

# An outranking-based sorting method for partially ordered categories

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## Abstract

Found on the ELECTRE methodology, a new sorting method is proposed for the assignment of actions to partially ordered categories. The categories are defined by central profiles evaluated on a coherent set of criteria. The action to be sorted is first, pair-wise compared to the central profiles by computing outranking degrees. The assignment of an action is then based on its relative position in regards to the reference profiles in an optimistic and pessimistic “outranking graph”. Furthermore, we analyze this procedure for two particular subproblems where either the categories are completely ordered or defined in a strict nominal way. For the latter case, we will point out the advantage of the use of such an approach.

**Key words :** Multicriteria Classification, Sorting, Ranking

## 1 Introduction

Grouping or classification problems have been extensively studied in the literature and we usually distinguish different types of grouping problems. The groups may be defined a priori or not. In the former case we speak about supervised classification problems whereas in the latter case about unsupervised classification problems. Moreover, there may be a “preference” order on the groups or not ([6]). On the basis of these characteristics, grouping problems are differentiated as given in Tab.1.

In multicriteria decision aid we usually distinguish the choice, ranking and sorting problematic ([12]). The latter consists in assigning a set of decision actions, evaluated with

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Groups	Defined a priori	Not predefined
Ordered	Sorting	Ranking
Not ordered	Nominal Classification	Clustering

Table 1: Different grouping problems.

respect to a coherent set of criteria, to predefined groups. On the basis of this assignment, recommendations may be given.

Several assignment rules have been designed in multicriteria decision aid to take explicitly into account the decision maker’s preferences. Particularly in the case where the classes (often called in this context categories) are defined in an ordinal way (i.e. where there is a complete order on the categories). Let us cite amongst others the sorting methods based on outranking or preference degrees (with for example Electre-Tri: [13], [5]; *FlowSort*: [10]; [1]; [11], etc.), the multi-attribute theory (with for example UTADIS, [14]) or the rough sets ([7]). On the other hand, for nominal classification methods let us cite amongst others filtering methods ([11]), PROAFTN ([3]), the k-Nearest Neighbor Classifier ([8]), etc. For a more exhaustive survey on classification and sorting methods, we refer the reader to [6].

To the best of our knowledge, sorting methods, defining the categories by reference profiles (e.g. Electre-tri, *FlowSort*, etc.) suppose that there is a complete order on the categories. Nevertheless, in some cases, the decision maker may not be willing to define completely ordered categories or to consider them all as incomparable. The categories may then be partially ordered.

In this paper, we propose a method for tackling assignment problems in which the categories, defined by central profiles, are partially ordered. For that purpose, the method will at first compute outranking degrees between the actions to be assigned and the reference profiles. On that basis, an optimistic and pessimistic outranking graph, in which the action to be assigned and the reference profiles are represented, is proposed. The relative position of the action, in regards to the central profiles in these optimistic and pessimistic graphs, will be the basis of the assignment rules.

The paper is organized as follows. First, we give a brief reminder on the nominal classification method PROAFTN (Section 2). In Section 3, we give a review of Electre-Tri-Central ([9]). Electre-Tri-Central is a recently proposed extension of Electre-Tri in the case where the categories are defined by central profiles, instead of limiting one. The assignment rules of Electre-Tri-Central will be given on the basis of the outranking graphs. In Section 4, we propose to extend these graphical rules to partially ordered categories. We give moreover an illustrative example for this type of problem. In Section 5, we analyze the proposed assignment procedure in a nominal classification problem. The paper ends with some conclusions and further directions of research.

## 2 Brief reminder of PROAFTN

PROAFTN (PROcédure d' Affectation Floue dans le cadre de la problématique de Tri Nominal) has been developed by Nabil Belacel to address nominal classification problems. It has been used in many applications, especially in the medical sector. For more information, we refer the interested reader to [2, 3, 4].

Let us note  $\mathcal{A}$  the set of actions to be classified. The classes, noted as  $C^h$  ( $\forall h = 1, \dots, K$ ) are defined by incomparable central profiles, denoted by  $r^h$ , to which an action of  $\mathcal{A}$  is compared. The assignment rule is based on the following idea: "Any action which is considered as indifferent or sensibly equivalent to a reference profile, will be assigned to the corresponding class" ([3]). To evaluate the similarity between a reference profile  $r^h$  and an action  $a$ , a symmetric similarity index  $\mathcal{I}(r^h, a)$  is computed. This index results from the aggregation of the partial indifference and discordance indexes corresponding to the different attributes ([4]).

