUYSSOU, ROY (1987), ROY, BOUYSSOU (1986), ROY, VINCKE (1987)) can be linked to the calculation of the expected value of a utility (or to other criteria). We can thus take into consideration the non-significant character of a slight indifference g(a') - g(a) with which it seems legitimate to associate the indifference a' I a or, indeed, to introduce the possibility of a zone of hesitation between indifference and strict preference when this difference g(a') - g(a) is neither small enough to justify indifference nor large enough to correspond to a clearly established preference.

### 3. THE DATA ARE NOT THE RESULT OF EXACT MEASUREMENT

## 3.1 Type I and Type II data

Let us suppose now that the options required by the two preceding sources of uncertainty, imprecision and inaccurate determination have been se-

lected. This implies that we can reason here knowing precisely what simplifications and types of aggregations have been made, what description of the future and type of punctualization technique have been chosen to modelize each element a of A. All of these options are indispensable in bringing to the fore or causing to "emerge" from reality what are universally called "data". It is important to keep in mind that here, as elsewhere (see BATESON (1975)), that data are not like luggage or letters which objectively exist in various places where we can go to collect them. Data are the product of many options determining the nature of information, the form of the rules to be followed to compute different figures required to build the final map of a (for instance, the value of a single performance g(a) in the traditional mono-criterion approach).

For each datum considered, we should ask if it is a Type I or a Type II datum, as defined below.

# a) $\underline{\text{Type I}}$ : Data required to describe the attitudes, consequences or outcomes of a

Data of this type are closely linked to omissions, simplifications and ways of characterizing possible futures. They may be viewed as defining an intermediate-stage map, which is generally too complex to be used directly for decision- or negotiation-aid.

The following can be cited as examples: a list of figures for receipts and expenditures, the number of passengers, decibel levels, death statistics, probability distributions.

## b) Type II: Data required to reduce the complexity of an intermediatestage map by aggregation procedures and punctualization techniques

Data of this type are closely linked to the system of values of a given actor. They may be viewed as defining some individual characteristics required by a punctualization technique or some common unit needed for a synthesis of heterogeneous factors.

The following can be cited as examples: discounting rate, value of time, value of noise, value of life, weights in a weighted sum of heterogeneous performances, utility function describing an attitude towards risk, ...

### 3.2 Imprecision often masks inaccurate determination

Data lead to figures. Those figures are commonly viewed as the product of measuring procedures. Reference is often made to the precision of a datum, the existence of biases, or errors, the degree of approximation, ... In fact, for at least some data, this vocabulary is illusory because what is measured (namely, what we must refer to in order to speak in terms of precision, bias, errors, approximations, etc.) is ill-defined. To demonstrate this, we shall consider first Type I and then Type II data.

Type I data are, by definition, considered to be rough data, i.e. they appear directly as traces or as descriptions closely linked to objects or events. The instruments used to produce these traces and descriptions are always, to some extent, imperfect. For example,

- the accounting nomenclature is never quite appropriate to the cost we want to measure ;
- counting people one by one at a railway station or basing statistics on samples gives only approximate figures.

This explains why it seems natural to discuss the quality of a datum in terms of high or low precision. If we want to increase our degree of precision, we must unavoidably ask the question what do we want to measure: precisely what cost, exactly what traffic flow. In both instances, these questions are much more awkward than they might seem at first glance. For the "what cost" question, we shall refer the reader to RIVELINE (1985), limiting ourselves in the present paper to a brief investigation of the second example concerning traffic flow (for more details, see ROY et al. (1986)).

As far as traffic is concerned, it is obvious that our purpose is not to apprehend something like the exact number of passengers entering a given station between 12.00 and 2.00 p.m. on the 1st of January 1987. The traffic we are interested in is an average of such figures. Yet, for defining such an average, it is necessary to provide answers to questions such as the following:

- the average on a given platform, for the whole station or for a single line ?
- the average for which period of the year ? for the work-week, the week-end, or holidays ?
- the average at what time of day : rush hour, hight-time or all of these taken as a whole ?

