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OUTRANKING-BASED INTERACTIVE PROCEDURE FOR MULTIPLE OBJECTIVE PROGRAMS

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APPLICATION DU SURCLASSEMENT DANS UNE PROCEDURE INTERACTIVE POUR LA PROGRAMMATION MULTICRITERE

RESUME

Nous proposons une nouvelle approche pour la résolution des problèmes d'optimisation formulés en termes de programmation mathématique multicritère. Elle est fondée sur l'utilisation d'une relation de surclassement modélisant les préférences du décideur dans les premières étapes d'un processus interactif. Ce modèle des préférences est traduit au cône des poids d'une fonction scalarisante engendrant un sous-ensemble de points non-dominés du plus grand intérêt à l'aide d'une régression ordinale. La méthodologie présentée s'applique aussi bien à la programmation linéaire que non-linéaire (discrète incluse). Elle peut être classifiée comme une méthode de réduction de l'espace des vecteurs de poids avec une interaction constructive visuelle.

MOTS-CLES

Programmation mathématique multicritère, procédure interactive, surclassement, régression ordinale, réduction de l'espace des vecteurs de poids

OUTRANKING-BASED INTERACTIVE PROCEDURE FOR MULTIPLE OBJECTIVE PROGRAMS

ABSTRACT

We propose a new approach to solving optimization problems formulated in terms of multiple objective mathematical programming. It is based on the use of an outranking relation for modelling decision maker's preferences in early stages of an interactive process. This model of preferences is translated into a cone of scalarizing function weights generating a subset of nondominated points of greatest interest, using an ordinal regression method. The presented methodology applies both to linear and to nonlinear (including discrete) programming problems. It could be classified as a weighting vector space reduction method with a visual constructive interaction.

KEYWORDS

Multiple Objective Mathematical Programming, Interactive Procedure, Outranking, Ordinal Regression, Weighting Vector Space Reduction

INTRODUCTION

In practice, interactive procedures have proved to be most effective in searching tradeoff space of multiple objective programs for the best compromise solution. Existing interactive procedures represent either descriptive or constructive conception of interaction (Bouyssou, 1984; Vanderpooten and Vincke, 1989). The first one postulates that decision maker's (DM's) preferences are stable and that he can control them in a logical and coherent way. The interactive process consists then in description and exhibition of a pre-existing global utility function. Thus, rather than speaking about interaction, one should speak here about iterative acquisition of information. In the latter approach, the DM's preferences are not supposed to pre-exist but they can evolve in the interactive process. The system of DM's preferences is constructed basing on some regularities of preferential attitudes, accepting however their instability and incompleteness. The interactive process is a learning process in the *trial and error* spirit.

In our opinion, the constructive conception of interaction is more realistic. Indeed, wavering, incoherency and hesitancy of the DM in the interactive process do not follow from the difficulty of expressing something what is known but from imprecision, uncertainty and inaccurate determination inherent to decision models. Roy (1989a) claims that the elements of arbitrariness which have an important impact on the model come essentially from four sources. The first three are related to the quality of data which are used to formulate objective functions and constraints. The last source is related to the complexity of the model interactions with the modelling process.

A key element of an interactive procedure is construction of successive proposals. When elaborating a construction process, two basic components are usually defined (Vanderpooten, 1989):

- preference parameters* which convey preference information,
- a *scalarizing function* whose purpose is to make use of preference information in order to evaluate alternatives.

However, the use of a single scalarizing function signifies that the available preference information is sufficient to compare alternatives using *indifference* and *strict preference* relations only. The elements of arbitrariness inherent to the model bring this assumption in question.

In MCDA concerning a finite set of alternatives, it is recommended (Roy, 1985) to distinguish the three following relations for any ordered pair (a,b) of alternatives:

a P b, i.e. a is *significantly preferred* to b,

a I b, i.e. a and b are *equivalent*,

a Q b, i.e. a is *weakly preferred* to b.

Introduction of Q corresponds to the definition of indifference and preference thresholds q_g and p_g , respectively, for criterion g. If g is to be maximized,

a I b $\Leftrightarrow -q_g(g(a)) \leq g(a) - g(b) \leq q_g(g(b))$,

a Q b $\Leftrightarrow q_g(g(b)) < g(a) - g(b) \leq p_g(g(b))$,

a P b $\Leftrightarrow p_g(g(b)) < g(a) - g(b)$.