An action  $a \in \mathcal{A}$  will be assigned to a class  $C^h$  according to one of the following rules:

1.  $a \in C^h \Leftrightarrow \mathcal{I}(a, r^h) = \max\{\mathcal{I}(a_i, r^k), \forall k = 1, \dots, K\}$
2.  $a \in \Omega_i = \{C^h \mid \mathcal{I}(a, r^h) > \lambda_I; \forall h = 1, \dots, K\}$

where  $\lambda_I$  represents a similarity threshold.

The difference between these two assignment rules lies in the fact that the first rule will always assign an action to a category. When there are some ex-aequo, the assignment will not be unique. On the other hand, it may happen that an action is assigned to none or to several categories according to the second rule. Fig.1 is an illustrative representation when working with the second rule in a two-dimensional space where the points inside the circles (considered as similar to a reference profile) are assigned to the corresponding category.

Moreover, there exists a relation between the similarity index and the outranking relations ([3]):

$$\mathcal{I}(r^h, a) = \min(S(a, r^h), S(r^h, a)) = \mathcal{I}(a, r^h) \quad (1)$$

However, let us remark, that [9] has pointed out that PROAFTN may not be well-adapted when the categories are completely ordered.

## 3 Electre-Tri-Central

In this section, we present briefly the Electre-Tri-Central sorting method proposed in [9]. This method is analogous to Electre-Tri proposed by [13] for the case of categories defined by central profiles.

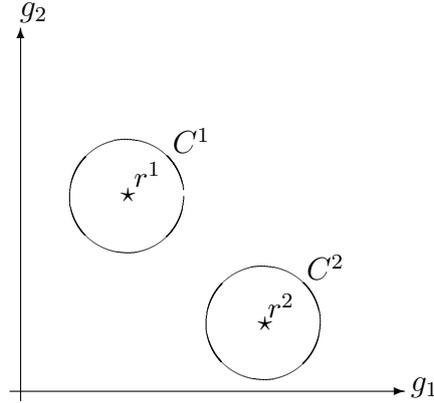


Figure 1: Representation of the second assignment rule of PROAFTN in a two-dimensional space.

We note  $C_m$  (with  $1 \leq m \leq K$ ) the ordered categories. Moreover, we suppose that category  $C_m$  is better than category  $C_n$  with  $m < n$ . The central profile of category  $C_h$  is noted  $r_h$ .

To assign an action  $a$  of the set  $\mathcal{A}$ , pair-wise comparisons are made on the basis of the outranking degrees  $S(a, r_h)$  and  $S(r_h, a)$ ,  $\forall h = 1, \dots, K$  ([12]), which take into account a coherent set of criteria  $\mathcal{G} = \{g_1, \dots, g_q\}$ .

Since the categories are completely ordered, the following conditions are imposed<sup>0</sup> :  $\forall m < n$  :

1.  $\forall j \in \mathcal{G} : g_j(r_n) \leq g_j(r_m)$  and  $\exists j \in \mathcal{G} : g_j(r_n) < g_j(r_m)$  (“dominance relation between the profile”)
2.  $r_n \prec r_m$  (“preference relation between the profiles”)

### 3.1 Assignment rules

In an ideal situation, an action  $a$  is assigned to a category  $C_h$ , if  $a$  and  $r_h$  are indifferent. In this case, we can notice that  $r_h$  is the least good (worst) profile which is at least as good as  $a$ . Alternatively,  $r_h$  is the best profile which is at least as good as  $a$ . These two considerations are the basis of the optimistic and pessimistic assignment rules.

**Optimistic version:** An action  $a$  is assigned to the category  $C_h$ , if  $r_h$  is the “worst” central profile which is at least as good as  $a$ . Formally:

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<sup>0</sup>We will suppose without any loss of generality that the criteria have to be maximized.

- Compare successively  $a$  and  $r_i$  with  $i$  from  $K$  to 1 by computing  $S(r_i, a)$ .
- If  $r_h$  is the first central profile such that  $r_h S a$ <sup>1</sup>, we then have that:  $C_{opt}(a) = C_h$

These rules may be illustrated by the hand of a “optimistic  $S$ -graph” as in Fig.2 where the arcs represent an outranking relation between two actions (denoted by squares). This graph may be reduced by representing only the transitive outranking relations since the profiles satisfy previous conditions. If we consider the oriented path  $\mathcal{C}_{1a}$  from  $r_1$  to  $a$ :  $\mathcal{C}_{1a} \equiv r_1 \rightarrow \dots \rightarrow r_j \rightarrow a$ , we may write the assignment rule as follows:

$$\begin{cases} \text{If } \nexists \mathcal{C}_{1a} : & \Rightarrow C_{opt}(a) = C_1 \\ \text{Else :} & \Rightarrow C_{opt}(a) = C_j \end{cases}$$

