On the interest of interacting criteria in MCDA

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Outline

1. Motivation
   - Introduction
   - MCDA models representing interacting criteria
2. Definition of independence
3. Shall we have interacting criteria?
   - Value-Focused Thinking
   - Some experiments
   - Case with reference points
4. How to learn interactions among criteria?
   - Supervised learning techniques
   - Unsupervised learning techniques
5. Conclusion

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THE motivating example of the Choquet integral

evaluation of students with 3 criteria: mathematics (M), statistics (S), languages (L)

The strategy of evaluation is defined by 2 rules:

> ***(R1)***: *For a student good at mathematics (M), L is more important than S.*
> ***(R2)***: *For a student bad in mathematics (M), S is more important than L.*

Using the above rules on the following table (evaluations in scale $[0, 20]$)

	Math.	Stat.	Lang.
student A	16	13	7
student B	16	11	9
student C	6	13	7
student D	6	11	9

we have:

- $A \prec B$ by Rule **R1**
- $C \succ D$ by Rule **R2**.

Analysis of the example

What does this mean?

In the example, interaction comes from (statistical) dependencies among criteria:

- A student good in Math is in general also good in Physics;
- But it is much more rare to have a student good in both Math and Litterature.

Analysis of the example

What does this mean?

In the example, interaction comes from (statistical) dependencies among criteria:

- A student good in Math is in general also good in Physics;
- But it is much more rare to have a student good in both Math and Litterature.

Origins of interaction

- (statistical) dependencies among criteria
  - Ex. of the students
- preferential dependencies among criteria
  - tolerance/intolerance
  - fairness

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MCDA models representing interacting criteria

	Without commensurability	With commensurability
Independence among criteria	**Additive Utility** $U(x) = \sum_{i \in N} v_i(x_i)$	**Weighted Sum** $U(x) = \sum_{i \in N} w_i \, u_i(x_i)$
Interaction among criteria	**Generalized Additive Utility (GAI)** $U(x) = \sum_{A \in \mathcal{A}} v_A(x_A)$ $(\mathcal{A} \subseteq \mathcal{P}(N))$	**Choquet integral** $U(x) = \sum_{i \in N} w_i \, u_i(x_i) - \sum_{\{i,j\} \subseteq N} \frac{I_{i,j}}{2} \, \lvert u_i(x_i) - u_j(x_j) \rvert$

Aim of the talk

Aim

- Is interaction among criteria really useful? When can we encounter it?
- How to measure/elicit interaction?

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Independence concepts without uncertainty

Weak independence

Attribute i is weakly independent to $N \setminus \{i\}$ if

$$(x_i, z_{-i}) \succsim (y_i, z_{-i}) \iff (x_i, t_{-i}) \succsim (y_i, t_{-i})$$

Preferential independence

Attributes S are preferentially independent to $N \setminus S$ if

$$(x_S, z_{-S}) \succsim (y_S, z_{-S}) \iff (x_S, t_{-S}) \succsim (y_S, t_{-S})$$

Weak Difference Independence

Attribute i is weakly difference independent to $N \setminus \{i\}$ if

$$(x_i, z_{-i})\,(x'_i, z_{-i}) \succsim^\star (y_i, z_{-i})\,(y'_i, z_{-i}) \iff (x_i, t_{-i})\,(x'_i, t_{-i}) \succsim^\star (y_i, t_{-i})\,(y'_i, t_{-i})$$

Independence concepts based on uncertainty

Decision under uncertainty

- $\mathcal{P}_X$: set of probability distributions over X (also called lotteries or gambles)
- Particular case (discrete support): $\langle p_1, x^1; \ldots, p_r, x^r \rangle$
- Given $P \in \mathcal{P}_X$, the marginal of P over $S \in \mathcal{S}$ is defined by, for every $x_S \in X_S$

$$P_S(x_S) = \sum_{x_{N \setminus S} \in X_{N \setminus S}} P(x_S, x_{N \setminus S}).$$

- $\succsim^L \subseteq \mathcal{P}_X \times \mathcal{P}_X$: preference over lotteries