This very simple example demonstrates that a Type I datum frequently involves an especially complex phenomenom which is unstable (in time as well as in space) and that a spate of hypotheses is required to define what is to be measured. When these hypotheses are specified in such a way as to reduce inaccuracies in determination as much as possible, it is not at all uncommon for what is measured to appear then as an aggregate which brings Type II data into play.

Type II data are much more removed from the object or event they are supposed to reflect or describe. A more or less sophisticated encoding system or a more or less formalized model is needed to connect a datum of this type to an object or an event. It is then difficult to speak in terms of measurement.

Even more than with Type I data, with Type II data, the very nature of the observed phenomenon is not strictly defined. Do we really know what is measured by :

- a discounting rate used for aggregating amounts of money over time throughout several consecutive periods;
- a value of time, of noise, of life used to aggregate lost time, noise, pollution, loss of human life:
  - a utility function used in a punctualization technique.

Such entities are, in part, produced or created by the measuring process (see ROY (1985b)). Under these conditions, how can we speak in terms of approximation, errors or bias?

# 3.3 The object is to consider crucial objective data individually, not to amalgamate them

We might think, in a general way, that the more comprehensive the point of view that a criterion g claims to represent is, the more the calculation of performances g(a) must call upon Type II data. If a criterion g's axis of significance is restricted to a category of attributes or consequences perceived as being of the same type and apprehendable in concrete terms, then the numerical values assigned to performance g(a) depends essentially on Type I data. Even if, for the reasons cited above, we cannot claim to measure these data with extreme precision, it is nonetheless true that each performance g(a) calculated in this way will most often seem to be relatively objective. The extent to which performance can vary due to inaccurate determination will remain relatively restricted. If, on the other hand, the criterion g is supposed to amalgamate attributes or consequences of a very heterogeneous nature (for example, apprehend concretely in dollars, hours, decibels, accident or death probabilities, ...), then the value of the performance g(a) may depend, in a fundamental way, on Type II data. To the extent that, for such data, the relationship between figures and the reality they represent is opaque, artificial and often the source of controversy, the values they receive will be more the reflection of personal values, or indeed partisan manipulations, than the trace of an objective reality. Decision- or negotiation-aid based on this type of criterion (especially if it is the only criterion used) quickly loses its scientific value.

When we do not automatically try to construct a single criterion, we can endeavor to isolate (cf. § 1.3 above) homogeneous axes of significance concerning restricted points of view and perceived as concrete enough, by the actors involved in the decision-making and negotiation processes, to constitute a basis of agreement. With exact of these points of view we may then associate a criterion (or pseudo-criterion)  $\mathbf{g}_i$  inasmuch as there should be a consensus concerning the fact that, according to the ith point of view, the diverse actions can be compared as the figures  $\mathbf{g}_i(\mathbf{a})$  (possibly involving the use of thresholds). This consensus and, by the same token, the models' objectivity are strongly conditioned by the level of impact which the Type II data that enter into the definition of the  $\mathbf{g}_i$  criteria adopted may have.

# 4. A MODEL IS NOT THE DESCRIPTION OF A REAL ENTITY INDEPENDENT OF THAT MODEL

#### 4.1 Descriptive and constructionist points of view

Let us now come back to the two traditional kinds of assertions introduced in  $\S 1.2$ :

- "a' is indifferent to a" denoted by a' I a;
- "a' is strictly preferred to a" denoted by a' P a.

In order to provide a basis of legitimacy for such assertions, two kinds of attitudes can be envisaged. We shall characterize these as descriptive and constructionist respectively.

The descriptive attitude refers to opinions which are assumed to exist somewhere and which are linked to an identified actor Z. They may be opinions which exist in the mind of Z whan Z is an individual. They may be the opinions shared by members of a committee when Z is a committee, or a consensus opinion when Z is a community. The relational system of preferences (I, P) considered is then viewed as the reflection of such an existing reality. The validity of the description composed of the two binary relations I and P defined on A comes exclusively from its conformity with the pre-existing reality of opinions.

With the constructionist attitude, assertions are seen as a product resulting from hypotheses and ways of working. These hypotheses and ways of working are proposed as appropriate bases for building, arguing for or against bringing about changes in the opinions which are the subject matter of the assertions. The validity of the relational system of preferences (I, P) built on A comes, in this case, from the fact that a given actor Z recognizes that it is able to play a role in informing decisions or guiding negotiations.