The following logical conditions are imposed on q_g and p_g , supposing a minimum of coherence in DM's preferences:

$$\frac{q_g(g(b)) - q_g(g(a))}{g(b) - g(a)} \geq -1, \quad \frac{p_g(g(b)) - p_g(g(a))}{g(b) - g(a)} \geq -1.$$

Criterion g involving indifference and preference thresholds is called *pseudo-criterion*.

Roy (1985) proposes moreover to handle situations where the DM is not able or doesn't want to make distinction between a P b, a Q b and a I b. He uses a grouped relation S called *outranking relation*. a S b means that a is at least as good as b, *non* a S b and *non* b S a means that a and b are incomparable. The outranking relation can be valued between 0 and 1 to express the strength of affirmation "a outranks b". It is then called *fuzzy outranking relation*.

Moreover, at the beginning of the interactive process, the DM usually does not know much, neither about the shape of the feasible set, nor about discrimination power of particular objective functions. This fact, together with the above mentioned arbitrariness of the model, encouraged us to substitute a single

scalarizing function by the fuzzy outranking relation in early stages of interaction.

We propose an interactive procedure for multiple objective programming which uses an outranking relation to model preliminary preferences of the DM. This relation is translated into a cone of scalarizing function directions (weights) generating a subset of nondominated points of greatest interest to the DM, using an ordinal regression method. This subset is then scanned by the DM and if he selects the best compromise point the procedure stops, otherwise further contraction of the cone is performed which best fits the DM's preferences. A preliminary idea of this procedure has been outlined in (Słowiński, 1988).

A general scheme of the procedure will be presented after a formal statement of the problem. Then, particular steps of the procedure will be described in detail. The description will be completed by an illustrative example. In the final section, some concluding remarks will be made.

PROBLEM STATEMENT AND BASIC DEFINITIONS

The general multiple objective programming problem is formulated as

$$\begin{array}{ll} \max \{g_1(x)=z_1\} & \\ \dots & \\ \max \{g_k(x)=z_k\} & \text{(P1)} \\ \text{s.t. } x \in D & \end{array}$$

where $x=[x_1, \dots, x_n]$, functions g_j need not to be linear and set D need not to be convex. It is assumed that each objective is bounded over D and that there does not exist a point in D at which all objectives are simultaneously maximized.

Problem (P1) can also be stated in a more compact form

$$\begin{array}{ll} \max \{z\} & \\ \text{s.t. } z \in Z & \text{(P2)} \end{array}$$

where $z=[z_1, \dots, z_k]$ and Z is an image of set D in the objective space. $Z \subset \mathbb{R}^k$ is the set of potential outcomes, which will be assumed to be compact.

Point $z' \in Z$ is *nondominated* iff there is no $z \in Z$ such that $z_j \geq z'_j \forall j$ and $z_i > z'_i$ for at least one i . Point $z' \in Z$ is *weakly nondominated* iff there is no $z \in Z$ such that $z_j > z'_j \forall j$. The set of all nondominated points is the *nondominated set*, denoted by N . Solution $x \in D$ is *efficient* iff its corresponding objective vector is nondominated. The set of all efficient solutions is the *efficient set*. For other definitions concerning nondominance and efficiency see e.g. Wierzbicki (1986).

Another useful definition is that of the *augmented weighted Chebyshev metric* in the objective space:

$$s(z, \Lambda, \rho) = \max_{j=1, k} \{ \lambda_j (z_j^* - z_j) \} + \rho \sum_{j=1}^k (z_j^* - z_j) \quad (1)$$

where $z_j^* = \max \{ g_j(x) : x \in D \} + \varepsilon_j$, $\varepsilon_j \geq 0$ is moderately small, $\Lambda = [\lambda_1, \dots, \lambda_k]$ is a weighting vector, $\lambda_j \geq 0$, $\sum_{j=1}^k \lambda_j = 1$, and ρ is a sufficiently small positive number. The diagonal direction of $s(z, \Lambda, \rho)$ is defined by $-(1/\lambda_1, \dots, 1/\lambda_k)$. Direction $-(1/\lambda_1, \dots, 1/\lambda_k)$ can be seen as generator of a cone in $\mathbb{R}^k$ with the origin in z^* .