Utility independence

Attribute i is utility independent to $N \setminus \{i\}$ if for every $i \in N$

$$\left\langle p_1, (x_i^1, z_{-i}^1); p_2, (x_i^2, z_{-i}^2); \ldots \right\rangle \succsim^L \left\langle p_1, (y_i^1, z_{-i}^1); p_2, (y_i^2, z_{-i}^2); \ldots \right\rangle$$
$$\iff \left\langle p_1, (x_i^1, t_{-i}^1); p_2, (x_i^2, t_{-i}^2); \ldots \right\rangle \succsim^L \left\langle p_1, (y_i^1, t_{-i}^1); p_2, (y_i^2, t_{-i}^2); \ldots \right\rangle$$

Independence concepts based on uncertainty

Additive independence

Attributes N are additively independent if for every $P, Q \in \mathcal{P}_X$, with $P_{\{i\}} \equiv Q_{\{i\}}$ for every $i \in N$, then $P \sim^L Q$.

Illustration

$$\left\langle 0.5, (a_1^\perp, a_2^\perp); 0.5, (a_1^\top, a_2^\top) \right\rangle \sim^L \left\langle 0.5, (a_1^\perp, a_2^\top); 0.5, (a_1^\top, a_2^\perp) \right\rangle.$$

Results

Results

- [Keeney 1972] Under utility independence, preferences are represented by

$$u(x) = \sum_{S \subseteq N,\ S \neq \emptyset} k_S \prod_{i \in S} u_i(x_i)$$

- [Fishburn 1965] Under additive independence, preferences are represented by

$$u(x) = \sum_{i \in N} k_i\ u_i(x_i)$$

- [Keeney 1974] Under utility independence and preferential independence, preferences are represented by (i.e. $1 + k\ u(x) = \prod_{i \in N} (1 + k\ k_i\ u_i(x_i))$)

$$u(x) = \sum_{S \subseteq N} k^{|S|-1} \prod_{i \in S} k_i\ u_i(x_i)$$

- [Dyer, Sarin 1979] Under Weak Difference Independence, preferences are represented by

$$u(x) = \sum_{S \subseteq N,\ S \neq \emptyset} k_S \prod_{i \in S} u_i(x_i)$$

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Value-Focused Thinking [Keeney 1992]

Fundamental vs. means objectives

- Fundamental objective = what the decision maker really cares about
- Means objective = ways to comply with the fundamental values

VFT steps

- Identify the values, derive from that the alternatives
- Check independence conditions (preference, utility, additive independence) to derive the form of the utility model
- Elicit the utility model

VFT dogma

- If attributes are not independent, this means that
  - either we do not have the appropriate set of fundamental objectives,
  - or means objectives are used as fundamental objectives
- In this case, rework to find the very fundamental objectives.
- This ensures (statistical, causal) independence among criteria

Value-Focused Thinking [Keeney 1992]

Complementarity among attributes

- Ressource allocation to individuals
- Attribute i = amount of ressource allocated to agent i
- The attributes are not additive independent as $\left\langle 0.5, (1,1); 0.5, (0,0) \right\rangle$ is strictly preferred to $\left\langle 0.5, (1,0); 0.5, (1,0) \right\rangle$
  - In the extreme case, $(0,0)$ is more fair than $(1,0)$ or $(0,1)$
- The attributes are the appropriate fundamental objectives
- Violation of additive independence because there is another fundamental objective – namely equity.
- Hence the model

$$u(x_1, x_2) = k_1 u_1(x_1) + k_2 u_2(x_2) + k u_1(x_1) u_2(x_2)$$

  where $k > 0$ because of equity/equal treatment
- *Criteria* 1 *complements criterion* 2 *as the better the achievement of* x_1*, the more significant it is to improve achievement of* x_2

Value-Focused Thinking [Keeney 1992]

Substitutability among attributes

- Risk management
- Attribute i = achievement on sector i
- The attributes are not additive independent as $\left\langle 0.5, (1,1); 0.5, (0,0) \right\rangle$ is strictly less preferred to $\left\langle 0.5, (1,0); 0.5, (1,0) \right\rangle$
  - $(0,0)$ represents a very large risk, whereas $(1,0)$ or $(0,1)$ yield in-between consequences.
- Violation of additive independence because there is another fundamental objective – namely risk aversion.
- Hence the model

$$u(x_1, x_2) = k_1 u_1(x_1) + k_2 u_2(x_2) + k u_1(x_1) u_2(x_2)$$

  where $k < 0$ because of risk aversion
- *Criteria 1 substitutes criterion 2 as the better the achievement of x_1, the less significant it is to improve achievement of x_2*

Value-Focused Thinking [Keeney 1992]

Findings on VFT (1/2)

- VFT does not exactly say that additive independence shall always hold;
- VFT acknowledges that the decision maker may violate preferential independance (in case of equity, complementary, redundancy,. . .)
  - VFT seems to explicitly extract interaction situation through new fundamental objectives
  - But this is very difficult when we have many attributes.
- When there are many interactions, it is more convenient to directly elicit a (e.g. 2-additive) capacity.