### 4.2 Conflictive and ambiguous situations

Let us consider an actor Z (to start with, we will suppose that Z is an individual) who has to compare two given alternatives, actions or

objects a' and a. He might find himself in one or the other of two categories of situations, namely:

## a) Situations devoid of hesitation or ambiguity

The attributes or consequences which argue in favor of a' or a in comparing the two actions are such that Z encounters no hesitation or amgibuity in choosing between a' P a, a P a' and a' I a. In other words, Z, by virtue of his own intense convictions, thinks he is capable of coming to a clear conclusion. He is sure his judgment in matters of preference is sound (at least with reference to his own system of values).

### b) Conflictive or ambiguous situations

Z experiences great difficulties in arriving at a conviction due to what he sees as conflicting arguments or inaccurate determination, uncertainties and imprecision resulting in ambiguity. In such situations, Z may be unable to arrive at a conclusion. Nevertheless, if Z is obliged to reach a conclusion, then he may arrive at a conclusion which could change later and/or could, seen in a certain light, seem to contradict other conclusions (by including some intransitivities, for example).

The conclusion arrived at by the individual Z (in each of these two categories of situations) could be in disagreement with those arrived at by other individuals who, with Z, make up the entity whose preferences are the subject of the model. When disagreement exists, we will consider that, for this entity (an "actor" made up of more than one individual), the situation is conflictive or amgibuous. Here, too, the situation may generate instability and/or contradictions.

### 4.3 Impact of modelling on the subject matter of the model

In management sciences, as in all the behavioral sciences, it seems impossible to deny the existence of confictive and ambiguous situations and the issues their very presence raises (cf. KARNI, SAFRA (1987), LESOURNE (1977), MCCORD, LEOTSARAKOS (1987), SOBEL (1987)). However, the presence

of such situations gives a privileged position to the influence the work involved in modelling can have on the way we arrive at judgments in matters of preference. This presence thus restricts the capacity of the descriptive as well as of the constructionist attitudes. We would like to illustrate these considerations by placing ourselves within the framework of the traditional mono-criterion approach.

Let us consider the relational system of preferences (I, P) defined on A by means of a single criterion g according to the traditional rules of performance comparisons (see  $\S 1.2$ ). When g is the product of a descriptive attitude, we expect complete conformity from the system (I, P)vis-a-vis the reality we want to describe. The meaning of this conformity is clear when we are confronted with situations devoid of hesitation and ambiguity. Yet, how can we give conformity a meaning when we are faced with situations of the conflictive or ambiguous type? To achieve this goal, it is recommended (cf. KEENEY, RAIFFA (1976)) that we resolve any instabilities or contradictions in order to structure preferences according to a weak-order (I\*, P\*) defined on A. This weak-order can then be represented by a function g. We should ask, however, what the conformity thus obtained signifies, since it refers to a reality which is itself, in part, shaped by a model which claims only to describe, but which, in fact, contributes at least partially to constructing that which i represents. In other words, there is a measure of inaccurate determination that the model helps to resolve in the interaction between reality and an attitude whose descriptive capacity is necessarily limited.

When we automatically adopt a constructionist attitude, it is important for the hypotheses and ways of working to be made explicitly clear and accepted as bases for building, justifying and bringing about changes in the preferences under considerration. This is only possible if the conclusions they lead to, in relation to situations initially considered to be devoid of hesitation and ambiguity, are seen to be acceptable (either because they conform to our initial convictions or because they have brought about changes in them). If this obtains, the single criterion g can be viewed as a tool for eliminating all conflictive and ambiguous situations. The way in which the model resolves conflicts, stabilizes fluctuating positions or decides between contradictions no longer claims to be descriptive, but it may be

easily contested, especially if certain conclusions seem shocking or simply not very favorable to certain actors. The latter are thus naturally led to question the hypotheses and/or ways of working. Here again, we cannot readily avoid interaction.