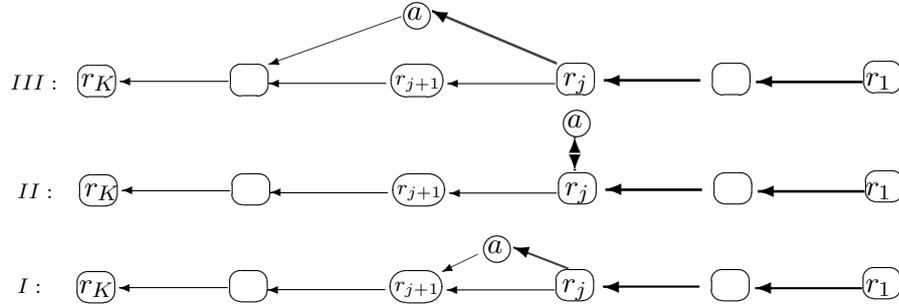


Figure 2: The reduced “optimistic S-graph”:  $xSy \Leftrightarrow x \rightarrow y$

Let us remark that if  $\mathcal{C}_{1a}$  does not exist, it means that  $r_1 \neg S a$ <sup>2</sup> and thus that  $a$  will be assigned to  $C_{opt}(a) = C_1$ .

**Pessimistic version:** An action  $a$  is assigned to the category  $C_h$ , if  $r_h$  is the “best” central profile which is at least as good as  $a$ . Formally:

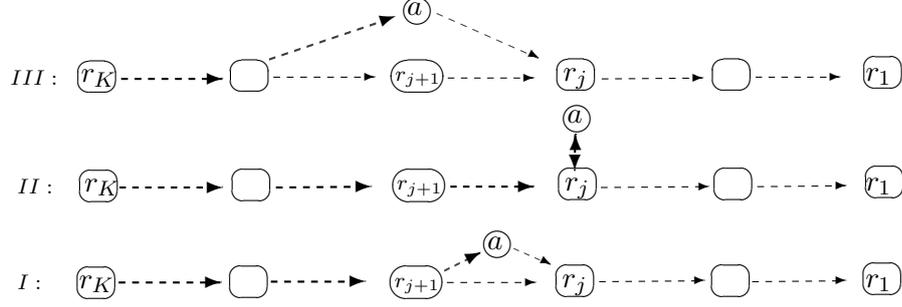
- Compare successively  $a$  and  $r_i$  with  $i$  from 1 to  $K$  by computing  $S(a, r_i)$ .
- If  $r_h$  is the first central profile such that  $a S r_h$ , we then have that:  $C_{pess}(a) = C_h$

In this case, the reduced “pessimistic S-graph” (see Fig.3) may be used by defining the path  $\mathcal{C}_{Ka}$  from  $r_K$  to  $a$ :  $\mathcal{C}_{Ka} \equiv r_K \dashrightarrow \dots \dashrightarrow r_j \dashrightarrow a$ :

$$\begin{cases} \text{If } \nexists \mathcal{C}_{Ka} : & \Rightarrow C_{pess}(a) = C_K \\ \text{Else:} & \Rightarrow C_{pess}(a) = C_j \end{cases}$$

<sup>1</sup>We will note  $a S b \Leftrightarrow S(a, b) \geq \lambda$

<sup>2</sup> $\neg$  stands for the logic negation operator.


 Figure 3: The reduced “pessimistic S-graph”:  $xSy \Leftrightarrow x \leftarrow\!\!\!-\! y$ 

In order to analyze the features of these assignment rules let us look at the assignment results in the three situations defined by [12] (with  $k \in \mathcal{N}$ ,  $1 \leq k \leq j$ ):

1.  $r_1 \succ a, r_2 \succ a, \dots, r_j \succ a, a \succ r_{j+1}, a \succ r_{j+2}, \dots, a \succ r_K$  (I)  
We have  $C_{opt} = C_j$  and  $C_{pess} = C_{j+1}$
2.  $r_1 \succ a, r_2 \succ a, \dots, r_{j-1} \succ a, a \mathcal{I} r_j, a \succ r_{j+1}, \dots, a \succ r_K$  (II)  
We have  $C_{opt} = C_j$  and  $C_{pess} = C_j$
3.  $r_1 \succ a, r_2 \succ a, \dots, r_{j+1} \succ a, a \mathcal{J} r_j, \dots, a \mathcal{J} r_{j+k-1}, a \mathcal{J} r_{j+k}, a \succ r_{j+k+1}, \dots, a \succ r_K$  (III)  
We have  $C_{opt} = C_{j-1}$  and  $C_{pess} = C_{j+k+1}$

The assignments are rather straightforward in situations *I* and *II*. Let us consider the situation *III* with  $k = 1$ . The number  $k$  represents the number of central profiles which are incomparable with  $a$ . If  $k = 1$ , action  $a$  is incomparable to  $r_j$  and will for that reason be assigned to the nearest better and worse category according respectively to the optimistic and pessimistic assignment rule. This can be motivated by the fact that since it is not indifferent to  $r_j$ , we may exclude category  $C_j$  from the possible categories and thus  $r_j$  from the set of reference profiles. If we eliminate  $r_j$ , we are then in a situation *I*.

