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Delayed differentiation in chemical supply chains by Reverse Blending

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Abstract. Mass customisation of products is a significant trend in modern production. In continuous production (flow process), delayed differentiation is not possible at the factory level, as in discrete production (production of objects). Reverse Blending (RB) was designed to delay the differentiation of fertiliser production, in small blending units located in agricultural areas, as they already exist in some regions. The RB models and a significant example of its use were recently published in *International Journal of Production Economics* (https://doi.org/10.1016/j.ijpe.2019.107603). Without going back on the positioning of the RB approach in the scientific literature, this paper aims to study some of its managerial implications, based on four case studies. The RB models and two other ones involved by these case studies are described in synthesis tables, allowing to understand their differences and their use quickly. These case studies show the value of this delayed differentiation approach in chemical supply chains.

Keywords: Delayed differentiation, supply chain, chemical industry, blending, composite design

1. Introduction

In recent decades, mass production of customised products has been a reality in the manufacturing industry, with the automotive industry being the most emblematic example. It was made possible by the combination of efforts on standardisation, commonality, modular design and platforms to quickly assemble modules from exclusive sets of alternative modules (e.g. engines or gearboxes). In the manufacturing industry, delayed differentiation, which allows the customers to be supplied with the product they want (Make-To-Order), in a set that can include hundreds of thousands of possible products, is carried out at the assembly lines.

This solution is not possible in the continuous production that process streams of raw materials to obtain batches of products in the form of powders, granules, or liquids. The variety can only be obtained in the plant by a succession of productions of batches of different products, leading either to a Make-To-Order of a limited number of different products within a year (fertiliser production, for example) or to a production-to-stock of a small set of different products (e.g. yoghurt production). For this type of production, delayed differentiation may only be possible at the level of the packaging of the product intended for a market, especially for linguistic or regulatory reasons, without any modification of the product. The only notable exception of delayed differentiation on the product is in the distribution of paint in specialised stores, where the customer chooses, a colour reference among several possible hundreds of colours displayed in a colour chart, the product being "assembled-to-order" on-site from a paint base and a reduced set of dyes, scrupulously respecting the proportions of the nomenclature associated with the chosen reference. In this case, the product is personalised in stores and not in the factory.

The Reverse Blending (RB), described in detail in an article published in the *International Journal of Production Economics* (IJPE) in August 2020, was designed to ensure a sustainable and reasonable agriculture. The huge required variety of fertiliser requirements (which is several thousand) is mainly due to the combination of the characteristics of a parcel of land and the nutrient requirements of the crop chosen for this parcel (the main ones being Nitrogen N, Phosphorus P and Potassium K). The use of one of the commercially available fertilisers necessarily leads to excess or insufficient of certain nutrients. The European regulation EU PE-CONS 76/18 (2019) defines acceptable deviations in the composition of a fertiliser to have the expected performance if its composition is that required for the "crop-soil" pair. The world's top fertiliser producers produce less than 200 different fertilisers. One of the old used solution, when fertilisers do not match nutriments requirements, is to mix two or more fertilisers to bring the blend composition closer to these needs; this blend is defined by a simple linear program to minimise an indicator of the difference between the required composition and the composition obtained by the "optimal" mixture of these fertilisers. Although this solution is often used, it rarely meets compositional requirements, within the limits of permitted tolerances, as defined by EU regulation.

The production nomenclature of a paint corresponding to a reference in a colour chart is defined from a paint base and a reduced list of pre-existing dyes. In the example of fertilisers, the list of needs does not exist (first problem), and the same applies to the equivalent of the dyes set (second problem). For the first problem, one can hope to rely on a relatively large sample of needs; in the IJPE article, this representative sample is 700 fertiliser needs, corresponding to 482 custom NPK fertiliser formulas. From this sample, the RB approach was able to be defined simultaneously by a quadratic program: *i*) composition specifications of ***new*** inputs belonging to the smallest possible size set, considered a Canonical Base (CB), to use classical mathematical terminology (these CB inputs being identified by the CBI acronym); this is the answer to the second problem; *ii*) the production nomenclature of fertilisers in this sample, that use the found CBIs, with an acceptance of very low deviance (this is the answer to the first problem). In the fertiliser context, the delayed differentiation, corresponding to a blend of CBIs, takes place in small remote blending units that can serve a sufficient set of customers.

This article does not cover the RB foundations and its original positioning in the scientific literature, that are described in the IJPE article. However, the technical appendix of this paper, structured around two synthesis tables comparing six blending models, allows to quickly understand the used models, point out their differences and see the originality of the RB approach. Two of these six blending models are introduced to meet the needs of this paper. It seems preferable to focus on some RB managerial consequences, not included in the IJPE article, that are described through four case studies.

- The first case study shows that the 8 CBIs found in the IJPE study, from a sample of 700 NPK fertiliser requirements (covered by 482 NPK fertiliser formulas), not only deliver these 482 formulas, which is a direct consequence of the RB use, but may be sufficient to meet a wide variety of other NPK fertiliser needs. This possibility is illustrated by a study of a random sample of about 50 new NPK fertilisers.
- The second case study analyses the consequences of the scenario of replacing the 2019 fertiliser production in the Jorf site (Morocco's largest fertiliser production centre), with a CBIs production whose blends and quantities allow to obtain exactly these fertilisers produced during that year. The focus turns to the consequences on the organisation of production and storage, at the monthly mesh, of a significant reduction in the diversity to be produced. This study does not intend to propose the replacement of existing fertilisers but to define a plausible

scenario of a CB that technically responds to an existing commercial offer, the replacement by the CBIs being of little interest. Starting from a CB able to respond precisely to the actual commercial offer, the impact study is based on plausible diversity of CBIs production and therefore on realistic working assumptions. However, with the same number of CBIs, the RB allows meeting hundreds, if not thousands, of different fertiliser requests, starting from the treatment of a different and huge sample of fertiliser requests.

- The next case study focuses on reducing the diversity of fertilisers available on the international market. Starting from all 142 different fertiliser formulas sold by the world's top fertiliser producers, a significant part of these 142 formulas can be obtained by a simple blend of some of these formulas, thus showing that the diversity offered on the market is not as broad as one might think. The RB approach allows us to go further in the reduction of diversity produced for the same offered diversity of 142 formulas. As with the previous case study, it is not a question of replacing these 142 fertilisers with blends of CBIs as again, for the same number of CBIs, it is possible to meet hundreds, if not thousands, of different fertiliser requests, by applying the RB approach to an extensive sample of fertiliser requests.
- The last case study aims to expand the RB scope through a study showing its possible interest in delayed differentiation of cosmetics, made at the point-of-sale level.