Whatever the attitude (descriptive or constructionist), whether it involves the mono-criterion approach or not, we observe that, due to the existence of conflictive or ambiguous situations, we cannot, in dealing with concrete problems, easily avoid the type of interaction between reality and the modelling process which bases and forms these hypotheses and procedures on this reality, while, at the same time, contributing to enriching and/or altering this same reality. In order for the work involved in modelling to play a role in the processes of decision-making or negotiation, it would seem essential for the interaction it entails (whether its aim is to describe what it contributes to producing or to get the hypotheses and ways of working it generated accepted) not to occur, due to the model, within a framework which would appear to the actors as a petrified yoke of arbitrary elements.

The traditional mono-criterion approach often elicits such a reaction. This is due not only to certain reasons cited in § 1.3, 2.3 and 3.3, but also to the very restrictive structure of the weak-order, which is the only one considered in this type of approach. The way in which it forces us to resolve conflicts or remove contradictions can give rise to arbitratiness. This explains the interest (cf. ROUBENS, VINCKE (1985), ROY (1985a), ROY, VINCKE (1987), VALADARES-TAVARES (1987)), which it increasingly focuses on other structures which leave room for incomparability and/or hesitation between indifference and strict preference. These structures seem less coercive in nature. The new structures lead directly to taking thresholds and, more importantly, several criteria into account. This is our final point, which highlights to what extent the way of taking imprecision, uncertainty and inaccurate determination into account can be changed by taking more than a single criterion into consideration.

## **REFERENCES**

- BATESON G. (1975): Step to an Ecology of Mind, Ballantine.
- BOUYSSOU D. (1984) : Approches descriptives et constructives d'aide à la décision : Fondements et comparaisons, Université de Paris-Dauphine, Thèse de 3e Cycle.
- BOUYSSOU D., ROY B. (1987) : "La notion de seuils de discrimination en analyse multicritère",  $\underline{\mathsf{INFOR}}$  (forthcoming).
- COHEN M., JAFFRAY J.Y. (1987): "Preponderence of the Certainty Effect over Probability Distorsion in Decision Making under Risk", in MUNIER B. (ed.), Risk, Decision and Rationality, D. Reidel Publishing Company, Dordrecht/Boston (forthcoming).
- DUBOIS D. (1983): Modèles mathématiques de l'imprécis et de l'incertain en vue d'applications aux techniques d'aide à la décision, Université Scientifique et Médicale de Grenoble, Thèse d'Etat.
- FISHBURN P.C. (1970): <u>Utility Theory for Decision Making</u>, John Wiley and Sons, New York.
- FISHER G.W., KAMLET M.S., FIENBERG S.E., SCHKADE D. (1986): "Risk Preferences for Gains and Losses in Multiple Objective Decision Making", <u>Management Science</u>, Vol. 32, No. 9, September, 1065-1086.
- GRASSIN N. (1986): "Construction 'Population' Criteria for the Comparison of Different Options of a High Voltage Route", <u>European Journal of Operational Research</u> 26, 42-47.
- KARNI E., SAFRA Z. (1987: "'Preference Reversals' and the Theory of Decision Making under Risk", in MUNIER B. (ed.), <u>Risk</u>, <u>Decision and Rationality</u>, D. Reidel Publishing Company, Dordrecht-Boston (forthcoming).
- KEENEY R.L., NAIR K. (1976): "Evaluating Potential Nuclear Power Plant Sites in the Pacific Northwest using Decision Analysis", <u>IIASA Professional Paper</u> no. 76-1; also in BELL D.E., KEENEY R.L., RAIFFA H. (eds.) (1977): <u>Conflicting Objectives in Decisions</u>, Chapter 14, John Wiley and Sons, and in KEENEY R.L. (1980): <u>Siting Energy Facilities</u>, Chapter 3, Academic Press.
- KEENEY R.L., RAIFFA H. (1976): <u>Decisions with Multiple Objectives: Preferences and Value Tradeoffs</u>, John Wiley and Sons.
- LESOURNE J. (1972) : <u>Le calcul économique</u> ; <u>Théorie et Applications</u>, Dunod, Collection Finances et Economie Appliquée.
- LESOURNE J. (1977): <u>A Theory of the Individual for Economic Analysis</u>, North-Holland Publishing Company.