Previous assignment rules are rather intuitive and are completely analogous to those defined in Electre-Tri-Limit. Moreover, the the outranking graphs enable a decision maker to understand them easily.

As proven in [9], Electre-Tri-Central presents the usual properties of a sorting method: properties of monotonicity, stability, independence, etc. Moreover,  $\forall a \in \mathcal{A}$  we have that

$C_{opt}(a) \geq C_{pess}(a)$ . Finally, let us remark that a relationship exists between Electre-Tri-Central and the classification method PROAFTN which links the similarity notion and the indifference relation.

## 4 Partially ordered categories

In [9], it has been pointed out that neither Electre-Tri is suited for classification problems nor PROAFTN for sorting problems. Nevertheless, these two problems may be considered as subproblems of a more general problem where there is a partial order on the categories or classes. In this section we will present assignment rules for this more general problem which may also be applied, as we will see, in the two previous particular subproblems.

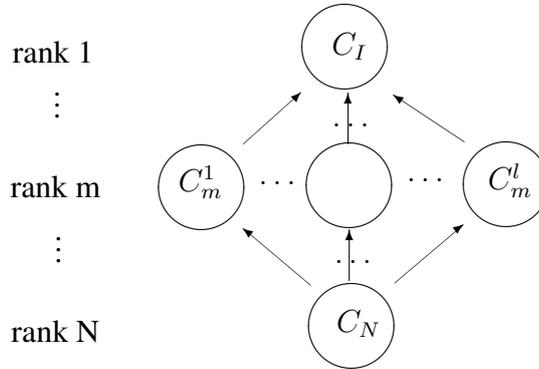


Figure 4: Representation of partially ordered categories.

Given a partial preference structure on the categories, we may define these by central profiles. We will note  $r_m^k$  the central profile of the  $k^{th}$  category of rank  $m$ , noted as  $C_m^k$ , and where  $C_m^k$  is preferred to  $C_n^l, \forall k, l; m < n$ . This is illustrated in Fig. 4.

We will suppose that there is only one category of the best rank,  $C_I$ , and one of the worst rank,  $C_N$ , which reference profiles will be noted  $r_I$  and  $r_N$ . If this is not the case, we may always define the ideal and nadir (virtual) categories,  $C_I$  and  $C_N$ , defined by the (virtual) reference profiles  $r_I$  and  $r_N$ , obtained for example as follows:  $\forall g_j \in \mathcal{G}$  :

$$g_j(r_I) = \max_{\forall m,k} [g_j(r_m^k)] \tag{2}$$

$$g_j(r_N) = \min_{\forall m,k} [g_j(r_m^k)] \tag{3}$$

Given the partial order on the categories, we may impose some reasonable conditions on the reference profiles:



Figure 5: Example of partially ordered reference profiles in the optimistic (left) and pessimistic (right) reduced “S-graph” where  $r_1^1 = r_I$  and  $r_3^1 = r_N$

1.  $\forall m; \forall k, l : r_m^l \mathcal{J} r_m^k$ : all the reference profiles of categories of the same rank are incomparable.
2.  $\forall m < n; \forall k, l; \forall g_j \in \mathcal{G} : g_j(r_n^l) \leq g_j(r_m^k)$  and  $\exists j \in \mathcal{G} : g_j(r_n^l) < g_j(r_m^k)$ : the profiles of categories of a better rank dominate all the profiles of a lower rank on each criterion.
3.  $\forall m < n; \forall k, l : r_n^l \prec r_m^k$ : the profiles of categories of a better rank are preferred to all the profiles of a lower rank on each criterion.

These outranking relations between the reference profiles may be represented by the reduced optimistic and pessimistic “S-graph” as for instance in Fig.5.

## 4.1 Assignment rules

An action  $a$  of  $\mathcal{A}$  will be pair-wise compared to the reference profiles by computing the outranking degrees  $S(a, r_m^k)$  and  $S(r_m^k, a)$ ,  $\forall m, k$ . These outranking relations will, as in Electre-Tri-Central, be represented in the the reference profiles’ reduced optimistic and pessimistic “S-graph” of (as for example in Fig.6) and then exploited by the assignment rules. The assignment will be based on its relative position.