2. Use of a CB to produce new fertilisers of the same type as those used for the creation of this CB

The 700 NPK needs, corresponding to the 482 custom formulas, were defined on a reasoned sample of parcels (whose description is given in https://doi.org/10.1016/j.ijpe.2019.107603) from a few provinces in Morocco where wheat cultivation is considered attractive (see http://dx.doi.org/10.17632/z3sbn5j9z7.1). These needs are calculated by the difference between the nutritional requirements of crops, compatible with sustainable agriculture, and the possible contributions of these parcels, given their soil characteristics. The RB application to this set of needs simultaneously define *i*) the CB, of which each input contains, in addition to major nutrients, the "filler" which is an additional neutral component used for chemical stabilisation of granule composition (see Table 1); *ii*) and the nomenclatures of these 482 fertilisers using the 8 CBIs found to satisfy each of the 482 formulas.

Table 1. The composites of the canonical base found from the example of the 700 NPK needs

		CBI *i*							
		filler	*i* =2	*i* =3	*i* =4	*i* =5	*i* =6	*i* =7	*i* =8
Component *c*	N (*c* =1)	0.00%	46.00%	0.00%	22.53%	29.78%	12.83%	52.20%	20.32%
	P (*c* =2)	0.00%	0.00%	46.00%	0.15%	23.53%	0.00%	42.32%	0.49%
	K (*c* =3)	0.00%	0.00%	0.00%	0.00%	0.00%	47.09%	0.49%	74.19%
	filler (*c* =4)	100%	54.00%	54.00%	77.31%	46.69%	40.08%	5.00%	5.00%

This CB has been defined to meet the requirements of Moroccan wheat, but it is not, strictly speaking, dedicated to this crop and this country. Indeed, the variety of fertiliser that can be obtained by a linear combination of these CBIs far exceeds the 482 fertilisers of the sample: a vast number of these linear combinations may correspond precisely to a fertiliser formula, others deviating a little but within the limits of tolerances set by European regulations.

To exploit this possibility, a linear programming model was created to find the CBIs nomenclature that meets the specifications of a new fertiliser. This model, called "Use of CB (UCB)" described in Table 7 (Column 3) of Appendix: *i*) minimises an indicator of the difference between the required composition and the optimal composition found; *ii*) prevent the use of a CBI in the nomenclature of the fertiliser to be produced, in case of insufficient percentage (1% lower bound is retained by default); *iii*) considers that there are no restrictions on the availability of CBIs and that the demand for the fertiliser studied is conventionally 100 tons to facilitate analysis of the results. The EU's regulatory constraints have not been integrated, and compliance is verified from the solution. This model is implemented under Excel, to allow easy use in deported blending units, where a new customised fertiliser is to be provided.

A random sample of 50 NPK fertilisers was formed. An extract from this sample is given in table 2 (full data is provided in the sheet "CS1.1" of the Excel file included in the Mendeley link[1] https://data.mendeley.com/datasets/2jdwk3pnjd/draft?a=4952dfdb-c261-4049-896c-e9b502327424). The results of these 50 optimisations (see the sheet "CS1.2"), an extract of which is presented in Table 2, lead to the exact obtaining of the required composition for 48 fertilisers. For the remaining two fertilisers, there are absolute differences well below the tolerances of the European standard: for fertiliser 8: 0% for N and K and -0.05% for P; for fertiliser 17: 0.01% for N and 0% for P and K.

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[1] This Mendeley link has been created to store a large amount of data so as to save space in the text. It consists of a single Excel workbook containing all the input data and results of the four treated case studies. In what follows, the data is called up by referring to the sheet that contains it.

Table 2. Excerpt from the optimal CBIs nomenclature to be used to meet the chemical characteristics of the 50 fertilisers

	The chemical specifications β_c					Optimal quantities x_i to be withdrawn from each CBI to produce each output								Demand (tons)
	%N (c=1)	%P (c=2)	%K (c=3)	%*filler* (c=4)		*filler*	i=2	i=3	i=4	i=5	i=6	i=7	i=8	
Fert.1	20.52%	8.94%	0.00%	70.54%	Fert.1	42.0	20.0	0.0	0.0	38.0	0.0	0.0	0.0	100
Fert.2	30.30%	9.07%	27.26%	33.38%	Fert.2	16.5	25.9	0.0	0.0	0.0	0.0	21.0	36.6	100
Fert.3	20.64%	27.15%	0.00%	52.21%	Fert.3	5.1	8.3	30.1	0.0	56.5	0.0	0.0	0.0	100
...	…				...	…								…
Fert.8	27.90%	12.00%	21.79%	38.31%	Fert.8	23.9	13.4	0.0	5.6	0.0	0.0	27.9	29.2	100
...	…				...	…								…
Fert.17	11.00%	12.87%	16.92%	59.21%	Fert.17	22.2	13.9	28.0	0.0	0.0	35.9	0.0	0.0	100
...	…				...	…								…
Fert.48	8.27%	15.93%	0.00%	75.80%	Fert.48	29.8	***1.0***	34.5	34.7	0.0	0.0	0.0	0.0	100
Fert.49	21.13%	19.50%	12.22%	47.15%	Fert.49	2.7	38.7	42.2	0.0	0.0	0.0	0.0	16.5	100
Fert.50	13.19%	6.70%	47.98%	32.13%	Fert.50	21.5	0.0	13.9	0.0	0.0	0.0	0.0	64.7	100

It is reasonable to believe, in light of these results, that the creation of a reasoned sample of a few thousand fertiliser needs that cross pedological characteristics and those of crops, can create a CB to cover the essential needs of NPK nutrients, to which we can add the Sulfur S that is also an essential nutrient for fertilisers. Other nutrients are sometimes requested, as the following case study will show. To meet these new needs, simply start from the original CB and add new composites to be created to cover these new needs.

3. The impact of Reverse Blending on managing the diversity produced by OCP in Jorf

From 2000 to 2020, OCP Group, one of the world leaders in the fertiliser sector, increased the diversity of its products from 5 to about 50 fertilisers. OCP is still pursuing this strategy of increasing diversity, which enables it to meet needs better and win new customers. The counterpart is an increasing complexity of the management of production and storage. The potential benefits of RB are examined here by exploring how production and storage would be organised if OCP's production for 2019 would have been organised using RB. The monthly production covers 28 fertiliser references produced on 7 production lines (see "CS2.1"). Figure 1 shows the shares of these fertilisers in the total production for each month. The diversity of the products (illustrated by the different colours in each stick) and the production volumes vary from one month to another, and these volumes correspond to small batches.