Value-Focused Thinking [Keeney 1992]

Findings on VFT (2/2)

- VFT does not give examples of statistical dependencies that cannot be solved by finding more appropriate fundamental objectives. . .

  Ex. quantity & quality in service disruption:

  - # persons affected by disruption is the quantity
  - the duration of the disruption is the quality
  - We encapsulate this *violation of additive independence* by creating an impact matrix of these two variables: $u_{1,2}(x_1, x_2)$.

Impact matrix		Number of persons				
		≈ 10	≈ 100	≈ 1000	≈ 10000	≈ 100000
Duration	≈ 1 week	0.1	0.1	0.3	0.5	0.7
	≈ 1 month	0.1	0.3	0.5	0.7	0.9
	≈ 2-9 months	0.3	0.5	0.7	0.9	0.9
	≈ 1 year	0.5	0.7	0.9	0.9	0.9

  - When we guess some dependencies among attributes, this amounts to using a GAI model (guess the subsets structure $\mathcal{S}$).

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Some experiments [Pirlot, Schmitz, Meyer, 2010]

Pirlot, Schmitz, Meyer, URPDM 2010

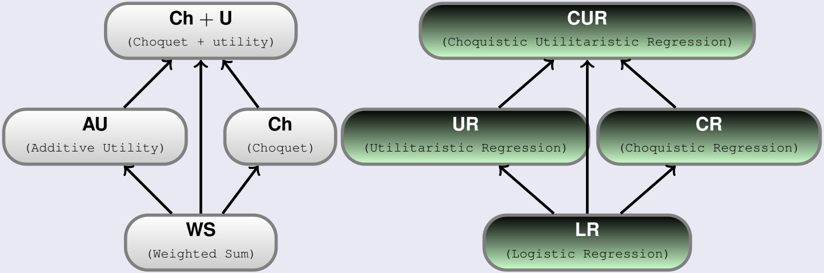
An empirical comparison of the expressiveness of the additive value function and the Choquet integral models for representing rankings:

- Comparison of three models:
  - Weighted Sum (WS): $U(x) = \sum_{i \in NM} \omega_i \, f_x(i)$
    $\Longrightarrow$ learn ω
  - Additive Utility (AU): $U(x) = \sum_{i \in N} v_i(x_i)$
    $\Longrightarrow$ learn $v_i : X_i \to \mathbb{R}$ (e.g. UTA)
  - Choquet Integral (Ch): $U(x) = C_v(f_x)$
    $\Longrightarrow$ learn v
- Representation of WS, AU and Ch on randomly generated datasets
- WS is the less general
- AU better represents randomly generated datasets than Ch.

Some experiments [Fallah Tehrani et al, 2014]

Fallah Tehrani, Labreuche, Hüllermeier, DA2PL 2014

MCDA: **Preference Learning (PL):**



Some experiments [Fallah Tehrani et al, 2014]