**Optimistic version:** On the basis of the reduced optimistic “S-graph”, we may define the optimistic oriented path  $\mathcal{C}_{Ia}$  as follows:  $\mathcal{C}_{Ia} \equiv r_I \rightarrow \dots \rightarrow r_m^k \rightarrow a$ . Therefore, we may define the optimistic assignment rule such that:

$$\begin{cases} \text{If } \nexists \mathcal{C}_{Ia} : & \Rightarrow C_{opt}(a) = C_I \\ \text{Else} & \Rightarrow C_{opt}(a) = C_m^k \end{cases}$$



Figure 6: case I: Example of the optimistic and pessimistic reduced “S-graph” with  $a$ :  $C_{opt}(a) = C_3^1$  and  $C_{pess}(a) = C_2^2$

From Fig.6 we may deduce for example that  $C_{Ia} \equiv r_I \rightarrow a$  and thus that  $C_{opt}(a) = C_I = C_1^1$ .

**Pessimistic version:** On the basis of the reduced pessimistic “S-graph”, we may define the pessimistic oriented path  $C_{Na}$  as follows:  $C_{Na} \equiv r_N \rightarrow \dots \rightarrow r_m^k \rightarrow a$ . Therefore, we may define the pessimistic assignment rule such that:

$$\begin{cases} \text{If } \nexists C_{Na} : & \Rightarrow C_{opt}(a) = C_N \\ \text{Else} & \Rightarrow C_{pess}(a) = C_m^k \end{cases}$$

From Fig.6 we may deduce that  $C_{Na} \equiv r_N \rightarrow r_2^2 \rightarrow a$  and that  $C_{pess}(a) = C_2^2$ .

## 4.2 Illustrative example

As an example, let us consider the following sorting problem which is an adaptation of the example given by P. Perny (p.159 in [11]).

The human resource department would like to evaluate the personnel of a computer company. For that purpose, they define four different categories: the less good people, the engineers, the technical salespeople and the managers. Obviously, the class of the less good people,  $C_N$ , is preferred by all the other categories. On the contrary, the class of managers may be seen as the best category (noted  $C_I$ ). No preference between the engineers and the technical salespeople may be expressed: these are considered as incomparable. We are thus in presence of partially ordered categories:  $C_1^1 = C_I$ ;  $C_2^1$  and  $C_2^2$  and finally  $C_3^1 = C_N$ .

Table 2: Evaluation of the performances of the central reference profiles.

$r_n^m$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$
$r_3^1$	8	8	8	8	10
$r_2^1$	10	10	15	15	12
$r_2^2$	15	15	10	10	12
$r_1^1$	18	17	18	17	17

Table 3: Evaluation of the performances of the actions of  $\mathcal{A}$ .

$a_i$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$
$a_1$	13	12	13	12	13
$a_2$	13	11	9	9	12
$a_3$	18	18	12	15	16
$a_4$	15	15	9	13	12
$a_5$	12	13	10	10	12
$a_6$	8	7	5	5	12
$a_7$	10	18	10	18	17
$a_8$	10	20	10	20	20
$a_9$	20	20	20	20	20
$a_{10}$	0	0	0	0	0
$a_{11}$	14	13	4	10	12

The categories will be defined by central reference profiles which are evaluated according to the following set of criteria (which have to be maximized):

1.  $g_1$ : software knowledge
2.  $g_2$ : programming experience
3.  $g_3$ : commercial aptitude
4.  $g_4$ : potential mobility
5.  $g_5$ : leadership attitude

The evaluations of the reference profiles on the different criteria are given in Tab.2 and represented in Fig.7. The parameters associated to the criteria are as follows:  $\forall g_j \in \mathcal{G} : q_j = 1; p_j = 2, v_j = 5$  and  $w_j = 0.2$ . The  $\lambda$ -threshold is fixed at 0.6. The evaluations of the actions are given in Tab.3.

The actions to be sorted are pair-wise compared with the reference profiles by computing the outranking degrees which are given in Tab.4. On the basis of these, the optimistic and pessimistic assignment rules sort the actions into the corresponding categories. The

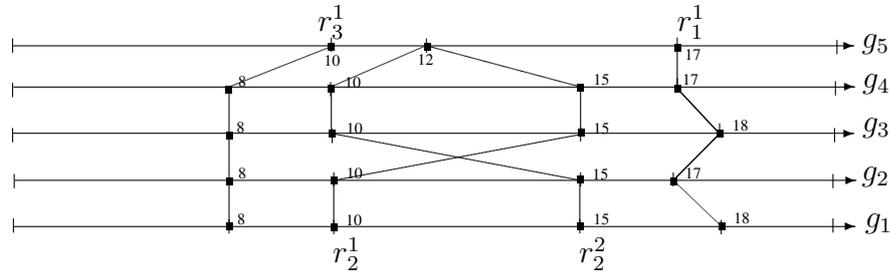


Figure 7: Representation of the performances of the central profiles  $r_1^1$ ,  $r_2^1$ ,  $r_2^2$  and  $r_3^1$ .