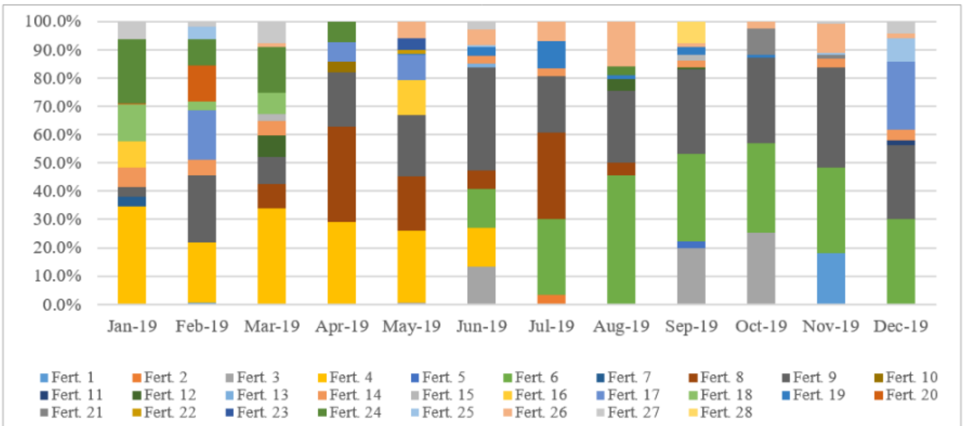

Figure 1: Individual shares of fertilisers in each month's production

This figure reflects the significant number of fertiliser reference changes that had to be made each month. A more detailed analysis, in the daily mesh, shows that over this year, 175 changes of references to be produced were made on the 7 production lines. Even considering the shortest launch time, which is in the order of two hours (and can go up to eight hours), production had to be stopped for at least 350 hours. Taking into account the average production rates, this translates into a minimum loss of production capacity of 33,783 tons in 2019. With a diversity of 28 fertilisers and 9 storage sheds, this method of production, which relies on the irregular launch of small batches, leads to problems of space allocation and fertilisers contamination due to poor separation between the different references.

To analyse the impact of replacing the current organisation with the CBIs production, it was first necessary to define a CB, which is difficult to dispute. The CB has been selected to meet the OCP's 2019 production program, which gives 8 CBIs; their optimal composition in terms of N, P, K, B_2O_3 (boron), Zn (zinc) and filler is given in Table 3. The CBIs quantities to be used to obtain the monthly produced volume of each fertiliser are shown in "CS2.2" and displayed in figure 2, to show the share of each CBI in each month's production.

Table 3: The optimal composition of the CBIs

		Canonical Basis Inputs (CBI i)								
		CBI 1	CBI 2	CBI 3	CBI 4	CBI 5	CBI 6	CBI 7	CBI 8	*filler*
Component c	N(c=1)	46.00%	11.86%	12.70%	19.00%	0%	2.34%	2.14%	0%	0%
	P(c=2)	0%	56.08%	16.11%	38.00%	0%	56.00%	56.00%	51.24%	0%
	K(c=3)	0%	0%	16.11%	0%	63.60%	0%	0%	0%	0%
	S(c=4)	0%	0%	0%	7.00%	25.27%	11.78%	7.06%	19.67%	0%
	B_2O_3(c=5)	0%	0%	0%	0%	6.13%	3.15%	0%	0%	0%
	Zn(c=6)	0%	0%	0%	0%	0%	0.00%	0%	7.79%	0%
	filler (c=7)	54.00%	32.06%	55.07%	36.00%	5.00%	26.72%	34.81%	21.30%	100%

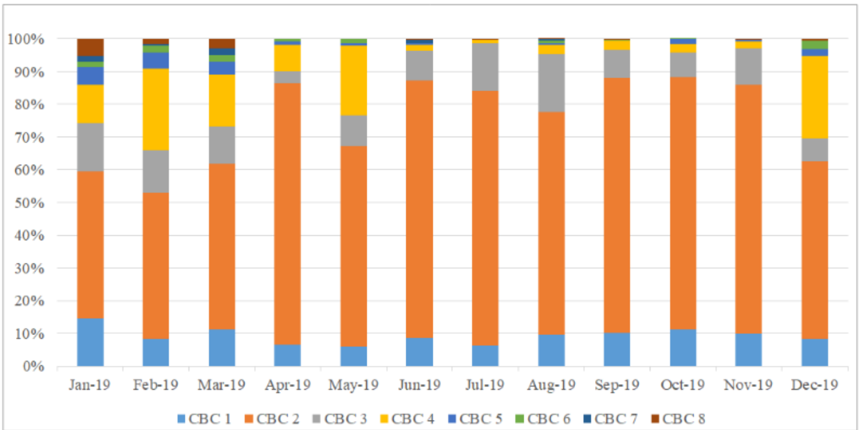

Figure 2: Share of CBIs in each month's production

Comparison of Figures 1 and 2 shows how this transformation considerably simplifies production management since, with only 8 CBIs, OCP's monthly production can be precisely ensured. Moreover, at least 90% of this monthly production involves only the first 4 CBIs, CBI 2 (orange colour) representing more than 50%. This flows consolidation, which would have been even more important if the treated variety had been more than 28 fertilisers, shows the opportunity to design a new production system based on a Make-To-Stock" policy. Indeed, with seven production lines, three of which have an annual throughput of around 897,000 tons and four of which have an annual throughput of around 669,000 tons, managing the production of 8 CBIs is relatively simple. Considering the required total quantity for each CBI and the throughput of these production lines, it is chosen to arbitrarily allocate the most requested CBIs to the most productive (and incidentally the most economical) lines. Consequently, since CBI 2 accounts for more than 66 % of the annual production volume, three production lines can be entirely devoted to it. For CBIs 1, 3 and 4, each representing about 10 %, one production line must be provided for each of them. Finally, as the remaining CBIs represents less than 4 % of the total production, they can all be allocated to one production line.

By producing continuously (without having to change to another reference), "mono-CBI" lines would eliminate the shutdowns resulting from launch times, thus boosting production capacity. The only remaining "multi-CBIs" production line, may be managed with "an order-point / order quantity" policy.

The management of storage sheds would also become easier since, as the storage of 8 CBIs, compared to 28 fertilisers, in 9 sheds, would present practically no problem. Each shed could be assigned to one or two CBIs, which would eliminate storage problems caused mainly by high diversity such as vacant space due to small production batches and production stoppages due to stock saturation.

Finally, let us look back to the relevance of the selected CB. The implications of producing CBIs of a CB that can reconstitute all of the OCP's produced fertilisers were shown. Based on an analysis of fertiliser needs on a large reasoned sample, it is possible, with another CB having the same number of CBIs, to meet hundreds, if not thousands, of fertiliser requirements.