GENERAL SCHEME OF THE INTERACTIVE PROCEDURE

The general scheme of the proposed interactive procedure is the following.

- Step 1. Generation of a small (finite) subset A of nondominated points (from 6 to 30) as representative as possible of the nondominated set N .
- Step 2. Construction of fuzzy outranking relation S in subset A .
- Step 3. Construction of two complete preorders $\bar{P}$, $\underline{P}$ in subset A , using so-called descending and ascending distillations of S .
- Step 4. Assessment of two scalarizing functions (augmented weighted Chebyshev metrics) as compatible as possible with $\bar{P}$ and $\underline{P}$, respectively, using an ordinal regression method.
- Step 5. Construction of a convex cone in the objective space on the basis of diagonal directions of the scalarizing functions assessed in step 4.
- Step 6. Interactive exploration of a nondominated subset $\tilde{N} \subset N$

delimited by the cone. The procedure terminates in this step if a best-compromise point has been found, or generates a new subset $A_{c\tilde{N}}$ and returns to step 2, otherwise.

The above procedure involves man-computer interaction of two kinds:

- interactive construction of fuzzy outranking relation S , modelling preliminary preferences of the DM (step 2),
- interactive exploration of an efficient region delimited by the cone defined from S (step 6).

DESCRIPTION OF STEPS

Step 1

The generation of sample A of nondominated points can be performed using one of several existing methods, for example the method by Choo and Atkins (1980) which we applied already in a different context (cf. Jacquet-Lagrèze et al., 1987; Słowiński, 1986, 1990). The techniques by Morse (1980), Törn (1980) and Steuer (1986, ch.14) can also be used in this step. The last one is the most general and can compute properly nondominated points of integer and nonlinear multiple objective programming problems.

In order to obtain a sample of the most representative points for a given region, it is beneficial to generate first a larger finite set of points and then, to filter them using a *forward filtering* technique (Steuer, 1986, ch.11). In result of forward filtering, one gets a required number of points that are the most different from one another. It is also recommended to equalize the ranges of the coordinates of nondominated points prior to the generation.

Step 2

Sample A is then presented to the DM who is taking part in construction of a fuzzy outranking relation S in set A . The construction of S proceeds as in ELECTRE III (Roy, 1978).

Let a be a nondominated point belonging to A . The j -th coordinate of this point in the objective space is called the j -th *performance*

of a , denoted by z_j^a ($j=1, \dots, k$). In other words, point a is represented by the vector of performances z^a in the objective space. The elements of arbitrariness inherent to the definition of the performances lead the DM to compare points $a, b \in A$ using pseudo-criteria, i.e. z_j with indifference and preference thresholds, q_j and p_j , respectively, $j=1, \dots, k$. Fuzzy outranking relation S is characterized by the definition of an outranking degree associating with each ordered pair of points (a, b) a number $0 \leq d(a, b) \leq 1$; d is a measure of credibility of the outranking $a S b$.

Calculation of d proceeds in the following way. For each pseudo-criterion z_j , two indices are calculated first for all pairs (a, b) of points belonging to A : concordance index $c_j(a, b)$ and discordance index $D_j(a, b)$. The former expresses to what extent the evaluation of a and b on z_j is concordant with assertion "a is at least as good as b". The latter indicates the strength of its opposition against this assertion. $D_j(a, b)$ involves a veto threshold v_j , i.e. the bound beyond which the opposition to the hypothesis $a S b$ is sufficiently motivated. The definition of both indices is given graphically in Fig. 1. The partial concordance indices are then aggregated taking