0/1 loss for the experiments

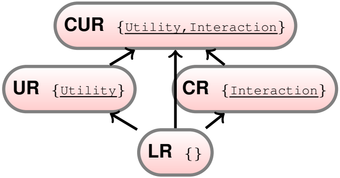
Datasets	**CUR**, $p_j = 2$	**CUR**, $p_j = 3$	**UR**, $p_j = 2$	**UR**, $p_j = 3$	**LR**	**CR**
ERA	.3191 $\pm$ .0185	.3015 $\pm$ .0197	**.2894 $\pm$ .0239**	.2953 $\pm$ .0365	.2932 $\pm$ .0261	**2891 $\pm$ .0241**
LEV	.1352 $\pm$ .0236	**.1302 $\pm$ .0126**	.1415 $\pm$ .0190	.1563 $\pm$ .0271	.1662 $\pm$ .0171	.1500 $\pm$ .0207
CEV	.0623 $\pm$ .0521	**.0240 $\pm$ .0160**	.0583 $\pm$ .0153	.0461 $\pm$ .0130	.1643 $\pm$ .0184	.0719 $\pm$ .0091
CPU	**.0285 $\pm$ .0301**	**.0244 $\pm$ .0252**	.1390 $\pm$ .0630	.1171 $\pm$ .0549	.0336 $\pm$ .0068	.0276 $\pm$ .0229
DenBosch	.1630 $\pm$ .0859	.1524 $\pm$ .0653	.1826 $\pm$ .0788	.1884 $\pm$ .0807	.1409 $\pm$ .0336	**.1283 $\pm$ .0683**
ESL	.0660 $\pm$ .0196	.0680 $\pm$ .0210	.0785 $\pm$ .0260	.0670 $\pm$ .0312	**.0602 $\pm$ .0264**	.0694 $\pm$ .0218
Mammo	.1642 $\pm$ .0271	**.1553 $\pm$ .0317**	.1685 $\pm$ .0302	.1600 $\pm$ .0303	.1683 $\pm$ .0231	.1693 $\pm$ .0285
Auto-MPG	**.0038 $\pm$ .0084**	.0054 $\pm$ .0120	**.0038 $\pm$ .0073**	**.0034 $\pm$ .0067**	.0538 $\pm$ .0282	.0654 $\pm$ .0266
Breast Cancer	.2773 $\pm$ .0348	.2989 $\pm$ .0550	.3079 $\pm$ .0635	.3042 $\pm$ .0501	**.2669 $\pm$ .0483**	.2861 $\pm$ .0482

CUR returns the best predictions in 5 out of the 9 datasets

CUR returns the worst predictions for ERA (more expressive models are not necessarily advantageous from a learning point of view)

very significant improvement for AUTO-MPG with **CUR** and **UR**

Some experiments [Fallah Tehrani et al, 2014]



$\frac{1}{2}\left[(\eta_{\mathbf{CUR}} - \eta_{\mathbf{CR}}) + (\eta_{\mathbf{UR}} - \eta_{\mathbf{LR}})\right]$

$\frac{1}{2}\left[(\eta_{\mathbf{CUR}} - \eta_{\mathbf{UR}}) + (\eta_{\mathbf{CR}} - \eta_{\mathbf{LR}})\right]$

$\eta_{\mathbf{LR}} - \eta_{\mathbf{CR}} - \eta_{\mathbf{UR}} + \eta_{\mathbf{CUR}}$

Marginal contributions and interactions

Datasets	$\eta_{\mathbf{CUR}}$	$\eta_{\mathbf{UR}}$	$\eta_{\mathbf{LR}}$	$\eta_{\mathbf{CR}}$		Marginal contribution of Utility	Marginal contribution of Interaction		Interaction among Utility and Interaction
ERA	.3015	.2894	.2932	.2891		-.0043	-.004		-.0162
LEV	.1302	.1415	.1662	.15		.02225	.01375		-.0049
CEV	.024	.0461	.1643	.0719		.08305	.05725		-.0703
CPU	.0244	.1171	.0336	.0276		-.04015	.04935		.0867
DenBosch	.1524	.1826	.1409	.1283		-.0329	.0214		.0176
ESL	.066	.067	.0602	.0694		-.0017	-.0041		.0102
Mammo	.1553	.16	.1683	.1693		.01115	.00185		.0057
Auto-MPG	.0038	.0034	.0538	.0654		.056	-.006		.0112
Breast Cancer	.2773	.3042	.2669	.2861		-.01425	.0038		.0461

- A slight advantage of interaction over utility.
- Interaction is negative when doing only utility or interaction is beneficial, but not both.
- Interaction is positive to 6 datasets (beneficial to do both utility and interaction).

Some experiments [T. Lust. ADT 2015]

T. Lust. ADT 2015

Choquet integral versus weighted sum in multicriteria decision contexts:

- Comparison of WS and Ch in a multi-objective optimization context.
- Given the efforts needed to set the parameters of the Choquet integral, it is important to measure, for a given decision problem, if it is really worth defining the Choquet integral or if a simple weighted sum could have been used to determine the best alternative.
- Computation of the probability that a recommendation of a decision maker could only been obtained with the Choquet integral and not with a weighted sum.
  - Concept of supported solution: point of the Pareto front that is the best according to a given utility model.
- When the number of criteria increases, the results show that this probability tends to one.