4. Analysis of the potential reduction of fertilisers variety produced by the world's leading producers

The diversity of fertilisers sold by the world's largest fertiliser producers (OCP Group, Yara, Agrium, K+S, Potashcorp, Mosaic, Uralkali, Belraruskali and CF Industries) consists of a total of 175 fertilisers, but if only those with different formulas are taken into account, a total of 142 fertiliser formulas is reached. These formulas can contain from 2 to 10 components; the first nine one corresponding respectively to N, P, K, S, B_2O_3, Zn, Mg (magnesium), Na (sodium) and Ca (calcium) and the last one corresponding to the filler. Note that fertilisers containing at least one of the major components (NPK) account for 56% of these formulas and if Sulfur (S) is added, a percentage of 82% is reached. The chemical characteristics of these formulas are given in "CS3.1".

This diversity of the offer is far from being able to cover the needs of sustainable agriculture. To avoid increasing this variety by creating new fertilisers, some distributors opt for mixing existing fertilisers in small blending units located in agricultural areas. This blending leads to the creation of a new fertiliser whose composition differs from that of existing fertilisers. As a result, the "diversity of potential use" of existing fertilisers is much higher than the diversity of the commercial offer. In this context, it is interesting to see how far the diversity of commercial offer can be reduced without affecting the "diversity of potential use ".

To do this, we will start by looking at the extent to which some existing fertilisers can be obtained by mixing other fertilisers, which leads to the search for the minimum set of fertilisers (inputs) needed to reconstitute the 142 fertilisers. In this search, the absolute deviation between the target value of the share of a component in the total fertiliser weight and the value obtained by the optimal solution must not exceed 0.0005 (for example, for fertiliser 1, if N must represent 20.52% of the total weight, the solution is only acceptable if the N share is between 20.47% and 20.57%). This margin of error is far less than what European regulations tolerate.. To further reduce this set of inputs able to reconstitute these 142 fertiliser formulas, the RB approach will then be applied to determine the canonical basis enabling the obtaining of these formulas. This analysis will be concluded by explaining the limits of the interest of reducing the commercial offer.

Reducing the commercial offer diversity of fertilisers by combining existing fertilisers

This analysis was conducted by a model called "Diversity Reduction (DR)" (described in column 4 of Table 7), which is a blending model corresponding to a parameteric mixed linear program. This model is "mix" due to the introduction of two binary variables, the first associated with the production of an output and the second with the use of an input. This program is "parametric" because we seek to produce all the outputs using a search process that maximises the number of outputs that can be produced from a combination of inputs belonging to a subset S (parameter) extracted from the set of outputs. The resolution of this model starts with S=J-1 then S is progressively decremented (S=J-2, J-3,...) until the required minimum number of inputs so that all the outputs can be obtained is found.

The use of the RD model reveals that instead of producing all of 142 fertilisers by chemical transformation, which is what is currently practised, it is indeed possible with 33 existing fertilisers to produce the remaining 109 fertilisers, by using the nomenclature given in sheet "CS3.2". As the tolerance to the target specifications is set at ±0.0005, a further test was made by decreasing the tolerance, and it was found that an exact optimum solution exists. It was preferred to keep this solution in the Mendeley folder.

Reducing the commercial offer diversity of fertilisers through Reverse Blending

The objective is to determine the CBIs that make it possible to recreate the 142 fertilisers currently sold, in compliance with European regulations (EU PE-CONS 76/18, 2019), as well as to satisfy a very large number of other nutritional needs. These CBIs must comply with certain compositional constraints that require a chemical element to be contained in a CBI in minimum and maximum proportions not to be exceeded. These proportions are listed, for each CBI, in Table 4.

The optimisation model to be used is the standard Reverse Blending (RB) one. Its mathematical formulation can be found in the second column of Table 7. As already explained, small deviations from the target specifications are allowed in the RB and RD as long as these deviations remain within the acceptable tolerances. It should be noted that in the regulations (EU PE-CONS 76/18, 2019), the nutritional specifications of fertiliser must be respected while taking into account a margin of tolerance that varies according to the number and type of the fertiliser. For example, for secondary nutrients, the tolerances allowed concerning the declared values of Ca, Mg, Na and S are set at one-quarter of the declared contents of these elements with a maximum of 0.9% in absolute value for CaO, MgO, Na2O and SO_3, i.e. 0.64% for Ca, 0.55% for Mg, 0.67% for Na and 0.36% for S. In order not to complicate the models, regardless of the number of components and whether they are major, secondary or trace elements, it was decided to opt for a very low tolerance for each nutrient, such that the regulatory tolerances are always respected.

The application of the RB model to the 142 fertiliser shows that the totality of these fertilisers, involving ten different components, can be satisfied with only 16 CBIs, one of which is a filler. The optimum composition of the CBIs is detailed in Table 4 and the quantities to be taken from them to produce each fertiliser are given in the sheet

"CS3.3". In analysing these quantities, it is noted that to produce 100 tons of the outputs *j=84, j=109, j=64* and *j=59*, 100 tons of the CBIs *i=2, i=3, i=8* and *i=15* are required, respectively. This means that these CBIs are fertilisers, and since they are used in many other blends, they must be treated as both CBIs and fertilisers.

Table 4. Optimal composition of the 16 CBIs required to meet the 142 fertilisers on the market (in %)

			% min	% max	CBI *i*															
					filler	*i* =2	*i* =3	*i* =4	*i* =5	*i* =6	*i* =7	*i* =8	*i* =9	*i* =10	*i* =11	*i* =12	*i* =13	*i* =14	*i* =15	*i* =16
Component *c*	*c* = 1	N	0	46.0	0	46.0	0	11.5	0	3.7	5.5	0	40.7	9.7	0	16.5	22.8	0	0	15.7
	c = 2	P	0	57.0	0	0	46.0	56.6	0	7.0	32.8	0	0	56.0	0	26.4	23.1	0	0	0
	c = 3	K	0	64.0	0	0	0	0.0	63.9	14.0	58.6	52.0	0	0	0	0	0	0	11.0	0
	c = 4	S	0	60.0	0	0	0	0	0	0	0	45.0	56.3	31.3	45.0	7.8	0	53.2	10.0	0
	c = 5	B_2O_3	0	2.0	0	0	0	0	0	0	0	0	0	0	0	1.6	0	0	0	0
	c = 6	Zn	0	6.0	0	0	0	0	0	0	0	0	0	0	0	0	5.8	0	0	0
	c = 7	Mg	0	27.0	0	0	0	0	0	0	0	0	0	0	0	0	0	26.6	5.0	0
	c = 8	Na	0	27.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27.0	0
	c = 9	Ca	0	27.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26.6
	c = 10	*filler*	3.0	100	100	54.0	54.0	31.8	36.1	75.3	3.0	3.0	3.0	3.0	55.0	47.8	48.4	20.2	47.0	57.8