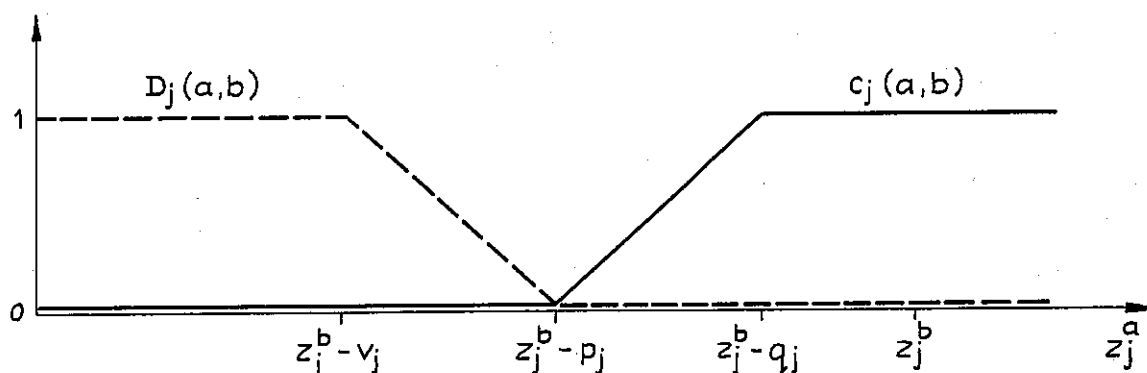


Fig.1. Concordance and discordance indices for $a, b \in A$ and pseudo-criterion z_j

into account relative importance of criteria defined by intrinsic coefficients $k_j (\geq 0)$,

$$C(a, b) = \frac{\sum_{j=1}^k k_j c_j(a, b)}{\sum_{j=1}^k k_j}$$

The degree of credibility $d(a, b)$ is obtained from the global concordance index, weakened by discordance indices (up to the point of its annulment),

$$d(a,b) = C(a,b) \prod_{j \in J} \frac{1 - D_j(a,b)}{1 - C(a,b)}, \quad J = \{j : D_j(a,b) > C(a,b)\}$$

Interesting remarks about how to give numerical values to thresholds and importance coefficients, and how to test the robustness of the outranking relation, have been made by Roy (1989b).

Step 3

The aim of this step is to derive two complete preorders in A , as different as possible, from the fuzzy outranking relation. Preorder $\bar{P}$ is obtained in a descending way, i.e. selecting first the best points, then the following, until the worst. Preorder P is obtained in an ascending way, i.e. the selection process starts with the worst points and ends with the best ones. In ELECTRE III, this procedure is called *distillation*. Let us sketch the procedure.

First, a crisp outranking relation S^{γ_1} is derived from the fuzzy relation represented by d :

$$a S^{\gamma_1} b \text{ iff } d(a,b) > \gamma_1 \text{ and } d(a,b) > d(b,a) + s[d(a,b)]$$

where γ_1 is close to 1 and $s[d] = \alpha + \beta d$ (typically, $\alpha = 0.3$, $\beta = -0.15$). Then, for every point a , the following discriminating indices are calculated:

$$p_A^{\gamma_1}(a) = |\{b \in A : a S^{\gamma_1} b\}|, \text{ called } \gamma_1\text{-power of } a,$$

$$f_A^{\gamma_1}(a) = |\{b \in A : b S^{\gamma_1} a\}|, \text{ called } \gamma_1\text{-weakness of } a,$$

$$\text{and } q_A^{\gamma_1}(a) = p_A^{\gamma_1}(a) - f_A^{\gamma_1}(a), \text{ called } \gamma_1\text{-qualification of } a.$$

Using γ_1 -qualification, the sets of best ($\bar{C}_1$) and worst (C_1) points are constructed:

$$\bar{C}_1 = \{a \in A : q_A^{\gamma_1}(a) = \bar{q}_A = \max_{c \in A} [q_A^{\gamma_1}(c)]\}$$

$$C_1 = \{a \in A : q_A^{\gamma_1}(a) = \underline{q}_A = \min_{c \in A} [q_A^{\gamma_1}(c)]\}$$

The procedure continues with $\bar{C}_1$ (descending distillation) and C_1 (ascending distillation), and $\gamma_2 < \gamma_1$ such that for $\bar{C}_1$:

$$\gamma_2 = \max_{d(a,b) \in \bar{\mathcal{P}}_1} [d(a,b)]$$

where $\bar{\mathcal{P}}_1 = \{d(a,b) : d(a,b) < \gamma_1 - s(\gamma_1), (a,b) \in \bar{C}_1 \text{ and } s(\gamma_1) = \alpha + \beta \gamma_1\}$. In some iteration r of the descending, and t of the ascending

distillation, $\gamma_r = \gamma_t = 0$. Then $\bar{C}_r$ and $\underline{C}_t$ become the first and the last equivalence classes in descending and ascending preorders, respectively. The procedure is repeated for sets $\bar{A} = A \setminus \bar{C}_r$, $\underline{A} = A \setminus \underline{C}_t$, and continued until $\bar{A} = \underline{A} = \emptyset$.