- LOOTSMA F.A., MEISNER J., SCHELLEMANS F. (1986): "Multi-Criteria Decision Analysis as an Aid to the Strategic Planning of Energy R & D", <u>European Journal of Operational Research</u> 25, 216-234.
- MCCORD M., LEOTSARAKOS Ch. (1987): "Investigation Utility and Value Functions with an 'Assessment Cube'", in MUNIER B. (ed.), <u>Risk, Decision and Rationality</u>, D. Reidel Publishing Company, Dordrecht/Boston (forthcoming).
- de NEUFVILLE R., DELQUIE Ph. (1987): "Exploration of the Influence of Certainty and Probability 'Effects' on the Measurement of Utility", in MUNIER B. (ed.), Risk, Decision and Rationality, D. Reidel Publishing Company, Dordrecht/Boston (forthcoming).
- NIJKAMP P., LEITNER H., WRIGLEY N. (eds.) (1985): Measuring the Unmeasurable, NATO ASI Series D.: Behavioural and Social Sciences no 22, Martinus Nijholl Publishers.
- PONSARC C. (1974) : "L'imprécision et son traitement en analyse économique", Université de Dijon, Institut de Mathématiques Economiques, <u>Document de Travail n° 4</u>, septembre.
- PONCARS C. (1986): "Spatial Fuzzy Consumer's Decision Making: A Multicriteria Analysis", <u>European Journal of Operational Research</u> 25, 235-256.
- RENARD F. (1986): "Utilisation d'ELECTRE dans l'analyse des réponses à un appel d'offres: Le cas de la machine de tri paquets à la Direction Générale des Postes", Université de Paris-Dauphine, <u>Cahier du LAMSADE</u> n° 73, juin, 30 p.
- RIVELINE C. (1985a) : "Evaluation des coûts Eléments d'une théorie de la gestion", Cours, Ecole nationale Supérieure des Mines de Paris.
- RIVELINE C. (1985b) : "Essai sur le dur et le mou", La Jaune et la Rouge,  $n^{\circ}$  406, juin-juillet, 101-108.
- ROUBENS M., VINCKE Ph. (1985): <u>Preference Modelling</u>, Lectures Notes in Economics and Mathematical Systems 250, Springer-Verlag.
- ROY B. (1985a) : <u>Méthodologie multicritère d'aide à la décision</u>, Economica, Paris.
- ROY B. (1985b): "Meaning and Validity of Interactive Procedures as Tools for Decision Making", Université de Paris-Dauphine, <u>Cahier du LAMSADE</u> n° 62, juillet, 35 p., and <u>European Journal of Operational Research</u> (forthcoming, 1987).
- ROY B., BOUYSSOU D. (1986): "Comparison of Two Decision-Aid Models Applied to a Nuclear Plant Siting Example", <u>European Journal of Operational Research</u> 25, 200-215.
- ROY B., PRESENT M., SILHOL D. (1986): "A Programming Method for Determining which Paris Metro Stations should be Renovated", European Journal of Operational Research 24, 318-334.
- ROY B., VINCKE Ph. (1987): "Pseudo-Orders: Definition, Properties and Numerical Representation", <u>Mathematical Social Sciences</u>, Volume 14, Issue 2, October (forthcoming).

SAVAGE L.J. (1954): The Foundations of Statistics, John Wiley and Sons, New York.

SKALA H.J., TERMINI S., TRILLAS E. (eds.) (1984): <u>Aspects of Vagueness</u>, D. Reidel Publishing Company, Dordrecht.

SOBEL J. (1987): "Metatickles and Ratification", in MUNIER B. (ed.), <u>Risk</u>, <u>Decision and Rationality</u>, D. Reidel Publishing Company, Dordrecht/Boston (forthcoming).

VALADARES-TAVARES L. (1987) : "Proposition d'u système relationnel de préférences multicritères avec transitivité généralisée", Université de Paris-Dauphine, Cahier du LAMSADE (forthcoming).

VON NEUMANN J., MORGENSTERN O. (1954): Theory of Games and Economic Behavior, John Wiley and Sons.