As the three most important nutrients for any soil, NPK is the most widely produced and sold fertiliser in the world. To manufacture the 79 NPK fertilizers (accounting for 56% of all references), the solver proposed, in addition to the filler (*i*=1), 6 CBIs (*i*=2 →*i*=7) whose composition makes it possible to satisfy all the nutritional needs of all 79 NPKs. In the example of the 700 NPK requirements, only one more composite was needed (7 CBIs). It confirms that the diversity of the sample considered is not proportional to the size of the CB. Sulphur is the 4th most essential soil nutrient, which is why NPKS accounts for 82% of the references considered, i.e. 117 fertilisers. To produce them, the RB requires a minimum number of 10 CBIs (*i*=2 →*i*=11). Finally, the RB makes it possible to meet all the fertilisers sold by the market leaders in the fertiliser market with only 15 CBIs, and this while meeting the nutritional requirements almost exactly (to within ±0.05%). It can be added that, having initially opted for zero tolerance, the output *j*=79 was not feasible and that, in order to be able to manufacture it, RB proposed it as a CBI, thus increasing the number of CBIs from 15 CBIs to 16 CBIs (not counting the filler). In order to avoid this increase in CBIs, it was chosen to tolerate a minimum deviation of 0.05% (minimum deviation to be accepted in order for the fertiliser *j*=79 to become feasible); especially since it remains largely in line with European regulations.

Limits of the interest of reducing the variety of the commercial offer

Whether the reduction of commercial diversity is achieved by blending existing fertilisers or by RB, it is shown that it is possible to reduce the variety of production without reducing the "diversity of potential use" associated with this existing commercial offer. It is reasonable to assume that the RB solution increases the "diversity of potential use" compared to the previous one, because CBIs have, on average, fewer components. However, if there are many economic justifications for this form of redundancy in the commercial offer, the question arises of the relevance of the "diversity of potential use" yielded by this commercial variety. It seems appropriate to us to take the problem in reverse, starting from a 'diversity of potential use' covering a majority of needs, and then determine the minimum required production variety. This exactly corresponds to the RB approach whose application is recommended in a perspective of reasoned and sustainable agriculture. The application of RB to a very large representative sample of fertiliser needs makes it possible to create a CB of numerical importance similar to the one found to produce the existing fertilisers, but with a wider and more relevant spectrum of use.

5. Possible application of RB to achieve a delayed differentiation of cosmetics

The potential application of RB was analysed for the fertiliser supply chain. Delayed differentiation is of interest to other chemical industries wishing to increase their diversity by offering their customers customised products.

RB can be applied to the manufacture of miscible products (granule, cream, paste, powder, liquid...) by mixing inputs, provided that these inputs are characterised by the same set of chemical components as the outputs, whose composition can be defined by the proportion of each component in their total weight. Such is the case for certain customisable cosmetic products, which are differentiated within the factory in response to customers' demand, leading to a classic approach of manufacturing small batches. An alternative approach inspired by the paint industry consists of a basic formula and a few colourants (predetermined nomenclature) in order to offer cosmetic products (e.g. Lancôme foundation) whose shade only is personalised according to the customer's skin colour that is determined by a skin scan. On the other hand, in addition to the customisation of shades, the RB could be used to propose a complete tailor-made product based on a delayed differentiation, that would be carried out at the store counter using a reduced number of CBIs. To illustrate this, RB is tested on a simplified example from the cosmetics industry: facial powders.

For a complete tailor-made product, it is necessary to start from a reasonably wide variety of facial powders that should be customised in terms of several dermocosmetic properties, namely: coverage, absorption, hydration and adhesiveness. According to Bennett (2017), the coverage of a powder refers to its ability to visually cover the skin in such a way as to reduce shine and hide any imperfections (e.g. dilated pores, acne, irregular pigmentation, etc.). The chemical components that can be used in different doses, depending on the degree of severity of skin imperfections, are TiO_2, Z_nO, $MgSIO_4$ or $Mg_3Si_4O_{10}(OH)_2$, $C_{36}H_{70}O_4Z_n$ and $CaCO_3$ or $MgCO_3$. Absorption, on the other hand, refers to the ability of the powder to absorb perspiration and excess sebum so that the face is no longer shiny. It depends on the following chemical elements: $MgCO_3$, Silica Fume and Kaolin. As far as hydration is concerned, it plays an essential role in the process of skin renewal and the work of epidermal cells and the prevention of premature ageing. The different levels of hydration can be provided by the chemical components $(C_{14}H_{20}NO_{11}Na)_n$ et $C_3H_8O_3$ et $C_{30}H_{50}$ (Bennett 2017). Finally, the property of adhesiveness refers to the ability of the powder to adhere and stick to the skin and depends on the complex: $C_{36}H_{70}MgO_4$. In the end, depending on the required level of each property on which the dose of one or more components depends, custom-made powders would be customised in terms of 12 chemical components. These components are shown in Table 5.

Table 5. Components considered in customising face powders

		Designation	Molecule	Property
Component c	$c=1$	Titanium dioxide	TiO_2	Couverage
	$c=2$	Zinc oxide	Z_nO	
	$c=3$	Talc	$MgSIO_4$	
	$c=4$	Zinc stearate	$C_{36}H_{70}O_4Z_n$	
	$c=5$	Prepared chalk	$CaCO_3$	
	$c=6$	Silica fume	-	Absorption
	$c=7$	Magnesium carbonate	$MgCO_3$	
	$c=8$	Kaolin	Al_2SiO_5	
	$c=9$	Sodium hyaluronate	$(C_{14}H_{20}NO_{11}Na)_n$	Hydration
	$c=10$	Glycerol	$C_3H_8O_3$	
	$c=11$	Squalene	$C_{30}H_{50}$	
	$c=12$	Magnesium stearate	$C_{36}H_{70}MgO_4$	Adhesiveness

In order to define the exact needs of each skin in these 12 elements, a complete diagnosis, based on a series of dermatological tests, must be carried out in order to identify the optimal formula (i.e. the optimal percentage of each ingredient in the final product for each client). For this case study, all of the outputs considered relate to a sample of 100 women's skins that have been theoretically defined (without control by dermatological specialists), which means that some individuals in the sample may not exist in reality, but as these are input parameters, they will be replaced by real needs once further investigation is done with the specialists.

It is assumed that a used CBI must represent at least $\kappa=1\%$ of the weight of 100 grams of powder. For the tolerated absolute deviation, an exact structure is chosen.

Applying the RB model to the above example, the resulting problem contains 896 variables and 2116 constraints, 1200 of which are non-convex quadratic. Solving this problem, under the Xpress solver, shows that only 8 CBIs are needed to satisfy 100 outputs (see the detailed solution in sheet “CS4.2”). The optimal composition of these CBIs is provided in Table 6.