Figure 8: Case II:  $C_{opt}(a) = C_3^1$  and  $C_{pess}(a) = C_1^1$

results are given in Tab.5.

Fig.6 - Fig.18 represent several possible situations when comparing  $a$  to the partially ordered reference profiles.

Let us remark that it might happen, like for instance in case III and V of the example from section 4.2, that more than one optimistic and pessimistic paths exist. This simply means that action  $a$  may be assigned, according to a procedure, to several categories. It doesn't respect thus the property of uniqueness proposed by [13].

On the basis of Eq.1, we may use the second assignment rule of PROAFTN and fix  $\lambda_I = 0.6$ . We can thus remark that the actions  $a_2, a_3, a_5, a_7, a_8, a_9, a_{11}$  will be assigned to no category. On the other hand, we have that  $C^{PRO}(a_1) = C_2^2 \cup C_2^1$ ,  $C^{PRO}(a_4) = C_2^2$ ,  $C^{PRO}(a_6) = C_3^1$ . It's obvious, on basis of Eq.1, that these last assignments are similar to the previous one.

Table 4: Pair-wise comparisons between the actions and the reference profiles.

	$r_3^1$	$r_2^1$	$r_2^2$	$r_1^1$
$S(a_1, r_n^k)$	1	0.6	0.6	0
$S(r_n^k, a_1)$	0	0.6	0.6	1
$S(a_2, r_n^k)$	1	0	0.5	0
$S(r_n^k, a_2)$	0	0.8	1	1
$S(a_3, r_n^k)$	1	0.8	1	0
$S(r_n^k, a_3)$	0	0	0	1
$S(a_4, r_n^k)$	1	0	1	0
$S(r_n^k, a_4)$	0	0	0.8	1
$S(a_5, r_n^k)$	1	0	0.6	0
$S(r_n^k, a_5)$	0	0.6	1	1
$S(a_6, r_n^k)$	0.6	0	0	0
$S(r_n^k, a_6)$	0.8	1	1	1
$S(a_7, r_n^k)$	1	0	0	0
$S(r_n^k, a_7)$	0	0	0	1
$S(a_8, r_n^k)$	1	0	0	0
$S(r_n^k, a_8)$	0	0	0	0.4
$S(a_9, r_n^k)$	1	1	1	1
$S(r_n^k, a_9)$	0	0	0	0
$S(a_{10}, r_n^k)$	0	0	0	0
$S(r_n^k, a_{10})$	1	1	1	1
$S(a_{11}, r_n^k)$	0.8	0	0	0
$S(r_n^k, a_{11})$	0	0.5	1	1

Table 5: Classification result of the actions.

$a_i$	$C_{opt}$	$C_{pess}$	case
$a_1$	$C_2^2, C_2^1$	$C_2^2, C_2^1$	V
$a_2$	$C_2^1, C_2^1$	$C_3^1$	III
$a_3$	$C_1^1$	$C_2^2, C_2^1$	XI
$a_4$	$C_2^2$	$C_2^2$	IV
$a_5$	$C_2^1$	$C_2^2$	XI
$a_6$	$C_3^1$	$C_3^1$	VII
$a_7$	$C_1^1$	$C_3^1$	II
$a_8$	$C_1^1$	$C_3^1$	VIII
$a_9$	$C_1^1$	$C_1^1$	IX
$a_{10}$	$C_3^1$	$C_3^1$	X
$a_{11}$	$C_2^2$	$C_3^1$	XII



Figure 9: Case III:  $C_{opt}(a) = C_2^2 \cup C_2^1$  and  $C_{pess}(a) = C_1^1$



Figure 10: Case IV:  $C_{opt}(a) = C_2^2$  and  $C_{pess}(a) = C_2^2$



Figure 11: Case V:  $C_{opt}(a) = C_2^2 \cup C_2^1$  and  $C_{pess}(a) = C_2^2 \cup C_2^1$