The resulting complete preorders $\bar{P}$ and $\underline{P}$ are different in general - this difference reflects the range of DM's hesitancy in the comparison of nondominated points at the present stage of problem solving.

Step 4

Given two complete preorders $\bar{P}$ and $\underline{P}$ in set A , two scalarizing functions are assessed, as compatible as possible with $\bar{P}$ and $\underline{P}$, respectively. As the scalarizing function we use the augmented weighted Chebyshev metric. We have chosen this metric because of two main advantages: it can be used to find properly nondominated points in a nonconvex set, and its assessment according to ordinal regression reduces to linear programming.

The idea of ordinal regression was already used to assess a piecewise-linear utility function for multiple objective linear programming (cf. Jacquet-Lagréze et al., 1987; Siskos and Despotis, 1989).

Let $\bar{s}'$ denote the scalarizing function perfectly compatible with $\bar{P}$, $\bar{s}$ - the scalarizing function being assessed and σ - an approximation error. For every $a \in A$,

$$\bar{s}'(z^a, \Lambda, \rho) = \bar{s}(z^a, \Lambda, \rho) + \sigma(a)$$

Assuming a small threshold $\delta > 0$, one can express all relations which constitute $\bar{P}$:

$$a P b \Leftrightarrow \bar{s}(z^b, \Lambda, \rho) - \bar{s}(z^a, \Lambda, \rho) + \sigma(b) - \sigma(a) \geq \delta \quad (2)$$

$$a I b \Leftrightarrow -\delta < \bar{s}(z^b, \Lambda, \rho) - \bar{s}(z^a, \Lambda, \rho) + \sigma(b) - \sigma(a) < \delta \quad (3)$$

Substituting (1) to right-hand side of (2) and (3), we get

$$y_b - y_a + \rho \sum_{j=1}^k (z_j^* - z_j^b) - \rho \sum_{j=1}^k (z_j^* - z_j^a) + \sigma(b) - \sigma(a) \geq \delta \quad (4)$$

$$-\delta < y_b - y_a + \rho \sum_{j=1}^k (z_j^* - z_j^b) - \rho \sum_{j=1}^k (z_j^* - z_j^a) + \alpha(b) - \alpha(a) < \delta \quad (5)$$

$$y_b \geq \lambda_j (z_j^* - z_j^b), \quad j=1, \dots, k \quad (6)$$

$$y_a \geq \lambda_j (z_j^* - z_j^a), \quad j=1, \dots, k \quad (7)$$

Parameters Λ and ρ of the scalarizing function $\bar{s}$ follow from the linear program:

$$\min \left(\sum_{a \in A} \alpha(a) \right) \quad (P3)$$

$$\text{s.t. } \left. \begin{array}{l} (4) \text{ if } a P b \\ (5) \text{ if } a I b \\ (6), (7) \end{array} \right\} \text{ for all pairs } a, b \in A \text{ such that } a \text{ and } b \text{ are 'consecutive' in } \bar{P}$$

$$\alpha(a) \geq 0 \quad \text{for all } a \in A$$

$$0 \leq \rho \leq 0.001, \quad \lambda_j \geq 0, \quad j=1, \dots, k, \quad \sum_{j=1}^k \lambda_j = 1$$

LP problem (P3) has $2|A|+k+1$ variables and at most $(k+2)|A|$ constraints.

Analogical LP problem can be set up for $\underline{s}$. The parameters of $\bar{s}$ and $\underline{s}$ calculated in this step are denoted by $\bar{\Lambda}$, $\bar{\rho}$ and $\underline{\Lambda}$, $\underline{\rho}$, respectively.