Table 6. Optimal composition of the 8 CBIs required to produce 100 facial powders

		CBI i							
		i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8
Component c	c = 1	10.0%	20.0%	0.0%	0%	29.0%	12.0%	0%	12.0%
	c = 2	0%	5.0%	25.0%	35.0%	0%	8.0%	39.0%	0%
	c = 3	38.6%	45.0%	50.0%	0%	65.0%	0%	0%	70.0%
	c = 4	21.5%	0%	0%	13.0%	0%	0%	21.0%	0%
	c = 5	0%	0%	0%	0%	0%	31.0%	0%	0%
	c = 6	2.0%	0%	8.0%	0%	0%	0%	0%	0%
	c = 7	0%	10.0%	0%	18.0%	4.0%	0%	0%	0%
	c = 8	12.3%	0%	0%	5.0%	0%	27.0%	0%	0%
	c = 9	5.4%	0%	13.0%	0%	0%	0%	0%	18.0%
	c = 10	0%	12.0%	2.0%	18.0%	0%	0%	33.0%	0%
	c = 11	10.2%	0%	0%	0%	0%	16.0%	0%	0%
	c = 12	0%	8.0%	2.0%	11.0%	2.0%	6.0%	7.0%	0%

Based on a new large sample of needs and the acceptation of slight tolerance on composition respect, one can think that the delayed differentiation provided by the RB may meet several hundreds of needs with a CB of a dozen of CBIs, with the help of chemists and dermatologist in this study.

6. Conclusion

The analysis of these case studies identifies three areas of reflection if delayed differentiation in chemical supply chains is seen as the solution to increasing a variety of products considered indispensable, as is the case with fertilisers. This mobilisation requires a strong and sustained commitment of the top management. In the pharmaceutical industry, only the first two (or three) groups to launch a new active ingredient survive on this therapeutic window. This example may encourage the top management of some big groups to invest in the RB.

- First of all, the disruptive nature of the RB approach implies that such a transformation is only possible from a multidisciplinary approach mobilising not only industrial engineering skills but also chemistry skills for the manufacture of CBIs and specialists for the analysis of potential needs to be met (agronomists, dermatologists...).
- The second focus is on "diversity of potential use." Discrete production industry favours a diversity of the offer. In the automotive industry, the combinatorial diversity of cars that can be produced from the same platform is potentially in the millions. The commercial diversity offered is reduced by the use of a car configurator that limits the combination to offer only combinations considered technically and commercially consistent. In the chemical industry, the commercial diversity offered is generally defined to meet precisely a limited number of needs, defined from market studies. With the RB approach, the reasoning is reversed: the same canonical basis can be used to meet hundreds or thousands of needs, most of them being unidentified.
- Finally, the question arises of the definition of the sample leading to the creation of a CB. This question is to be positioned in a global supply chain approach, involving production centers that can serve one or more commercial areas, of various characteristics, through a distribution network (that treats higher volume flows, thanks to the RB). If we take the example of fertilisers, the question of the usefulness of a large global sample of fertiliser needs arises. It is likely that it is

more interesting to define some samples associated with vast markets, served by one or two factories. Such an approach leading to so many disjointed sets of CBIs, one wonders whether it is not wise to pool some CBIs, especially from a risk management perspective. This type of thinking should be based on economic studies mobilising relevant cost systems.

It is by working simultaneously in these three directions that the RB is likely to be able to be adopted.

7. Appendix

7.1 The three formulations of blending problems

Table 7 below provides a comparative synthesis of four models: the classic blending problem (identified by B in the table 7), the model of Reverse Blending (identified by RB), the model of CB use to produce fertilisers not included in the sample used to determine the CB (identified by UCB), and the one of diversity reduction in a set of existing fertilisers (identified by DR).

The "classic" formulation of the Blending Problem is that of the determination of the quantities of inputs to be mixed to produce the required quantities of several outputs whose compositional constraints must be respected. Inputs are available in limited quantities, and their unit costs are known. In the single-period problem, the solution sought is one that minimise the total cost of obtaining inputs.

In the formulation of Reverse blending, cost considerations are absent as neither the fertilisers demands nor the CBIs production costs are known.

In this problem, the composition of the CBIs (inputs) and the blend formulas of these CBIs, are simultaneously sought to achieve with sufficient precision the structure of the outputs to be produced. The lack of knowledge of demands leads to working with identical demands set at an arbitrary level, as the problem is to find a technical solution without worrying, at this stage, about cost considerations. This problem, which aims to meet all the fertiliser demands of a set, with the smallest canonical basis (optimisation criterion), pertains to a steady-state formulation. This problem includes quadratic constraints where CBIs quantities used to produce an output are multiplied by components percentages of CBIs, both being determined by the optimisation. At this stage, there is no concern about the technical problems of the manufacturability of these new composites and their manufacturing costs.

The production of an output that did not belong to the sample that led to the creation of the CB, is a new nomenclature problem where that product is to be produced by blending CBIs. The components structure of that blend must be as close as possible to that required by the output to be produced. To do this, one tries to minimise the sum of the absolute deviations between the target-composition and the one obtained in the blend. The corresponding model is identified by UCB in Table 7.

The latest model aims to determine the smallest subset of outputs belonging to an existing set of outputs, allowing the outputs of the complementary subset to be produced, as well as the definition of their mixing nomenclatures. The considerations of actual costs and demands do not intervene because we are only interested here in the technical possibility of making fertilisers by combining other fertilisers.

Table 7. Comparative modelling of Blending (B), Reverse Blending (RB), Use of CB (UCB) and Reduction of output Diversity to be manufactured (DR).

B	RB	UCB	DR	***Indexes***	
X	X	-	X	j	Output ($j \in J$; $\lvert J \rvert$ = J)
X	X	X	X	c	Chemical component ($c \in C$; $\lvert C \rvert$ = C)
X	-	-	-	i	Existing input (raw material) ($i \in I$; $\lvert I \rvert$ = I)
-	X	-	-	i	Input of the Canonical Basis to create ($i \in I$; $\lvert I \rvert$ = I)
-	-	X	-	i	Input of the Canonical Basis **created previously** ($i \in I$), used to produce the required output
-	-	-	X	i	**Existing** input (= output) ($i \in I \wedge I \subset J \wedge \lvert I \rvert$ = S).
B	**RB**	**UCB**	**DR**	***Parameters***	
X	-	-	-	β_{cj}^{min}	Minimal proportion weight of component c in the weight of output j
X	-	-	-	β_{cj}^{max}	Maximal proportion weight of component c in the weight of output j
-	-	X	-	β_c	Target proportion of component c in the weight of the requested output
-	X	-	X	β_{cj}	Target proportion of component c in the weight of output j
X	X	-	X	D_j	Demand for output j, expressed in weight, actual for B and fictitious for RB et RD
-	-	X	-	D	Demand for the studied output, expressed in weight
X	-	-	-	A_i	Availability of input i, expressed in weight. Guaranteed avaibility for RB et UCB
X	-	X	X	α_{ci}	Proportion of component c in the total weight of input i (B and DR) and of CBI i (UCB)
X	-	-	-	ω_i	Unit cost of input i
-	X	-	X	η_{cj}	Maximal absolute deviation of component c in the total weight of output j
-	X	-	-	M	High constant (Big M)
-	X	-	-	τ_{ci}^{min}	Minimal proportion weight of component c in the total weight of input j
-	X	-	-	τ_{ci}^{max}	Maximal proportion weight of component c in the total weight of input j
-	X	X	-	κ	Minimal weight percentage of an input used to produce an output, if that input is used to produce that output
B	**RB**	**UCB**	**DR**	***Variables***	
X	-	-	X	x_{ij}	Quantity of input i used in the production of output j
-	X	-	-	x_{ij}	Quantity of CBI i used in the production of output j
-	-	X	-	x_i	Quantity of CBI i used to produce the new requested output
-	X	-	-	α_{ci}	Proportion of component i in the total weight of CBI i
-	X	-	-	w_{ij}	Binary variable which is equal to1 if CBI i is used to produce fertilizer j and 0 otherwise
-	-	-	X	w_j	Binary variable which is equal to1 if output j is producted and 0 otherwise
-	X	-	X	y_i	Binary variable which is equal to1 if input i (CBI i for RB) is used and 0 otherwise
-	-	X	-	η_c	Maximum absolute difference between the proportion of component c obtained in the CBIs blend and the target-proportion βc