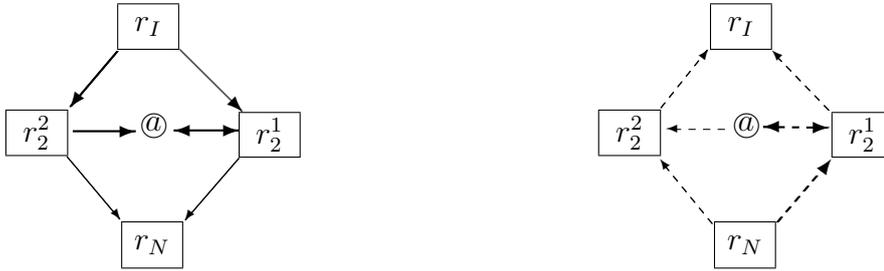


Figure 12: case VI:  $C_{opt}(a) = C_2^2 \cup C_2^1$  and  $C_{pess}(a) = C_2^1$



Figure 13: case VII:  $C_{opt}(a) = C_3^1$  and  $C_{pess}(a) = C_3^1$



Figure 14: case VIII:  $C_{opt}(a) = C_1^1$  and  $C_{pess}(a) = C_3^1$

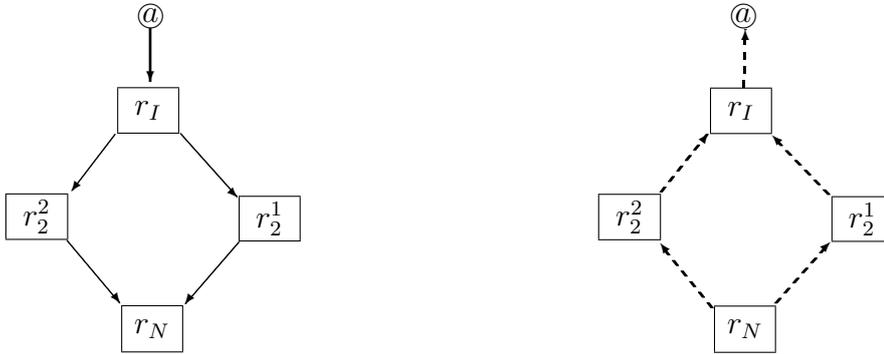


Figure 15: case IX:  $C_{opt}(a) = C_1^1$  and  $C_{pess}(a) = C_1^1$

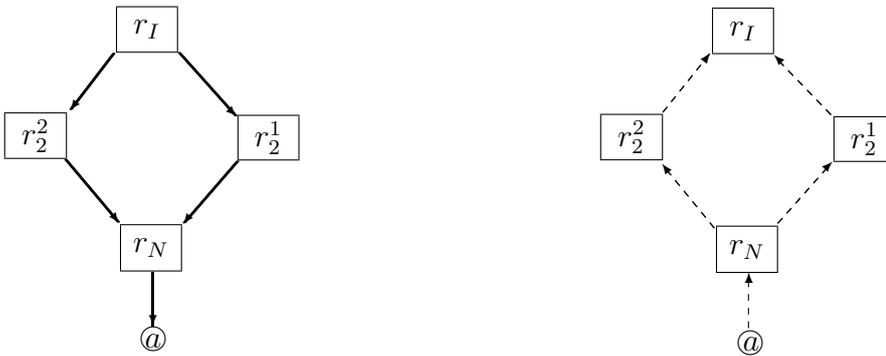


Figure 16: case X:  $C_{opt}(a) = C_3^1$  and  $C_{pess}(a) = C_3^1$



Figure 17: case XI:  $C_{opt}(a) = C_1^1$  and  $C_{pess}(a) = C_2^1$  and  $C_2^2$



Figure 18: case XII:  $C_{opt}(a) = C_2^1$  and  $C_{pess}(a) = C_3^1$

## 5 Nominal classification problems

When the categories are completely ordered, the assignment rules are, by construction, similar to the one of Electre-Tri-Central. This assignment procedure may thus also be used in this subproblem.

When there is no order on the classes, we may define a virtual ideal and nadir reference profile in order to apply previous assignment rules (see Fig.19). Actions, considered as similar<sup>3</sup> to a reference profile, are indifferent to that reference profile and will thus be assigned to the corresponding class (cf. Eq.1).

Nevertheless, an action which is not similar to any reference profile may be assigned (e.g.  $b_1, b_2, b_3, b_4$  in Fig.19).

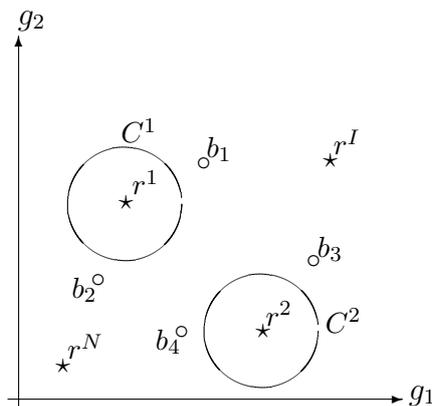


Figure 19: Representation of the ideal and nadir reference profile in case of nominal classification problems.