Let us notice that solutions of LP problems (P3) may not be unique. So, it is recommended to perform post-optimal analysis in order to capture the stability (imprecision) of estimated functions $\bar{s}$ and $\underline{s}$. The sets of weighting vectors $\bar{\Lambda}$ and $\underline{\Lambda}$ corresponding to equivalent solutions of (P3) are denoted by $\bar{\mathcal{L}}$ and $\underline{\mathcal{L}}$, respectively.

Step 5

Diagonal directions of Chebyshev metrics $\bar{s}$ and $\underline{s}$,

$$-(1/\bar{\lambda}_1, \dots, 1/\bar{\lambda}_k) \quad \text{and} \quad -(1/\bar{\lambda}_1, \dots, 1/\bar{\lambda}_k)$$

can be used in different ways to define a convex cone emanating from z^* in direction of the nondominated set N . For example, it can be obtained from rotation of the diagonal directions round the axis $-(2/(\bar{\lambda}_1 + \underline{\lambda}_1), \dots, 2/(\bar{\lambda}_k + \underline{\lambda}_k))$, or using a simple construction procedure presented in (Słowiński, 1988). The cone generates a nondominated subset $\tilde{N} \subset N$ corresponding to current preferences of the DM, modelled by S .

Delimitation of the nondominated subset $\tilde{N}$ should be considered

jointly with the technique of scanning this subset (step 6). We propose to organize step 5 in the following way:

Step 5.1. Create union $\mathcal{L}$ of the sets of weighting vectors obtained in step 4: $\mathcal{L} = \bar{\mathcal{L}} \cup \underline{\mathcal{L}}$.

Step 5.2. Calculate $l_j = \min_{\lambda_j \in \mathcal{L}} \langle \lambda_j \rangle$ and $u_j = \max_{\lambda_j \in \mathcal{L}} \langle \lambda_j \rangle$, $j=1, \dots, k$.

Step 5.3. Define set $\mathfrak{L}$ of weighting vectors:

$$\mathfrak{L} = \{ \Lambda \in \mathbb{R}^k : \lambda_j \in [l_j, u_j], j=1, \dots, k, \sum_{j=1}^k \lambda_j = 1 \}$$

Since vector $-(1/\lambda_1, \dots, 1/\lambda_k)$ is a generator of the cone originating in z^* , the set of all $\Lambda \in \mathfrak{L}$ creates the convex cone generating subset $\tilde{N} \subset N$. Precisely, minimization of $s(z, \Lambda, \rho)$ on set Z , yields nondominated point $\tilde{z} \in \tilde{N}$, for every $\Lambda \in \mathfrak{L}$.

Step 6

This step is again an interactive one. In order to explore subset $\tilde{N}$, we can use the scanning technique of Steuer and Choo (1983). The whole step is then organized in the following way:

Step 6.1. Randomly generate a large number ($\approx 50 \times \pi$) of weighting vectors from $\mathfrak{L}$. Filter this set to obtain a fixed number ($2 \times \pi$) of representative weighting vectors.

Step 6.2. For each representative weighting vector Λ , solve the augmented weighted Chebyshev program: $\min \{s(z, \Lambda, \rho)\}$, s.t. $z \in Z$, where ρ is a mean of ρ 's obtained for $\bar{s}$ and $\underline{s}$ in step 4. Filter the ($2 \times \pi$) resulting nondominated points to obtain π points ($\pi \geq k$).

Step 6.3. Present the π points to the DM and ask him to select the best compromise point. If it has been selected then STOP, otherwise continue and go either to step 6.4a or to step 6.4b, upon request of the DM.

Step 6.4a. Gather the π points in set A and go back to step 2, possibly with finer thresholds q_j and p_j .

Step 6.4b. Ask the DM to select from among the π points (possibly, plus some other nondominated points known from previous iterations), at most k points with the least but satisfactory scores on particular objectives. Create a new set $\mathcal{L}$ of weighting vectors corresponding to these points and go back to step 5.2.