Table 7 (continue)

B	RB	UCB	DR	***Objective function***
X	-	-	-	$\text{Min}(\Sigma_j \Sigma_i \omega_i x_{ij})$ Minimisation of the production cost
-	X	-	-	$\text{Min}(\Sigma i\, y\, i)$ Minimisation of the number of CBIs in the CB
-	-	X	-	$\text{Min}(\Sigma_c \eta_c)$ Minimisation of the sum of absolute differences between the obtained composition and the target-one
-	-	-	X	$\text{Max}(\Sigma_j w_j)$ Maximisation of the number of outputs that can be produced with the set $I \subset J \wedge \lvert I \rvert = S$)

B	RB	UCB	DR	***Satisfaction of composition constraints of an output***	
X	-	-	-	$\beta_{cj}^{\min} \leq \sum_i \alpha_{ci} \cdot x_{ij} / D_j \leq \beta_{cj}^{\max}, \forall c, j$	**Linear** constraints
-	X	-	-	$\beta_{cj} - \eta_{cj} \leq \sum_i \alpha_{ci} \cdot x_{ij} / D_j \leq \beta_{cj} + \eta_{cj}, \forall c, j$	**Quadratic** constraints
-	-	X	-	$\beta_c - \eta_c \leq \sum_i \alpha_{ci} \cdot x_i / D \leq \beta_c + \eta_c, \forall c$	**Linear** constraints
-	-	-	X	$(\beta_{cj} - \eta_{cj}) \cdot w_j \leq \sum_i \alpha_{ci} \cdot x_i / D_j \leq (\beta_{cj} + \eta_{cj}) \cdot w_j, \forall c, j$	**Linear** constraints

B	RB	UCB	DR	***Satisfaction of demand***
X	X	-	-	$\sum_i x_{ij} = D_j, \forall j$
-	-	X	-	$\sum_i x_i = D$
-	-	-	X	$\sum_i x_{ij} = D_j \cdot w_j, \forall j$

B	RB	UCB	DR	***Main specific constraints***	
X	-	-	-	$\sum_j x_{ij} \leq A_i, \forall i$	Constraint of inputs availibility
-	X	-	-	$\left\lvert \begin{array}{l} \sum_c \alpha_{ci} = 1, \forall i \\ \tau_{ci}^{\text{Min}} \leq \alpha_{ci} \leq \tau_{ci}^{\text{Max}} \end{array} \right. , \forall i, c$	Constraints on components structure of CBI i
-	X	-	-	$\sum_j x_{ij} \leq M \cdot y_i, \forall i$	Determination of the Binary variable of using CBI i
-	X	-	-	$\left\lvert \begin{array}{l} x_{ij} \leq M \cdot w_{ij} \\ x_{ij} \geq \kappa \cdot w_{ij} \cdot D_j \end{array} \right. , \forall i, j$	Constraint of minimum use of CBI i in ouput j if that CBI i is used
-	-	-	X	$\sum_j x_{ij} \leq M \cdot y_i, \forall i$ $\sum_i y_i = S$	To force $y_i = 1$ if i is used Progressive decrementation of S (starting from J) until $\Sigma_j w_j = J$

Additional remarks on obtaining the canonical basis of the 482 NPK fertiliser formulas

Fertilisers are made from several existing composites (urea, monammonic phosphate, potassium chloride...). According to experts, the processes of making the composites to be created to obtain the CB are technically close to those of fertiliser manufacturing. One can then think of using the two-stage blending approach, known as ***Pooling***, in which inputs are intermediate inputs produced by mixing primary inputs. Transposing this approach to our problem lead to consider CBIs as *intermediate inputs* and the existing composites as *primary inputs*. In this class of problems, used for example in hydrocarbons to enable a diversity of final supply of outputs of different characteristics, the final storage tanks can be fed directly by primary inputs and/or by a mixture of a few intermediate inputs from a few tanks specialised in the storage of these intermediate inputs. These intermediate inputs are obtained by blending primary inputs. Modelling of this pooling problem has quadratic constraints and is described in Table 2 (identified by P).

The RB is not concerned by the CBI manufacture. As the manufacture of these composites is technically similar to the fertilisers one, it is naturally thought that the problem posed can be addressed by a Pooling formulation, in which the primary inputs would be the composites used to make fertilisers. However, it is not so simple because, in Pooling, at the three levels studied, the products are of the same nature, which implies a lack of technical constraints on the mixtures. In the manufacture of fertilisers, the combination of specific primary composites is prohibited for reasons of safety or product stability. An adapted version of Pooling (identified by AP for Adapted Pooling) has been proposed to integrate: *i*) these manufacturing constraints of intermediate inputs from primary inputs; *ii*) and to take into account the propagation of these constraints in the blending of intermediate inputs to produce an output, as the blend of intermediate inputs cannot include incompatible primary inputs.

To form the CB described in the IJPE article, the AP model was initially used to obtain as many outputs as possible that can be made from existing primary inputs. The RB model was then used to complete the CB so that the remaining outputs could be manufactured.

Table 8. Comparative modelling of Blending (B), Pooling (P) and Adapted Pooling (AP) problems.