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<sup>3</sup>according to the second rule of PROAFTN

Actually, the relative position in the partial order will be exploited to refine the result when there is no similarity. In the example of Section 4.2, let us consider that  $r_I$  and  $r_N$  are virtual reference profiles which do not represent a particular category. They are thus introduced for the assignment procedure. In case I, action  $a$  will be assigned to class  $C_2^2$  and  $C_I$ . The same results may be obtained for  $b_3$  in Fig.19 if we consider that the two criteria  $g_1$  and  $g_2$  have to be maximized. This should be interpreted as follows: action  $a$  actually belongs to the “upper” (better) part of category  $C_2^2$ . On the contrary, action  $a$  should belong to the “lower” (worse) part of category  $C_2^1$  in situation XII (since it will be assigned to  $C_2^1$  and  $C_N$ ). The same results may be obtained for  $b_2$  in Fig.19.

We can thus remark that the use of the partial order may be useful in situations where there is no strict similarity or indifference. It permits thus to refine the assignment.

## 6 Conclusions

In this paper, we developed a new multicriteria sorting method inspired by the ELECTRE methodology for assignment problems where the classes are partially ordered. The categories are defined by central profiles. The assignment rules are based on a graphical representation of the outranking relations between the reference profiles and the actions to be sorted.

The proposed assignment procedure may be used in the particular subproblems where either the categories are completely ordered or defined in a nominal way. Furthermore, when there is no order on the classes, we illustrated that such a method permits to precisely point out the issues involved and to refine the assignments. The graphical representations contribute to an easy comprehension of the assignment results.

Some effort should be spent on facilitating the determination of the parameters of the model such as for instance the central profiles or the thresholds. More generally, a natural extension of this model should be the adaptation of the assignment rules when the categories are defined by several central profiles.

## References

- [1] C. Araz and I. Ozkarahan. Supplier evaluation and management system for strategic sourcing based on a new multicriteria sorting procedure. *International Journal of Production Economics*, 106:585–606, 2007.

- [2] N. Belacel. Méthodes de Classification Multicritère : Méthodologies et applications à l'aide au diagnostic médical. Thèse de doctorat, Université Libre de Bruxelles, 2000.
- [3] N. Belacel and M. Boulassel. PROAFTN: a fuzzy assignment method to grade bladder cancer malignancy using features generated by computer-assisted image analysis. *Foundations of Computing and Decision Sciences*, 25(1):37 – 47, 2000.
- [4] N. Belacel and M. Boulassel. Multicriteria fuzzy assignment method: a useful tool to assist medical diagnosis. *Artificial Intelligence in Medicine*, 21(1-3):201 – 207, 2001.
- [5] J. Almeida Dias, J.R. Figueira, and B. Roy. Electre Tri-C: A multiple criteria sorting method based on central reference actions. Working paper 5/2008, CEG-IST, Instituto Superior Técnico, Lisboa, Portugal, 2008.
- [6] M. Doumpos and C. Zopounidis. A multicriteria classification approach based on pairwise comparisons. *European Journal of Operational Research*, 158:378–389, 2004.
- [7] S. Greco, B. Matarazzo, and R. Slowinski. Rough sets theory for multicriteria decision analysis. *European Journal of Operational Research*, 129:1–47, 2001.
- [8] J. Han and M. Kamber. *Data Mining: Concepts and Techniques*. 2001.
- [9] Ph. Nemery. Utilisation des relations de surclassement dans des problèmes de classification. *Université Libre de Bruxelles, CoDE, SMG, Techincal Report*, 001, 2008.
- [10] Ph. Nemery and C. Lamboray.  $\mathcal{F}$ lowSort: a flow-based sorting method with limiting and central profiles. *TOP*, 16:90–113, 2008.
- [11] P. Perny. Multicriteria filtering methods based on concordance and non-discordance principles. *Annals of Operations Research*, 80:137–165, 1998.
- [12] B. Roy and D. Bouyssou. *Aide Multicritère à la décision : Méthodes et cas, Economica, Paris*. Paris, 1993.
- [13] W. Yu. ELECTRE TRI : Aspects méthodologiques et manuel d'utilisation. *Document du Lamsade, Université Paris-Dauphine, Paris*, 74, 1992.
- [14] C. Zopounidis and M. Doumpos. Business failure predictions using the utadis multicriteria analysis method. *Journal of Operational Research Society*, 50:1138–1148, 2002.