Both versions of step 6.4 result in reduction of the weighting vector space (cone contraction) for the next round of exploration. In the case of MOLP, a useful option in step 6.3 consists in display of objective value trajectories between any two nondominated points (cf. Korhonen and Laakso, 1986). Other scanning methods can also be used in step 6 (cf. Lewandowski and Wierzbicki, 1988).

ILLUSTRATIVE EXAMPLE

Let us consider the following MOLP problem (cf. Steuer, 1986):

$$\begin{aligned}
 & \max \{x_1 = z_1\} \\
 & \max \{x_2 = z_2\} \\
 & \max \{x_3 = z_3\} \\
 & \text{s.t. } 5x_1 + 8x_2 + 3x_3 \leq 30 \\
 & \quad \quad x_1 + 3x_2 + 6x_3 \leq 6 \\
 & \quad \quad 5x_1 + 3x_2 + 6x_3 \leq 30 \\
 & \quad \quad x_1, x_2, x_3 \geq 0
 \end{aligned}
 \tag{P4}$$

The nondominated set N is shown in Fig. 2. It is composed of three triangular pieces.

Step 1. Sample A of nondominated points (cf. Table 1 and Fig. 2) has been obtained using the method by Choo and Atkins (1980).

Step 2. The DM specifies the thresholds and the relative importance of criteria required for construction of fuzzy outranking relation S in set A (Table 2).

Table 1. Set A

Point	z_1	z_2	z_3
a	1.24	1.87	1.87
b	3.21	1.40	1.40
c	1.68	2.93	1.40
d	1.68	1.4	2.91
e	4.15	0.93	0.93
f	1.11	3.96	0.93
g	1.11	0.93	3.61
h	5.08	0.46	0.46
i	0.55	4.54	0.46
j	0.55	0.46	4.31

Table 2. Data required for construction of S in A

Thresholds	z_1	z_2	z_3
indifference	0.2	0.2	0.2
preference	0.4	0.4	0.4
veto	0.6	0.6	0.6
importance coefficient	1	1	1

Step 3. The result of the distillation procedure applied on the fuzzy outranking relation S is shown in Fig.2