B	P	AP	*Indexes*	
X	X	X	j	Output ($j \in \mathcal{J}$; $\lvert \mathcal{J} \rvert$ = J)
X	X	X	c	Chemical component ($c \in$ C ; $\lvert C \rvert$ = C)
X	-	-	i	Existing input (raw material) ($i \in \mathcal{I}$; $\lvert \mathcal{I} \rvert$ = I)
-	X	-	i	Intermediate input ($i \in \mathcal{I}'$; $\lvert \mathcal{I}' \rvert$ = I'), blend of primary inputs. I' is a known parameter
-	-	X	i	Intermediate input ($i \in \mathcal{I}'$; $\lvert \mathcal{I}' \rvert$ = I'), composite created by a blend of primary composites. I' is a parameter to determine
-	X	X	k	Primary input ($k \in \mathcal{K}$; $\lvert \mathcal{K} \rvert$ = K) : raw material (P) or existing composite (AP)
B	**P**	**AP**	***Parameters***	
X	X	-	β_{cj}^{min}	Minimal proportion weight of component c in the weight of output j
X	X	-	β_{cj}^{max}	Maximal proportion weight of component c in the weight of output j
-	X	X	β_{cj}	Target proportion of component c in the weight of output j
X	X	X	D_j	Demand for output j, expressed in weight, actual for B, maximal for RB and fictitious for RD
X	X	-	A_i , A_k	Availability (expressed in weight) of existing (B) or primary input k (P)
X	X	X	α_{ci}, γ_{ck}	Proportion of component c in the total weight of input i (B) or of primary input k (P) and (AP)
-	X		ϕ_{ki}, ϕ_{kj}, ϕ_{ij}	Sending cost of a unit flow from k to i, from i to j and from k to j
-	-	X	η_{cj}	Maximum absolute deviation from component requirements c in the total weight of the output j
-	-	X	M	High constant (Big M)
-		X	κ	Minimal weight percentage of an intermediaite input used to produce an output, if that intermediaite input is used
-	-	X	κ'	Minimal weight percentage of a primary input used to produce an intermediate input, if that primary input is used
-	-	X	$\zeta_{kk'}$	Boolean parameter equal to 1 if composite materials k and k' are incompatible, and 0 otherwise
B	**P**	**AP**	***Variables***	
X	X	X	x_{ij}	Quantity of input i (B) or intermediate input i (P and AP) used to produce output j
-	X	X	y_{ki}	Quantity of primary input k used to produce intermediate input i
-	X	-	z_{ki}	Quantity of primary input k used to produce output j
-	X	X	α_{ci}	Proportion of component c in the total weight of intermediate input i
-		X	w_{ij}	Binary variable which is equal to1 if intermediaite input i is used to produce output j and 0 otherwise
-	-	X	w_j	Binary variable which is equal to1 if output j is producted and 0 otherwise

Table 8 (continue).

B	P	AP	***Objective function***	
X	-	-	$\text{Min}(\Sigma_{j,i}\ \omega_i\ .\ x_{ij})$	Minimisation of the production cost
-	X	-	$\text{Min}(\Sigma_{j,k}\ \phi_{kj}\ .\ z_{kj} + \Sigma_{k,i}\ \phi_{ki}\ .\ y_{ki} + \Sigma_{i,j}\ \phi_{ij}\ .\ x_{ij})$	Minimisation of the supply cost
-	-	X	$\text{Max}(\Sigma_j\ w_j)$	Maximisation of the number of outputs that can be produced
B	**P**	**AP**	***Satisfaction of composition constraints of an output***	
X	-	-	$\beta_{cj}^{\min} \le \sum_i \alpha_{ci} \cdot x_{ij} / \text{D}_j \le \beta_{cj}^{\max}, \forall c, j$	**Linear** constraints
-	X	-	$\beta_{cj}^{\min} \le \left[\sum_k \gamma_{ck} z_{kj} + \sum_i \alpha_{ci} x_{ij}\right] / \left[\sum_k z_{kj} + \sum_i x_{ij}\right] \le \beta_{cj}^{\max}, \forall\ c,j$	**Quadratic** constraints
-	-	X	$\beta_{cj} - \eta_{cj} \le \sum_i \alpha_{ci} \cdot x_{ij} / \text{D}_j \le \beta_{cj} + \eta_{cj}, \forall c, j \mid \beta_{cj} > 0$ $\sum_i \alpha_{ci} \cdot x_{ij} / \text{D}_j = 0, \forall c, j \mid \beta_{cj} = 0$	**Quadratic** constraints
B	**P**	**AP**	***Satisfaction of demand***	
X	-	-	$\sum_i x_{ij} = \text{D}_j, \forall j$	
-	X	-	$\sum_i x_{ij} + \sum_k z_{kj} \le \text{D}_j, \forall j$	
-	-	X	$\sum_i x_{ij} = \text{D}_j \cdot w_j, \forall j$	
B	**P**	**AP**	***Main specific constraints***	
X	-	-	$\sum_j x_{ij} \le \text{A}_i, \forall i$	Constraint of inputs availability
-	X	-	$\sum_j z_{kj} + \sum_i y_{ki} \le \text{A}_k, \forall k$	Constraint of primary inputs availability
-	X	X	$\sum_j x_{ij} = \sum_k y_{ki}, \forall i$	Flows conservation contraints of intermediate inputs
-	X	X	$\sum_k \gamma_{ck} \cdot y_{ki} = \alpha_{ci} \cdot \sum_j x_{ij}, \forall c, i$	Flows conservation contraints of components
-	-	X	$\left\lvert \begin{array}{l} x_{ij} \le \text{M} \cdot w_{ij} \\ x_{ij} \ge \kappa \cdot w_{ij} \cdot \text{D}_j \end{array} \right., \forall i, j$	Constraint of minimum use of intermediate input i in output j if that intermediate input i is used
-	-	X	$\left\lvert \begin{array}{l} y_{ki} \le \text{M} \cdot z_{ki} \\ y_{ki} \ge \kappa' \cdot z_{ki} \cdot \sum_j x_{ij} \end{array} \right., \forall k, i$	Constraint of minimum use of primary input k in intermediate input i if k is used
-	-	X	$\left\lvert \begin{array}{l} \sum_i x_{ij} \le \text{M} \cdot w_j \\ \sum_i x_{ij} \ge (1/\text{M}) \cdot w_j \end{array} \right., \forall j$	Constraint forcing $w_j = 1$ if output j is produced
-	-	X	$z_{ki} + z_{k'i} \le 1\ , \forall i, k, k' \mid k \ne k' \wedge \zeta_{kk'} = 1$	Constraints of incompatibility between primary inputs in intermediate input production
-	-	X	$w_{ij} + w_{i'j} \le 3 - \zeta_{kk'} \cdot (z_{ki} + z_{k'i'}), \forall j, \forall i, i' \mid i' \ne i, \forall k, k' \mid k \ne k' \wedge \zeta_{kk'} = 1$	Propagation of the incompatibility constraints between primary inputs on the blending of intermediate inputs to produce an output