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Case with reference points

Capacities: 2 reference levels

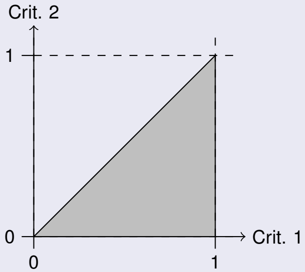

Figure: Set of vectors $t = (t_1, t_2)$ such that $t_1 \geq t_2$.

Case with reference points

k-ary capacities: $k+1$ reference levels

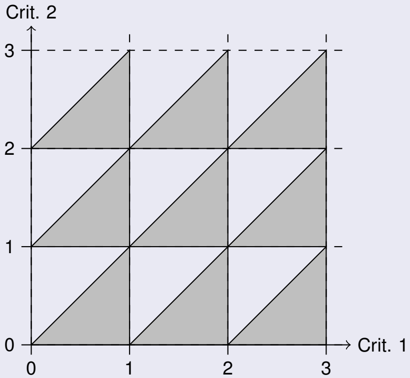

Figure: Example with $k = 3$.

Case with reference points

Ex. of multiple interaction strategies with *k*-ary capacities

Figure: Ex. of decision strategies [Labreuche, Grabisch'2017].

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Some experiments

[Grabisch, Kojadinovic, Meyer'2008]

- Operation Research style: LP, Quadratic Programming

[Fallah Tehrani, Cheng, Dembczynski, Hüllermeier 2012]

- Machine Learning: Choquistic Regression

[Mayag, Grabisch, Labreuche'2008]

- Extension of MACBETH

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Based on information theory

Based on information theory [Kojadinovic, EJOR'2003]

Unsupervised learning given:

- $x^k = (x_1^k, \ldots, x_n^k) \in X$ for $k \in \{1, \ldots, K\}$

Compute:

- P_i: random variables of the scores $x_i^1, \ldots, x_i^K$ on attribute i
- $H(P_S) = -E\left\{\log\left(f_{P_S}(\rho_S)\right)\right\}$: measure of information brought by $\Pi = [\{P_i\}_{i\in S}]$, where $E\{\cdot\}$ is the expectation operator and $f_{P_S}(\rho_S)$ is the probability density function (pdf) of the multidimensional random variable $\Pi = [\{P_i\}_{i\in S}]$.
- $v(S) = \frac{H(P_S)}{H(P_N)}$.

Remarks:

- Def $H(P_S)$ is conceptually related to the notion of mutual information.
  Hence it is thus a natural measure of the degree of statistical dependence between the criteria within S.
- But estimating $H(P_S)$ is complex, especially for large values of n
- Moreover, if n is large and K is small, the estimator for $H(P_S)$ yields a large variance

Based on second-order statistics

Based on second-order statistics [Rowley, Geschke, Lenzen, FSS'2015]

Unsupervised learning given:

- $x^k = (x_1^k, \ldots, x_n^k) \in X$ for $k \in \{1, \ldots, K\}$

Compute:

- P_i: random variables of the scores $x_i^1, \ldots, x_i^K$ on attribute i
- $\mathrm{Cov}(P_i, P_j) = E\left\{(P_i - E\{P_i\})\,(P_j - E\left\{P_j\right\})\right\}$: covariance between P_i and P_j
- Covariance matrix $R_\sqcap = \left[\mathrm{Cov}(P_i, P_j)\right]_{i,j}$
- $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$: Eigenvalues of Covariance matrix $R_\sqcap$, with associated eigenvectors $v_1, v_2, \ldots, v_n$
- $J(P_S) = \sum_{i \in S} \min(\lambda_i, 1)$
- $v(S) = \frac{J(P_S)}{J(P_N)}$.

Remarks:

- The analysis is limited to pairwise dependencies among attributes

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Conclusion

Is interaction really useful?

- In practice, DMs naturally express interaction among criteria
- Interaction can be explicitly guessed (as in VFT) or learnt

Open problems

- Better understand the origin of interaction: from (statistical) dependencies or preferential dependencies
- Better discriminate between these two types of interaction
- Is is possible to perform “meaningful” unsupervised learning?