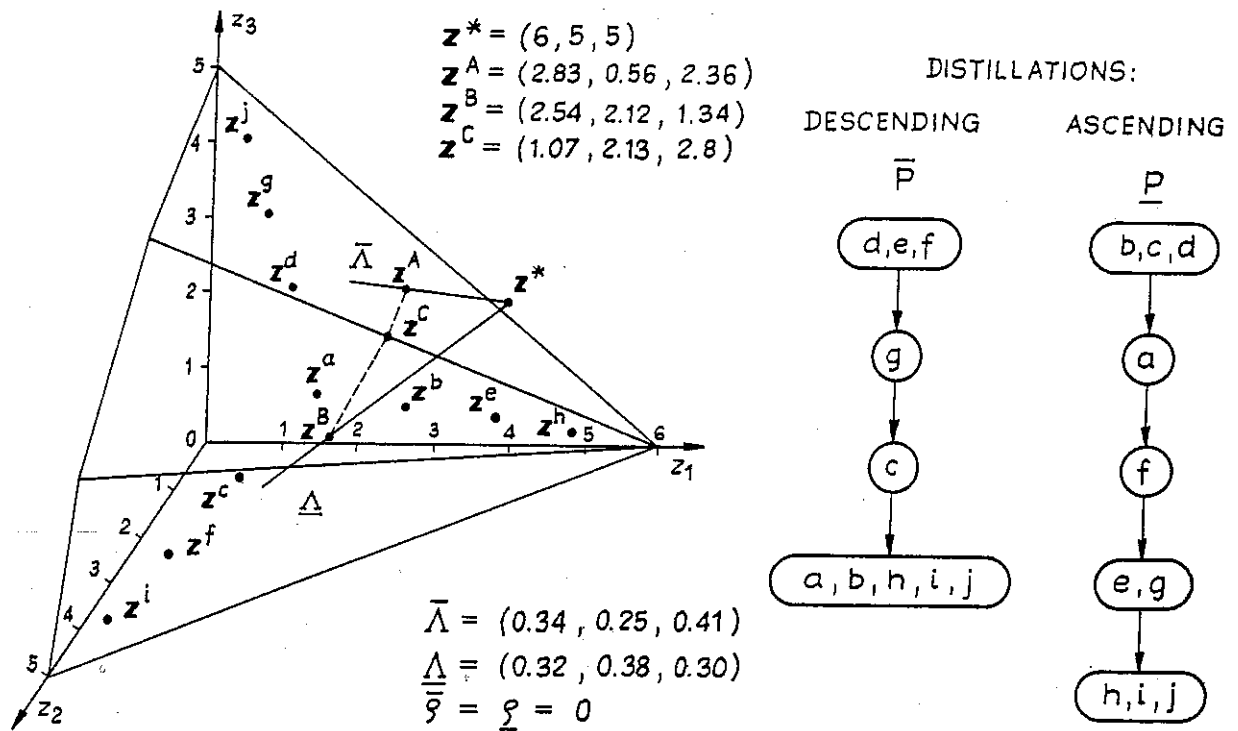


Fig. 2. Feasible set of problem (P4)

Step 4. Using the ordinal regression method, two scalarizing functions $\bar{s}$ and $\underline{s}$ have been obtained, perfectly compatible with the descending and ascending preorders, respectively. Diagonal directions of these functions are shown in Fig. 2.

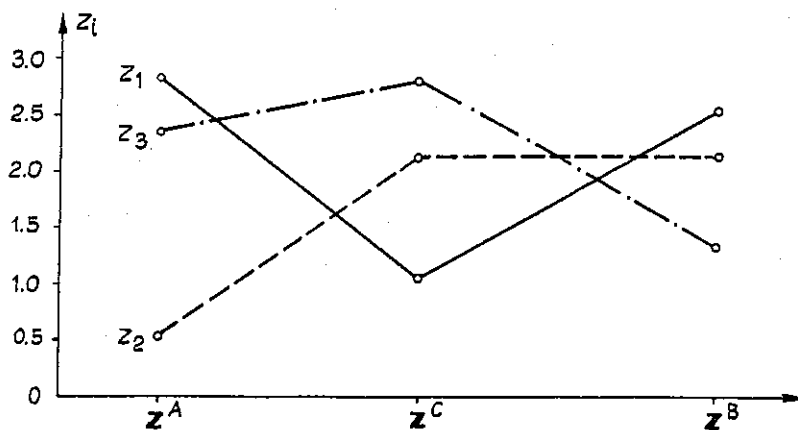


Fig. 3. Objective value trajectories

Step 5. Diagonal directions define set $\mathcal{Q}$ of weighting vectors:

$$\mathcal{Q} = \{ \Lambda \in \mathbb{R}^3 : \lambda_1 \in [0.32, 0.34], \lambda_2 \in [0.25, 0.38], \lambda_3 \in [0.3, 0.41], \sum_{j=1}^k \lambda_j = 1 \}$$

$\mathcal{Q}$ defines implicitly the cone originating in z^* which generates subset $\tilde{N} \subset N$.

Step 6. Objective value trajectories shown in Fig. 3 scan subset $\tilde{N}$ between points z^A and z^B .

The next iteration can start either from step 6.4a or from step 6.4b.

CONCLUDING REMARKS

The interactive method presented in this paper can be classified as a *weighting vector space reduction method with a (visual) constructive interaction*. Let us conclude with the following comments.

- (a) The method can be applied to linear and nonlinear (including discrete) problems.
- (b) No assumption is made about any implicit utility function.
- (c) The method involves interaction of two kinds:
 - interactive construction of fuzzy outranking relation S , modelling preliminary preferences of the DM (step 2),
 - interactive exploration of an efficient region delimited by the cone of scalarizing function directions (weights) defined from S (step 6).
- (d) In spite of monotonic reduction of the weighting vector space (cone contraction), the DM can retract to points abandoned in previous iterations (cf. step 6.4b).
- (e) The method is based on a learning-oriented perspective. The difference between preorders resulting from distillation of S reflects the range of DM's hesitancy in the comparison of nondominated points at the present stage of problem solving.
- (f) In the MOLP case, a graphical display of objective value trajectories enables a visual interaction.
- (g) The calculation steps widely depend on the problem. Several optimization steps are to be performed in order to produce samples of nondominated points (steps 1 and 6).

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